

# Contents

1. Topological Preliminaries .....	1
Nets, Generating Systems .....	1
Compactness .....	7
Extra: The Axiom of Choice .....	11
2. Metrizable Compact Spaces .....	14
Extra: The Cantor Set and Variations .....	20
3. The Stone-Weierstrass Theorem .....	22
Extra: The key is asking the “right” questions .....	27
4. Weak Topologies. The Alaoglu Theorem .....	29
Weak Topologies. Complete Regularity. ....	29
Normed spaces. The $w'$ -topology. ....	33
Extra: Alaoglu (1914 – 1981) .....	39
5. Riesz Spaces .....	41
Generalities .....	41
Homomorphisms. Ideals .....	48
Archimedean spaces. Units .....	52
Extra: Riesz (1880 – 1956) .....	55
6. Yosida’s Representation Theorem .....	58
Yosida’s Theorem .....	58
Stone’s Representation Theorem (special) .....	63
Extra: Kakutani (1911 – 2004) .....	65
7. The Stone-Čech compactification .....	68
Extra: Stone (1903 – 1989) .....	75
8. Evaluations .....	77
Generalities .....	77
Realcompactness .....	81
Extreme Points .....	83
Postscript: Complex Values .....	86
Extra: Realcompactness of discrete topological spaces .....	87
9. $C(X)$ determines $X$ .....	92
Postscript: Without linearity .....	94
Extra: Extremely disconnected spaces .....	97

10. The Riesz Representation Theorem .....	103
$\sigma$ -Algebras. Measures .....	103
Riesz' Theorem .....	108
Baire and Borel .....	113
Example: Stieltjes integrals .....	118
Extra: Noncompact Spaces .....	121
11. Banach Algebras .....	125
Extra: The Origins of Banach Algebras .....	131
12. Other Scalars .....	133
The Banaschewski Compactification .....	135
Homomorphisms .....	137
Algebras .....	139
Stone's Representation Theorem (general) .....	142
Extra: Non-Archimedean Valued Fields .....	146
Appendix A: Normed Vector Spaces .....	149
Appendix B: Lattices .....	153
Appendix C: Integration .....	155
Solutions and hints for selected exercises .....	160
About the Authors .....	170
Symbols .....	171
Index .....	172



<http://www.springer.com/978-94-6239-200-7>

Spaces of Continuous Functions

Groenewegen, G.L.M.; van Rooij, A.C.M.

2016, IX, 173 p. 23 illus., Hardcover

ISBN: 978-94-6239-200-7

A product of Atlantis Press