Chapter 1
General Problem of Dynamics of Crank-Piston Mechanism with Clearances in Kinematic Pairs

1.1 Analysis of Machines and Mechanisms Used for Oil and Gas Transportation

Piston machines (PM) are widely applied in all branches of industries, including aviation, motor and railway transport, refrigeration and construction industry, etc. PMs are also used for production and transportation of natural resources, including oil and gas transportation in Black Sea Region. These machines represent the heavy duty systems, on which are applied large external forces. Under the action of these forces are deteriorating actuating elements and often failing machines.

Kinematic and dynamic characteristics of PM joint mechanisms mainly depend on clearances in kinematic pairs. The existence of these clearances is due to the manufacturing process of covering and enclosing elements of pairs and their deterioration, caused by friction in pair and machines as well as external technological forces.

Under the impact of forces in kinematic pairs of crank-piston mechanisms (CPM), large loadings are developed, leading to increased reactions in them. High deterioration in kinematic pairs leads to increased clearances.

Additional displacements of links in tolerance zone of clearances under the impact of various forces lead to collisions of kinematic pair elements. Increased dynamic stresses in separate links cause the whole mechanism to vibrate. This results in links additional displacements in clearances used for selecting optimal values, in which basic dynamic characteristics of CPM as well as reliability and durability of PM functions are within the permissible ranges and represent the actual problem. The above is of great importance in design and dynamic calculations of PM.

In addition, in the PM operation process, the clearances in kinematic pairs impact each other. These impacts make up large values, and systematical repetition causes premature deterioration, components breakdown and machines' failure.

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CPM links have another significant property, which determines the durability and reliability of piston machines' operation.

Thus, improvement of reliability and durability of PM represents the actual problem as well as political demand for resources production, which reduces the environmental impact in transportation process.

1.2 Review of Some Scientific Works on Joint Mechanisms with Clearances and Crank-Piston Mechanisms

If clearance does not exist in kinematic pairs of joint mechanisms, the mechanism is known as "ideal"; but if it exists, then the mechanism is "real".

First type mechanisms are comparatively easy. For second type mechanisms, the clearances in the kinematic pairs increase the degree of freedom. Some parameters of mechanisms are defined only after the solution of differential equations of additional motion is related to the increased generalized coordinates. For dynamic analysis of real joint mechanisms of piston machines, it is necessary to apply well developed dynamic models of researched mechanisms.

The issues of the impact of clearance on dynamics of mechanisms and machines are seen often in the works of scientists.

In this work, issues of dynamics relating to clearances are considered [1]. But, due to the problem of graphical methods, this work lacked further development. Also, [2, 3] the problem of analyzing the impact of clearances on kinematic and dynamic accuracy of mechanisms was looked into; the kinetostatic analysis of mechanism with clearance was stated [4].

Great attention was given to issues of constructing mathematical models of mechanisms with clearances [5–9]. Further, dynamic model [10–12] of mechanical systems with clearances was developed.

The motion of mechanisms with clearances is described using systems of essentially non-linear differential equations. Its solution in analytical form is impossible. Thus, the tasks would be solved with computers that can possibly do dynamics analysis of mechanism with clearances without essentially simplifying the mathematical model of the mechanism. Consequently, it is possible to imagine not only qualitative assessment of phenomenon, but also the qualitative assessment of process, carried out in mechanism with clearances.

Dynamic research of joint mechanisms with clearances in kinematic pairs is described considering additional motion of mechanisms in tolerance zone of clearances. In the work of [13], the model grounded on principle of conservation of moment for colliding bodies (internal and external element) of kinematic pair with clearances is offered. The system of colliding bodies is considered in two states: free bodies and impact form.

References [14–17] studied the development of four- and five bar spherical joint mechanisms with clearances in kinematic pairs. In this work, firstly, dynamic research of spherical four-bar mechanisms with clearances was done [14, 15], taking into account all applications of link forces; and in the work of [16, 17], besides the four-bar mechanisms, five-bar spherical mechanism with two degree of freedom was investigated in conjunction with the clearances in kinematic pair.

The above analysis of scientific works was basically on planar and spherical joint four- and five-bar mechanisms, with clearances in kinematic pairs.

Current methods of reciprocating machinery design [18] are based on partial empirical relations and do not consider factors that affect the mechanical and thermodynamic processes of these machines. Till date, no applied formulae are available for durability and reliability of the parts, for mechanical losses in the bearings, and the impact of these phenomena on these couplers. On the other hand, very detailed model-leading to computer intensive simulations has recently been developed for piston ring dynamics in the context of blow-by estimation [19], lubrication conditions of the ring/liner contact [20], dynamical loaded bearings and stress and failure analysis of assembled parts. However, until now, the insight gained with these models has not found way into more detailed models from reciprocating machines such as piston compressor. To this end, a modular system modeling approach as described below combined with model reduction is necessary to keep the computational efforts in reasonable magnitudes.

There are a lot of works on piston machines [21–25], dynamics of piston machines, mechanical losses in joints of links, accuracy of crank-and-rod mechanism and moments of friction on bearing of piston machine.

The research on joint mechanisms with clearances in kinematic pairs confirms that the study of piston machines and their mechanisms with the forces applied on their parts represents the actual problem.

1.3 Dynamic Model of Crank-Piston Mechanism with Clearances in Kinematic Pairs

The dynamic characteristics of crank-piston mechanism mainly depend on clearances in kinematic pairs.

The motion of "real" mechanism (mechanism with clearances) is essentially different from motion of corresponding "ideal" mechanism (mechanism without clearances).

The feature of dynamics of mechanisms with clearances represents incommensurable small values of clearances in comparison with nominal dimensions of separate links of mechanism. However, the existence of these clearances significantly affects laws on its links motion as well as operability of the whole mechanism.

In the development of dynamic model of CPM (Fig. 1.1) the followings are stipulated: clearances in rotational kinematic pairs crank-rod (2-3), rod-piston

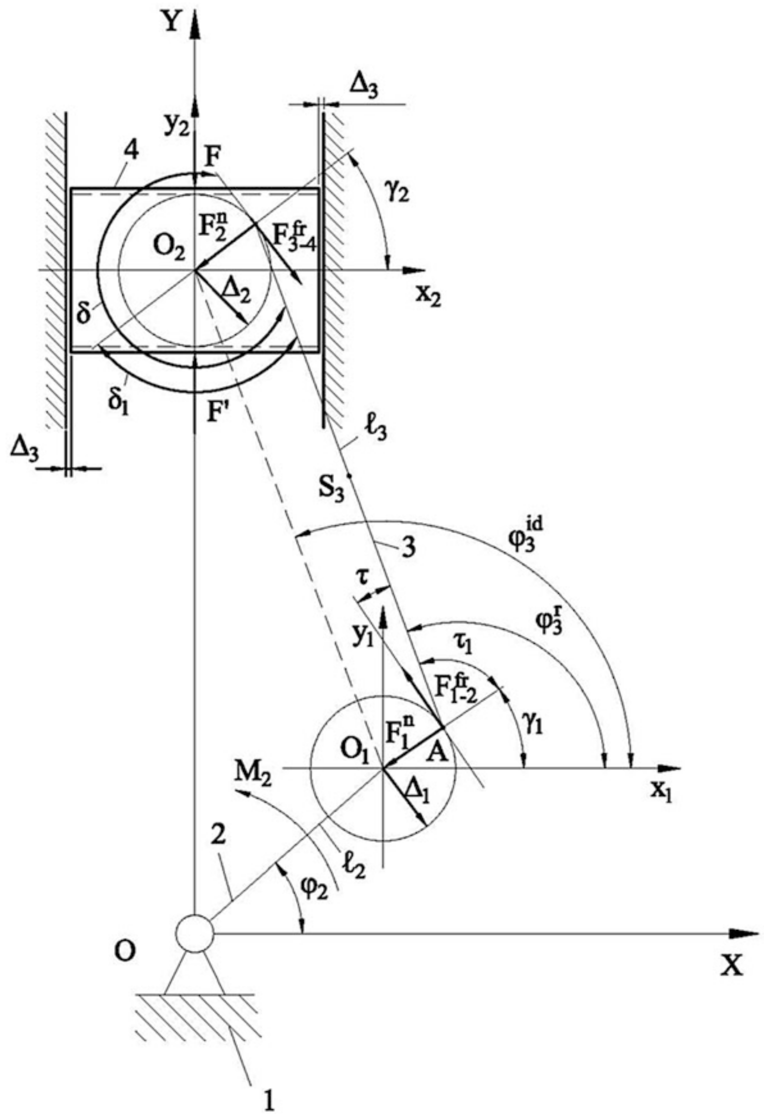


Fig. 1.1 Dynamic model of crank-piston mechanism with clearances in kinematic pairs

(slider) (3-4) and sliding pair 4-1, which are accordingly Δ_1, Δ_2 and Δ_3. The values of clearances are defined by the difference of radiuses

$$\Delta_i = r_i - r_j, \tag{1.1}$$

where
Δ_I are the values of *i*-kinematic pair clearance;
r_i radius of external element of *i*-kinematic pair;
r_j radius of internal element of *i*-kinematic pair.

Such schemes of mechanism are applied in internal combustion engines, various pumps and other mechanisms, designated for transformation of rotational motion in translation motion contrarily.

In the development of dynamic model of mechanism with three clearances in kinematic pairs the following permissions are stipulated: (a) links of mechanism are absolutely rigid; (b) links are homogenous, their masses are uniformly distributed and are concentrated in their geometrical centers; (c) in order to simplify the mathematical transformation let us assume that in pairs 2-3 and 3-4 radiuses $r_j = 0$, i.e., these radiuses are designed as points.

The phenomenon of slider motion in the cylinder of tolerance zone of clearance Δ_3 is presented as the main factor of dynamic model formed.

1.4 Classification of Slider Motion in Cylinder with Clearances

Let us assume that supporting surfaces of cylinders (guide) 1 (Fig. 1.2) are elastic, then pressure on these surfaces is distributed through complex law, that is, defined by complex loadings and elastic properties of piston (slider) and surfaces of guides.

The solution of such task is rather complex; thus let us assume certain simplifications.

As there always exists production clearance between piston and cylinder, the piston has complex motion: sliding—along cylinder (movement along the Oy axis with velocity V_4) and rotational—around their center of mass O_1 (Fig. 1.2a). It is accepted that desaxial e = 0. With these properties, the central mechanism would

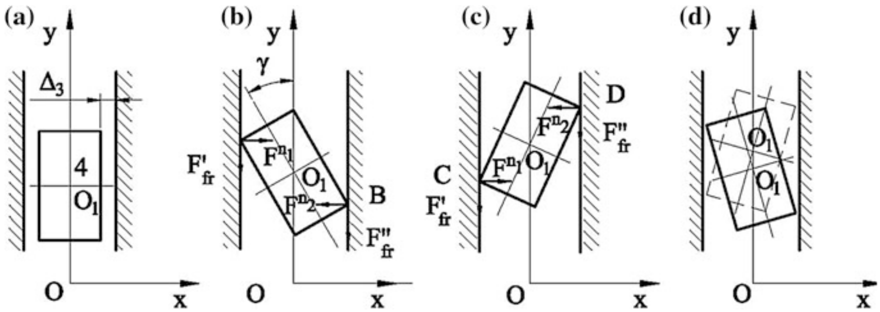


Fig. 1.2 Modes of piston motion in cylinder

become non-central (desaxial). In piston 4 movement of Oy axis of cylinder, the piston would occupy either the position mentioned in Fig. 1.2b or the one mentioned in Fig. 1.2c. In both cases, the two points of the piston—accordingly A and B, and C and D are in contact with the cylinder.

Besides, it is possible to also have free contact-less movement of piston in tolerance zone of cylinder in the presence of clearance (Fig. 1.2d).

The above gives the possibility to conclude that piston movement would be considered strictly on direction along the Oy axis of the cylinder, and accordingly, the friction force in this case would be defined by the expressions,

$$F_{fr} = fF_{41}^{n} \quad \text{and} \quad F_{fr} = \beta V \tag{1.2}$$

as counterclockwise rotation related to OY axis (Fig. 1.2b) as well as clockwise direction (Fig. 1.2c).

In the expressions (1.2) f is the coefficient of friction; F_{41}^{n} is the normal component of total reaction F_{41}; $\beta = \mu s/h$ is the constant coefficient, called viscous friction coefficient; μ is the coefficient of proportionality called dynamic viscous (N s/m^2); V is the value of sliding velocity; S is the liquid flow area; h is the value of clearance.

It is necessary to mention that in arbitrary mode of piston movement, its position is always defined if its O_1 center position is related to OY axis and coordinates of arbitrary vertex axis cross-section. At this time it is possible to define angle of piston rotation γ (Fig. 1.2b) and $\gamma = \operatorname{arctg}\frac{x}{y}$.

In free parallel movement of piston related to cylinder (Fig. 1.1a), we will have

$$\left.\begin{array}{cc} F_{41}^{n} = 0; & F_{fr} = 0; \\ x = a; & y = b; \\ \gamma = 0; & desaxial\, e = 0. \end{array}\right\} \tag{1.3}$$

In contact motion with break contacts (Fig. 1.2b, c), we will have,

$$\left.\begin{array}{cc} F_{41}^{n} > 0; & F_{fr} > 0; \\ x = a + \Delta_3; & y < b; \\ \gamma = \operatorname{arctg}\frac{x}{y}; & e = 0. \end{array}\right\} \tag{1.4}$$

$$\left.\begin{array}{cc} F_{41}^{n} > 0; & F_{fr} > 0; \\ x < a; & y > b + \Delta_3; \\ \gamma = \operatorname{arctg}\frac{x}{y}; & e = 0. \end{array}\right\} \tag{1.5}$$

In free contactless motion (Fig. 1.2d), we will have

$$\left.\begin{array}{cc} F_{41}^{n} = 0; & F_{fr} = 0; \\ x < a + \Delta; & y < b; \\ \gamma = \operatorname{arctg}\frac{x}{y}; & e = 0. \end{array}\right\} \tag{1.6}$$

$$\left.\begin{array}{ll} F_{41}^{n} = 0; & F_{fr} = 0; \\ x = a + \Delta_3; & y = a; \\ \gamma = \operatorname{arctg}\frac{x}{y}; & e = 0. \end{array}\right\} \tag{1.7}$$

The expressions (1.3)–(1.7) are characterized by the piston's movement related to OY axis of the cylinder, when e = 0, and in a given Δ_3 clearance represents the initial conditions of piston's transformation from one mode of motion to another.

In the pistons, such mode of motion in cylinder, under the conditions of given Δ_3, clearance center O_1 of mass of piston is located on one side of the cylinder and XOY coordinates system of OY axis (Fig. 1.3); they are distanced by desaxial. The mechanism is called desaxial mechanism. At same time, the piston's guide related to cylinder has six different positions and accordingly in the area of Δ_3 clearance related to cylinder will perform six modes of motion.

1. Free parallel motion of piston is related to OY cylinder. The piston's position is defined by desaxial and x and y coordinates of A point (Fig. 1.3a);
2. Parallel motion of piston is in contact with cylinder (Fig. 1.3b). At this time, the value of desaxial $e = a + \Delta_3 - x$. The normal component of reaction is uniformly distributed on the length of generatrix, and tangential component

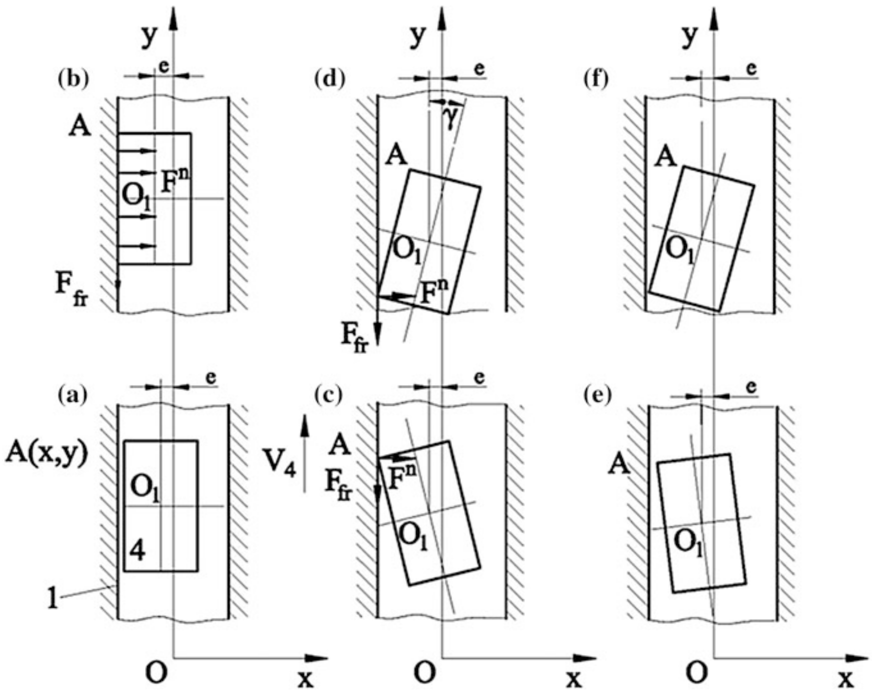


Fig. 1.3 The second mode of piston motion in cylinder

(friction force F_{fr}) is directed in a reverse position along the velocity V_4 of piston (Fig. 1.3b);
3. Related to OY axis counterclockwise rotational motion (Fig. 1.3c), we have contact motion with break contact in XOY plane. At this time, the position is defined by desaxial and piston's rotation γ angle;
4. Related to OY axis clockwise rotational motion (Fig. 1.3d), when piston's position is defined by desaxial and piston's rotation γ angle, we have contact motion with break contact;
5. For free contactless movement of piston in Δ_3, clearance area has counter-clockwise rotational motion (Fig. 1.3e) and clockwise rotational motion (Fig. 1.3f).

In cases of piston's movement related to cylinder, when e $\neq$ const, it is possible to have the following expressions:

1. Free parallel motion of piston is related to cylinder (Fig. 1.3a)

$$\left.\begin{array}{cc} F^n = 0; & F_{fr} = 0; \\ x < a + \Delta_3; & y = b; \\ \gamma = 0; & e > 0. \end{array}\right\} \tag{1.8}$$

2. Contact motion of piston with linear with contour break (Fig. 1.3b)

$$\left.\begin{array}{cc} F^n > 0; & F_{fr} > 0; \\ x = a + \Delta_3 - e; & y = b; \\ \gamma = 0; & e > 0. \end{array}\right\} \tag{1.9}$$

3. Contact motion of piston counterclockwise rotational with break contact (Fig. 1.3c)

$$\left.\begin{array}{cc} F^n > 0; & F_{fr} > 0; \\ x = a + \Delta_3; & y < b; \\ \gamma = \operatorname{arctg}\frac{x}{y}; & e > 0. \end{array}\right\} \tag{1.10}$$

4. Contact motion of piston clockwise rotational with break contact (Fig. 1.3d)

$$\left.\begin{array}{cc} F^n > 0; & F_{fr} > 0; \\ x < a + \Delta_3; & y > b; \\ \gamma = \operatorname{arctg}\frac{x}{y}; & e > 0. \end{array}\right\} \tag{1.11}$$

5. Free parallel motion of piston related to cylinder (Fig. 1.3e, f)

$$\left.\begin{array}{cc} F^n = 0; & F_{fr} = 0; \\ x < a + \Delta_3; & y < b; \\ \gamma = \operatorname{arctg}\frac{x}{y}; & e > 0; \end{array}\right\} \tag{1.12}$$

$$\left.\begin{array}{ll} F^n = 0; & F_{fr} = 0; \\ x < a + \Delta_3; & y > b; \\ \gamma = \operatorname{arctg}\frac{x}{y}; & e > 0. \end{array}\right\} \quad (1.13)$$

The (1.8)–(1.13) expressions represent initial conditions related to cylinder in the transformation from one mode of piston's motion to another.

It is possible to consider also the third mode of piston's motion according to the second mode geometrical and dynamic parameters. In such case value, desaxial would be related to OY axis of the cylinder.

1.5 Dynamic Model Piston-Cylinder of Sliding Kinematic Pair with Clearances of Crank-Piston Mechanism

Carried out classification of piston's motion in cylinder with consideration of clearance Δ_3 and desaxial e gives the possibility to compile dynamic model piston-cylinder of sliding pair that would be applied for describing of arbitrary motion of piston in the tolerance zone of clearance.

Let us notice that in same time the piston and cylinder are considered as absolute rigid with distributed masses and geometrical center of piston coincides with its center of mass.

With consideration of these conditions dynamic model of piston-cylinder with consideration of clearance would had the form, presented on the Fig. 1.4.

According of Fig. 1.4 the sliding kinematic pair of piston-cylinder is located in fixed XOY orthogonal coordinates system in such order that cylinder's axis of symmetry coincides with OY axis, and the piston carried out reciprocating motion related to this axis by e desaxial.

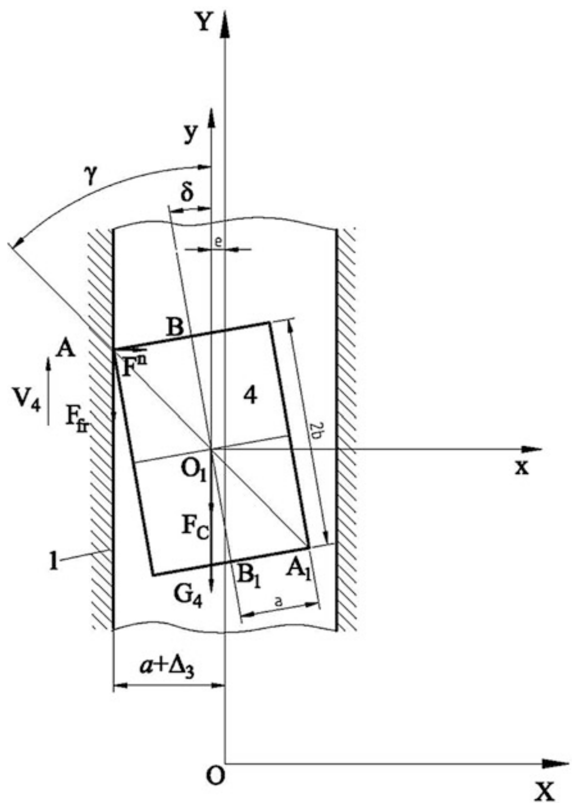
For the piston's motion describing let's introduce utility orthogonal xo1y coordinates system that origin is located in the geometrical and mass center O1. The movable O1x and O1y axis are in parallel to according OX and OY axis.

The rotation of piston's AA_1 diagonal and rotation of piston's longitudinal axis BB_1 of symmetry related to OY axis let's designate as γ and δ angles accordingly. On the Fig. 1.4, *a*—is the radius of piston, 2b—is the length. The radius if guide cylinder is a + Δ_3, and e desaxial I considered on the left side of OY axis.

In the contact A point are obtained normal component of reaction force F^n and friction F_{fr} force that is directed in reverse to piston movement velocity V_4. In the center of mass O_1 of cylinder is applied gravity G_4 force and resistance F_R forces of piston. At piston upward movement on it is allied resistance F force, and at it downward movement resistance F′ force.

The classification of piston-cylinder sliding kinematic pair of crank-piston machine's movement, dynamic model and analysis gives the possibility to solve the task of investigation of given mechanism with taking into account clearances in the all three kinematic pairs and variable desaxial in sliding pair.

Fig. 1.4 Dynamic model of sliding kinematic pair piston-cylinder with clearance



1.6 Real Dynamic Model of Crank-Piston Mechanism with Clearances in Three Kinematic Pairs

At development of dynamic model of crank-piston mechanism are considered Δ_1, Δ_2 and Δ_3 clearances accordingly in the 2-3, 3-4 and 4-1 kinematic pairs (Fig. 1.1).

For construction of real dynamic model is considered piston-cylinder kinematic pair's factor, when the piston in the tolerance zone Δ_3 of cylinder performs the complex motion, due that the mechanism by origination of variable desaxial e transforms to desaxial mechanism.

With taking into account the mentioned as well as Figs. 1.1 and 1.4 the real dynamic model of crank-piston mechanism with clearances in three kinematic pairs looks as presented on the Fig. 1.5.

The crank-piston OAB mechanism is located on the fixed XOY orthogonal coordinates system in such order that the origin of system O will coincides to O pivot of crank, and the reciprocal movement of slider—to OY axis of coordinates system.

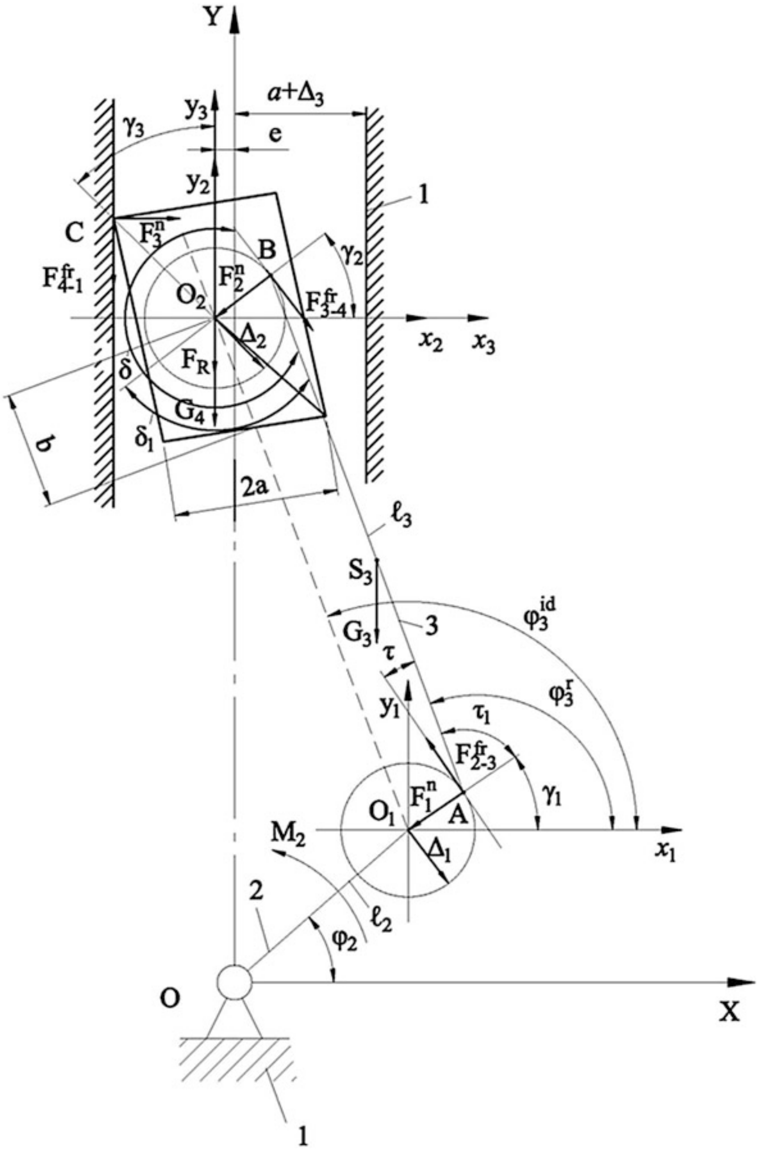


Fig. 1.5 Real dynamic model of crank-piston mechanism with clearances in three kinematic pairs

In order to study caused due the existing in the kinematic pairs of mechanism clearances additional movements of links let's in the center of each kinematic pair apply movable orthogonal coordinates system: $x_1o_1y_1$, $x_2o_2y_2$ and $x_3o_3y_3$.

In the contact A and B points of internal and external elements of 2-3 and 3-4 rotational kinematic pairs of mechanism are applied normal components F_1^n and F_2^n

of reaction forces that are directed to centers of kinematic pairs. In the contact C point of piston and cylinder is applied normal component of F_3^n reaction force and it is directed in perpendicular of cylinder's axis.

In the contact A and B points of internal and external elements of 2-3 and 3-4 rotational kinematic pairs of mechanism also are applied tangential forces of reaction (friction $F_{2\text{-}3}^{fr}$ and $F_{3\text{-}4}^{fr}$ forces) that are directed in direction of tangential lines (in perpendicular to normal components). At the same time as direction of tangential forces action is applied reverse direction of internal elements rotation.

In the reciprocating 4-1 kinematic pair of mechanism the friction force is applied in the contact C point and is directed in reverse of piston movement. This force is characterized by two directions of action along the OY axis that would be explained due reciprocating movement of piston related to OY axis.

The friction and normal components of reaction forces related to AB rod of mechanism makes τ and τ_1 angles. In the kinematic pairs angles of rotation of internal and external elements contact A, B and C points let's accordingly designate as γ_1, γ_2 and γ_3. The gravity forces G_3 and G_4 of rod and piston are applied in the centers of masses.

The rod's rotation angle related to OX axis is φ_3. The radius of piston and half of its length let's designate as a and b accordingly.

As it is mentioned at existing of Δ_3 clearance in the 4-1 sliding kinematic pair the mechanism with e desaxial transforms as desaxial mechanism that is possible to makes the values from 0 up to Δ_3.

$$e = \begin{cases} 0, & if\ x_3 = a + \Delta_3; \\ \Delta_3, & if\ x_3 = 0. \end{cases} \tag{1.14}$$

At consideration of crank-piston mechanism in the kinematic pairs the x_1, y_1; x_2, y_2; and x_3, y_3 linear and γ_1, γ_2 and γ_3 angular coordinates of internal and external elements contact A, B and C points are variable linear and angular values (generalized coordinates). The values of these coordinates are depending on arrangement of contact points.

For the solution of tasks of dynamic research of mechanisms with clearances in the most cases were accepted that angular velocity of crank rotation is constant value

$$\dot{\varphi}_2 = \dot{\varphi}_2(t) = const. \tag{1.15}$$

At research of real mechanisms with clearances the consideration of angular velocity of crank as constant value didn't fully characterized this process that is caused due additional movements of links. The impact of links additional movements in tolerance zones of clearances on basic movements of mechanism requires deep studying and investigations. Let's accordingly consider the case when

$$\dot{\varphi}_2 = \dot{\varphi}_2(t) \neq const. \tag{1.16}$$

In the rotational 2-3 and 3-4 Kinematic pairs of mechanism for the generalized coordinates x_1, y_1; x_2, y_2 let's write down (Fig. 1.5):

$$\begin{aligned} x_1 &= \Delta_1 \cos\gamma_1; \quad y_1 = \Delta_1 \sin\gamma_1; \\ x_2 &= \Delta_2 \cos\gamma_2; \quad y_2 = \Delta_2 \sin\gamma_2. \end{aligned} \tag{1.17}$$

Similarly for x_3 and y_3 generalized coordinates of reciprocating 4-1 kinematic pair we will have (Fig. 1. 5):

$$\begin{aligned} x_3 &= (\sin\gamma_3)^{-1}\Big[\big(a^2+b^2\big)\sin^2\gamma_3 \\ &\qquad + (a+\Delta_3-e)^2\cos^2\gamma_3\Big]^{1/2} \\ y_3 &= (a+\Delta_3-e)ctg\gamma_3. \end{aligned} \tag{1.18}$$

For the variable velocities of (1.17) and (1.18) generalized coordinates we will have:

$$\left.\begin{aligned} \dot{x}_1 &= -\Delta_1\dot{\gamma}_1 \sin\gamma_1; \quad \dot{y}_1 = \Delta_1\dot{\gamma}_1\cos\gamma_1; \\ \dot{x}_2 &= -\Delta_2\dot{\gamma}_2 \sin\gamma_2; \quad \dot{y}_2 = \Delta_2\dot{\gamma}_2\cos\gamma_2. \end{aligned}\right\} \tag{1.19}$$

$$\left.\begin{aligned} \dot{x}_3 &= -(a+\Delta_3-e)^2\dot{\gamma}_3\cos\gamma_3\Big(\big[(a^2+b^2)\sin^2\gamma_3 \\ &\qquad -(a+\Delta_3-e)^2\cos^2\gamma_3\big]\Big)^{1/2}\sin^2\gamma_3; \\ \dot{y}_3 &= -\dot{\gamma}_3(a+\Delta_3-e)\big(\sin^2\gamma_3\big)^{-1}. \end{aligned}\right\} \tag{1.20}$$

By the differentiation of (1.19) and (1.20) values will be obtained generalized x_1, y_1; x_2, y_2; x_3, y_3 coordinates values of acceleration.

1.7 Movements of Crank-Piston Mechanism with Consideration of Clearances in Three Kinematic Pairs

The movements of crank-piston mechanism (additional and basic) with taking into account the clearances in three kinematic pairs are stipulated by additional movements in joint connections as well as basic movements of mechanism.

At research of joint mechanism with clearances almost always is carried out research of additional motions of mechanism and is considered that angular velocity of input link (crank) represents the constant values.

It is necessary to mention that at research of real mechanism is necessary to investigate not only additional movements, but also the basic movements. In this case would be known impact of additional movements of mechanism on its basic movements. Accordingly the definition of real values of input link's laws of motion represents the significant problem.

Let's assume that at given moment of movement in considered crank-piston mechanism with clearances in 2-3, 3-4 and 4-1 kinematic pairs simultaneously is keeping contact (C). In this case we have additional contact motion.

If at the movement of mechanism in all kinematic pairs the contact will be failed and elements of pairs will be moved in the tolerance zone of clearances, we will have the break (B) movement of mechanism. If in the different pairs of mechanism the motion will be carried out by alternation of break-contact or contact-break motion, then accordingly we will obtain break-contact (BC) or contact-break (CB) motion that will be affected on basic (b) movements of mechanism.

In the kinematic pairs of mechanism with $\Delta_1, \Delta_2, \Delta_3$ clearances at various combination of contact (C) and break (B) is possible to differ eight modes of additional motion that will be considered with basic (b) movements of mechanism. In the case of additional motion of mechanism will be obtained 36 differential equations, and for the basic movements—eight differential equations. Totally are the 44 differential equations. In the Table 1.1 is presented quantity of additional and basic movements, when the mechanism has the clearances in three kinematic pairs.

The analysis of crank-piston mechanism with clearances in three kinematic pairs indicates that for studying of additional and basic movements are necessary the solution of 44 differential equations, that will not be carried out even by current computer equipment.

Preceding from the mentioned the dynamic research of crank-piston mechanism with taking into account the clearances in kinematic pairs will be given for the case, when the mechanism has clearance in two (2-3 and 3-4) kinematic pairs. At the same time would be considered also the basic (b) movements of mechanism (Table 1.2).

Table 1.1 Analysis of crank-piston mechanism with clearances in three kinematic pairs

Mode of motion	Modes of motion in tolerance zones of clearance	Coordinates of additional movements	Coordinates of basic movements
I	CCC + b	γ_1; γ_2; γ_3	φ_2
II	CCB + b	γ_1; γ_2; x_3, y_3	φ_2
III	CBB + b	γ_1; x_2, y_2; x_3, y_3	φ_2
IV	CBC + b	γ_1; x_2, y_2; γ_3	φ_2
V	BCC + b	x_1, y_1; γ_2; γ_3	φ_2
VI	BCB + b	x_1, y_1; γ_2; x_3, y_3	φ_2
VII	BBC + b	x_1, y_1; x_2, y_2; γ_3	φ_2
VIII	BBB + b	x_1, y_1; x_2, y_2; x_3,y_3	φ_2

Table 1.2 Additional movements of crank-piston mechanism

Mode of motion	Modes of motion in tolerance zones of clearance	Coordinates of additional movements	Coordinates of basic movements
I	CC + b	γ_1; γ_2	φ_2
II	CB + b	γ_1; x_2, y_2	φ_2
III	BC + b	x_1, y_1; γ_2	φ_2
IV	BB + b	x_1, y_1; x_2, y_2	φ_2

For the additional movements of crank-piston mechanism will be received 12 differential equations and for basic movements—four differential equations. Totally are the 16 differential equations.

Therefore, for the certain crank-piston mechanism we will have four modes of motion:

1. In all kinematic pairs we have contact (CC). In such case the additional and basic movements of mechanism would be defined by three generalized coordinates γ_1, γ_2 and φ_2.
2. In the kinematic pair 2-3 is keeping contact (C) and in the 3-4 pair is existing break (B). In such case the additional and basic movements of mechanism would be defined by four generalized coordinates γ_1, x_2, y_2 and φ_2.
3. In the kinematic pair 2-3 we have break (B) and in the 3-4 pair is existing contact (C). In such case the additional and basic movements of mechanism would be defined by four generalized coordinates x_1, y_2; γ_2 and φ_2.
4. In the 2-3 and 3-4 kinematic pair is keeping break (B). In such case the additional and basic movements of mechanism would be defined by generalized coordinates x_1, y_1; x_2, y_2 and φ_2.

The transformation from one mode of motion of crank-piston mechanism on another is carried out when existing in the contact point normal component of F_i^n reaction force will be equal to zero. At the same time occurs break in kinematic chain of mechanism and definition of positions of A and B points in tolerance zone of clearance would be carried out only by generalized—additional x_i, y_i linear coordinates. With taking into account these changes would be changed also the basic movement. From the conditions of transformation from contact movement to free movement we will have

$$\begin{aligned} F_i^n &= 0; \\ x_i &= \Delta_i \cos\gamma_i; \\ y_i &= \Delta_i \sin\gamma_i. \end{aligned} \tag{1.21}$$

where

γ_i is the of kinematic pairs internal element's angle of rotation related to external elements in the initial moment of break;

Δ_i is the clearance in I kinematic pair.

The transformation from mechanism's break mode of motion on contact is characterized by impacts of freely moved internal and external elements, at which normal component of F_i^n reaction force was instantaneous growing. Simultaneously begins the contact movement. For the conditions of transformation from break movement on contact movement we will have:

$$\begin{aligned} F_i^n &\neq 0; \\ \gamma_i &= \arccos \frac{x_i}{\Delta_i}; \\ y_i &= \arcsin \frac{y_i}{\Delta_i}. \end{aligned} \tag{1.22}$$

The differential equations of mechanism's movement for arbitrary modes of additional and basic motions would be obtained according of Lagrangian second order equations. At the same time would be applied expressions (1.17)–(1.22).

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