Preface

This book is written in the spirit of Z. Semadeni's Banach Spaces of Continuous Functions and Rings of Continuous Functions by L. Gillman and M. Jerison, but on a lower level.

It is intended to be more accessible than Semadeni's book – at the cost of being considerably less thorough. It covers a wider range than the Gillman-Jerison book, putting more stress on lattice structure, less on multiplication.

Basically, our text deals with the connection between the topology of a space X (mostly a compact Hausdorff space) and the algebraic structure of $C(X)$, the set of all continuous real valued functions on X. We consider $C(X)$ as a ring, as a lattice and as a metric space. A typical result is that X (compact Hausdorff) is metrizable if and only if $C(X)$ is separable. Another: if X and Y are compact Hausdorff spaces and if $C(X)$ and $C(Y)$ are isomorphic rings, then X and Y must be homeomorphic.

A second theme is the occurrence of $C(X)$ (with compact X) in representation theorems. Not only is $C(X)$ an algebra (ring + vector space) and a Riesz space (lattice + vector space), several beautiful theorems characterize among the algebras and the Riesz spaces those that are "isomorphic" to $C(X)$ for some X. This leads to the Stone-Čech compactification and the Stone space of a Boolean algebra.

Concerning presupposed knowledge:

We expect the reader to be acquainted with the basic theory of *normed vector spaces* (norms, the dual Banach space, the Hahn-Banach Theorem) and of *ordered sets* (the lattice operations). Subjects such as the Alaoglu Theorem are treated in the text itself. For the reader's convenience and to establish notations we have added Appendix A on normed spaces and Appendix B on lattices.

Our attitude with regard to *topology* is a bit ambivalent. The reader is supposed to have knowledge of the elements of the theory (continuity, the Hausdorff property, Urysohn's Lemma). Terms like "completely regular" and "zerodimensional" are introduced. Our first chapter discusses convergence of nets and generating systems in some detail because not all standard books use the same terminology.

No knowledge of *integration theory* is assumed beyond the Riemann integral. There is an Appendix C on abstract integration theory, but in character it differs from the other two appendices. Our text is self-contained as far as integration theory is concerned. However, what we need is so rudimentary that the knowledgeable reader may feel that we are cheating, giving a false idea of what integration is all about. Indeed, our treatment (in Chapter 10) is no natural preparation for the more advanced theory. Therefore, we devote Appendix C to building the bridge from our text to the general literature.

About the set-up of the book:

Each chapter is followed by an *Extra*, either a short piece about history or an extension of the theory, an excursion.

There are many *exercises*. Some are decorated with a star; these contain results that are part of the theory and may be used farther on in the main text. Other exercises present examples or are meant for the reader to pick up routines.

At the end of the book there is a section with solutions for all starred exercises and hints for some others.

We make no claim for originality. Everything in this book can be found in the standard literature. Indeed, most is in Semadeni's book. Accordingly, our lists of references to mathematics are very brief. The references to the historical notes and the stories about mathematicians are more detailed.

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