Preface

The uniqueness of this book lies in describing generalized metric spaces by means of mappings. Back in 1961, Alexandroff [3] proposed the idea of using the mapping method to study spaces in the first Prague Topological Symposium. The survey paper "Mappings and spaces" written by Arhangel'skiĭ [31] in 1966 inherited and developed the idea. We were greatly interested in this paper. Professors L. Wu and B. Chen then translated it into Chinese (originally in Russian) and published it in "Mathematics" (1981–1982), and wanted to arouse the interest of Chinese scholars. For a comprehensive introduction to the theory of generalized metric spaces, we recommend the books written by Burke, Lutzer [88] and Gruenhage [162], and two chapters written by Nagata [378] and Tamano [449] in the book "Topics in General Topology" [364].

There are roughly three perspectives of investigating spaces by using mappings:

(1) Which classes of generalized metric spaces can be represented as images or preimages of metric spaces under certain mappings? For example, the M-spaces introduced by Morita [360] for investigating the normality of product spaces can be expressed as preimages of metric spaces under quasi-perfect mappings. This has opened up a new way of investigating M-spaces and established connection between this class of spaces and metric spaces.

(2) What are the intrinsic characterizations of images of metric spaces under certain mappings? For example, the closed images of metric spaces (usually called Lašnev spaces) are characterized, by Foged [130], as regular Fréchet-Urysohn spaces with a σ-hereditarily closure-preserving k-network. Thus, it can be compared with the Burke-Engelking-Lutzer metrization theorem [87] and one can connected these spaces with some generalized metric spaces defined by k-networks, for example $\aleph$-spaces, etc.

(3) Certain generalized metric spaces are preserved under what kinds of mappings? Take the class of metrizable spaces as an example, by the Hanai-Morita-Stone theorem [363, 441], we know that metrizability is invariant under perfect mappings. Michael [336] further proved that

metrizability is invariant under countably bi-quotient closed mappings, which shows the charm of bi-quotient mappings.

These simple examples in the above three aspects reveal great colorfulness and attractiveness of infiltrations of mapping method for investigating generalized metric spaces.

Following the ideas and methods of Alexandroff-Arhangel'skiĭ, this book presents the relevant results in the theory of generalized metric spaces in the past three decades from the perspective of mappings particularly the achievements of Chinese scholars in recent years. As an interesting scientific research reading material, it further elaborates these ideas and methods.

This book is also suitable as an elective course material or a teaching reference book for the graduates and senior undergraduates of mathematics major, or as a reading material for graduate students of general topology. It also can be used as a reference book for mathematicians and scientific researchers in other fields.

Suzhou, China Guoshi Gao
September 1992

Preface to the English Edition

The concept of paracompactness introduced by Dieudonné [112] in 1944 is a significant sign of general topology to enter the peak period. The extraordinary Bing-Nagata-Smirnov metrization theorem [62, 370, 428], established in the years 1950 and 1951, created a fundamental change for the full exploration of the nature of metrizability. Meanwhile, the theorem disclosed a bright future of the investigation on properties of generalized metric spaces. The deep study on paracompactness and metrizability performs the prelude to the research on the theory of generalized metric spaces.

In 1961, at the international topological symposium named "General Topology and its Relationship with Modern Analysis and Algebra" in Prague, Alexandroff [3] put forward the idea of investigating spaces by mappings, namely, to connect various classes of spaces by using mappings as a linkage. In this way, the fundamental research framework and whole structure of the theory of topological spaces can be reflected by the relationships between mappings and spaces. Mappings thus became a powerful tool for revealing the internal properties of various classes of spaces. In 1966, Arhangel'skiĭ [31] published the historical literature titled "Mappings and Spaces" which presented a series of constructive concrete steps of how to operate the Alexandroff idea. As a milepost in the road of the vigorous development of general topology, it ushered in an innovation era for investigating spaces by mappings. Since then, the Alexandroff idea became an indispensable method for the research. It promoted a rapid development of general topology, particularly the theory of generalized metric spaces. In a word, the Alexandroff idea of mutual classifications of spaces and mappings has constituted an important part of contemporary general topology [4].

According to Alexandroff and Arhangel'skiĭ, the core of investigating spaces by means of mappings is to establish the extensive connections, with the aid of mappings, between the class of metric spaces and classes of spaces having specific topological properties, to study the intrinsic characterizations of images of metric spaces under various mappings, and to discuss which kinds of mappings can preserve certain classes of spaces. This framework determines the goals of this book: to

comprehensively describe characterizations of images of metric spaces under various mappings and to establish mapping theorems for several important generalized metric spaces.

The most prominent feature of this book is to discuss the theory of generalized metric spaces in a systematic way from the perspective of mappings. Through the mapping theorems of generalized metric spaces, this book tries to point out that the principle of classifying spaces by mappings is undeniably a powerful research tool that has decisive significance in general topology, by which one can peep the full linkage between spaces and mappings. Most of the contents of this book are from hundreds of research papers on spaces and mappings published in the recent 50 years. While paying attention to the innovation and unique features of the contents, this book highlights the outstanding achievements of Chinese scholars in recent years. The authors hope this book can guide readers to grasp the principle of classifying spaces by mappings and catch up the new developments of the theory of generalized metric spaces. We also hope this book can provides solid support for readers in their further research work and can magnify the impacts from China's general topological circles to the world.

This book is composed of three chapters, two appendices and a list of more than 400 references. To provide necessary preparation for the descriptions of relationships between spaces and mappings in the following two chapters, Chap. 1 briefly introduces some basic concepts of generalized metric spaces, the properties of fundamental operations on these spaces and their simple characterizations. By means of several families of sets with specific properties and the concepts of bases or their generalizations, Chap. 2 presents the intrinsic characterizations of images or preimages of metric spaces under quotient mappings, pseudo-open mappings, countably bi-quotient mappings, open mappings, closed mappings and compact-covering mappings or under these mappings with some additional conditions, for example the fibers are separable or compact. Chapter 3 introduces several classes of generalized metric spaces defined by specific bases, weak bases, k-networks, networks and (mod k)-networks etc., such as M-spaces, p-spaces, g-metrizable spaces, $\aleph$-spaces, k-semi-stratifiable spaces, σ-spaces, Σ-spaces and so on. Characterizations and mapping theorems of these spaces are also obtained in this chapter. There are two appendices at the end of this book. The first one aims to help readers to better understand some results on the covering properties used in the text. The second one aims to help readers to obtain a more comprehensive vision of the theory of generalized metric spaces, particularly to help them clearly recognize the place of Alexandroff's idea in contemporary general topology.

The first two editions of this book were written in Chinese and published in 1995 and 2007, respectively [271, 282]. They have played an active role on the development of the theory of spaces and mappings. The primary goal of this new edition is to update recent developments in the area and to continue to revise the text for the sake of clarity and accessibility for readers. In order to arouse interest about the theory of generalized metric spaces and promote contributions of Chinese mathematicians in a broader international society, this edition is published in English.

We benefited from the input of many talented people in writing this book. The most important person in our academic career is Professor G. Gao.[1] He is our mentor. His outstanding work in spaces and mappings and his actively promoting Alexandroff-Arhangel'skiĭ ideas have created opportunities for us to deeply engage in the study of this subject [140, 142]. We are very grateful to Prof. Gao's thorough coaching, invaluable advice, altruistic helps and constant encouragements since 1979. He has laid down the learning and research foundations for us, pointed out the direction of further exploration to us and given us courage and confidence in our research. This book reflects Professor Gao's research style and academic thinking to a certain extent. We also thank Professor Alexander V. Arhangel'skiĭ for his valuable comments on improving the writing of this book.

This edition is supported by the fund projects "The three spaces problems in paratopological groups" and "The coverage problems in sensor networks based on local information and rough set theory" of the National Natural Science Foundation of China (Project Numbers 11471153 and 61472469).

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Suzhou, China — Ziqiu Yun
May 2016

[1]G. Gao, see "Biographies of Modern Chinese Mathematicians in the 20th Century, V. 3", edited by M. Chen, Jiangsu Education Press, Nanjing, 1998, 287–297.

http://www.springer.com/978-94-6239-215-1

Generalized Metric Spaces and Mappings

Lin, S.; Yun, Z.
2016, XVI, 328 p., Hardcover
ISBN: 978-94-6239-215-1
A product of Atlantis Press