

Preface

Many problems arising in applied mathematics or mathematical physics, can be formulated in two ways namely as differential equations and as integral equations. In the differential equation approach, the boundary conditions have to be imposed externally, whereas in the case of integral equations, the boundary conditions are incorporated within the formulation, and this confers a valuable advantage to the latter method. Moreover, the integral equation approach leads quite naturally to the solution of the problem as an infinite series, known as the Neumann expansion, the Adomian decomposition method, and the series solution method in which the successive terms arise from the application of an iterative procedure. The proof of the convergence of this series under appropriate conditions presents an interesting exercise in an elementary analysis.

This book encompasses recent developments of integral equations on time scales. For many population models biological reasons suggest using their difference analogues. For instance, North American big game populations have discrete birth pulses, not continuous births as is assumed by differential equations. Mathematical reasons also suggest using difference equations—they are easier to construct and solve in a computer spreadsheet. North American large mammal populations do not have continuous population growth, but rather discrete birth pulses, so the differential equation form of the logistic equation will not be convenient. Age-structured models add complexity to a population model, but make the model more realistic, in that essential features of the population growth process are captured by the model. They are used difference equations to define the population model because discrete age classes require difference equations for simple solutions. The discrete models can be investigated using integral equations in the case when the time scale is the set of the natural numbers. A powerful method introduced by Poincaré for examining the motion of dynamical systems is that of a Poincaré section. This method can be investigated using integral equations on the set of the natural numbers. The total charge on the capacitor can be investigated with an integral equation on the set of the harmonic numbers.

This book contains elegant analytical and numerical methods. This book is intended for the use in the field of integral equations and dynamic calculus on time

scales. It is also suitable for graduate courses in the above fields. This book contains nine chapters. The chapters in this book are pedagogically organized. This book is specially designed for those who wish to understand integral equations on time scales without having extensive mathematical background.

The basic definitions of forward and backward jump operators are due to Hilger. In Chap. 1 are given examples of jump operators on some time scales. The graininess function, which is the distance from a point to the closed point on the right, is introduced in this chapter. In this chapter, the definitions for delta derivative and delta integral are given and some of their properties are deduced. The basic results in this chapter can be found in [2]. Chapter 2 introduces the classification of integral equations on time scales and necessary techniques to convert dynamic equations to integral equations on time scales. Chapter 3 deals with the generalized Volterra integral equations and the relevant solution techniques. Chapter 4 is concerned with the generalized Volterra integro-differential equations and also solution techniques. Generalized Fredholm integral equations are investigated in Chap. 5. Chapter 6 is devoted on Hilbert–Schmidt theory of generalized integral equations with symmetric kernels. The Laplace transform method is introduced in Chap. 7. Chapter 8 deals with the series solution method. Nonlinear integral equations on time scales are introduced in Chap. 9.

The aim of this book was to present a clear and well-organized treatment of the concept behind the development of mathematics and solution techniques. The text material of this book is presented in highly readable, mathematically solid format. Many practical problems are illustrated displaying a wide variety of solution techniques. Nonlinear integral equations on time scales and some of their applications in the theory of population models, biology, chemistry, and electrical engineering will be discussed in a forthcoming book “Nonlinear Integral Equations on Time Scales and Applications.”

The author welcomes any suggestions for the improvement of the text.

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