

Chapter 2

Fundamentals of 3M&I-Body System

Abstract This chapter describes the mathematical foundations of the 3M&I-body system. The concept of pair matrix was first proposed in 1983 and has been explored and developed into the IO-matrix system with input (revenue) and output (cost/profit). This generalization formed the axiom system of body in vector space, and was applied to a class of Matsui's equation and Matsui's matrix equation (Matsui's ME). Based on this pair matrix, the Sandwich model ($S = W$ model) was proposed in 2009, and was formulated as a type of Matsui's ME for the cases of enterprise, motor and particle physics bodies. The Matsui's ME of strategic plan (IDTCBG) type could be generally transformed into the corresponding characteristic equation, and was solved analytically by using the matrix computing method. Finally, the problem of sandwich control was considered by using the progressive method, and 3D dynamism was explored by using the vector cell approach for solving the Matsui equation. The progressive method was extended to the state equation of progressive control with Matsui's logic of second-order (acceleration) type. Also, the 3D dynamism was obtained by mapping the p -dimension to the $p/2$ (Z and L domain) control problem in the multidimensional space.

Keywords Pair matrix • Input/output • Matsui's equation • Sandwich • Characteristic equation • 3D management

2.1 Algebra: Pair Matrix Body and Line Versus OE

2.1.1 Introduction

The modeling problem of bodies is probably the inverse approach to the reductionism proposed by Rene Descartes (1596–1650), and this has been called, not the system-centered, but the body-centered approach since 1977 [1]. Here, we discuss the pair relations (move/flow) of the input-output system of bodies.

For each body, there is one input (inflow) and two outputs (outflow and over-flow) in the move/flow system (line). The pair relations are of two types: (inflow,

outflow) and (inflow, overflow), and these are denoted as the pair matrix of line versus OE (ordered entry) type.

The latter (the OE type) is new and was first treated as the pair matrix, whereas the former (the line type) is ordinal and was later compared with this type as the opposite of pair matrices. This notation was introduced in 1983 [1, 2], and was developed into a novel approach for describing the input-output system, which was different from the production and control system in IEOR.

In Sect. 2.2, the sandwich proposition [3] of a body is discussed by using pair (matrix) approach, and is applied to the enterprise, motor and physics (body) scenarios. For this proposition, the formulations by the pair map geometrics, vector space, or Matsui's ME [4, 5] were implemented and found to be useful.

In Sect. 2.3, the problem of sandwich control is considered by using the progressive method, and the issue of 3D dynamism is explored by using the vector cell approach to Matsui's equation [2, 6]. Matsui's logic of second-order (acceleration) type and the mapping control to $p/2$ (Z and L domain) from p -dimension space is also discussed.

2.1.2 Pair Matrix: Definition and Axiom

Let us denote the set of pair matrices by K , and their elements by $A(a, c)$ and $B(b, d) \in K$, in which a, b are inputs and c, d are outputs. Then, the mathematical construct of pair matrices is introduced as follows:

(1) Additivity axiom

For any $(A, B) \in K$, the operation of addition is given as follows:

$$A + B = ((a + b, c + d)). \quad (2.1)$$

wherein the associative and commutative laws hold. Also, for any $(A, B) \in K$,

$$\exists X \in K \quad \text{s.t.} \quad A + X = B, \quad (2.2)$$

and the zero element is $[0, 0]$.

(2) Multiplicative axiom

Several multiplicative axioms are formulated. For $x = (a, c) \in A$ and $y = (b, d) \in B$, one axiom considers the conditional reward as posterior probability. Now, the following relations hold:

$$r(x \cap y) = r(x)r(y|x) \quad (2.3)$$

$$r(y \cap z) = r(z)r(y|z) \quad (2.4)$$

Then, the respective rewards are as follows:

$$r(y|x) = y/x, \quad r(z|y) = z/y \quad (2.5)$$

These additive and multiplicative laws hold respectively as follows:

$$r((y + z|x)) = r(y|x) + r(z|x), \quad (2.6)$$

$$r(z|x) = r(y|x)r(y|z). \quad (2.7)$$

Another axiom considers the type of pair comparison, as seen in the case of Matsui's matrix equation case [4, 5]. This type transforms the pair matrix into the weighting factor by using the analytic hierarchy process (AHP) [7].

2.1.3 Vector Case and Matsui's ME

Next, let us assume the elements, x and $y \in K$, as pair vectors. Then, the inner and outer products are defined as follows:

$$(A, B) = ([x, y]), \quad \text{inner product} \quad (2.8)$$

$$[A \times B] = (|x \times y|), \quad \text{outer product.} \quad (2.9)$$

For Matsui's equation ($W = ZL$), this notation could give the respective generalization to $W' = Z'L'$ with angle, θ . Then, this angle could be used to strategically manipulate the sandwich body.

That is, the vector and scalar products in Fig. 2.1 are as follows:

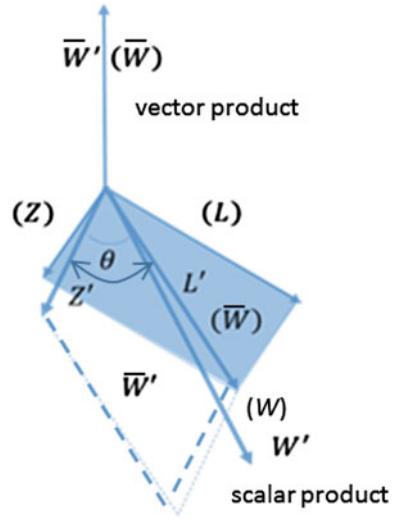
$$\overline{W'} = |x \times y| = |Z' \times L'| = |Z'| |L'| \sin \theta. \quad (2.10)$$

$$W' = (x, y) = (Z', L') = |Z'| |L'| \cos \theta. \quad (2.11)$$

Especially for the vector product, when $\theta = \pi/2$, $\overline{W'}$ coincides with W in Matsui's equation. Also, when $\theta = 0$ ($Z = 1/\lambda$), the latter becomes Little's formula: $\lambda W = L$.

For example, let us denote the vectors as: Z' and L' , by the revenue: as ER , and lead time: as LT , respectively, in Fig. 2.1. Then, the vector product, W' , denote the assets (force in physics). Also, let us denote the profit, EN , by the revenue, ER , and cost, EC . This scalar product is the alternative (multiplicative) definition of the profit equation, $EN = ER - EC$.

Fig. 2.1 Matsui's equation ($W = ZL$) and its generalization ($W' = Z'L'$) with $\theta = \angle Z'L'$



2.1.4 Pair Body: Line Versus OE

Here, the set of pair matrices is called the pair body. There are two types of series bodies in the ordinal line (a) and OE bodies in the ordered-entry line (b) in Fig. 2.2. The former has the arrival in input and departure in output, while the latter has the arrival in input and overflow in output.

Fig. 2.2 Pair body: line versus OE (ordered entry)

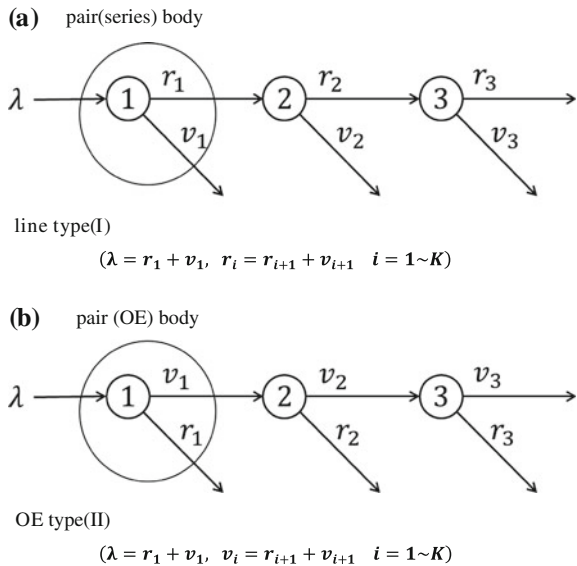


Table 2.1 Two pair bodies and their duality

Pair body	Series		OE (ordered entry)	
Input-output	Arrival-departure		Arrival-overflow (delay)	
Overflow (ratio)	Overflow (delay)	Overflow rate	Departure	Departure rate (production rate)
Evaluation	Addition	Multiplicative	Subtraction (inner product)	Division
Combination	Modular		Integral	

Both represent the interchange relation of production (r) and overflow (v) rates, and are called the dual system. Table 2.1 lists the dual relation of line versus OE.

2.1.5 Duality of Series Versus OE

From Fig. 2.3, the case where the overflow rate is $v_i = 0$ consists of the transformed bodies in the modular class (additive and multiplicative). For the case in which the overflow rate $v_i \neq 0$ consists of the transformed bodies in the integral class (subtraction and division) in Fig. 2.4.

In Fig. 2.3, the revenue, ER , in the input, and operating (variable) cost, EC , in the output, are shown. In Fig. 2.4, the revenue, ER , in the input, and the profit, EN , in the output, are shown. Then, the probability of processing, $P(0 < P < 1)$, is here regarded as the EN to ER ratio, and the probability of loss, $B(0 < B < 1)$, is regarded as the EC to ER ratio.

The probability B is given by

$$B = (ER, EC) = R(ER \cap EC | ER) = EC/ER, \quad (2.12)$$

where $ER \cap EC = EC(\subset ER)$. The probability P is given by

$$P = (ER, EN) = R(ER \cap EN | ER) = EN/ER, \quad (2.13)$$

where $ER \cap EN = EN(\subset ER)$.

Fig. 2.3 Each unit of pair (series) body

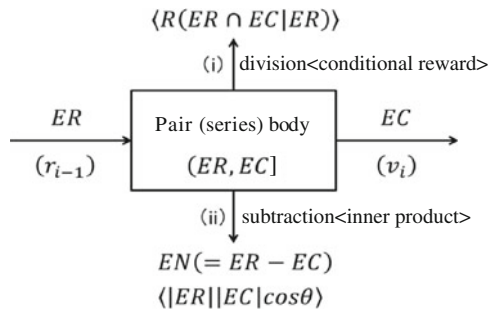
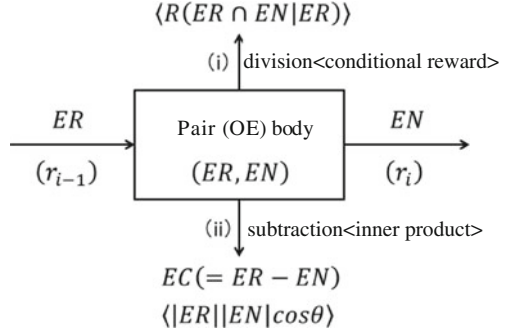


Fig. 2.4 Each unit of pair (OE) body



Both classes can be unified in a modular class after proper subtraction and division. For the modular class, the production (r_I) and overflow (r_{II}) rates are given as follows [8]:

$$r_I = r_k = \lambda \prod_{i=1}^k P_i, \quad \text{line type} \quad (2.14)$$

$$v_{II} = v_k = \lambda \prod_{i=1}^k P_i, \quad \text{OE type} \quad (2.15)$$

Also, the flow times, F_I and F_{II} , correspond to the total cost and profit, respectively, and are given as follows [8]:

$$F_I = \sum_{i=1}^k EC_i, \quad \text{cost (linetype)} \quad (2.16)$$

$$F_{II} = \left(\sum_{i=1}^k EN_i \right) / r_{II}, \quad \text{profit (OEtype)} \quad (2.17)$$

2.2 Analytics: Sandwich Theory and Matsui's Equation

2.2.1 Outline of the Sandwich Problem

Using pair matrix notation, which we have already developed in 1983, the management game model (MGM), consists of two centers (sales and manufacturing) [1, 9]. The MGM pair matrix table has yielded several (pair) strategic maps to the main three types of manufacturing and service enterprises, and is summarized in Fig. 2.5 [1, 3].

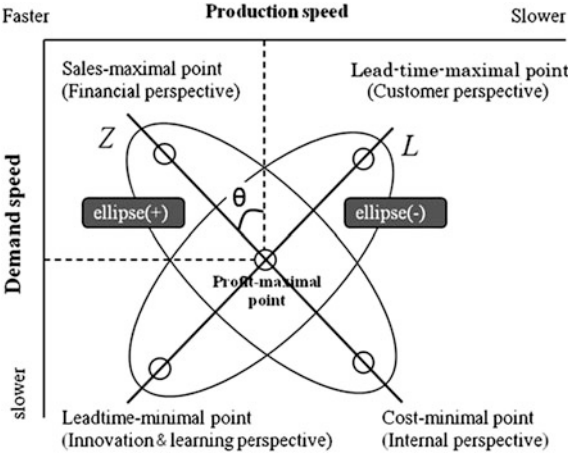


Fig. 2.5 The balancing solution for the sandwich problem (pair map)



Fig. 2.6 The sandwich model for an enterprise (toward nature vs. artifacts)

Moreover, for these pair maps, we have proposed the sandwich concept of enterprises in 2009, and have presented the sandwich model of enterprises in Fig. 2.6 (2013) [3]. This sandwich model will be developed here for the sandwich proposition of nature versus artifacts body.

For the class of 3M&I-body, our sandwich proposition is defined by the following considerations:

Definition 2.1 The input-output system that holds the sandwich body is defined by the M-system, and the pair bodies on M-system are denoted by M-pair body.

Proposition 2.1 *It seems sufficiently plausible that*

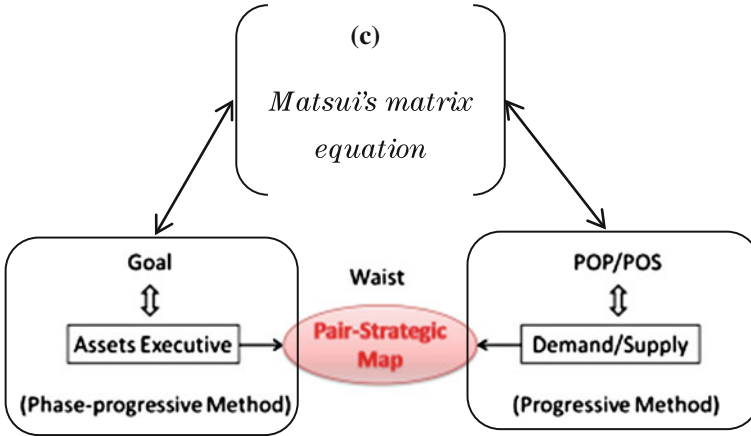
- (i) *if some input-output system is assumed under a 3M&I-body, there is at least one pair body on it,*
- (ii) *any sandwich body could be composed of an M-pair body or decomposed into an M-pair body and other, and*
- (iii) *the sandwich body could be formulated as the type of Matsui’s ME.*

The meaning and validity of this proposition will be discussed for the three types of enterprise, motor and physics from the three formulations of pair map geometrics, vector space, and Matsui's ME. Then, the duality of Matsui's, Newton's, and Jewell's equation in Table 1.4 will be formalized and visualized from the mathematical aspect.

2.2.2 Sandwich Proposition: Enterprise Type

For the system in Fig. 2.6, the recent results are summarized in Fig. 2.7. The formulation in Fig. 2.7a was provided in [3], and the formulation in Fig. 2.7b was described in Sect. 2.1.3 (Fig. 2.1). Finally, the formulation in Fig. 2.7c, the *IDTCBG* logics, is a type of Matsui's ME based on the independent component analysis (ICA) [10], as shown in Fig. 2.8, and is partially coincident with the Chinese "Kishotenketsu" (strategy story in logics, *IDTC*).

The basic form of the ICA is referred to in Fig. 2.8. The respective ICA stages proceed from the Introduction (S_1 , S_2), Development (X) and Turn (Z) to Conclusion (Y), and the U and V are rotation matrices. Then, the solution space, Y , is extended to W as follows:



$$(b) \text{ Outer space } \longleftrightarrow \left[\begin{array}{c} \text{Intermediate} \\ \text{angle } (\theta) \end{array} \right] \longleftrightarrow \text{ Inner space}$$

$$(a) \quad \sqrt{\tilde{Z}} \times \tilde{L} \leq (\text{Nash's solution}) \leq (\tilde{Z} + \tilde{L})/2$$

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Fig. 2.7 Three treatments of the sandwich model

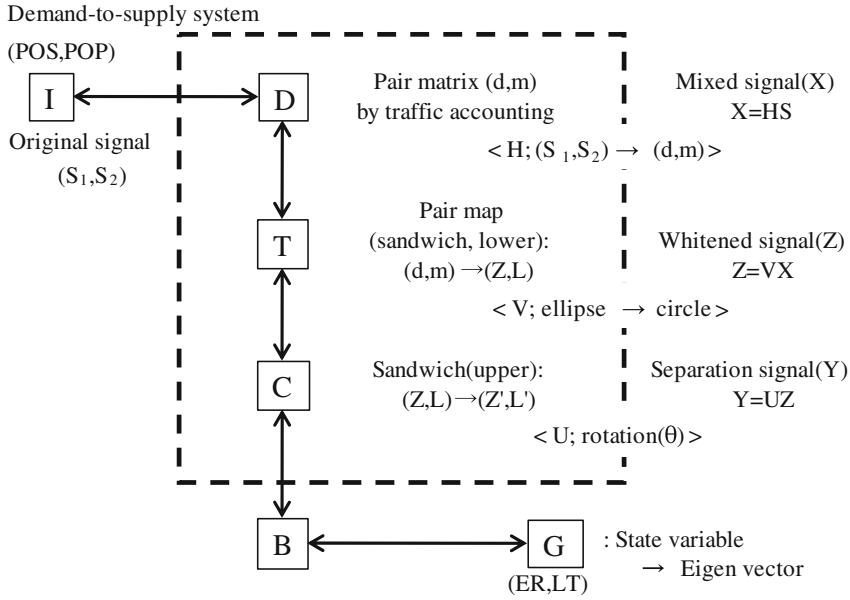


Fig. 2.8 Matsui's matrix equation (Matsui's ME) with Independent component analysis (ICA). *I* Introduction, *D* Development, *T* Turn, *C* Conclusion, *B* Balancing, *G* Goal

$$W(=UTV) = H^{-1}, \quad (2.18)$$

in which T is the intermediate matrix of pair strategy (map) type.

These formulations have a common property; the rotation of the intermediate angle (θ) is available as a strategic variable. This sandwich property of the enterprise type could support the sandwich proposition in nature versus artifacts body.

2.2.3 Type of Enterprise-Motor Body

The dualism of Matsui's, Newton's, and Jewell's equations can be visualized as shown in Fig. 1.3 This would support the sandwich proposition of an electric body. This example is provided for the electric motor of PMSM type in Fig. 2.9 [11, 12], and is similar to the sandwich property shown in Fig. 2.7.

This example is an engineering type of an artifact, and is composed of two rotations: Clark ($\alpha\beta$) and Park (dq) transformations. The formulation is similar to the type of Matsui's ME in Fig. 2.10. These findings could provide the clues to the auto-control enterprises, and might enable the realization of the enterprise-motor robot (body) of analog type.

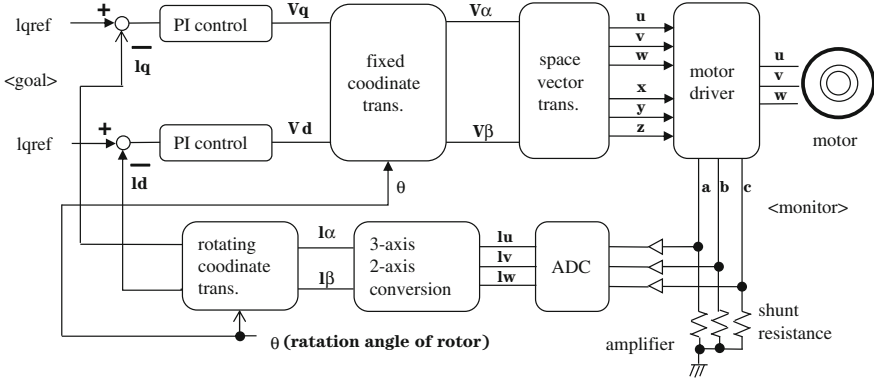


Fig. 2.9 Vector control motor of PMSM type

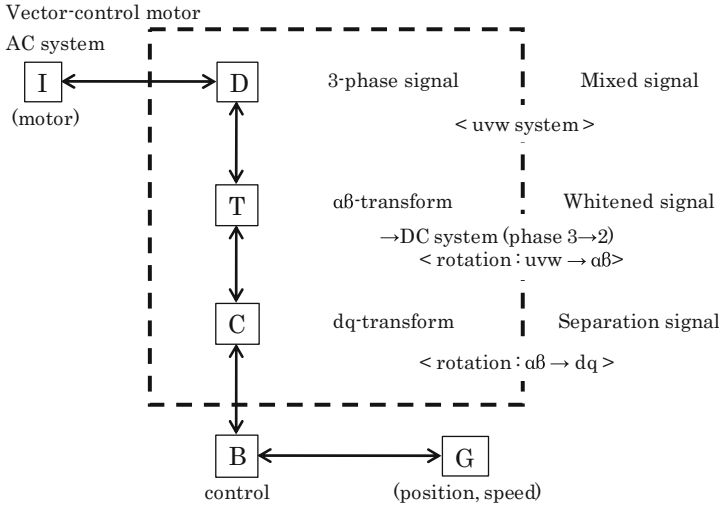


Fig. 2.10 Vector control motor with Matsui's ME

2.2.4 Type of Particle Physics Bodies

The type of particle physics bodies corresponds to the 3M&I-body in enterprise, as shown in Fig. 2.11. In this figure, the weight in Higgs denotes the amount of value, and the force in Graviton means the lead time in enterprise.

This formulation of a particle physics body is imaged as a type of Matsui's ME, and is proposed in Eq. (2.19) or Fig. 2.12, Chap. 2. In Fig. 2.11, the development (D) is the stage from the 3M&I to particle life cycle, and the conclusion (C) is the stage from the particle life cycle to pair map.

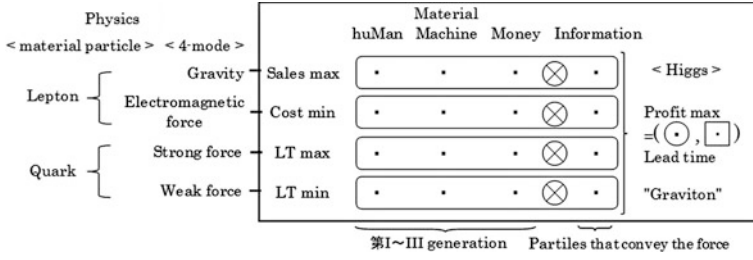


Fig. 2.11 3M&I-body of Particle physics type. Higgs boson: a particle that gives weight to the substance, graviton: a particle that mediates the gravitational interaction

$$\begin{aligned}
 & \begin{matrix} \boxed{I} & \boxed{D} & \boxed{T} & \boxed{C} \end{matrix} \\
 & (M_1, M_2, M_3) \times \begin{pmatrix} M_1, I_n & & & \\ & M_2, D & & \\ & & M_3, M_t & \\ & & & I, D_c \end{pmatrix} \times (TM) \times \begin{pmatrix} I, LT_{max} & & & \\ & D, EC_{min} & & \\ & & M_t, LT_{min} & \\ & & & D_c, ER_{max} \end{pmatrix} \times \\
 & \begin{matrix} \boxed{B} & \boxed{G} \end{matrix} \\
 & \begin{pmatrix} H_1 & G_1 \\ H_2 & G_2 \\ H_3 & G_3 \\ H_4 & G_4 \end{pmatrix} = (H \quad G)
 \end{aligned}$$

M1, M2, M3: 3M, I: Information, In: Introduction, D: Development, Mt: maturity, Dc: Decline, B: Balancing, G: Goal
H, H1, H2, H3: Higgs state <EN>, G, G1, G2, G3: Gravition state <LT>

(2.19)

Fig. 2.12 Type of 3M&I-body with Matsui's ME (2.19)

Generally, the type of Matsui's ME could be transformed into the corresponding characteristic equation as follows: For any 3M&I-body, the generalized equation is given by

$$Ax = \Lambda x \quad (2.20)$$

where A is an operator, x is the proper state and Λ is the proper value.

In Figs. 2.8, 2.10 and 2.12, the *IDTC* states are represented by

$$Ax_i = \lambda_i x_i, \quad i = 1, 2, 3 \quad (2.21)$$

where $i = 1(I), 2(D)$ and $3(T)$. Then, Eq. (2.20) is composed of those of (2.21), and is as follows:

$$A = \begin{pmatrix} D & 0 \\ 0 & T \\ 0 & C \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad (2.22)$$

2.3 Control: Progressive Method and 3D-Geometry

2.3.1 3D-Control Problem of 3M&I-Body

The administrative world of 3M&I-body is shown in Fig. 2.13 for the multidimensional space (Fig. 1.2 [13], Chap. 1). Then, it is suggested that the lower level of the body with p -dimension is projected to the superplane of the pair map and the upper level of the body is the vector space in the vertical axis of the superplane.

Usually, the p -dimension control might be not feasible. By using Matsui's equation, the $p/2$ -vector control of the superplane could be feasible, as shown in Fig. 2.14. There, the $p/2$ denotes the contraction from p to the two dimensions of vectors, Z and L . By using this contraction, the 3D control of 3M&I-body could be made feasible under the strategy (θ, φ) .

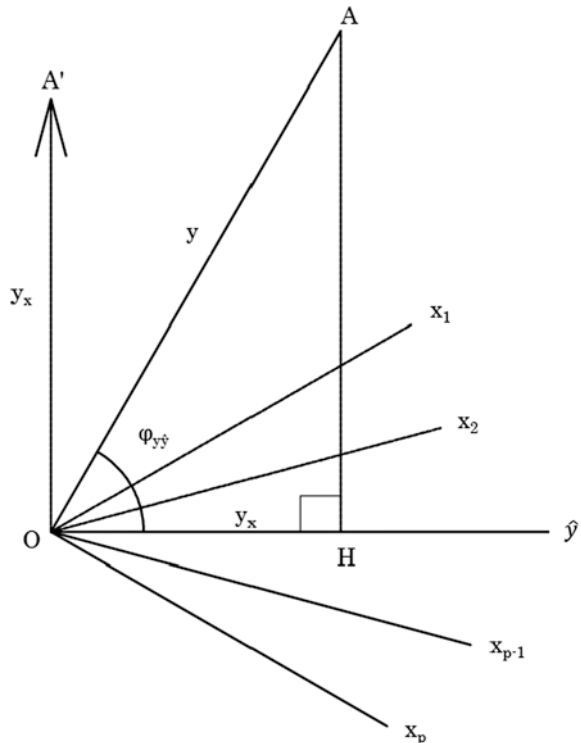
For example, let the x_i 's be denoted as Z and L in $W = ZL$. Then, the strategic (θ, φ) is from the undertarget goal (AH) (1.1) as follows.

$$\exists(a^*|\theta, \varphi) \text{ s.t. } \min f(a) = AH(\theta, \varphi), \quad (2.23)$$

in which $f(a)$ is the function of least squares:

$$f(a) = \|Fa - y\|^2. \quad (2.24)$$

Fig. 2.13 3D diagram:
superplane (pair map) and its
vector space



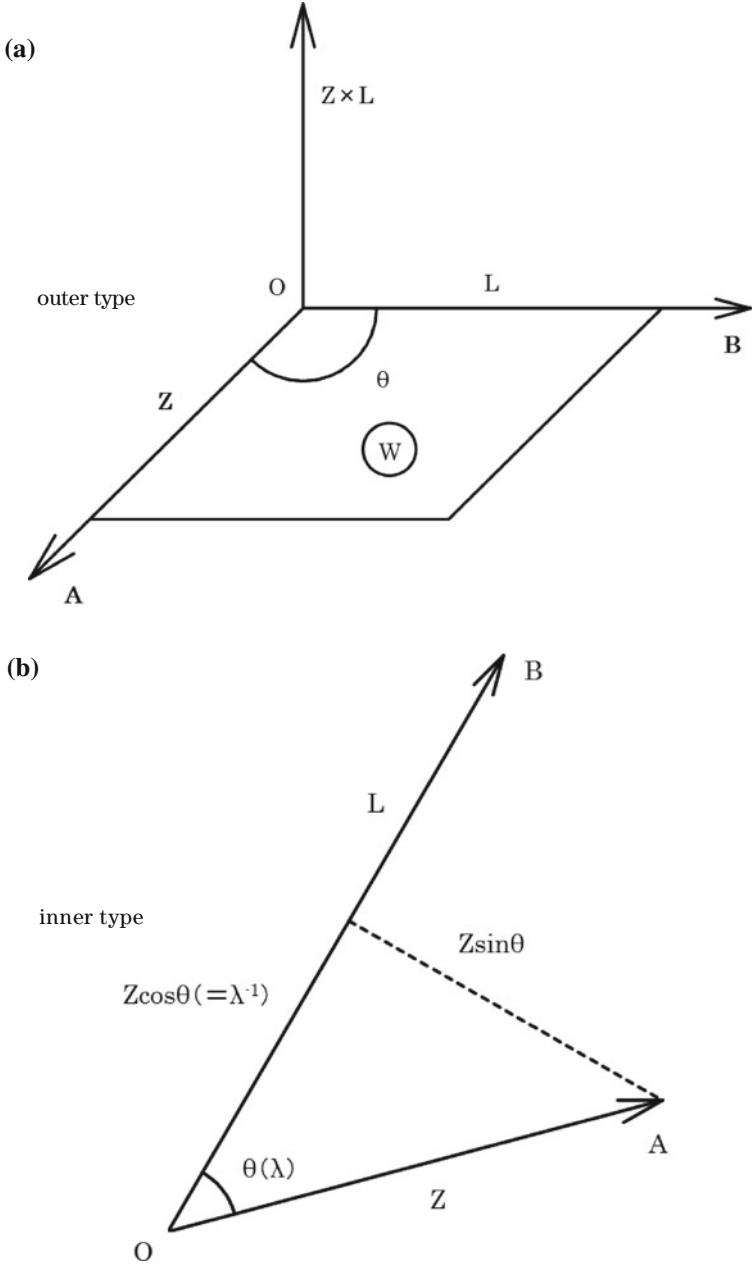


Fig. 2.14 Outer (a) versus inner (b) product at Matsui’s equation

2.3.2 Progressive and Matsui's Logic

Generally, the equation of progressive control is from [4] as follows:

$$O_{t+1} = \bar{D}_{t+1} + MI_{t+1} - I_t, \quad t = 0, 1, 2, \dots \quad (2.25)$$

where O_t is the next order (supply) quantity, $\bar{D}_{t+1}$ is the look-ahead demand indicator, MI_t is the moving-standard (marginal) inventory, and I_t is the present amount of inventory.

The MI_t can be regarded as the state of progressive control, and is represented as the node of the network in Fig. 2.15. This progressive method is also available for 3D control, similar to the new inventory control, ODICS, as in [4].

For ODICS, Matsui's logic [4] is useful and effective in the on-demand progressive control of the look-ahead type. This will be also applied to the progressive control of $p/2$ type in 3D.

Now, let us denote the demand speed and Chameleon's criteria by $Z(>0)$ and $\bar{\beta}(0 < \bar{\beta} < 1)$. If the following relation is satisfied by the principle of the lever (1.3):

$$Z_t \bar{\beta}_t = Z_{t+1} \bar{\beta}_{t+1}, \quad t = 0, 1, 2, \dots \quad (2.26)$$

The $(t+1)$ -epoch Chameleon's criteria, $\bar{\beta}_{t+1}$ is given by

$$\bar{\beta}_{t+1} = (Z_t/Z_{t+1})\bar{\beta}_t. \quad (2.27)$$

Also, Matsui's logic of the second order is obtained for acceleration as follows:

$$\bar{\beta}_{t+1} = (Z_{t+1}/Z_{t-1})\bar{\beta}_{t-1}. \quad (2.28)$$

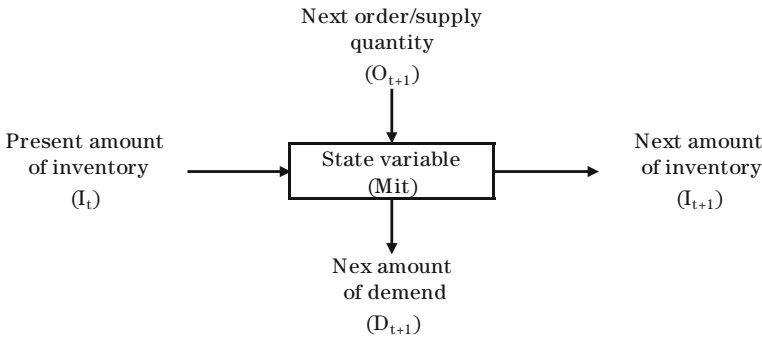


Fig. 2.15 State variable of progressive control: $MT_{t+1} = O_{t+1} + I_t - \bar{D}_{t+1} \rightarrow I_{t+1}$

From (2.26) and (2.28), the second order $\bar{\beta}_{t+1}$ becomes

$$\bar{\beta}_{t+1} = (Z_t Z_{t+1} / Z_{t-1}^2) \bar{\beta}_t. \quad (2.29)$$

2.3.3 Medium Estimation of State

Generally, the medium criterion is important and is satisfied to the newsvendor [14] or revenue control types [15] in IEOR. Here, we consider the medium estimation of state.

Now, the inequality for the medium criterion, Y , is as follows:

$$(X - N)^+ > Y > (N - X)^+, \quad (2.30)$$

where N is the number of units in demand or no-shows, and X is the units ordered or overbooked.

This stochastic inequality, $P(X > N)$, becomes the trade-off (medium) of the underestimate cost and overestimate cost of demand in Fig. 2.16. In this figure, the penalties (costs) of the three types are introduced as follows:

$\beta_1(C_h)$: maintenance cost of criterion

$\beta_2(C_o)$: cost of overestimating demand or no-shows (opportunity cost)

$\beta_3(\rho)$: cost of underestimating demand or no-shows (price)

By Fig. 2.16, the following inequality can be expressed:

$$P(X < N)p \geq P(X > N)C_o + C_h. \quad (2.31)$$

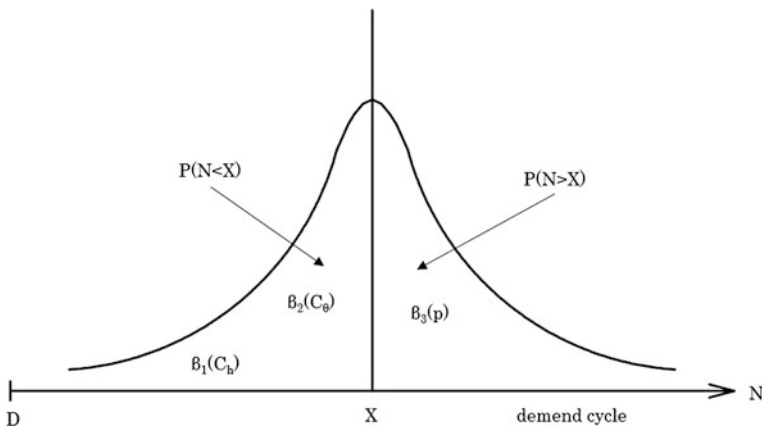


Fig. 2.16 Newsvendor versus revenue control

For revenue control type, and for newsvendor type, the inequality is obtained:

$$\beta_1 + \beta_2 P(N < X) \geq \beta_3 P(N > X). \quad (2.32)$$

Thus, the lower and upper bound for $P(X > N)$ is given by

$$\frac{p - C_h}{p + C_o} \geq P(X > N) \geq \frac{\beta_3 - \beta_1}{\beta_2 + \beta_3}. \quad (2.33)$$

The inequality (2.32) shows the medium estimation of state.

2.3.4 3D Cell and Graphics Modeling

For the control of 3M&I-body, we should administer the sandwich body in real time in a sustainable manner as possible. Then, the effective 3D graphics would be transferred to the $p/2$ control problem, and this would become a comprehensive and visual method to the Matsui's ME.

The cell modeling is well known at the cell automaton model. Figure 2.17 shows the one- and two-dimensional discrete conveyor types [16]. These are the scalar type with Z or $Z \times Z$, and can be extended to the type of 3D modeling by a 3D cell, $Z \times Z \times Z$.

Then, the 3D cell in Matsui's equation can be visualized in the (Z, L, W) -space, and is shown in Fig. 2.18. In the (Z, L, W) -space, as shown in Fig. 2.18, the unit of the 3D cell is $Z \times Z \times Z$. For example, Z'_i s are accounts, L'_i s are dates, and W'_i s are amounts (Z, L) with different units, Z 's.

2.3.5 Two Types of 3D Dynamism

The two types of 3D modeling are an accounting system (lower level in sandwich), as shown in Fig. 2.18, and finance system (upper level in sandwich), as shown in

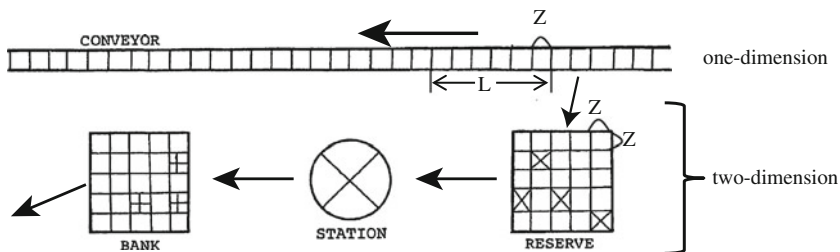


Fig. 2.17 One- and two-dimensional cells in discrete conveyor type

Fig. 2.18 3D cell in Matsui's equation: W (value) = Z (unit) $\times$ L (amount)

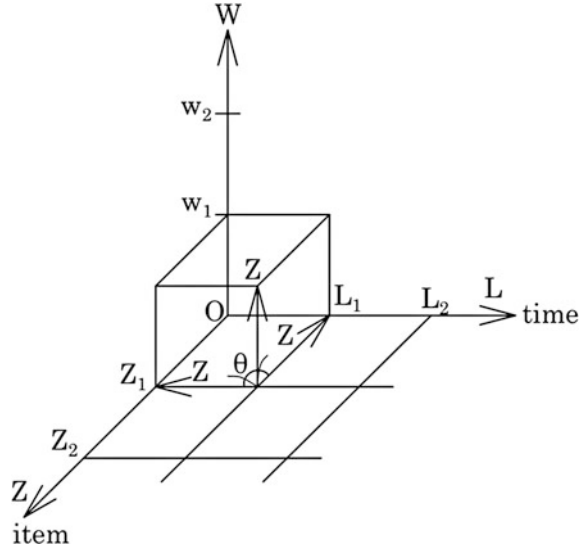
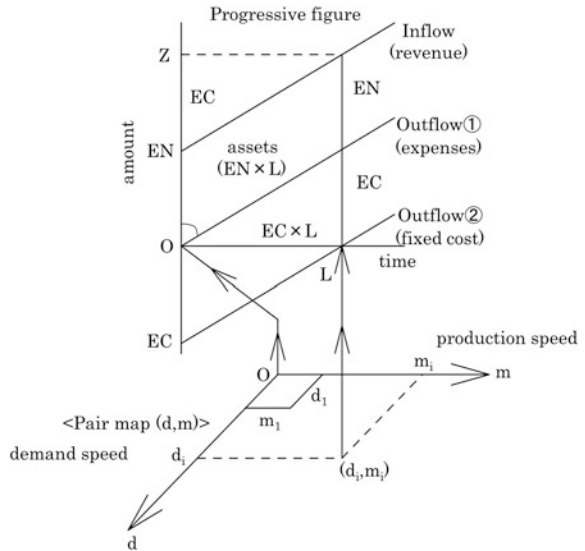


Fig. 2.19. The former is based on progressive figures, and the latter is based on the progressive phase in (2.34) and (2.35), respectively.

$$\text{position } (d, m) \rightarrow \text{progressive figure } (Z, L), \quad (2.34)$$

$$\text{plane } (Z, L) \rightarrow \text{progressive phase } (\overline{W}, \overline{L}). \quad (2.35)$$

Fig. 2.19 Dynamism of the 3D accounting system at the lower level



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Fundamentals and Principles of Artifacts Science

3M&I-Body System

Matsui, M.

2016, VIII, 74 p. 50 illus., 37 illus. in color., Softcover

ISBN: 978-981-10-0472-8