

## Chapter 2

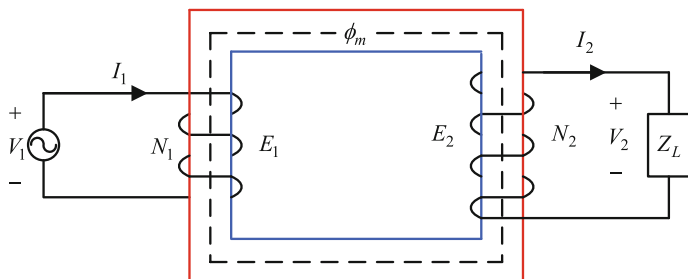
# Transformer: Principles and Practices

### 2.1 Introduction

There are many devices such as three-phase ac generator, transformer etc., which are usually used in a power station to generate and supply electrical power to a power system network. In the power station, the three-phase ac generator generates a three-phase alternating voltage in the range between 11 and 20 kV. The magnitude of the generated voltage is increased to 120 kV or more by means of a power transformer. This higher magnitude of voltage is then transmitted to the grid substation by a three-phase transmission lines. A lower line voltage of 415 V is achieved by stepping down either from the 11 or 33 kV lines by a distribution transformer. In these cases, a three-phase transformer is used in either to step-up or step-down the voltage. Since a transformer plays a vital role in feeding an electrical network with the required voltage, it becomes an important requirement of a power system engineer to understand the fundamental details about a transformer along with its analytical behavior in the circuit domain. This chapter is dedicated towards this goal. On the onset of this discussion it is worth mentioning that a transformer, irrespective of its type, contains the following characteristics (i) it has no moving parts, (ii) no electrical connection between the primary and secondary windings, (iii) windings are magnetically coupled, (iv) rugged and durable in construction, (v) efficiency is very high i.e., more than 95 %, and (vi) frequency is unchanged.

### 2.2 Working Principle of Transformer

Figure 2.1 shows a schematic diagram of a single-phase transformer. There are two types of windings in a single-phase transformer. These are called primary and secondary windings or coils. The primary winding is connected to the alternating



**Fig. 2.1** Single-phase transformer

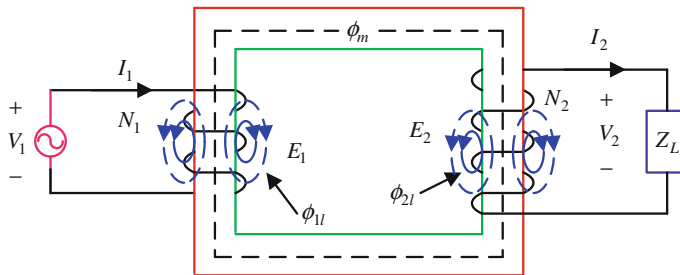
voltage source and the secondary winding is connected to the load. The primary and secondary winding parameters are represented by the suffix  $p$  or 1 and  $s$  or 2, respectively. A sinusoidal current flows in the primary winding when it is connected to an alternating voltage source. This current establishes a flux  $\phi$  which moves from the primary winding to the secondary winding through low reluctance magnetic core.

About 95 % of this flux moves from the primary to the secondary through the low reluctance path of the magnetic core and this flux is linked by the both windings and a small percent of this flux links to the primary winding. According to the Faradays laws of electromagnetic induction, a voltage will be induced across the secondary winding as well as in the primary winding. Due to this voltage, a current will flow through the load if it is connected with the secondary winding. Hence, the primary voltage is transferred to the secondary winding without a change in frequency.

## 2.3 Flux in a Transformer

The current in the primary winding establishes a flux. The flux that moves from primary to secondary and links both the windings is called the mutual flux and its maximum value is represented by  $\phi_m$ .

Flux which links only the primary winding and completes the magnetic path through the surrounding air is known as primary leakage flux. The primary leakage flux is denoted by  $\phi_{1l}$ . Similarly, secondary leakage flux is that flux which links only the secondary winding and completes the magnetic path through the surrounding air. The secondary leakage flux is denoted by  $\phi_{2l}$ . Mutual and leakage fluxes are shown in Fig. 2.2.

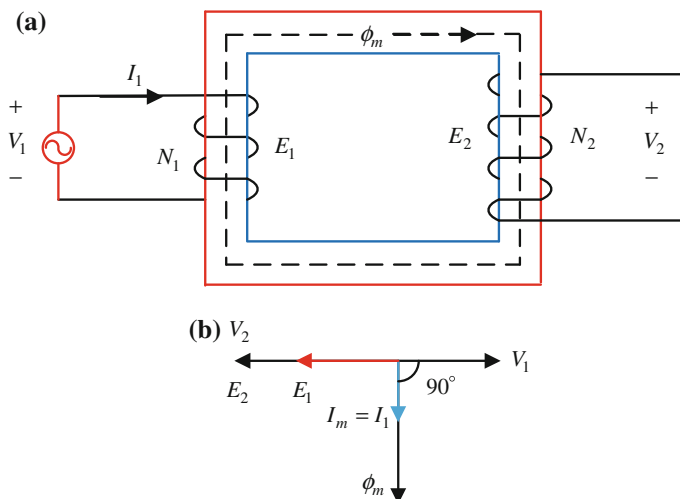


**Fig. 2.2** Mutual and leakage fluxes

## 2.4 Ideal Transformer

An ideal transformer is one which does not supply any energy to the load i.e., the secondary winding is open circuited. The main points of an ideal transformer are (i) no winding resistance, (ii) no leakage flux and leakage inductance, (iii) self-inductance and mutual inductance are zero, (iv) no losses due to resistance, inductance, hysteresis or eddy current and (v) coefficient of coupling is unity.

Figure 2.3a shows an ideal transformer where the secondary winding is left open. A small magnetizing current  $I_m$  will flow in the primary winding when it is connected to the alternating voltage source,  $V_1$ . This magnetizing current lags behind the supply voltage,  $V_1$  by  $90^\circ$  and produces the flux  $\phi$ , which induces the primary and secondary emfs. These emfs lag behind the flux,  $\phi$  by  $90^\circ$ . The magnitude of primary induced emf  $E_1$  and supply voltage  $V_1$  is the same, but are  $180^\circ$  out of phase as shown in Fig. 2.3b.



**Fig. 2.3** Ideal transformer and phasor diagram

## 2.5 E.M.F. Equation of Transformer

The primary winding draws a current when it is connected to an alternating voltage source. This primary sinusoidal current produces a sinusoidal flux  $\phi$  that can be expressed as,

$$\phi = \phi_m \sin \omega t \quad (2.1)$$

Instantaneous emf induced in the primary winding is,

$$e_1 = -N_1 \frac{d\phi}{dt} \quad (2.2)$$

Similarly, instantaneous emf induced in the secondary winding is,

$$e_2 = -N_2 \frac{d\phi}{dt} \quad (2.3)$$

Substituting Eq. (2.1) into the Eq. (2.2) yields,

$$e_1 = -N_1 \frac{d}{dt}(\phi_m \sin \omega t) \quad (2.4)$$

$$e_1 = -N_1 \omega \phi_m \cos \omega t \quad (2.5)$$

$$e_1 = N_1 \omega \phi_m \sin(\omega t - 90^\circ) \quad (2.6)$$

The maximum value of  $e_1$  is,

$$E_{m1} = N_1 \omega \phi_m \quad (2.7)$$

The rms value of the primary emf is,

$$E_1 = \frac{E_{m1}}{\sqrt{2}} \quad (2.8)$$

Substituting Eq. (2.7) into Eq. (2.8) yields,

$$E_1 = \frac{N_1 2\pi f \phi_m}{\sqrt{2}} \quad (2.9)$$

$$E_1 = 4.44 f \phi_m N_1 \quad (2.10)$$

Similarly, the expression of the secondary emf is,

$$E_2 = 4.44f\phi_m N_2 \quad (2.11)$$

The primary and secondary voltages can be determined from Eqs. (2.10) and (2.11) if other parameters are known.

## 2.6 Turns Ratio of Transformer

Turns ratio is an important parameter for drawing an equivalent circuit of a transformer. The turns ratio is used to identify the step-up and step-down transformers. According to Faraday's laws, the induced emf in the primary ( $e_1$ ) and the secondary ( $e_2$ ) windings are,

$$e_1 = -N_1 \frac{d\phi}{dt} \quad (2.12)$$

$$e_2 = -N_2 \frac{d\phi}{dt} \quad (2.13)$$

Dividing Eq. (2.12) by Eq. (2.13) yields,

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} \quad (2.14)$$

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} = a \quad (2.15)$$

Similarly, dividing Eq. (2.10) by (2.11) yields,

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a \quad (2.16)$$

where  $a$  is the turns ratio of a transformer. In case of  $N_2 > N_1$ , the transformer is called a step-up transformer. Whereas for  $N_1 > N_2$ , the transformer is called a step-down transformer. The losses are zero in an ideal transformer. In this case, the input power of the transformer is equal to its output power and this yields,

$$V_1 I_1 = V_2 I_2 \quad (2.17)$$

Equation (2.17) can be rearranged as,

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = a \quad (2.18)$$

The ratio of primary current to the secondary current is,

$$\frac{I_1}{I_2} = \frac{1}{a} \quad (2.19)$$

Again, the magnetomotive force produced by the primary current will be equal to the magnetomotive force produced by the secondary current and it can be expressed as,

$$\mathfrak{F} = \mathfrak{F}_1 - \mathfrak{F}_2 = 0 \quad (2.20)$$

$$N_1 I_1 = N_2 I_2 \quad (2.21)$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \quad (2.22)$$

From Eq. (2.22), it is concluded that the ratio of primary to secondary current is inversely proportional to the turns ratio of the transformer.

The input and output power of an ideal transformer is,

$$P_{in} = V_1 I_1 \cos \phi_1 \quad (2.23)$$

$$P_{out} = V_2 I_2 \cos \phi_2 \quad (2.24)$$

For an ideal condition, the angle  $\phi_1$  is equal to the angle  $\phi_2$  and the output power can be re-arranged as,

$$P_{out} = \frac{V_1}{a} a I_1 \cos \phi_1 \quad (2.25)$$

$$P_{out} = V_1 I_1 \cos \phi_1 = P_{in} \quad (2.26)$$

From Eq. (2.26), it is seen that the input and the output power are the same in case of an ideal transformer. Similarly, the input and output reactive powers are,

$$Q_{out} = V_2 I_2 \sin \phi_2 = V_1 I_1 \sin \phi_1 = Q_{in} \quad (2.27)$$

From Eqs. (2.26) and (2.27), the input and output power and reactive power can be calculated if other parameters are given.

*Example 2.1* The number of turns in the secondary coil of a 22 kVA, 2200 V/220 V single-phase transformer is 50. Find the (i) number of primary turns, (ii) primary full load current, and (iii) secondary full load current. Neglect all kinds of losses in the transformer.

**Solution**

The value of the turns ratio is,

$$a = \frac{V_1}{V_2} = \frac{2200}{220} = 10$$

(i) The value of the primary turns can be determined as,

$$a = \frac{N_1}{N_2}$$

$$N_1 = aN_2 = 10 \times 50 = 500$$

(ii) The value of the primary full load current is,

$$I_1 = \frac{22 \times 10^3}{2200} = 10 \text{ A}$$

(iii) The value of the secondary full load current is,

$$I_2 = \frac{22 \times 10^3}{220} = 100 \text{ A}$$

*Example 2.2* A 25 kVA single-phase transformer has the primary and secondary number of turns of 200 and 400, respectively. The transformer is connected to a 220 V, 50 Hz source. Calculate the (i) turns ratio, and (ii) mutual flux in the core.

**Solution**

(i) The turns ratio is,

$$a = \frac{N_1}{N_2} = \frac{200}{400} = 0.5$$

(ii) The value of the mutual flux can be calculated as,

$$V_1 = E_1 = 4.44f\phi_m N_1$$

$$\phi_m = \frac{V_1}{4.44fN_1} = \frac{220}{4.44 \times 50 \times 200} = 4.95 \text{ mWb}$$

**Practice problem 2.1**

The primary voltage of an iron core single-phase transformer is 220 V. The number of primary and secondary turns of the transformer are 200 and 50, respectively. Calculate the voltage of the secondary coil.

**Practice problem 2.2**

The number of primary turns of a 30 kVA, 2200/220 V, 50 Hz single-phase transformer is 100. Find the (i) turns ratio, (ii) mutual flux in the core, and (iii) full load primary and secondary currents.

**2.7 Rules for Referring Impedance**

For developing equivalent circuit of a transformer, it is necessary to refer the parameters from the primary to the secondary or the secondary to the primary. These parameters are resistance, reactance, impedance, current and voltage. The ratio of primary voltage to secondary voltage is [1–4],

$$\frac{V_1}{V_2} = a \quad (2.28)$$

The ratio of primary current to secondary current is,

$$\frac{I_1}{I_2} = \frac{1}{a} \quad (2.29)$$

Dividing Eq. (2.28) by Eq. (2.29) yields,

$$\frac{\frac{V_1}{V_2}}{\frac{I_1}{I_2}} = \frac{a}{\frac{1}{a}} \quad (2.30)$$

$$\frac{\frac{V_1}{I_1}}{\frac{V_2}{I_2}} = a^2 \quad (2.31)$$

$$\frac{Z_1}{Z_2} = a^2 \quad (2.32)$$

Alternative approach: The impedances in the primary and secondary windings are,

$$Z_1 = \frac{V_1}{I_1} \quad (2.33)$$

$$Z_2 = \frac{V_2}{I_2} \quad (2.34)$$

Dividing Eq. (2.33) by Eq. (2.34) yields,

$$\frac{Z_1}{Z_2} = \frac{\frac{V_1}{I_1}}{\frac{V_2}{I_2}} \quad (2.35)$$



$$\frac{Z_1}{Z_2} = \frac{V_1}{V_2} \times \frac{I_2}{I_1} \quad (2.36)$$

$$\frac{Z_1}{Z_2} = a \times a \quad (2.37)$$

$$\frac{Z_1}{Z_2} = a^2 \quad (2.38)$$

From Eq. (2.38), it can be concluded that the impedance ratio is equal to the square of the turns ratio. The important points for transferring parameters are (i)  $R_1$  in the primary becomes  $\frac{R_1}{a^2}$  when referred to the secondary, (ii)  $R_2$  in the secondary becomes  $a^2 R_2$  when referred to the primary, (iii)  $X_1$  in the primary becomes  $\frac{X_1}{a^2}$  when referred to the secondary, and (iv)  $X_2$  in the secondary becomes  $a^2 X_2$  when referred to the primary.

**Example 2.3** The number of primary and secondary turns of a single-phase transformer are 300 and 30, respectively. The secondary coil is connected with a load impedance of  $4 \Omega$ . Calculate the (i) turns ratio, (ii) load impedance referred to the primary, and (iii) primary current if the primary coil voltage is 220 V.

### Solution

- (i) The value of the turns ratio is,

$$a = \frac{N_1}{N_2} = \frac{300}{30} = 10$$

- (ii) The value of the load impedance referred to primary is,

$$Z'_L = a^2 Z_L = 10^2 \times 4 = 400 \Omega$$

- (iii) The value of the primary current is,

$$I_1 = \frac{V_1}{Z'_L} = \frac{220}{400} = 0.55 \text{ A}$$

### Practice problem 2.3

A load impedance of  $8 \Omega$  is connected to the secondary coil of a 400/200 turns single-phase transformer. Determine the (i) turns ratio, (ii) load impedance referred to primary and (iii) primary current if the primary coil voltage is 120 V.

## 2.8 Equivalent Circuit of a Transformer

Windings of a transformer are not connected electrically. The windings are magnetically coupled with each other. In this case, it is tedious to do proper analysis. Therefore, for easy computation and visualization, the practical transformer needs to be converted into an equivalent circuit by maintaining same properties of the main transformer. In the equivalent circuit, the related parameters need to be transferred either from the primary to the secondary or vice versa. A two windings ideal transformer is shown in Fig. 2.4.

### 2.8.1 Exact Equivalent Circuit

Figure 2.5 shows an exact equivalent circuit referred to the primary where all the parameters are transferred from the secondary to the primary and these parameters are,

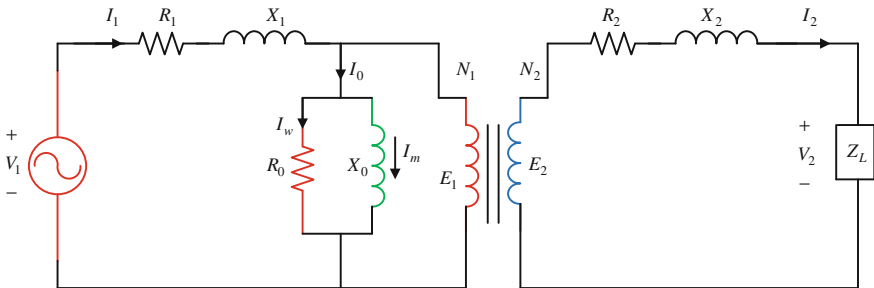


Fig. 2.4 Two windings transformer

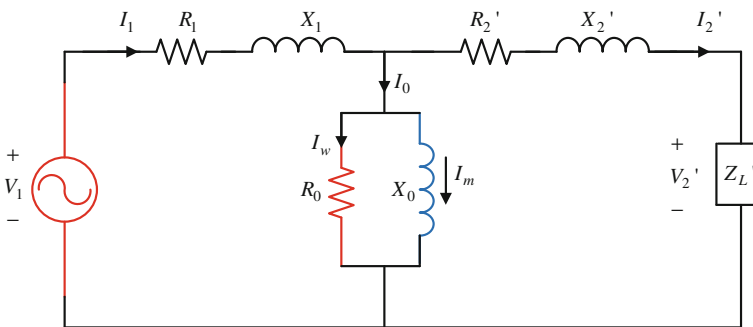


Fig. 2.5 Exact equivalent circuit referred to primary

$$R'_2 = a^2 R_2 \quad (2.39)$$

$$X'_2 = a^2 X_2 \quad (2.40)$$

$$Z'_L = a^2 Z_L \quad (2.41)$$

$$I'_2 = \frac{I_2}{a} \quad (2.42)$$

$$V'_2 = a V_2 \quad (2.43)$$

Figure 2.6 shows the exact equivalent circuit referred to the secondary where all the parameters are transferred from the primary to the secondary.

These parameters are,

$$R'_1 = \frac{R_1}{a^2} \quad (2.44)$$

$$X'_1 = \frac{X_1}{a^2} \quad (2.45)$$

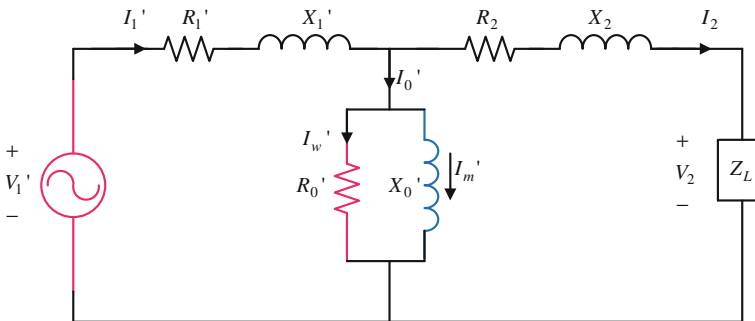
$$I'_1 = a I_1 \quad (2.46)$$

$$V'_1 = \frac{V_1}{a} \quad (2.47)$$

$$I'_w = a I_w \quad (2.48)$$

$$I'_m = a I_m \quad (2.49)$$

$$I'_0 = a I_0 \quad (2.50)$$



**Fig. 2.6** Exact equivalent circuit referred to secondary

### 2.8.2 Approximate Equivalent Circuit

The no-load current is very small as compared to the rated primary current. Therefore, there is a negligible voltage drop due to  $R_1$  and  $X_1$ . In this condition, it can be assumed that the voltage drop across the no-load circuit is the same as the applied voltage without any significant error. The approximate equivalent circuit can be drawn by shifting the no-load circuit across the supply voltage,  $V_1$ . Figure 2.7 shows an approximate equivalent circuit referred to the primary. The total resistance, reactance and impedance referred to the primary are,

$$R_{01} = R_1 + R'_2 = R_1 + a^2 R_2 \quad (2.51)$$

$$X_{01} = X_1 + X'_2 = X_1 + a^2 X_2 \quad (2.52)$$

$$Z_{01} = R_{01} + jX_{01} \quad (2.53)$$

The no-load circuit resistance and reactance are,

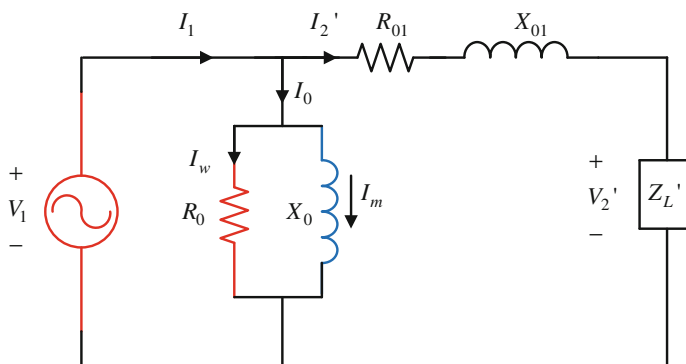
$$R_0 = \frac{V_1}{I_w} \quad (2.54)$$

$$X_0 = \frac{V_1}{I_m} \quad (2.55)$$

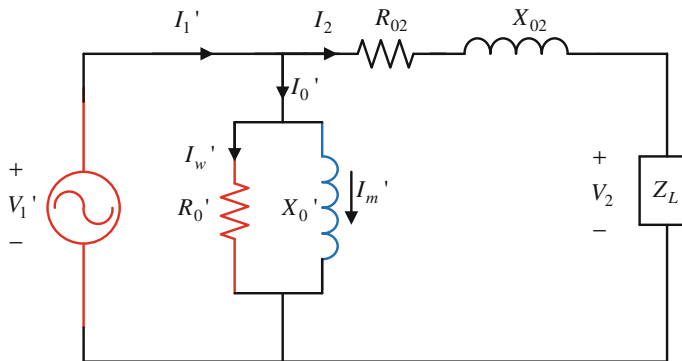
Figure 2.8 shows an approximate equivalent circuit referred to the secondary.

The total resistance, reactance and impedance referred to the secondary is,

$$R_{02} = R_2 + R'_1 = R_2 + \frac{R_1}{a^2} \quad (2.56)$$



**Fig. 2.7** Approximate equivalent circuit referred to primary



**Fig. 2.8** Approximate equivalent circuit referred to secondary

$$X_{02} = X_2 + X_1' = X_2 + \frac{X_1}{a^2} \quad (2.57)$$

$$Z_{02} = R_{02} + jX_{02} \quad (2.58)$$

The no-load circuit resistance and reactance referred to the secondary are,

$$R_0' = \frac{V_1'}{I_w'} \quad (2.59)$$

$$X_0' = \frac{V_1'}{I_m'} \quad (2.60)$$

**Example 2.4** A 2.5 kVA, 200 V/40 V single-phase transformer has the primary resistance and reactance of 3 and 12  $\Omega$ , respectively. On the secondary side, these values are 0.3 and 0.1  $\Omega$ , respectively. Find the equivalent impedance referred to the primary and the secondary.

### Solution

The value of the turns ratio is,

$$a = \frac{V_1}{V_2} = \frac{200}{40} = 5$$

The total resistance, reactance and impedance referred to the primary can be determined as,

$$R_{01} = R_1 + a^2 R_2 = 3 + 25 \times 0.3 = 10.5 \Omega$$

$$X_{01} = X_1 + a^2 X_2 = 12 + 25 \times 0.1 = 14.5 \Omega$$

$$Z_{01} = \sqrt{10.5^2 + 14.5^2} = 17.9 \Omega$$

The total resistance, reactance and impedance referred to the secondary are calculated as,

$$R_{02} = R_2 + \frac{R_1}{a^2} = 0.3 + \frac{3}{25} = 0.42 \, \Omega$$

$$X_{02} = X_2 + \frac{X_1}{a^2} = 0.1 + \frac{12}{25} = 0.58 \, \Omega$$

$$Z_{01} = \sqrt{0.42^2 + 0.58^2} = 0.72 \, \Omega$$

### Practice problem 2.4

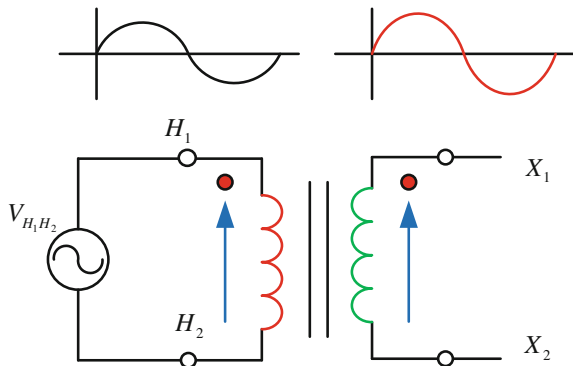
A 1.5 kVA, 220/110 V single-phase transformer has the primary resistance and reactance of 6 and 18  $\Omega$ , respectively. The resistance and reactance at the secondary side are 0.6 and 0.5  $\Omega$ , respectively. Calculate the equivalent impedance referred to the primary and the secondary.

## 2.9 Polarity of a Transformer

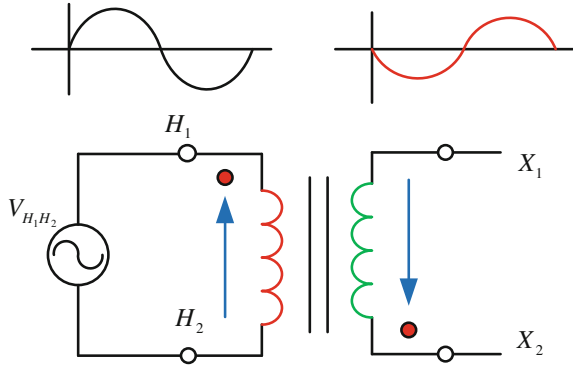
The relative directions of induced voltages between the high voltage and low voltage terminals is known as the polarity of a transformer. The polarity of a transformer is very important to construct three-phase transformer bank, parallel connection of transformer, connection of current transformer (CT) and potential transformer (PT) power with metering device. Two polarities namely additive and subtractive are used in the transformer.

A polarity of a transformer is said to be an additive if the measured voltage between the high voltage and the low voltage terminals is greater than the supply voltage at the high voltage terminals. The additive polarity of a transformer is marked by the orientation of dots as shown in Fig. 2.9. Whereas, a polarity is said to be a subtractive if the measured voltage between the high voltage and the low voltage terminals is lower than the supply voltage at the high voltage terminals. The

**Fig. 2.9** Additive polarity



**Fig. 2.10** Subtractive polarity



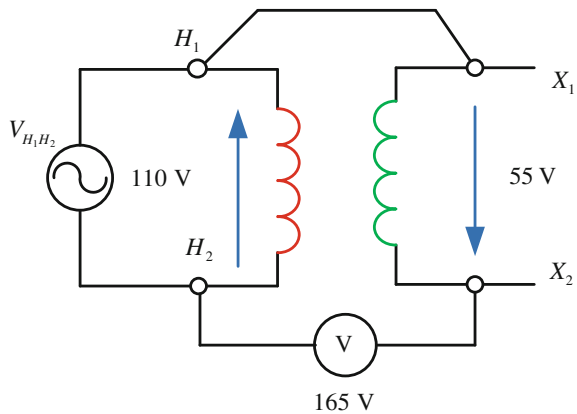
subtractive polarity of a transformer is marked by the orientation of dots as shown in Fig. 2.10. Consider a 220/110 V single-phase transformer with the high voltage and the low voltage terminals for testing polarities. The high voltage terminal  $H_1$  is connected to the low voltage terminal  $X_1$  by a cable. The voltmeter is connected between  $H_2$  and  $X_2$ . In this case, the turns ratio of the transformer is,

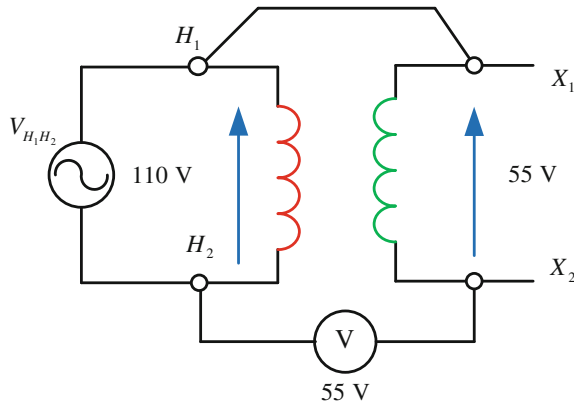
$$a = \frac{V_1}{V_2} = \frac{220}{110} = 2$$

For safety issue, a lower voltage needs to be applied to the primary side i.e., high voltage terminals. Suppose, a voltage of 110 V is applied to the primary side. In this case, a voltage of 55 V ( $110/2$ ) will appear at the secondary terminals. If the meter read out the voltage of 165 V ( $110 + 55$ ) then the transformer is said to be in additive polarity. This connection is shown in Fig. 2.11.

Whereas, if the voltmeter reads the voltage of 55 V ( $110 - 55$ ) then the transformer is said to be in subtractive polarity as shown in Fig. 2.12.

**Fig. 2.11** Testing of additive polarity

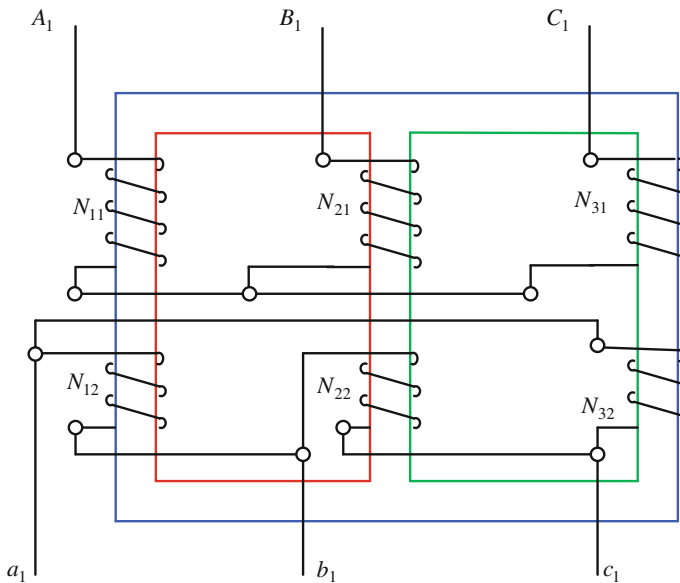




**Fig. 2.12** Testing of subtractive polarity

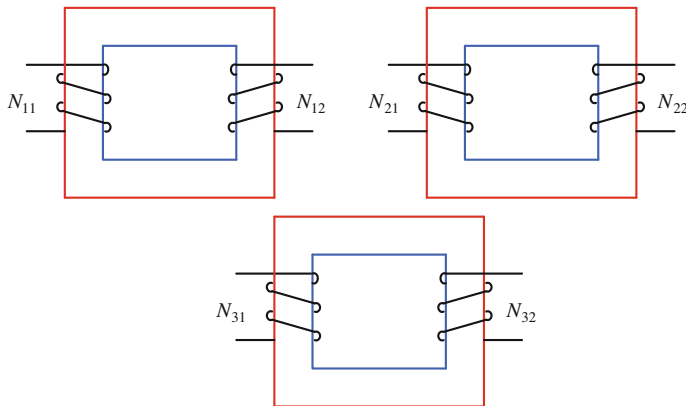
## 2.10 Three-Phase Transformer

A three-phase power transformer is used at the power generating station to step-up the voltage from 11 to 120 kV. Whereas in the power distribution substation, the three-phase voltage is again stepped down to 11 kV voltage through a three-phase distribution transformer.



**Fig. 2.13** Three windings on a common core and wye-delta connection





**Fig. 2.14** Three single-phase transformers

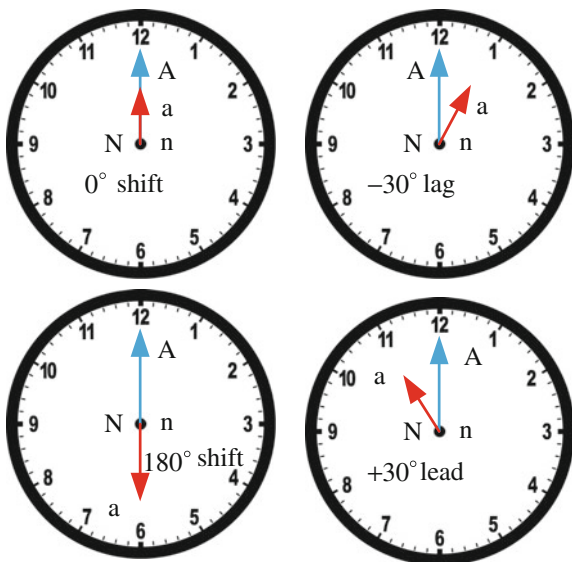
Therefore, a three-phase transformer can be made either by three windings wound on a common core or by three single-phase transformer connected together in a three-phase bank.

The first approach is a cheaper one that results in a transformer with smaller size and less weight. The main disadvantage of the first approach is that if one phase becomes defective, then the whole transformer needs to be replaced. Whereas in the second approach, if one of the transformers becomes defective then the system can be given power by an open delta at a reduced capacity. In this case, the defective transformer is normally replaced by a new one. A three-phase transformer with wye-delta connection is shown in Fig. 2.13 and the three single-phase transformers are shown in Fig. 2.14.

## 2.11 Transformer Vector Group

The most common configurations of a three-phase transformer are the delta and star configurations, which are used in the power utility company. The primary and secondary windings of a three-phase transformer are connected either in the same (delta-delta or star-star), or different (delta-star or star-delta) configuration-pair. The secondary voltage waveforms of a three-phase transformer are in phase with the primary waveforms when the primary and secondary windings are connected in the same configuration. This condition is known as ‘no phase shift’ condition. If the primary and secondary windings are connected in different configuration pair then the secondary voltage waveforms will differ from the corresponding primary

**Fig. 2.15** Representation of vector groups with clock



voltage waveforms by 30 electrical degrees. This condition is called a '30° phase shift' condition. The windings and their position to each other are usually marked by vector group. The vector group is used to identify the phase shift between the primary and secondary windings. In the vector group, the secondary voltage may have the phase shift of 30° lagging or leading, 0° i.e., no phase shift or 180° reversal with respect to the primary voltage. The transformer vector group is labeled by capital and small letters plus numbers from 1 to 12 in a typical clock-like diagram. The capital letter indicates primary winding and small letter represents secondary winding. In the clock diagram, the minute hand represents the primary line to neutral voltage, and its place is always in the 12. The hour hand represents the secondary line to neutral voltage and its position in the clock changes based on the phase shift as shown in Fig. 2.15.

There are four vector groups used in the three-phase transformer connection. These vector groups are (i) Group I: 0 o'clock, zero phase displacement (Yy0, Dd0, Dz0), (ii) Group II: 6 o'clock, 180° phase displacement (Yy6, Dd6, Dz6), (iii) Group III: 1 o'clock, -30° lag phase displacement (Dy1, Yd1, Yz1), and (iv) Group IV: 11 o'clock, 30° lead phase displacement (Dy11, Yd11, Yz11). Here, Y represents wye connection, D represents delta connection and z represents the zigzag connection. The connection diagrams for different combinations are shown in Figs. 2.16, 2.17, 2.18, 2.19, 2.20, 2.21, 2.22, 2.23, 2.24, 2.25, 2.26 and 2.27.

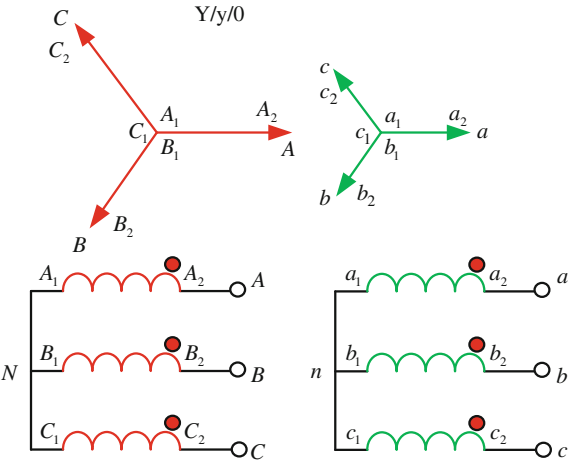


Fig. 2.16 Connection of Y-y-0

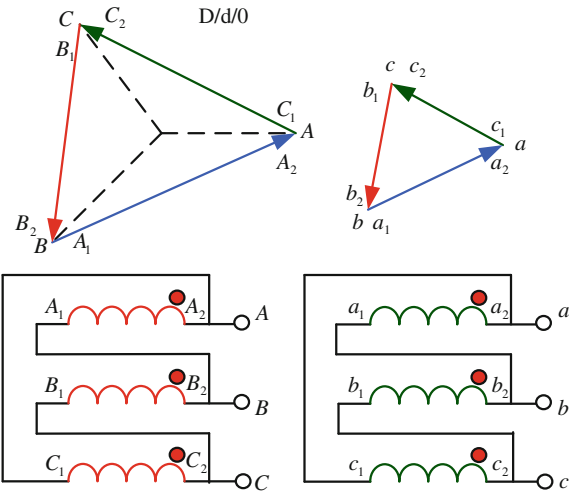


Fig. 2.17 Connection of D-d-0

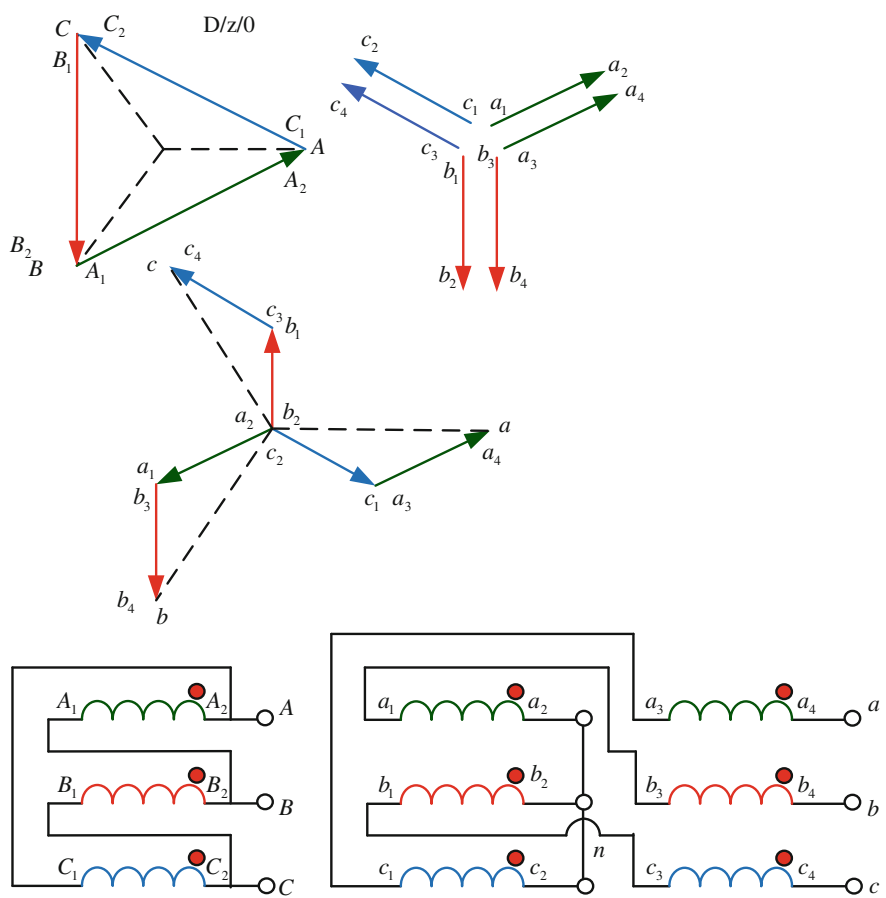
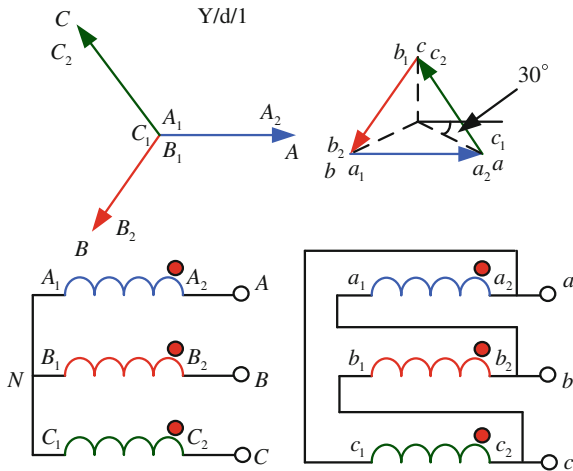


Fig. 2.18 Connection of D-z/0

Fig. 2.19 Connection of Y-d-1



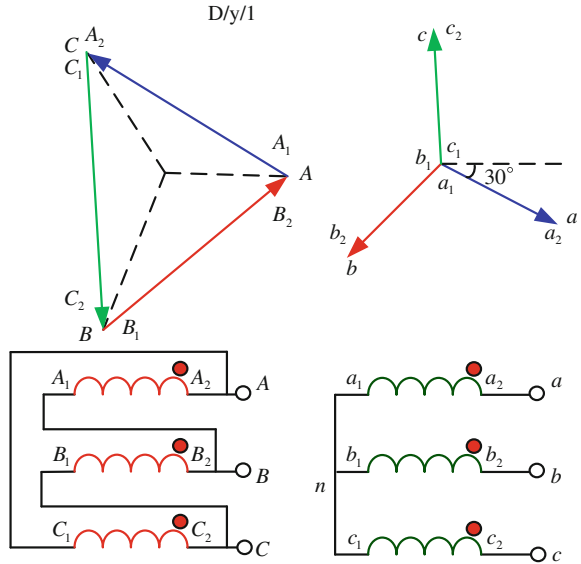


Fig. 2.20 Connection of D-y-1

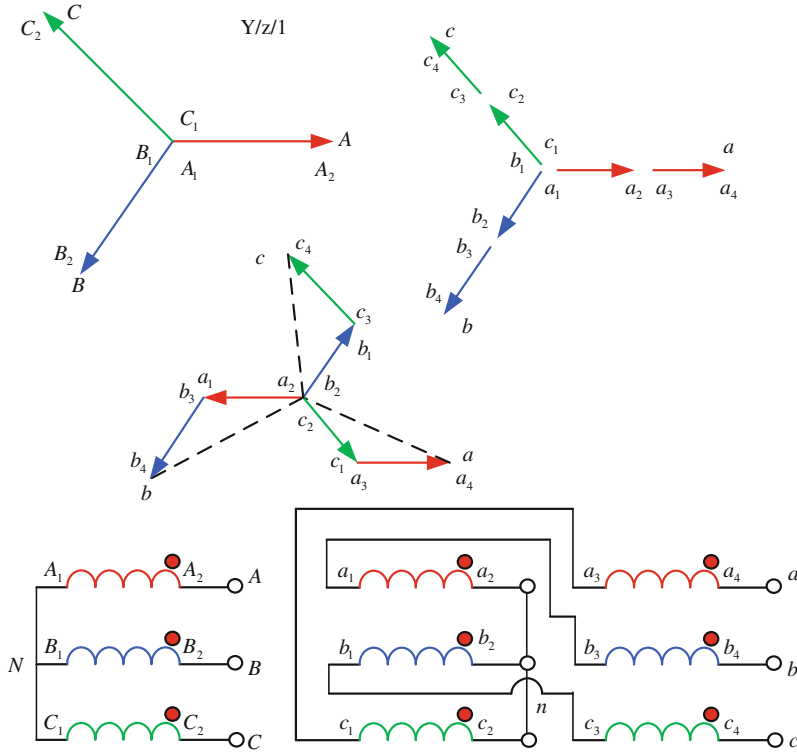


Fig. 2.21 Connection of Y-z-1



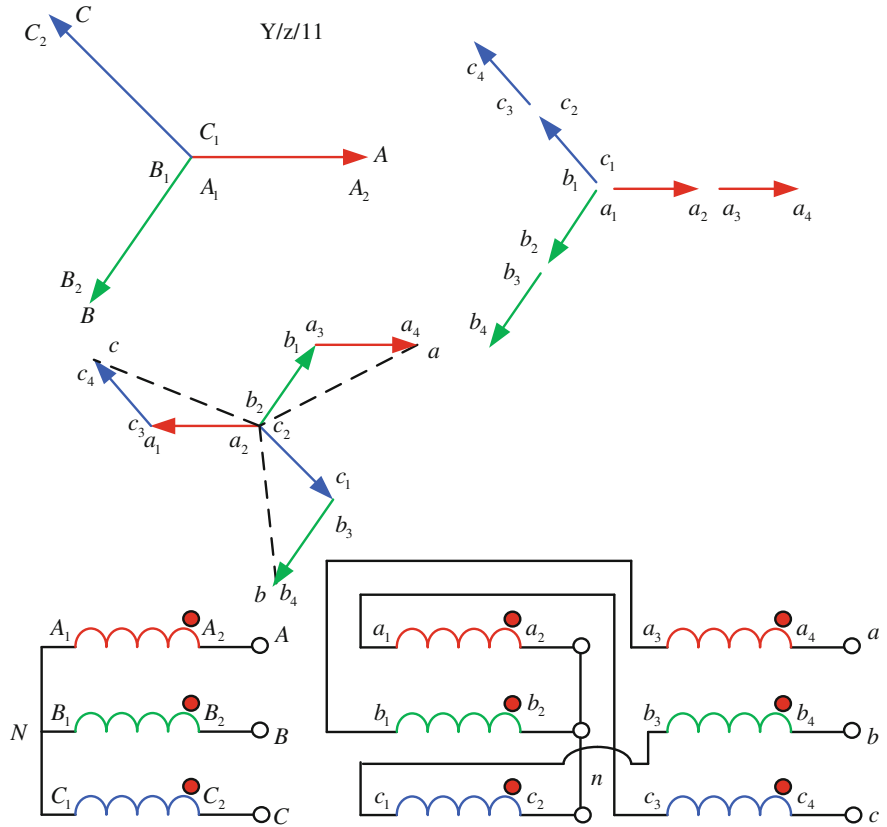
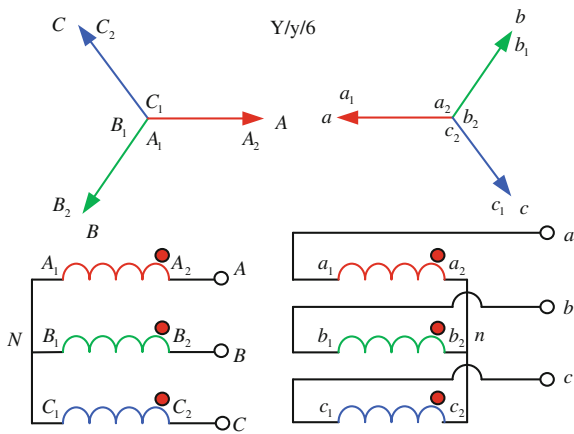


Fig. 2.24 Connection of Y-z-11

Fig. 2.25 Connection of Y-y-6



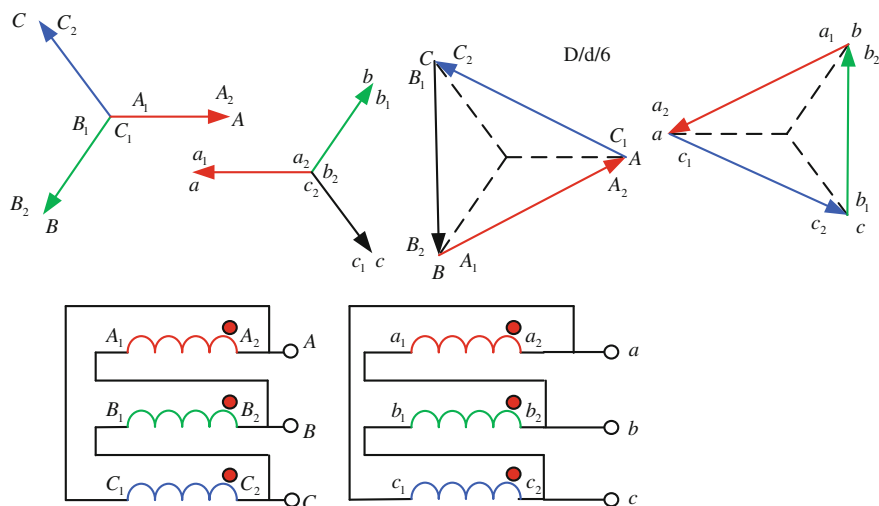


Fig. 2.26 Connection of D-d-6

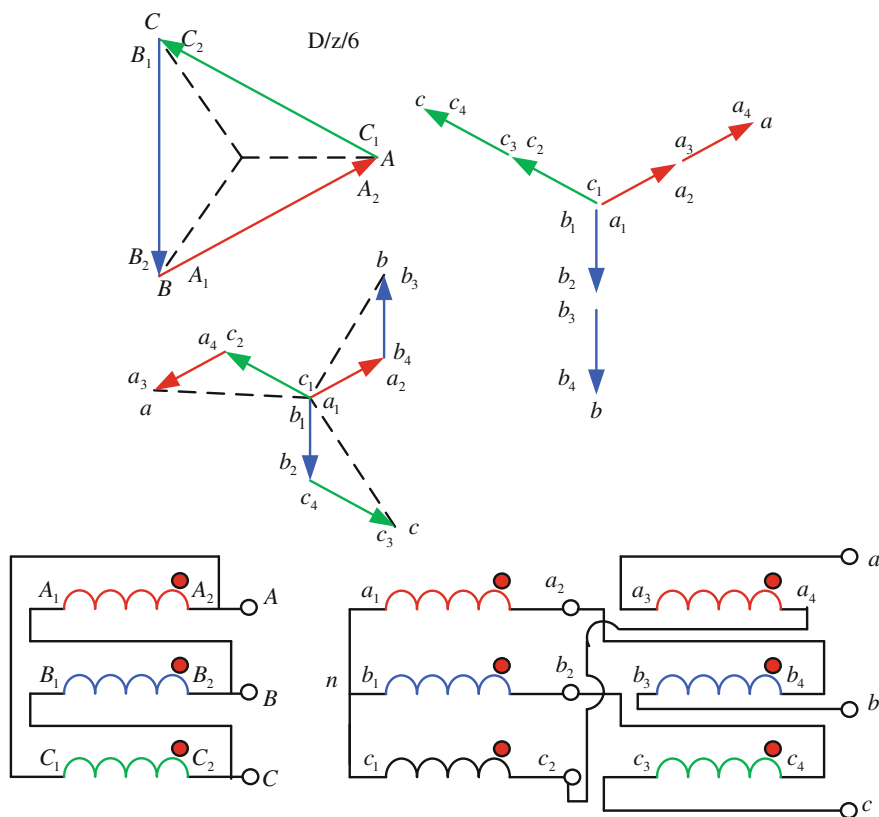


Fig. 2.27 Connection of D-y-6



## 2.12 Voltage Regulation of a Transformer

Different types of loads like domestic, commercial and industrial are usually connected with the secondary winding of a transformer. All these loads are operated with a constant magnitude of voltage. The secondary voltage of a transformer under operation changes due to voltage drop across the internal impedance and the load. The voltage regulation of a transformer is used to identify the characteristic of the secondary side voltage changes under different loading conditions. The voltage regulation of a transformer is defined as the difference between the no-load terminal voltage ( $V_{2NL}$ ) to full load terminal voltage ( $V_{2FL}$ ) and is expressed as a percentage of full load terminal voltage. It is therefore can be expressed as,

$$\text{Voltage regulation} = \frac{V_{2NL} - V_{2FL}}{V_{2FL}} \times 100\% = \frac{E_2 - V_2}{V_2} \times 100\% \quad (2.61)$$

Figure 2.28 shows an approximate equivalent circuit referred to the secondary without no-load circuit to find the voltage regulation for different power factors. Phasor diagrams for different power factors are shown in Fig. 2.29 where the secondary voltage  $V_2$  is considered as the reference phasor.

The phasor diagram with a unity power factor is shown in Fig. 2.29a and the phasor form of the secondary induced voltage for a unity power factor can be written as,

$$E_2 = V_2 + I_2(R_{02} + jX_{02}) \quad (2.62)$$

Figure 2.29b shows the phasor diagram for a lagging power factor, and from this diagram, the following expressions can be written,

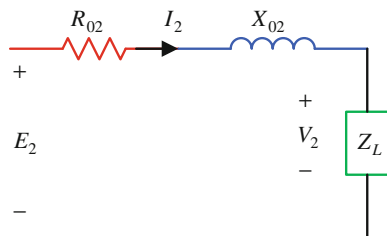
$$AC = V_2 \cos \phi_2 \quad (2.63)$$

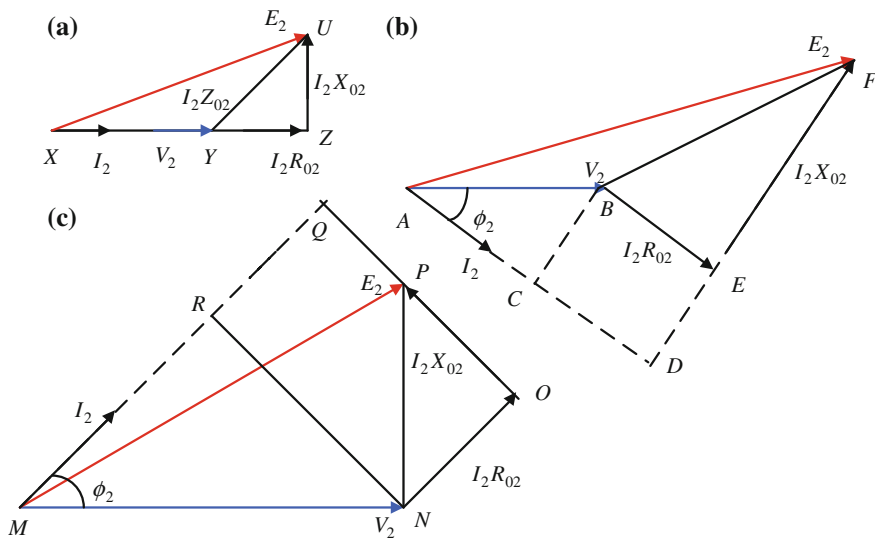
$$BC = DE = V_2 \sin \phi_2 \quad (2.64)$$

$$CD = BE = I_2 R_{02} \quad (2.65)$$

$$EF = I_2 X_{02} \quad (2.66)$$

**Fig. 2.28** Approximate equivalent circuit referred to secondary





**Fig. 2.29** Phasor diagram for different power factors

From the right angle triangle- $ADF$ , the expression of  $E_2$  can be derived as,

$$AF^2 = AD^2 + DF^2 \quad (2.67)$$

$$AF^2 = (AC + CD)^2 + (DE + EF)^2 \quad (2.68)$$

Substituting equations from (2.63) to (2.66) into Eq. (2.68) yields,

$$E_2 = \sqrt{(V_2 \cos \phi_2 + I_2 R_{02})^2 + (V_2 \sin \phi_2 + I_2 X_{02})^2} \quad (2.69)$$

In phasor form, Eq. (2.69) can be written as,

$$E_2 = (V_2 \cos \phi_2 + I_2 R_{02}) + j(V_2 \sin \phi_2 + I_2 X_{02}) \quad (2.70)$$

Figure 2.29c shows the phasor diagram for a leading power factor and from this diagram, the following expressions can be written,

$$MR = V_2 \cos \phi_2 \quad (2.71)$$

$$RN = OQ = V_2 \sin \phi_2 \quad (2.72)$$

$$RQ = NO = I_2 R_{02} \quad (2.73)$$

$$PO = I_2 X_{02} \quad (2.74)$$

The expression of  $E_2$  from the right angled triangle  $MQP$  can be derived as,

$$MP^2 = MQ^2 + QP^2 \quad (2.75)$$

$$MP^2 = (MR + RQ)^2 + (OQ - PO)^2 \quad (2.76)$$

Substituting equations from (2.71) to (2.74) into Eq. (2.76) yields,

$$E_2 = \sqrt{(V_2 \cos \phi_2 + I_2 R_{02})^2 + (V_2 \sin \phi_2 - I_2 X_{02})^2} \quad (2.77)$$

In phasor form, Eq. (2.77) can be written as,

$$E_2 = (V_2 \cos \phi_2 + I_2 R_{02}) + j(V_2 \sin \phi_2 - I_2 X_{02}) \quad (2.78)$$

**Example 2.5** The primary coil resistance and reactance of a 200/400 V single-phase transformer are 0.3 and 0.6  $\Omega$ , respectively. The secondary coil resistance and reactance are 0.8 and 1.6  $\Omega$ , respectively. Calculate the voltage regulation if the secondary current of the transformer is 10 A at a 0.8 pf lagging.

### Solution

The value of the turns ratio is,

$$a = \frac{V_1}{V_2} = \frac{200}{400} = 0.5$$

The value of the total resistance referred to the secondary is,

$$R_{02} = R_2 + \frac{R_1}{a^2} = 0.8 + \frac{0.3}{0.25} = 2 \Omega$$

The total reactance referred to the primary is,

$$X_{02} = X_2 + \frac{X_1}{a^2} = 1.6 + \frac{0.6}{0.25} = 4 \Omega$$

The no-load voltage is,

$$\begin{aligned} E_2 &= (V_2 \cos \phi_2 + I_2 R_{02}) + j(V_2 \sin \phi_2 + I_2 X_{02}) \\ E_2 &= (400 \times 0.8 + 10 \times 2) + j(400 \times 0.6 + 40) \\ E_2 &= 440.5 \angle 39.5^\circ \text{ V} \end{aligned}$$

The voltage regulation can be determined as,

$$\begin{aligned}\text{Voltage regulation} &= \frac{E_2 - V_2}{V_2} \times 100 \% \\ \text{Voltage regulation} &= \frac{440.5 - 400}{400} \times 100 \% = 10 \%\end{aligned}$$

### Practice problem 2.5

A 110/220 V single-phase transformer has the resistance of  $0.2 \Omega$  and a reactance of  $0.8 \Omega$  in the primary winding. The resistance and reactance in the secondary winding are  $0.9$  and  $1.8 \Omega$ , respectively. Calculate the voltage regulation, when the secondary current is 6 A at a 0.85 power factor leading.

## 2.13 Efficiency of a Transformer

The efficiency is an important parameter to identify the characteristics of any machine. The efficiency  $\eta$ , of any machine can be defined as the ratio of its output to the input. Mathematically, it can be expressed as,

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}} = 1 - \frac{\text{losses}}{\text{input}} \quad (2.79)$$

Let us consider the following:

$P_{out}$  is the output power, in W,

$P_{in}$  is the input power, in W,

$P_{losses}$  is the total losses, in W.

Equation (2.79) can be modified as,

$$\eta = \frac{P_{out}}{P_{in}} = 1 - \frac{P_{losses}}{P_{in}} \quad (2.80)$$

It is worth nothing that the efficiency of a transformer is generally higher than other electrical machines because the transformer has no moving parts.

## 2.14 Iron and Copper Losses

The iron loss of a transformer is often called as core loss, which is a result of an alternating flux in the core of the transformer. The iron loss consists of the eddy current loss and the hysteresis loss. In the transformer, most of the flux transferred from the primary coil to the secondary coil through low reluctance iron path. A few

portion of that alternating flux links with the iron parts of the transformer. As a result, an emf is induced in the transformer core. A current will flow in that parts of the transformer. This current does not contribute in output of the transformer but dissipated as heat. This current is known as eddy current and the power loss due to this current is known as eddy current loss. The eddy current loss ( $P_e$ ) is directly proportional to the square of the frequency ( $f$ ) times the maximum magnetic flux density ( $B_m$ ) and the eddy current loss can be expressed as,

$$P_e = k_e f^2 B_m^2 \quad (2.81)$$

where  $k_e$  is the proportionality constant.

Steel is a very good ferromagnetic material which is used for the core of a transformer. This ferromagnetic material contains number of domains in the structure and magnetized easily. These domains are like small magnets located randomly in the structure. When an mmf is applied to the core then those domains change their position. After removing mmf, most of the domains come back to their original position and remaining will be as it is. As a result, the substance is slightly permanently magnetized. An additional mmf is required to change the position of the remaining domains. Therefore, hysteresis loss is defined as the additional energy is required to realign the domains in the ferromagnetic material. The hysteresis loss ( $P_h$ ) is directly proportional to the frequency ( $f$ ) and 2.6th power of the maximum magnetic flux density ( $B_m$ ) and the expression of hysteresis loss is,

$$P_h = k_h f B_m^{2.6} \quad (2.82)$$

where  $k_e$  is the proportionality constant.

Now that the magnetic flux density is usually constant, Eqs. (2.81) and (2.82) can be modified as,

$$P_e \propto f^2 \quad (2.83)$$

$$P_h \propto f \quad (2.84)$$

Practically, hysteresis loss depends on the voltage and the eddy current loss depends on the current. Therefore, total losses of the transformer depend on the voltage and the current not on the power factor. That is why the transformer rating is always represented in kVA instead of kW.

In the transformer, copper losses occur due to the primary and the secondary resistances. The full load copper losses can be determined as,

$$P_{cu\text{loss}} = I_1^2 R_1 + I_2^2 R_2 \quad (2.85)$$

$$P_{cu\text{loss}} = I_1^2 R_{01} = I_2^2 R_{02} \quad (2.86)$$

## 2.15 Condition for Maximum Efficiency

The expression of the output power of a transformer is,

$$P_{out} = V_2 I_2 \cos \phi_2 \quad (2.87)$$

The expression of the copper loss is,

$$P_{cu} = I_2^2 R_{02} \quad (2.88)$$

The expression of an iron loss is,

$$P_{iron} = P_{eddy} + P_{hys} \quad (2.89)$$

According to Eq. (2.79), the efficiency can be expressed as,

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}} \quad (2.90)$$

Substituting Eqs. (2.87), (2.88) and (2.89) into Eq. (2.90) yields,

$$\eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_{iron} + P_{cu}} \quad (2.91)$$

$$\eta = \frac{V_2 \cos \phi_2}{V_2 \cos \phi_2 + \frac{P_{iron}}{I_2} + \frac{I_2^2 R_{02}}{I_2}} \quad (2.92)$$

The terminal voltage at the secondary side is considered to be constant in case of a normal transformer. For a given power factor, the secondary current is varied with the variation of load. Therefore, the transformer efficiency will be maximum, if the denominator of Eq. (2.92) is minimum. The denominator of Eq. (2.92) will be minimum, for the following condition.

$$\frac{d}{dI_2} \left( V_2 \cos \phi_2 + \frac{P_{iron}}{I_2} + \frac{I_2^2 R_{02}}{I_2} \right) = 0 \quad (2.93)$$

$$-\frac{P_{iron}}{I_2^2} + R_{02} = 0 \quad (2.94)$$

$$P_{iron} = I_2^2 R_{02} \quad (2.95)$$

From Eq. (2.95), it is concluded that the efficiency of a transformer will be maximum, when the iron loss is equal to the copper loss. From Eq. (2.95), the expression of secondary current can be written as,

$$I_2 = \sqrt{\frac{P_{\text{iron}}}{R_{02}}} \quad (2.96)$$

For maximum efficiency, the load current can be expressed as,

$$I_{2\eta} = \sqrt{\frac{P_{\text{iron}}}{R_{02}}} \quad (2.97)$$

Equation (2.97) can be re-arranged as,

$$I_{2\eta} = I_2 \sqrt{\frac{P_{\text{iron}}}{I_2^2 R_{02}}} \quad (2.98)$$

Multiplying both sides of Eq. (2.98) by the secondary rated voltage  $V_2$  yields,

$$V_2 I_{2\eta} = V_2 I_2 \sqrt{\frac{P_{\text{iron}}}{I_2^2 R_{02}}} \quad (2.99)$$

$$VI_{\text{max efficiency}} = VI_{\text{rated}} \times \sqrt{\frac{\text{iron loss}}{\text{full load copper loss}}} \quad (2.100)$$

**Example 2.6** A 30 kVA transformer has the iron loss and full load copper loss of 350 and 650 W, respectively. Determine the (i) full load efficiency, (ii) output kVA corresponding to maximum efficiency, and (iii) maximum efficiency. Consider that the power factor of the load is 0.6 lagging.

**Solution**

- (i) The total value of full load loss is,

$$P_{\text{loss}} = 350 + 650 = 1000 \text{ W} = 1 \text{ kW}$$

The output power at full load is,

$$P_{\text{out}} = 30 \times 0.6 = 18 \text{ kW}$$

The input power at full load is,

$$P_{\text{in}} = 18 + 1 = 19 \text{ kW}$$

The efficiency at full load is,

$$\eta = \frac{18}{19} \times 100 = 94.74 \%$$

(ii) The output kVA corresponding to maximum efficiency is,

$$\begin{aligned} &= kVA_{\text{rated}} \times \sqrt{\frac{\text{Iron loss}}{\text{Cu loss at full load}}} \\ &= 30 \times \sqrt{\frac{350}{650}} = 22 \text{ kVA} \end{aligned}$$

The output power is,

$$P_{o1} = 22 \times 0.6 = 13.2 \text{ kW}$$

(iii) For maximum efficiency, iron loss is equal to copper loss.  
The total value of the loss is,

$$P_{\text{loss-1}} = 2 \times 350 = 700 \text{ W}$$

The value of the input power is,

$$P_{in1} = 13.2 + 0.7 = 13.9 \text{ kW}$$

The efficiency is,

$$\eta = \frac{13.2}{13.9} \times 100 = 94.96 \%$$

### Practice problem 2.6

A 200 kVA transformer is having an iron loss of 1.5 kW and a copper loss of 5 kW at full load condition. Find the kVA rating at which the efficiency is maximum. Also, find the efficiency at a unity power factor.

## 2.16 Transformer Tests

The equivalent circuit parameters are very important to characterize the performance of transformer. The parameters of a transformer equivalent circuit can be determined by the open circuit and the short circuit tests.



### 2.16.1 Open Circuit Test

The main objectives of the open circuit test are to determine the no-load current and iron loss. The components of the no-load current are used to determine the no-load circuit resistance and reactance.

In an open circuit test, the secondary side is considered to be open circuit, and the primary coil is connected to the source as shown in Fig. 2.30a, where all measuring instruments are connected in the primary side. A specific alternating voltage is applied to the primary winding. Then the wattmeter will measure the iron loss and small amount of copper loss. The ammeter and voltmeter will measure the no-load current and the voltage, respectively. Since, the no-load current is very small, the copper losses can be neglected. Then the wattmeter reading can be expressed as,

$$P_0 = V_1 I_0 \cos \phi_0 \quad (2.101)$$

From Eq. (2.101), the no-load power factor can be determined as,

$$\cos \phi_0 = \frac{P_0}{V_1 I_0} \quad (2.102)$$

The working and magnetizing components of the current can be determined as,

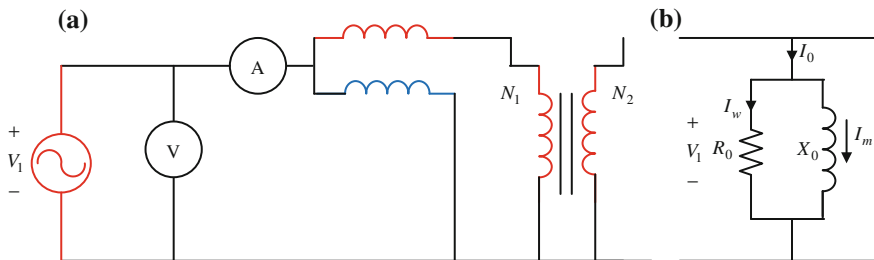
$$I_w = I_0 \cos \phi_0 \quad (2.103)$$

$$I_m = I_0 \sin \phi_0 \quad (2.104)$$

Then the no-load circuit resistance and reactance can be determined as,

$$R_0 = \frac{V_1}{I_w} \quad (2.105)$$

$$X_0 = \frac{V_1}{I_m} \quad (2.106)$$



**Fig. 2.30** Connection diagrams for open circuit test and no-load circuit

**Example 2.7** A 200/400 V, 50 Hz single-phase transformer has the no-load test data of 200 V, 0.6 A, 80 W. Calculate the no-load circuit resistance and reactance.

**Solution**

The power factor can be determined as,

$$\cos \phi_0 = \frac{P_0}{V_1 I_0} = \frac{80}{200 \times 0.6} = 0.67$$

$$\sin \phi_0 = 0.74$$

The values of the working and magnetizing components of the no-load current are,

$$I_w = I_0 \cos \phi_0 = 0.6 \times 0.67 = 0.4 \text{ A}$$

$$I_m = I_0 \sin \phi_0 = 0.6 \times 0.74 = 0.44 \text{ A}$$

The no-load circuit parameters can be determined as,

$$R_0 = \frac{V_1}{I_w} = \frac{200}{0.4} = 500 \Omega$$

$$X_0 = \frac{V_1}{I_m} = \frac{200}{0.44} = 454.5 \Omega$$

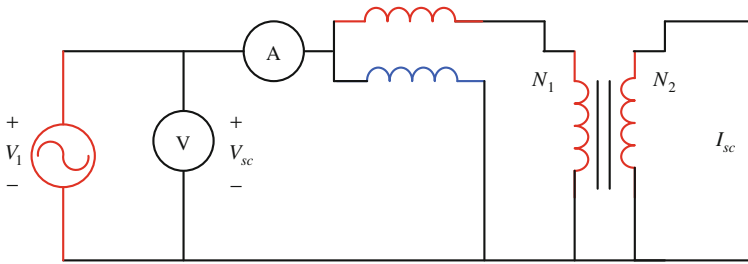
### 2.16.2 Short Circuit Test

The main objectives of the short circuit test are to determine the equivalent resistance, reactance, impedance and full load copper loss. In this test, the supply voltage and the measuring instruments (e. g., wattmeter, ammeter) are connected to the primary side and the secondary winding is shorted out by a wire as shown in Fig. 2.31, or connected with an ammeter. The primary voltage is adjusted until the current in the short-circuited winding is equal to the rated primary current. Under this condition, the wattmeter will measure the full load copper loss and it can be written as,

$$P_{sc} = I_{sc} V_{sc} \cos \phi_{sc} \quad (2.107)$$

From Eq. (2.107), the short circuit power factor can be calculated as,

$$\cos \phi_{sc} = \frac{P_{sc}}{I_{sc} V_{sc}} \quad (2.108)$$



**Fig. 2.31** Connection diagram for short circuit test

The equivalent impedance can be calculated as,

$$Z_{01} = Z_{eq} = \frac{V_{sc}}{I_{sc}} \quad (2.109)$$

In addition, the equivalent resistance and reactance can be calculated as,

$$R_{01} = Z_{01} \cos \phi_{sc} \quad (2.110)$$

$$X_{01} = Z_{01} \sin \phi_{sc} \quad (2.111)$$

*Example 2.8* A 25 kVA, 2200/220 V, 50 Hz single-phase transformer's low voltage side is short-circuited and the test data recorded from the high voltage side are  $P = 150$  W,  $I_1 = 5$  A and  $V_1 = 40$  V. Determine the (i) equivalent resistance, reactance and impedance referred to primary, (ii) equivalent resistance, reactance and impedance referred to secondary, and (iii) voltage regulation at unity power factor.

### Solution

(i) The parameters referred to primary are,

$$Z_{01} = \frac{V_1}{I_1} = \frac{40}{5} = 8 \, \Omega$$

$$R_{01} = \frac{P}{I_1^2} = \frac{150}{25} = 6 \, \Omega$$

$$\phi = \cos^{-1} \left( \frac{6}{8} \right) = 41.4^\circ$$

$$X_{01} = Z_{01} \sin \phi = 8 \sin 41.4^\circ = 5.2 \, \Omega$$

(ii) The turns ratio is,

$$a = \frac{V_1}{V_2} = \frac{2200}{220} = 10$$

The parameters referred to secondary are,

$$\begin{aligned} Z_{02} &= \frac{Z_{01}}{a^2} = \frac{8}{100} = 0.08 \, \Omega \\ R_{02} &= \frac{R_{01}}{a^2} = \frac{6}{100} = 0.06 \, \Omega \\ X_{02} &= \frac{X_{01}}{a^2} = \frac{5.2}{100} = 0.052 \, \Omega \end{aligned}$$

(iii) The secondary side current is,

$$I_2 = \frac{25,000}{220} = 113.6 \, \text{A}$$

The secondary induced voltage is,

$$E_2 = V_2 + I_2 Z_{02} = 220 + 113.6 \times 0.08 = 229 \, \text{V}$$

The voltage regulation is,

$$VR = \frac{E_2 - V_2}{V_2} = \frac{229 - 220}{220} = 4 \, \%$$

### Practice problem 2.7

The no-load test data of a 220/2200 V, 50 Hz single-phase transformer are 220 V, 0.4 A, 75 W. Calculate the no-load circuit resistance and reactance.

### Practice problem 2.8

A 30 kVA, 2300/220 V, 50 Hz single-phase transformer's low voltage side is short-circuited by a thick wire. The test data recorded from the high voltage side are  $P = 400 \, \text{W}$ ,  $I_1 = 8.5 \, \text{A}$  and  $V_1 = 65 \, \text{V}$ . Find the (i) equivalent resistance, reactance and impedance referred to primary and (ii) equivalent resistance, reactance and impedance referred to secondary, and (iii) voltage regulation at a 0.6 lagging power factor.

## 2.17 Autotransformer

A small rating transformer with a variable voltage output is usually used in the educational laboratory as well as in the testing laboratory. This type of small rating transformer with a variable output is known as autotransformer. An autotransformer has one continuous winding that is common to both the primary and the secondary. Therefore, in an autotransformer, the primary and secondary windings are connected electrically. The advantages of an autotransformer over a two-winding transformer include lower initial investment, lower leakage reactance, lower losses compared to conventional transformer and lower excitation current.

An autotransformer with primary and secondary windings is shown in Fig. 2.32. In this connection, the suffix  $c$  indicates the common winding and the suffix  $s$  indicates the series winding. From Fig. 2.32, the following equations can be written as,

$$\frac{N_s}{N_c} = \frac{I_c}{I_s} = a \quad (2.112)$$

$$\frac{V_s}{V_c} = \frac{N_s}{N_c} = a \quad (2.113)$$

The total voltage in the primary side is,

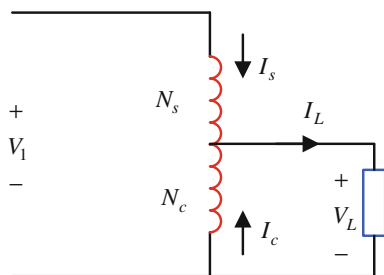
$$V_1 = V_s + V_c = V_c \left( 1 + \frac{V_s}{V_c} \right) \quad (2.114)$$

Substituting Eq. (2.113) into Eq. (2.114) yields,

$$\frac{V_1}{V_c} = (1 + a) \quad (2.115)$$

$$\frac{V_L}{V_1} = \frac{1}{1 + a} \quad (2.116)$$

**Fig. 2.32** Connection diagram for an autotransformer



Where the voltage at the load is equal to the voltage at the common terminals i.e.,  $V_L = V_c$ .

The expression of the load current can be written as,

$$I_L = I_s + I_c = aI_s + I_s \quad (2.117)$$

$$I_L = (1 + a)I_s \quad (2.118)$$

$$\frac{I_L}{I_s} = (1 + a) \quad (2.119)$$

**Example 2.9** Figure 2.33 shows a single-phase 120 kVA, 2200/220 V, 50 Hz transformer which is connected as an autotransformer. The voltages of the upper and lower parts of the coil are 220 and 2200 V, respectively. Calculate the kVA rating of the autotransformer.

### Solution

The current ratings of the respective windings are,

$$I_{pq} = \frac{120,000}{220} = 545.5 \text{ A}$$

$$I_{qr} = \frac{120,000}{2200} = 54.5 \text{ A}$$

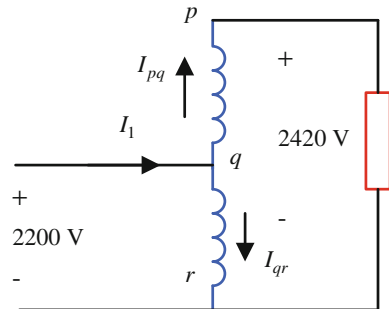
The current in the primary side is,

$$I_1 = 545.5 + 54.5 = 600 \text{ A}$$

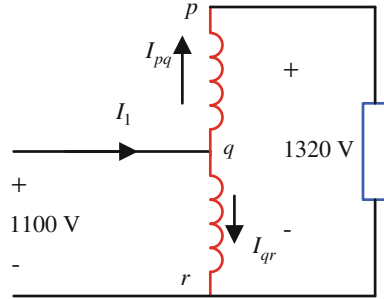
The voltage across the secondary side is,

$$V_2 = 2200 + 220 = 2420 \text{ V}$$

**Fig. 2.33** An autotransformer with a specific voltage



**Fig. 2.34** An autotransformer with a specific voltage



Therefore, the kVA ratings of an autotransformer,

$$\text{kVA}_1 = \frac{600 \times 2200}{1000} = 1320 \text{ kVA}$$

$$\text{kVA}_2 = \frac{545.5 \times 2420}{1000} = 1320 \text{ kVA}$$

### Practice problem 2.9

A single-phase 100 kVA, 1100/220 V, 50 Hz transformer is connected as an autotransformer as shown in Fig. 2.34. The voltages of the upper and lower parts of the coil are 220 and 1100 V, respectively. Determine the kVA rating of the autotransformer.

## 2.18 Parallel Operation of a Single-Phase Transformer

Nowadays, the demand of load is increasing with the increase in population and industrial sector. Sometimes, it is difficult to meet the excess demand of power by the existing single unit transformer. Therefore, an additional transformer is required to connect in parallel with the existing one. The following points should be considered for making parallel connections of transformers:

- Terminal voltage of both transformers must be the same.
- Polarity must be same for both transformers.
- For both transformers, the percentage impedances should be equal in magnitude.
- Ratio of  $R$  to  $X$  must be the same for both transformers.
- Phase sequences and phase shifts must be the same (for three-phase transformer).

## 2.19 Three-Phase Transformer Connections

The primary and secondary windings of the transformer may be connected in either by wye (Y) or delta ( $\Delta$ ). Three-phase transformer connections are classified into four possible types; namely, Y-Y(wye-wye), Y- $\Delta$  (wye-delta),  $\Delta$ -Y(delta-wye) and  $\Delta$ - $\Delta$  (delta-delta).

### 2.19.1 Wye-Wye Connection

Figure 2.35 shows the Y-Y connection diagram. This type of connection of a three-phase transformer is rarely used for large amount of power transmission.

Neutral point is necessary for both primary and secondary sides in some cases. In balanced loads, this type of connection works satisfactorily and provides neutral to each side for grounding. At the primary side, the phase voltage can be written as,

$$V_{P1} = \frac{V_{L1}}{\sqrt{3}} \quad (2.120)$$

The secondary phase voltage can be written as,

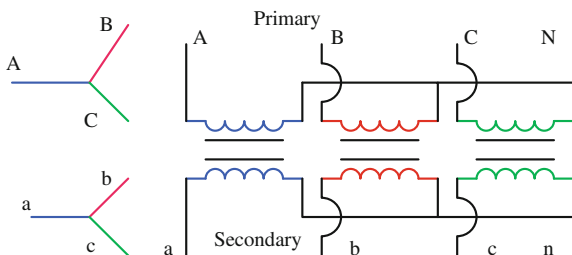
$$V_{P2} = \frac{V_{L2}}{\sqrt{3}} \quad (2.121)$$

The ratio of the primary line voltage to the secondary line voltage of this connection is,

$$a = \frac{V_{L1}}{V_{L2}} \quad (2.122)$$

$$a = \frac{\sqrt{3}V_{P1}}{\sqrt{3}V_{P2}} = \frac{V_{P1}}{V_{P2}} \quad (2.123)$$

**Fig. 2.35** Y-Y connection diagram





### 2.19.2 Wye-Delta Connection

The wye-delta connection is mainly used at the substation where the voltage is stepped down. In this connection, the current in the secondary coil is 57.7 % of the load current. At the primary side of this connection, a proper copper wire is used to ground the neutral point. Figure 2.36 shows the connection diagram of the wye-delta transformer. In this connection, the expression of the primary line voltage is,

$$V_{L1} = \sqrt{3}V_{P1} \quad (2.124)$$

At the secondary side, the line voltage is,

$$V_{L2} = V_{P2} \quad (2.125)$$

The ratio of primary phase voltage to secondary phase voltage is,

$$a = \frac{V_{P1}}{V_{P2}} \quad (2.126)$$

The ratio of primary line voltage to secondary line voltage is,

$$\frac{V_{L1}}{V_{L2}} = \frac{\sqrt{3}V_{P1}}{V_{P2}} = \sqrt{3}a \quad (2.127)$$

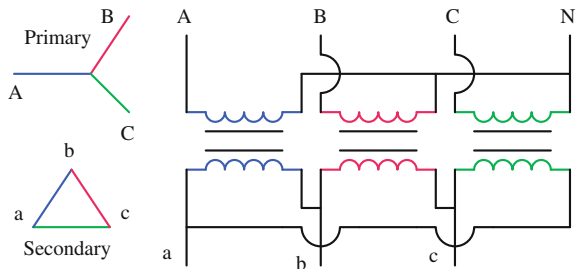
The primary phase current is,

$$I_{P1} = I_{L1} \quad (2.128)$$

In this case, the turns ratio is,

$$a = \frac{I_{P2}}{I_{P1}} \quad (2.129)$$

**Fig. 2.36** Wye-delta connection diagram



The expression of the secondary phase current is,

$$I_{P2} = aI_{P1} \quad (2.130)$$

The secondary line current is,

$$I_{L2} = \sqrt{3}I_{P2} = \sqrt{3}aI_{P1} \quad (2.131)$$

### 2.19.3 Delta-Wye Connection

The delta-wye connection is generally used at the power generating station to step the voltage. Figure 2.37 shows the connection diagram of a delta-wye transformer. In this connection, the expression of the primary line voltage is,

$$V_{L1} = V_{P1} \quad (2.132)$$

The line voltage at the secondary side is,

$$V_{L2} = \sqrt{3}V_{P2} \quad (2.133)$$

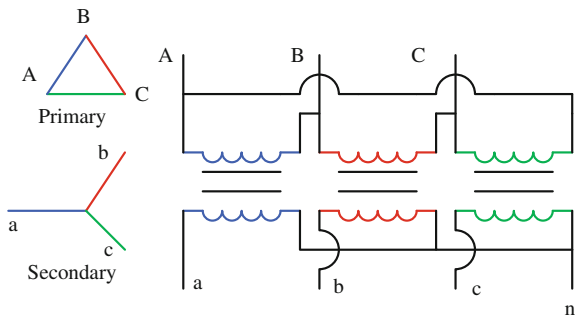
The ratio of primary line voltage to secondary line voltage is,

$$\frac{V_{L1}}{V_{L2}} = \frac{V_{P1}}{\sqrt{3}V_{P2}} = \frac{a}{\sqrt{3}} \quad (2.134)$$

The phase current at the primary side is,

$$I_{P1} = \frac{1}{\sqrt{3}}I_{L1} \quad (2.135)$$

**Fig. 2.37** Delta-wye connection diagram



In this connection, the turns ratio is,

$$a = \frac{I_{P2}}{I_{P1}} \quad (2.136)$$

In this case, the secondary phase current is,

$$I_{P2} = aI_{P1} \quad (2.137)$$

The secondary line current is,

$$I_{L2} = I_{P2} = aI_{P1} \quad (2.138)$$

### 2.19.4 Delta-Delta Connection

The delta-delta connection is generally used for both high voltage and low voltage rating transformers where insulation is not an important issue. The connection diagram for delta-delta transformer is shown in Fig. 2.38. In this case, the primary line voltage is,

$$V_{L1} = V_{P1} \quad (2.139)$$

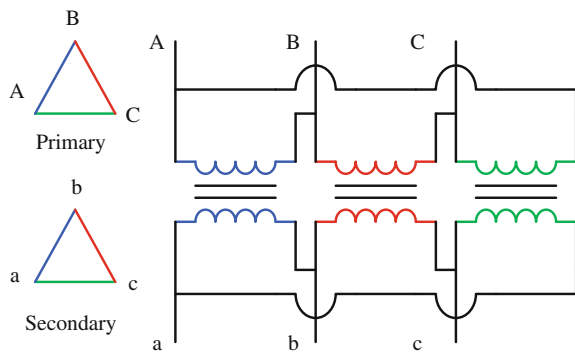
The secondary line voltage is,

$$V_{L2} = V_{P2} \quad (2.140)$$

The ratio of line-to-line voltage of this connection is,

$$\frac{V_{L1}}{V_{L2}} = \frac{V_{P1}}{V_{P2}} = a \quad (2.141)$$

**Fig. 2.38** Delta-delta connection diagram



The secondary line current is,

$$I_{L2} = \sqrt{3} I_{P2} \quad (2.142)$$

The output capacity in delta-delta connection can be expressed as,

$$P_{0\Delta-\Delta} = \sqrt{3} V_{L2} I_{L2} \quad (2.143)$$

Substituting Eq. (2.142) into Eq. (2.43) yields,

$$P_{0\Delta-\Delta} = \sqrt{3} V_{L2} \times \sqrt{3} I_{P2} = 3 V_{L2} I_{P2} \quad (2.144)$$

The remaining two transformers are able to supply three-phase power to the load terminal if one of the transformer is removed from the connection. This type of three-phase power supply by two transformers is known as open delta or V-V connection. This open delta connection is able to supply three-phase power at a reduced rate of 57.7 %. The connection diagram for open delta connection is shown in Fig. 2.39.

In the V-V connection, the secondary line current is,

$$I_{L2} = I_{P2} \quad (2.145)$$

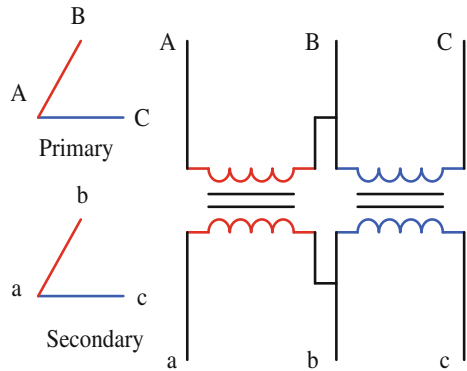
The output capacity in V-V connection is,

$$P_{0V-V} = \sqrt{3} V_{L2} I_{P2} \quad (2.146)$$

The ratio of delta-delta output capacity to the V-V output capacity is,

$$\frac{P_{0V-V}}{P_{0\Delta-\Delta}} = \frac{\sqrt{3} V_{L2} I_{P2}}{3 V_{L2} I_{P2}} = 0.577 \quad (2.147)$$

**Fig. 2.39** Open delta or V-V connection diagram



*Example 2.10* A three-phase transformer is connected to an 11 kV supply and draws 6 A current. Determine (i) line voltage at the secondary side, and also (ii) the line current in the secondary coil. Consider the turns ratio of the transformer is 11. Also, consider delta-wye and wye-delta connections.

**Solution**

**Delta-wye connection:**

The phase voltage at the primary side is,

$$V_{P1} = V_{L1} = 11 \text{ kV}$$

The phase voltage at the secondary side is,

$$V_{P2} = \frac{V_{P1}}{a} = \frac{11,000}{11} = 1000 \text{ V}$$

(i) The line voltage at the secondary is,

$$V_{L2} = \sqrt{3}V_{P2} = \sqrt{3} \times 1000 = 1732 \text{ V}$$

(ii) The phase current in the primary is,

$$I_{P1} = \frac{I_{L1}}{\sqrt{3}} = \frac{11}{\sqrt{3}} = 6.35 \text{ A}$$

The line current in the secondary is,

$$I_{L2} = I_{P2} = aI_{P1} = 11 \times 6.35 = 69.85 \text{ A}$$

**Wye-delta connection:**

The line voltage at the primary side is,

$$V_{L1} = 11 \text{ kV}$$

The phase voltage at the primary side is,

$$V_{P1} = \frac{V_{L1}}{\sqrt{3}} = \frac{11,000}{\sqrt{3}} = 6350.85 \text{ V}$$

- (i) The phase voltage at the secondary side is,

$$V_{P2} = \frac{V_{P1}}{a} = \frac{6350.85}{11} = 577.35 \text{ V}$$

The line voltage at the secondary is,

$$V_{L2} = V_{P2} = 577.35 \text{ V}$$

- (ii) The phase current in the secondary is,

$$I_{P2} = aI_{P1} = 11 \times 6 = 66 \text{ A}$$

The line current in the secondary is,

$$I_{L2} = \sqrt{3}I_{P2} = \sqrt{3} \times 66 = 114.32 \text{ A}$$

### Practice problem 2.10

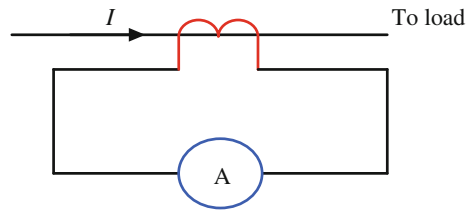
A three-phase transformer is connected to the 33 kV supply and draws 15 A current. Calculate (i) the line voltage at the secondary side, and also (ii) the line current in the secondary winding. Consider the turns ratio of the transformer is 8 and wye-delta connection.

## 2.20 Instrument Transformers

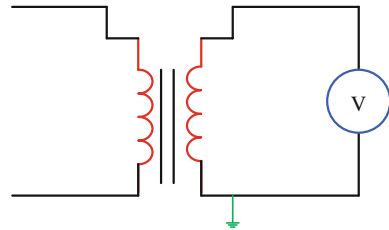
The magnitude of the voltage and the current are normally high in the power system networks. To reduce the magnitude of the voltage and current, instrument transformers are used. There are two types of instrument transformers; namely, current transformer (CT) and potential transformer (PT). If a power system carries an alternating current greater than 100 A, then it is difficult to connect measuring instruments like low range ammeter, wattmeter, energy meter and relay.

The current transformer is then connected in series with the line to step down the high magnitude of the current to a rated value of about 5 A for the ammeter and the current coil of the wattmeter. The diagram of a current transformer is shown in Fig. 2.40. If the system voltage exceeds 500 V, then it is difficult to connect measuring instruments directly to the high voltage. The potential transformer is then used to step down to a suitable value of the voltage at the secondary for supplying the voltmeter and the voltage coil of the wattmeter. The secondary of the instrument transformer is normally grounded for safety reason. The connection diagram of a potential transformer is shown in Fig. 2.41.

**Fig. 2.40** Connection diagram of current transformer



**Fig. 2.41** Connection diagram of potential transformer



## Exercise Problems

- 2.1 A single-phase transformer is having the primary voltage of 240 V. The number of turns at the primary and secondary coils are 250 and 50, respectively. Determine the secondary voltage.
- 2.2 A single-phase transformer is having the secondary voltage of 100 V. The number of turns at the primary and secondary windings are 500 and 50, respectively. Calculate the primary voltage.
- 2.3 The 100 A current flows in the primary coil of a single-phase transformer. Calculate the secondary current with the turns ratio of 0.05.
- 2.4 The number of turns at the primary and secondary windings of a single-phase transformer are 500 and 100, respectively. Find the primary current if the secondary current is 10 A.
- 2.5 The ratio of the primary current to the secondary current of a single-phase transformer is 1:10. Find the voltage ratio. Also, find the secondary voltage if the primary voltage is 100 V.
- 2.6 The turns ratio of a single-phase transformer is 1:4. The secondary coil has 5000 turns and 60 V. Find the voltage ratio, primary voltage and number of turns.
- 2.7 The voltage ratio of a single-phase transformer is 5:1. The voltage and number of turns at the primary coil are found to be 1100 V and 500, respectively. Calculate the voltage and secondary number of turns at the secondary coil.
- 2.8 A single-phase transformer is connected to the 120 V source draws 4 A current. Calculate the current in the secondary coil when the transformer steps up the voltage to 500 V.

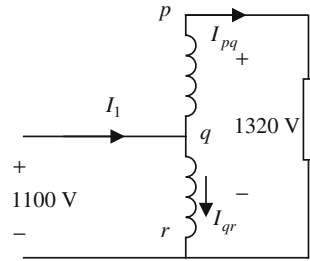
- 2.9 The number of turns at the primary and secondary coils of a single-phase transformer are found to be 480 and 60, respectively. The transformer draws a current of 0.6 A when connected to 120 V supply. Determine the current and the voltage at the secondary coil.
- 2.10 The turns ratio of a 5 kVA single-phase transformer is found to be 2. The transformer is connected to a 230 V, 50 Hz supply. If the secondary current of a transformer is 6 A, then find the primary current and secondary voltage.
- 2.11 The secondary number of turns of a 30 kVA, 4400/440 V single-phase transformer is found to be 100. Find the number of primary turns, full load primary and secondary currents.
- 2.12 A 5 kVA, 1100/230 V, 50 Hz single-phase transformer is installed near the domestic area of a country. Find the turns ratio, primary and secondary full load currents.
- 2.13 The number of turns of the primary winding of a single-phase 50 Hz transformer is found to be 50. Find the value of the core flux, if the induced voltage at this winding is 220 V.
- 2.14 The number of primary turns of a 60 Hz single-phase transformer is found to be 500. Calculate the induced voltage in this winding, if the value of the core flux is 0.05 Wb.
- 2.15 The maximum flux of a 3300/330 V, 50 Hz step-down single-phase transformer is found to be 0.45 Wb. Calculate the number of primary and secondary turns.
- 2.16 The cross sectional area of a 5 kVA, 2200/220 V, 50 Hz single-phase step-down transformer is found to be  $25 \text{ cm}^2$  and a maximum flux density is  $4 \text{ Wb/m}^2$ . Calculate the primary and secondary turns.
- 2.17 The number of turns at the primary and secondary of an ideal single-phase transformer are found to be 500 and 250, respectively. The primary of the transformer is connected to a 220 V, 50 Hz source. The secondary coil supplies a current of 5 A to the load. Determine the (i) turns ratio, (ii) current in the primary and (iii) maximum flux in the core.
- 2.18 The primary number of turns of a 4 kVA, 1100/230 V, 50 Hz single-phase transformer is 500. The cross sectional area of the transformer core is  $85 \text{ cm}^2$ . Find the (i) turns ratio, (ii) number of turns in the secondary and (iii) maximum flux density in the core.
- 2.19 The primary and secondary turns of a single-phase transformer are 50 and 500, respectively. The primary winding is connected to a 220 V, 50 Hz supply. Find the (i) flux density in the core if the cross sectional area is  $250 \text{ cm}^2$  and (ii) induced voltage at the secondary winding.
- 2.20 The primary coil number of turns of a single-phase 2200/220 V, 50 Hz transformer is found to be 1000. Find the area of the core if the maximum flux density of the core is  $1.5 \text{ Wb/m}^2$ .
- 2.21 The maximum flux of a single-phase 1100/220 V, 50 Hz transformer is found to be 5 mWb. The number of turns and the voltage at the primary winding are found to be 900 and 1100 V, respectively. Determine the frequency of the supply system.



- 2.22 The maximum flux density of a 6 kVA, 2200/220 V, 50 Hz single-phase transformer is found to be  $0.6 \text{ Wb/m}^2$ . The induced voltage per turn of the transformer is 10 V. Calculate the (i) primary number of turns, (ii) secondary number of turns, and (iii) cross sectional area of the core.
- 2.23 A single-phase transformer is having the primary voltage per turn is 20 V. Find the magnetic flux density if the cross sectional area of the core is  $0.05 \text{ m}^2$ .
- 2.24 The no load primary current of an 1100/230 V single-phase transformer is found to be 0.5 A and absorbs a power of 350 W from the circuit. Find the iron loss and magnetizing components of the no-load current.
- 2.25 A 3300 V/230 V single-phase transformer draws a no-load current of 0.5 A at a power factor of 0.4 in an open circuit test. Determine the working and magnetizing components of the no-load current.
- 2.26 A single-phase transformer draws a no-load current of 1.4 A from the 120 V source and absorbs a power of 80 W. The primary winding resistance and leakage reactance are found to be  $0.25 \Omega$  and  $1.2 \Omega$ , respectively. Determine the no-load circuit resistance and reactance.
- 2.27 Under no-load condition, a single-phase transformer is connected to a 240 V, 50 Hz supply and draws a current of 0.3 A at a 0.5 power factor. The number of turns on the primary winding is found to be 600. Calculate the (i) maximum value of the flux in the core, (ii) magnetizing and working components of the no load current, and (iii) iron loss.
- 2.28 The no-load current of a 440/110 V single-phase transformer is measured 0.8 A and absorbs a power of 255 W. Calculate the (i) working and magnetizing components of the no-load current, (ii) copper loss in the primary winding if the primary winding resistance is  $1.1 \Omega$ , and (iii) core loss.
- 2.31 The primary winding resistance and reactance of a 300 kVA, 2200/220 V single-phase transformer are 2 and  $9 \Omega$ , respectively. The secondary winding resistance and reactance are found to be  $0.02 \Omega$  and  $0.1 \Omega$ , respectively. Find the equivalent impedance referred to the primary and the secondary.
- 2.32 The primary winding resistance and reactance of a 200 kVA, 3300/220 V single-phase transformer are found to be 5 and  $12 \Omega$ , respectively. The same parameters in the secondary winding are found to be  $0.02 \Omega$  and  $0.05 \Omega$ , respectively. Find the equivalent impedance referred to the primary and the secondary. Also, determine the full load copper loss.
- 2.33 A 1100/400 V single-phase transformer having the resistance of  $5 \Omega$  and the reactance of  $9 \Omega$  in the primary winding. The secondary winding resistance and reactance are found to be  $0.6 \Omega$  and  $1.1 \Omega$  respectively. Find the voltage regulation when the secondary delivers a current of 5 A at a 0.9 lagging power factor.
- 2.34 A 2200/220 V single-phase transformer having the resistance of  $6 \Omega$  and the reactance of  $16 \Omega$  in the primary coil. The resistance and the reactance in secondary winding are found to be  $0.07 \Omega$  and  $0.15 \Omega$ , respectively. Find the voltage regulation when the secondary supplies a current of 6 A at a 0.86 power factor leading.

- 2.35 The iron loss and full load copper loss of a 40 kVA transformer are found to be 450 and 750 W, respectively. Calculate the (i) full load efficiency, (ii) output kVA corresponding to maximum efficiency, and (iii) maximum efficiency. Consider the power factor of the load is 0.95 lagging.
- 2.36 The iron loss of a 40 kVA transformer is 50 % of the full load copper loss. The full load copper loss is found to be 850 W. Calculate the (i) full load efficiency, (ii) output kVA corresponding to maximum efficiency, and (iii) maximum efficiency. Assume the power factor of the load is 0.9 lagging.
- 2.37 The iron loss and full load copper loss of a 25 kVA transformer are found to be 300 and 550 W, respectively. Find the (i) full load efficiency, (ii) output kVA corresponding to maximum efficiency, and (iii) maximum efficiency. Assume the power factor of the load is 0.6 lagging.
- 2.38 The primary and secondary windings parameters of a 100 kVA, 2200/220 V transformer are  $R_1 = 0.6 \Omega$ ,  $X_1 = 0.9 \Omega$ ,  $R_2 = 0.007 \Omega$ ,  $X_2 = 0.008 \Omega$ . The transformer is operating at a maximum efficiency of 75 % of its rated load with 0.9 lagging power factor. Calculate the (i) efficiency of transformer at full load, and (ii) maximum efficiency, if the iron loss is 350 W.
- 2.39 A 200 kVA, 2400/240 V transformer has the primary and windings parameters  $R_1 = 20 \Omega$ ,  $X_1 = 27 \Omega$ ,  $R_2 = 0.18 \Omega$ ,  $X_2 = 0.26 \Omega$ . The transformer is operating at a maximum efficiency of 80 % of its rated load with 0.8 pf lagging. Calculate the (i) efficiency of the transformer at full load, and (ii) maximum efficiency if the iron loss is 400 W.
- 2.40 The low voltage side of a 30 kVA, 2400/220 V, 50 Hz single-phase transformer is short circuited. The test data recorded from the high voltage side are 200 W, 6 A and 45 V. Find the (i) equivalent resistance, reactance and impedance referred to the primary, (ii) equivalent resistance, reactance and impedance referred to the secondary, and (iii) voltage regulation at 0.6 power factor lagging.
- 2.41 The low voltage winding of a 25 kVA, 1100/220 V, 50 Hz single-phase transformer is short circuited. The test data recorded from the high voltage side are 156 W, 4 A and 50 V. Calculate the (i) equivalent resistance, reactance and impedance referred to the primary, (ii) equivalent resistance, reactance and impedance referred to the secondary, and (iii) voltage regulation at 0.85 power factor leading.
- 2.42 The test data of a 10 kVA, 440/220 V, 50 Hz single-phase transformer are as follows:
- Open circuit test: 220 V, 1.4 A, 155 W;  
Short circuit test: 20.5 V, 15 A, 145 W;  
Calculate the approximate equivalent circuit parameters.
- 2.43 The open circuit test data of a single-phase transformer are recorded as the 220 V, 1.2 A and 145 W. Find the no-load circuit parameters.

**Fig. P2.1** An autotransformer with a specific voltage



- 2.44 A single-phase 100 kVA, 1100/220 V transformer is connected as an autotransformer, which is shown in Fig. P2.1. The voltage of the upper portion and lower portion of the coil are found to be 220 and 1100 V, respectively. Calculate the kVA rating of the autotransformer.
- 2.45 A 200 kVA three-phase, 50 Hz core-type transformer is connected as delta-wye and has a line voltage ratio of 1100/440 V. The core area and the maximum flux density of the transformer are found to be  $0.04 \text{ m}^2$  and  $2.3 \text{ Wb/m}^2$ , respectively. Calculate the number of turns per phase of the primary and secondary coils.
- 2.46 The number of turns in the primary and secondary coils are found to be 600 and 150, respectively. The transformer is connected to 11 kV, 50 Hz supply. Find the secondary line voltage when the windings are connected as (a) delta-wye, and (b) wye-delta.

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Power Systems Grounding

Salam, M.A.; Rahman, Q.M.

2016, XII, 309 p. 247 illus., 16 illus. in color., Hardcover

ISBN: 978-981-10-0444-5