A New Firefly Algorithm with Local Search for Numerical Optimization

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Abstract. Firefly algorithm (FA) is a recently proposed swarm intelligence optimization technique, which has shown good performance on many optimization problems. In the standard FA and its most variants, a firefly moves to other brighter fireflies. If the current firefly is brighter than another one, the current one will not be conducted any search. In this paper, we propose a new firefly algorithm (called NFA) to address this issue. In NFA, brighter fireflies can move to other positions based on local search. To verify the performance of NFA, thirteen classical benchmark functions are tested. Experimental results show that our NFA outperforms the standard FA and two other modified FAs.

Keywords: Firefly algorithm (FA) · Swarm intelligence · Local search · Numerical optimization

1 Introduction

Firefly algorithm (FA) is a new swarm intelligence algorithm developed by Yang in 2010 [1]. It is inspired by the social behavior of fireflies based on the flashing and attraction characteristics of fireflies. In the past five years, the research of FA has attracted much attention. Different versions of FA has been designed to solve benchmark or real-world optimization problems [2–6].

To enhance the performance of FA, Farahani et al. [7] proposed a Gaussian distributed FA (GDFA). Computational results on five benchmark functions show that GDFA outperforms PSO and the standard FA. Tilahun and Ong [8] modified the random movement of the brighter firefly by generating random directions in order to determine the best direction. If such a direction is not generated, it will remain its current position. Moreover, the assignment of attractiveness is modified in such a way that the effect of the objective function is magnified. Simulation results show that the

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modified FA performs better than the standard FA in finding the best solution with smaller CPU time. Fister et al. [9] proposed a memetic FA (MFA) to solve combinatorial optimization problems. In MFA, the parameter α is dynamically adjusted, and the parameter β is changed in the range [0.2, 1.0] based on the distance between two fireflies. Additionally, the random part $\alpha\varepsilon$ for the movement of the attraction is scaled by the size of the search range. Experimental results show that the MFA is significantly better than the standard FA. In our previous work [10], the MFA is used as the standard FA and combined with other strategies. Gandomi et al. [11] introduced chaos into FA to increase its global search ability for robust global optimization. Different chaotic maps are utilized to tune the attractive movement of fireflies. Results show that the chaotic FA (CFA) outperforms the standard FA. In [12], quaternion is used for the representation of individuals in FA so as to enhance the performance of the firefly algorithm and to avoid any stagnation. Yu et al. [13] designed a new FA with a wise step strategy (WSSFA), which considers the information of firefly's personal and the global best positions. Results show that the modified algorithm outperforms the standard FA on twenty benchmark functions. In [14], a variable step size FA (VSSFA) is proposed, where a dynamical method is used to update the parameter α. Computational results show that WSSFA and VSSFA achieve better solutions than the standard FA on a set of low-dimensional benchmark functions (D = 2). However, our experiments demonstrate that both of them can hardly obtain reasonable solutions for some high-dimensional problems (D = 30). Compared to WSSFA and VSSFA, MFA can achieve promising solutions.

In the FA, the fitness function for a given problem is associated with the light intensity. The brighter the firefly is, the better the firefly is. That means a brighter firefly has a better fitness value. The search process of FA depends on the attractions between fireflies. Based on these attractions, a firefly tends to move other brighter fireflies. If a firefly is brighter than another one, the brighter firefly will not be conducted any search. In this paper, we propose a new FA (called NFA) to avoid this case. When the current firefly is brighter than another one, a local search operation is conducted on the current one to provide more chances of finding more accurate solutions. It is noted that the proposed NFA is implemented based on the MFA. Therefore, the NFA is a hybrid algorithm by combining the MFA and the proposed local strategy. To verify the performance of NFA, a set of well-known benchmark function with D = 30 are tested. Experimental results show that NFA performs better than the standard FA, MFA, and VSSFA.

The rest paper is organized as follows. In Sect. 2, the standard FA is briefly introduced. In Sect. 3, the proposed NFA is described. Experimental results are presented in Sect. 4. Finally, the work is concluded in Sect. 5.

2 Firefly Algorithm

As mentioned before, the FA mimics the behavior of the social behavior of the flashing characteristics of fireflies. To simply the behavior of fireflies and construct the search mode of FA, three rules are used as follows [1]:

- All fireflies are unisex so that one firefly is attracted to other fireflies regardless of their sex;
- Attractiveness is proportional to their brightness. For any two fireflies, the less bright one is attracted by the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If no one is brighter than a particular firefly, it moves randomly;
- The brightness or light intensity of a firefly is affected or determined by the landscape of the objective function to be optimized. For a minimization problem, the brightness can be proportional to the objective function. It means that the brighter firefly has smaller objective function value.

As light intensity and thus attractiveness decreases as the distance from the source increases, the variations of light intensity and attractiveness should be monotonically decreasing functions. This can be approximated by the following Eq. [1]:

$$I(r) = I_0 e^{-\gamma r^2}. \tag{1}$$

where I is the light intensity, I_0 is the original light intensity, and γ is the light absorption coefficient. The attractiveness of a firefly is proportional to the light intensity. The attractiveness β of a firefly can be defined by [1]:

$$\beta(r) = \beta_0 e^{-\gamma r^2}. \tag{2}$$

where β_0 is a constant and presents the attractiveness at $r = 0$. The distance between r_{ij} between any two fireflies i and j can be calculated by [1]:

$$r_{ij} = \|X_i - X_j\| = \sqrt{\sum_{d=1}^{D} (x_{id} - x_{jd})^2}. \tag{3}$$

where D is the dimensional size of the given problem.

Based on the above definitions, the movement of this attraction is defined by [1]:

$$x_{id}(t+1) = x_{id}(t) + \beta_0 e^{-\gamma r_{ij}^2}\left(x_{jd}(t) - x_{id}(t)\right) + \alpha \varepsilon_{id}(t). \tag{4}$$

where x_{id} and x_{jd} is the dth dimension of firefly i and j, respectively, α is a random value with the range of [0,1], ε_{id} is a Gaussian random number for the dth dimension, and t indicates the index of generation.

3 Proposed Approach

Recently, some new FA variants were proposed to enhance the performance of FA. However, these algorithms only work well on some low-dimensional problems. For high-dimensional problems (such as $D = 30$), they can hardly find reasonable solutions. In [9], Fister et al. designed a memetic FA (MFA) by introducing multiple strategies. Experimental results show that MFA performs well when $D = 30$.

In MFA, a new updating search equation is defined as follows [9].

$$x_{id}(t+1) = x_{id}(t)(1-\beta) + x_{jd}(t)\beta + \alpha(r-0.5). \tag{5}$$

$$\alpha = \alpha\left(\frac{1}{9000}\right)^{\frac{1}{t}}(up - low). \tag{6}$$

$$\beta = \beta_{\min} + (\beta_0 - \beta_{\min})e^{-\gamma r_{ij}^2}. \tag{7}$$

where α is the generation index, r is a random number between 0 and 1, and $\beta_{\min}$ is a constant value. In [9], β_0 and $\beta_{\min}$ are set to 1.0 and 0.2, respectively. The initial a is set to 0.2. In the proposed NFA, we use the MFA as the basic algorithm. Then, we embed a local search strategy into the MFA.

Algorithm 1: The Proposed NFA

```
Begin
    Randomly initialize all fireflies in the swarm;
    while FEs <= MaxFEs do
        for i=1 to N do
            for j=1 to i do
                if firefly j is better than firefly i then
                    Generate a new firefly according to Eq. (5);
                    Evaluate the new solution;
                end if
                else
                    Conduct the local search according to Eq. (8);
                end else
            end for
        end for
    end while
End
```

In the standard FA and its most variants, a firefly can move to other brighter fireflies based on the attraction operations. However, if the current firefly is brighter than another one, the current one will not be conducted any search. To avoid this case, we design a new solution updating model. When the above case occurs, a local search is conducted on the brighter firefly as follows.

$$x_{id}^*(t) = Best_d(t) + (x_{id}(t) - x_{kd}(t))(2r-1). \tag{8}$$

where r is a random number within [0, 1], $Best$ is the global best solution found so far, and X_k is a randomly selected solution from the current population ($i \neq j$). If X_i^* is better than X_i, then replace X_i with X_i^*; otherwise keep the X_i.

The main steps of the proposed NFA are presented in Algorithm 1, where N is the population size, and FEs is the number of fitness evaluations, and Max_FEs is the maximum number of fitness evaluations.

4 Experimental Results

4.1 Test Functions

In order to verify the performance of the proposed NFA, there are thirteen classical benchmark functions used in the following experiments [15, 16]. According to their properties, they are divided into two groups: unimodal functions (f_1-f_7) and multimodal functions (f_8-f_{13}). All test functions are minimization problems. In this paper, we only consider the problems with D = 30. The mathematical descriptions of these functions are listed as follows.

(1) Sphere

$$f_1(x) = \sum_{i=1}^{D} x_i^2$$

where $x_i \in [-100, 100]$, and the global optimum is 0.

(2) Schwefel 2.22

$$f_2(x) = \sum_{i=1}^{D} |x_i| + \prod_{i=1}^{D} x_i$$

where $x_i \in [-10, 10]$, and the global optimum is 0.

(3) Schwefel 1.2

$$f_3(x) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} x_j\right)^2$$

where $x_i \in [-100, 100]$, and the global optimum is 0.

(4) Schwefel 2.21

$$f_4(x) = \max_i(|x_i|, 1 \leq i \leq D)$$

where $x_i \in [-100, 100]$, and the global optimum is 0.

(5) Rosenbrock

$$f_5(x) = \sum_{i=1}^{D-1} \left[100\left(x_i^2 - x_{i+1}\right)^2 + (x_i - 1)^2\right]$$

where $x_i \in [-30, 30]$, and the global optimum is 0.

(6) Step

$$f_6(x) = \sum_{i=1}^{D} (\lfloor x_i + 0.5 \rfloor)^2$$

where $x_i \in [-100,\ 100]$, and the global optimum is 0.

(7) Quartic with noise

$$f_7(x) = \sum_{i=1}^{D} ix_i^4 + rand[0,\ 1)$$

where $x_i \in [-1.28,\ 1.28]$, and the global optimum is 0.

(8) Schwefel 2.26

$$f_8(x) = \sum_{i=1}^{D} -x_i \sin\left(\sqrt{|x_i|}\right)$$

where $x_i \in [-500,\ 500]$, and the global optimum is -12569.5.

(9) Rastrigin

$$f_9(x) = \sum_{i=1}^{D} [x_i^2 - 10\cos(2\pi x_i) + 10]$$

where $x_i \in [-5.12,\ 5.12]$, and the global optimum is 0.

(10) Ackley

$$f_{10}(x) = -20 \cdot \exp\left(-0.2 \cdot \sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right) - \exp\left(\frac{1}{D}\sum_{i=1}^{D} \cos(2\pi x_i)\right) + 20 + e$$

where $x_i \in [-32,\ 32]$, and the global optimum is 0.

(11) Griewank

$$f_{11}(x) = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

where $x_i \in [-600,\ 600]$, and the global optimum is 0.

(12) Penalized 1

$$\begin{aligned} f_{12}(x) = & 0.1\{\sin^2(3\pi x_1) + \sum_{i=1}^{D-1} (x_i - 1)^2[1 + \sin^2(3\pi x_{i+1})] \\ & + (x_D - 1)^2[1 + \sin^2(2\pi x_D)]\} + \sum_{i=1}^{D} u(x_i, 10, 100, 4) \end{aligned}$$

where $x_i \in [-50,\ 50]$, and the global optimum is 0.

(13) Penalized 2

$$f_{13}(x) = \frac{\pi}{D}\{10\sin^2(3\pi y_1) + \sum\nolimits_{i=1}^{D-1}(y_i - 1)^2[1 + \sin^2(3\pi y_{i+1})] + (y_D - 1)^2[1 + \sin^2(2\pi x_D)]\} + \sum\nolimits_{i=1}^{D} u(x_i, 5, 100, 4)$$

where $x_i \in [-50, 50]$, and the global optimum is 0.

4.2 Involved Algorithms and Parameter Settings

In this section, the proposed NFA is compared with the standard FA and two other FA variants. The involved algorithms are listed as follows:

- The Standard FA.
- Memetic FA (MFA) [9].
- Variable step size FA (VSSFA) [14].
- The proposed NFA.

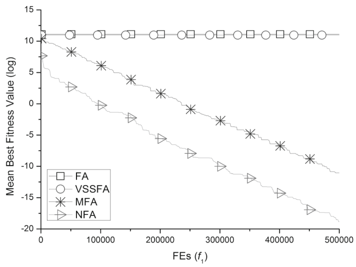
The parameter settings of the above four algorithms are listed as follows. To have a fair comparison, all algorithms use the same *Max_FEs* and *N* as the termination condition, and the *Max_FEs* and *N* are set to 5.0E + 05 and 20, respectively. For the standard FA and VSSFA, $\alpha = 0.2$, $\beta_0 = 1$, and $\gamma = 1$ are used [14]. For MFA and NFA, $\beta_0 = 1.0$, $\beta_{\min} = 0.2$, $\alpha = 0.2$, and $\gamma = 1$ are used. For each test function, each algorithm is run 30 times and the mean best fitness values are reported.

4.3 Results

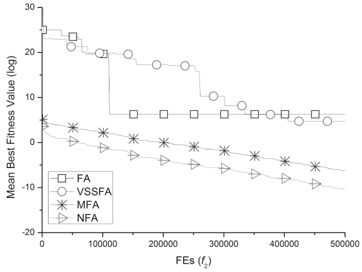
Table 1 presents the computational results of FA, VSSFA, MFA, and NFA on the test set, where "*Mean*" indicates the mean best fitness values. As shown, both FA and

Table 1. Results achieved by FA, VSSFA, MFA, and NFA on the test suite.

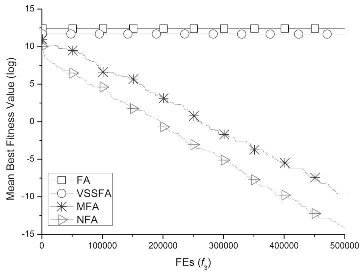
Functions	FA	VSSFA	MFA	NFA
	Mean	*Mean*	*Mean*	*Mean*
f_1	6.67E + 04	5.84E + 04	1.56E − 05	**6.59E − 09**
f_2	5.19E + 02	1.13E + 02	1.85E − 03	**3.21E − 05**
f_3	2.43E + 05	1.16E + 05	5.89E − 05	**7.29E − 07**
f_4	8.35E + 01	8.18E + 01	1.73E − 03	**2.29E − 04**
f_5	2.69E + 08	2.16E + 08	2.29E + 01	**1.57E − 03**
f_6	7.69E + 04	5.48E + 04	**0.00E + 00**	**0.00E + 00**
f_7	5.16E + 01	4.43E + 01	1.30E − 01	**7.68E − 04**
f_8	−1563.4	−1854.6	**−7634.35**	−7160.3
f_9	3.33E + 02	3.12E + 02	6.47E + 01	**4.97E + 01**
f_{10}	2.03E + 01	2.03E + 01	4.23E − 04	**1.68E − 05**
f_{11}	6.54E + 02	5.47E + 02	9.86E − 03	**7.39E − 03**
f_{12}	7.16E + 08	3.99E + 08	5.04E − 08	**3.43E − 11**
f_{13}	1.31E + 09	8.12E + 08	6.06E − 07	**5.02E − 10**



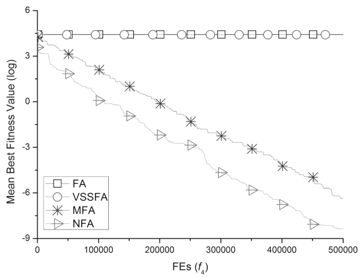
Sphere(f_1)



Schwefel 2.22(f_2)



Schwefel 1.2(f_3)



Schwefel 2.21(f_4)

Fig. 1. The search processes of FA, VSSFA, MFA, and NFA on four selected functions.

VSSFA could hardly achieve reasonable solutions on all test functions. Even on simple unimodal function Sphere (f_1), they still cannot converge to promising solutions. Compared to FA and VSSFA, MFA and NFA obtains much better solutions on all test functions. NFA outperforms MFA on 11 functions, while MFA only performs better than MFA on f_8. Both of them can find the global optimum on f_6.

Figure 1 lists the search processes of FA, VSSFA, MFA, and NFA on four selected functions. As seen, NFA shows the fastest convergence speed among all four algorithm. FA and VSSFA cannot improve the fitness value during the whole search process.

5 Conclusion

In this paper, we propose a new firefly algorithm (NFA) to improve the performance of the standard FA. Unlike the standard FA and its most modifications, the NFA defines a new operation for brighter fireflies. When a firefly is brighter than another one, the brighter firefly will be conducted on a local search. This is helpful to enhance the local search and improve the accuracy of solutions. Moreover, the NFA employs the modifications of MFA. By the hybridization of MFA and the proposed local search, NFA achieves much better results than the standard FA, VSSFA, and MFA on the majority of test functions. The proposed local search can be embedded in other FAs. This will be investigated in the future work.

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