

# Preface

This book is based on my graduate-course lectures given at the Graduate School of Mathematics of the University of Tokyo in October 2008 (at the invitation of T. Funaki and M. Jimbo), at the Department of Physics of the University of Tokyo in November 2010 (at the invitation of S. Miyashita), at the Department of Mathematics of Tokyo Institute of Technology in December 2010 (at the invitation of K. Uchiyama), at École de Physique des Houches (Les Houches Physics School) in May 2011 (organized by C. Donati-Martin, S. Péché and G. Schehr), at the Faculty of Mathematics of Kyushu University in June 2013 (at the invitation of H. Osada and T. Shirai), and at the Graduate School of Arts and Sciences of the University of Tokyo in July 2014 (at the invitation of A. Shimizu). First of all I would like to thank those organizers for giving me such opportunities.

The purpose of my lectures is to introduce recent topics in mathematical physics and probability theory, especially the topics on the Schramm–Loewner evolution (SLE) and interacting particle systems related to random matrix theory. A typical example of the latter systems is Dyson’s Brownian motion model. For this purpose I have considered one story to tell the SLE and the Dyson model as ‘children’ of the Bessel processes. The Bessel processes make a one-parameter family of one-dimensional diffusion processes with parameter  $D$ , in which the  $D$ -dimensional Bessel process,  $\text{BES}^{(D)}$ , is defined as the radial part of the  $D$ -dimensional Brownian motion, if  $D$  is a positive integer. This definition implies that Bessel processes are ‘children’ of the Brownian motion, and hence, the SLE and the Dyson model are ‘grandchildren’ of the Brownian motion.

The organization of this book is very simple. In Chap. 1 the parenthood of Brownian motion in diffusion processes is clarified and we define  $\text{BES}^{(D)}$  for any  $D \geq 1$ . There, the importance of two aspects of  $\text{BES}^{(3)}$  is explained. SLE is introduced as a complexification of  $\text{BES}^{(D)}$  in Chap. 2. We show that rich mathematics and physics involved in SLE are due to the nontrivial dependence of the Bessel flow on  $D$ . In Chap. 3 Dyson’s Brownian motion model with parameter  $\beta$  is introduced as a multivariate extension of  $\text{BES}^{(D)}$  with the relation  $D = \beta + 1$ . We will concentrate on the case where  $\beta = 2$ . In this case the Dyson model inherits the two

aspects of  $BES^{(3)}$  and has very strong solvability. That is, the process is proved to be determinantal in the sense that all spatio-temporal correlation functions are given by determinants, and all of them are controlled by a single function called the correlation kernel.

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