Chapter 2
Circuit Solutions

Old order changeth yielding place to new. And God fulfils himself in many ways lest one good custom should corrupt the world…

Alfred Tennyson

In this chapter we discuss solution of circuits with current and voltage sources and develop a new procedure which uses power source. Voltage/current sources are well-known in literature but *power* as source does not find mention in any of the books on electric circuits (e.g., see [1]). *Power source* however is a reality in power networks. To be able to do this, variables in our analysis will be *element* voltages and currents and not *nodal* voltages and currents. Since element variables have not been customarily used in the past for general power system analysis, it will be necessary to remind ourselves of this fact occasionally, if not frequently.

2.1 Multi-terminal Representation

Consider a system with nb buses. The conventional bus voltage and current vectors are related by the equation [2],

$$V = \begin{bmatrix} Z_{11} & Z_{12} & & Z_{1i} & & Z_{1,nb} \\ Z_{21} & Z_{22} & & Z_{2i} & & Z_{2,nb} \\ . & . & \cdots & . & \cdots & . \\ . & . & & . & & . \\ Z_{nb,1} & Z_{nb,2} & & Z_{nb,i} & & Z_{nb,nb} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_i \\ . \\ I_{nb} \end{bmatrix} \quad (2.1)$$

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$$= \begin{bmatrix} Z_{11} \\ Z_{21} \\ \cdot \\ \cdot \\ Z_{nb,1} \end{bmatrix} I_1 + \begin{bmatrix} Z_{12} \\ Z_{22} \\ \cdot \\ \cdot \\ Z_{nb,2} \end{bmatrix} I_2 + \cdots + \begin{bmatrix} Z_{1,nb} \\ Z_{2,nb} \\ \cdot \\ \cdot \\ Z_{nb,nb} \end{bmatrix} I_{nb} \tag{2.2}$$

We can write (2.2) as,

$$V = V(1) + V(2) + \cdots + V(nb) \tag{2.3}$$

where,

$$V(k) = \begin{bmatrix} V_{1k} \\ \hdashline V_{2k} \\ \hdashline \cdot \\ \hdashline \cdot \\ \hdashline V_{nb,k} \end{bmatrix} \tag{2.4}$$

In what follows, we will use Z-bus and consider a single source. This implies using only one column of the bus impedance matrix Z. This idea is different from that in basic circuit analysis that employs node analysis or mesh analysis to obtain circuit currents and node voltages [1]. For multiple sources superposition poses an interesting challenge.

2.2 Solution for Element Variables

2.2.1 Single Current Source

Consider only *one* source current injected at bus i. Set all other injections to zero. Let I_{ii} be the source current. (We will reserve single subscripted I_i and V_i for current and voltage when multiple sources are connected.) Voltages of buses m and n are given by,

$$V_{mi} = Z_{mi} I_{ii} \tag{2.5}$$

$$V_{ni} = Z_{ni} I_{ii} \tag{2.6}$$

Voltage *across* and the current *in* an element e (between buses m and n) are,

$$\begin{aligned} v_{ei} &= V_{mi} - V_{ni} = (Z_{mi} - Z_{ni}) I_{ii} \\ v_{ei} &= \xi_{ei} I_{ii} \end{aligned} \tag{2.7}$$

$$i_{ei} = y_e (V_{mi} - V_{ni}) = y_e \xi_{ei} I_{ii} \tag{2.8}$$

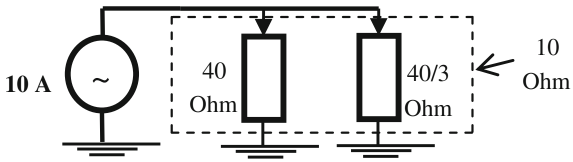

Fig. 2.1 Circuit with current source

Term ξ_{ei} has been defined in (1.17).
Power in element e due to generator i is,

$$\begin{aligned} p_{ei} &= \mathrm{Re}\{v_{ei} i_{ei}^*\} \\ &= |I_{ii}|^2 \mathrm{Re}\{\xi_{ei}\xi_{ei}^* y_e^*\} \end{aligned} \tag{2.9}$$

2.2.2 Example

Consider the circuit in Fig. 2.1. Net resistance offered to the source is,

$$40 \| \frac{40}{3} = 10\,\Omega$$

$$\begin{aligned} \textit{Source Current} &= 10\,\mathrm{A} \\ \textit{Voltage across loads} = \textit{common bus voltage} &= 10 \times 10 = 100\,\mathrm{V} \end{aligned}$$

Resistance values: *element* 1 $= 40\,\Omega$, *element* 2 $= \frac{40}{3}\,\Omega$
Bus impedance and node element matrices are,

$$Z = [10] : A = [1 \quad 1]$$

From (1.15) and (1.17),

$$\xi \underline{\underline{\Delta}} \begin{bmatrix} \xi_{11} \\ \xi_{21} \end{bmatrix} = A^T Z = \begin{bmatrix} 1 \\ 1 \end{bmatrix} [10] = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

Using (2.7) and (2.8),

$$\begin{aligned} \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} &= \begin{bmatrix} \xi_{11} \\ \xi_{21} \end{bmatrix} I_i = \begin{bmatrix} 10 \\ 10 \end{bmatrix} 10 = \begin{bmatrix} 100 \\ 100 \end{bmatrix} \mathrm{V} \\ \begin{bmatrix} i_{11} \\ i_{21} \end{bmatrix} &= \begin{bmatrix} y_1 & 0 \\ 0 & y_2 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} = \begin{bmatrix} \frac{1}{40} & 0 \\ 0 & \frac{3}{40} \end{bmatrix} \begin{bmatrix} 100 \\ 100 \end{bmatrix} \\ &= \begin{bmatrix} 2.5 \\ 7.5 \end{bmatrix} \mathrm{A} \end{aligned}$$

Using (2.9),

$$Power\,in\,40\,\Omega = (10)^2 \times (10 \times 10) \times \frac{1}{40} = 250\,\mathrm{W}$$

Similarly,

$$Power\,in\,\frac{40}{3}\,\Omega = (10)^2 \times (10 \times 10)\frac{3}{40} = 750\,\mathrm{W}$$
$$Total\,Power = 250 + 750 = 1000\,\mathrm{W}$$

From total current and net resistance,

$$Total\,Power = 10^2 \times 10 = 1000\,\mathrm{W}$$

2.2.3 Single Voltage Source

In this case,

$$|I_{ii}| = \left|\frac{V_{ii}}{Z_{ii}}\right| \tag{2.10}$$

$$v_{ei} = V_{mi} - V_{ni} = (Z_{mi} - Z_{ni})I_{ii} = \xi_{ei} I_{ii} \tag{2.11}$$

$$i_{ei} = y_e(V_{mi} - V_{ni}) = y_e \xi_{ei} I_{ii} \tag{2.12}$$

And element power expression (2.9) becomes,

$$p_{ei} = \left|\frac{V_{ii}}{Z_{ii}}\right|^2 \mathrm{Re}\{\xi_{ei}\xi_{ei}^* y_e^*\} \tag{2.13}$$

2.2.4 Example

Consider the circuit in Fig. 2.2. Let the voltage applied be 100 V.

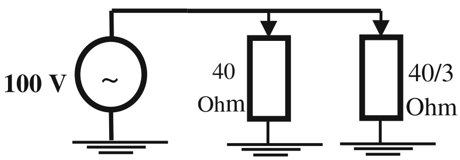

Fig. 2.2 Circuit with voltage source

Net resistance is, $40 \| \frac{40}{3} = 10\,\Omega$

$$
\begin{aligned}
Source\,Current &= \frac{100}{10} = 10\,\text{A} \\
Z = [10]; A &= [\,1 \quad 1\,] \\
\xi = \begin{bmatrix} \xi_{11} \\ \xi_{21} \end{bmatrix} &= A^T Z = \begin{bmatrix} 1 \\ 1 \end{bmatrix} [10] = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \\
\begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} &= \begin{bmatrix} \xi_{11} \\ \xi_{21} \end{bmatrix} I_i = \begin{bmatrix} 10 \\ 10 \end{bmatrix} 10 = \begin{bmatrix} 100 \\ 100 \end{bmatrix} \text{V} \\
\begin{bmatrix} i_{11} \\ i_{21} \end{bmatrix} &= \begin{bmatrix} y_1 & 0 \\ 0 & y_2 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} = \begin{bmatrix} \frac{1}{40} & 0 \\ 0 & \frac{3}{40} \end{bmatrix} \begin{bmatrix} 100 \\ 100 \end{bmatrix} \\
&= \begin{bmatrix} 2.5 \\ 7.5 \end{bmatrix} \text{A}
\end{aligned}
$$

Using (2.13),

$$
Power\,in\,40\,\Omega = \left(\frac{100}{10}\right)^2 \times (10 \times 10) \times \frac{1}{40} = 250\,\text{W}
$$

Similarly,

$$
\begin{aligned}
Power\,in\,\frac{40}{3}\,\Omega &= \left(\frac{100}{10}\right)^2 \times (10 \times 10) \frac{3}{40} = 750\,\text{W} \\
Total\,Power &= 250 + 750 = 1000\,\text{W}
\end{aligned}
$$

From total current and net resistance,

$$
Total\;Power = 10^2 \times 10 = 1000\,\text{W}
$$

2.2.5 Single Power Source

In this case,

$$
I_{ii}^2 = \frac{P_i}{R_{ii}}; R_{ii} = \text{Re}\{Z_{ii}\} \tag{2.14}
$$

Therefore,

$$I_{ii} = \sqrt{\frac{P_i}{R_{ii}}} \tag{2.15}$$

$$v_{ei} = V_{mi} - V_{ni} = \xi_{ei} I_{ii} \tag{2.16}$$

$$i_{ei} = y_e(V_{mi} - V_{ni}) = y_e \xi_{ei} I_{ii} \tag{2.17}$$

And power in element e due to generator i,

$$p_{ei} = \frac{P_i}{R_{ii}} \mathrm{Re}\{\xi_{ei}\xi_{ei}^* y_e^*\} \tag{2.18}$$

2.2.6 Example

Consider power source as shown in Fig. 2.3. Let the power injected be 1000 W. For it to be expended in the net 10 Ω, the current will be,

$$
\begin{aligned}
Source\ Current &= \sqrt{\frac{1000}{10}} = 10\mathrm{A} \\
Z &= [10]; A = [1 \quad 1] \\
\xi &= \begin{bmatrix} \xi_{11} \\ \xi_{21} \end{bmatrix} = A^T Z = \begin{bmatrix} 1 \\ 1 \end{bmatrix} [10] = \begin{bmatrix} 10 \\ 10 \end{bmatrix} \\
\begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} &= \begin{bmatrix} \xi_{11} \\ \xi_{21} \end{bmatrix} I_i = \begin{bmatrix} 10 \\ 10 \end{bmatrix} 10 = \begin{bmatrix} 100 \\ 100 \end{bmatrix} \mathrm{V} \\
\begin{bmatrix} i_{11} \\ i_{21} \end{bmatrix} &= \begin{bmatrix} y_1 & 0 \\ 0 & y_2 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} = \begin{bmatrix} \frac{1}{40} & 0 \\ 0 & \frac{3}{40} \end{bmatrix} \begin{bmatrix} 100 \\ 100 \end{bmatrix} \\
&= \begin{bmatrix} 2.5 \\ 7.5 \end{bmatrix} \mathrm{A}
\end{aligned}
$$

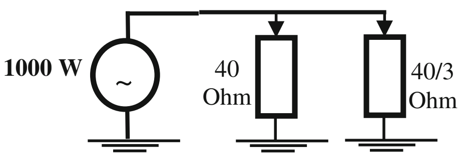

Fig. 2.3 Circuit with power source

Using (2.18),

$$Power\,in\,40\,\Omega = \left(\frac{1000}{10}\right) \times (10 \times 10) \times \frac{1}{40} = 250\,\mathrm{W}$$

Similarly,

$$Power\,in\,\frac{40}{3}\,\Omega = \left(\frac{1000}{10}\right) \times (10 \times 10)\frac{3}{40} = 750\,\mathrm{W}$$
$$Total\,Power = 250 + 750 = 1000\,\mathrm{W}$$

From total current and net resistance, we cross-check that,

$$Total\,Power = 10^2 \times 10 = 1000\,\mathrm{W}$$

Remark Suppose in the above example, the source capacity was limited to 900 W (such limits do arise in ac power systems, e.g., capability curves of turbine generators), then obviously the net current would be $\sqrt{900/10}$ = 9.4868 A, and the voltage, 94.868 V. This depressed voltage (as compared to 100 V with 1000 W) occurs when source is pegged at 900 W. Currents in 40 and 40/3 Ω will be 2.371 A and 7.115 A (for 900 W), as against 2.5 A and 7.5 A (for 1000 W), respectively. Such depressed voltage/current conditions before blackouts often result because of generators reaching thresholds of their *PQ* capabilities. (Capability charts are provided by manufacturers.)

Reflections

Standard solutions with voltage and current sources for passive networks are available in all books on electric circuits. In power network, that we describe here, the source—the synchronous generator—is a 'power' source! No textbook ever mentions constant power as source for electric circuits. We show that solution with 'constant power' source can indeed be obtained and wonder why this has been not been attempted so far..!

References

1. K.V.V. Murthy, M.S. Kamath, *Basic Circuit Analysis*, 8th edn. (Jaico Publishing House India, 2010)
2. M.A. Pai, *Computer Techniques in Power System Analysis* (Tata McGraw-Hill, New Delhi, 1979)

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