

Chapter 2

Market Areas of Retailers in Plane Space

This chapter examines the retailers' market areas and prices in a plane space. Assuming two retailers in a market space, the chapter first derives the shape of the boundary between the retailers' market areas. Second, relaxing the assumption for new retailers to enter the plane market space until every retailer's profit becomes zero, the retailers' market areas and prices are established in a free-entry competitive market space.

2.1 Shapes of Boundary Between Two Retailers' Market Areas

Assume that there are only two retailers, A and B, in a plane market space and consumers reside evenly on the market space. The same kind of goods are sold by the two retailers; each consumer purchases the goods at a retailer that offers the lowest delivered price to the consumer. The sites at which the delivered prices of the two retailers become the same form the market area boundary. This subsection explains the shapes of the market boundary, which vary according to the retailers' conditions.

2.1.1 *Derivation of the Shapes of Market Boundaries by Using Two Regular Cone Surfaces*

The situations of the two retailers A and B on a plane space are shown in Fig. 2.1a. Retailer A is located at origin O on the x - y plane space. Retailer B is located at point L away from retailer A along the x axis. The price of the good at each retailer's store is shown by p_A and p_B , respectively.

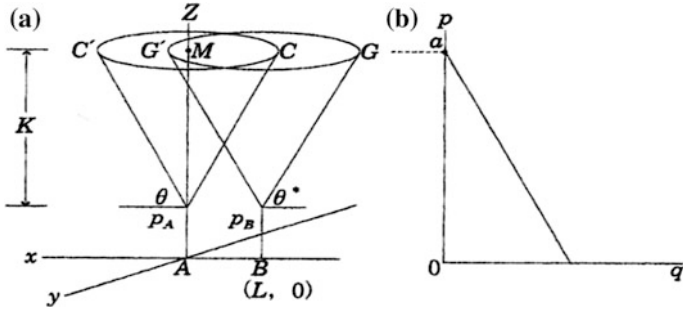


Fig. 2.1 Derivation of the market boundary by two regular cone surfaces. **a** Two regular cone surfaces, **b** Demand curve

The demand function of every consumer is given by Eq. (2.1), which is shown in Fig. 2.1b:

$$q_i = a - p_i - t_i u \quad (i = A, B) \quad (2.1)$$

Here, t_i ($i = A, B$) shows the transportation cost per mile for the retailers, A and B. The transportation costs per mile are described by θ, θ^* in Fig. 2.1a. The delivered prices of the goods are measured by the z axis in Fig. 2.1a. Thus, the level of the delivered price of retailer A to each consumer is shown by the left regular cone surface, $p_A CMC$. The delivered price of retailer B is indicated by the right regular cone surface, $p_B GMG'$. The market boundary by the two retailers' market areas is formed by the intersection of these two cone surfaces.¹

Let us derive the market boundary shape by formulating the two cone surfaces. The height of the cone of retailer A is noted by K , as shown in Fig. 2.1a. The difference between the heights of the two cones is noted by S . For simplicity, the bottom of retailer A's cone is assumed at the origin $(0, 0, 0)$ on the three-dimensional space. The regular cone surface of retailer A is shown by Eq. (2.2):

$$K^2(x^2 + y^2) = z^2 C^2 \quad (2.2)$$

where C is the radius of the circular market area of the retailer A. Because $K/C = \tan \theta$, Eq. (2.2) is transformed to Eq. (2.2a).

$$(x^2 + y^2) \tan^2 \theta = z^2 \quad (2.2a)$$

¹This method was introduced by Miyasaka (1970) into the market boundary analysis. He derived boundary shapes of two market areas by using two regular cone surfaces shown in Fig. 2.1a. Parr (1995) explained the market borders in a different way from that of this chapter.

In a similar way, the regular cone surface of retailer B is given by Eq. (2.3):

$$\left((x-L)^2 + y^2\right) \tan^2 \theta^* = (z-S)^2 \quad (2.3)$$

Hence, the market boundary of the two retailers' market areas can be determined by solving the simultaneous Eqs. (2.2a) and (2.3) by x and y . Next, subsections explain the shapes of the market area boundary according to the retailers' conditions.

2.1.2 Boundary Shape in the Case of the Same Prices and Transportation Costs

It is supposed that the two retailers' prices and transportation costs per mile are the same. In this case, the value of S , which indicates the difference between the prices of the retailers, is zero. Thus, solving the simultaneous Eqs. (2.2a) and (2.3) with respect to x and y gives Eq. (2.4):

$$2xL \tan^2 \theta - L^2 \tan^2 \theta = 0 \quad (2.4)$$

Here, x is derived as $(L/2)$. The market area boundary is shown as a straight line that divides the line combining the two retailers. This boundary is shown by $b_1 - b_2$ in Fig. 2.2a.

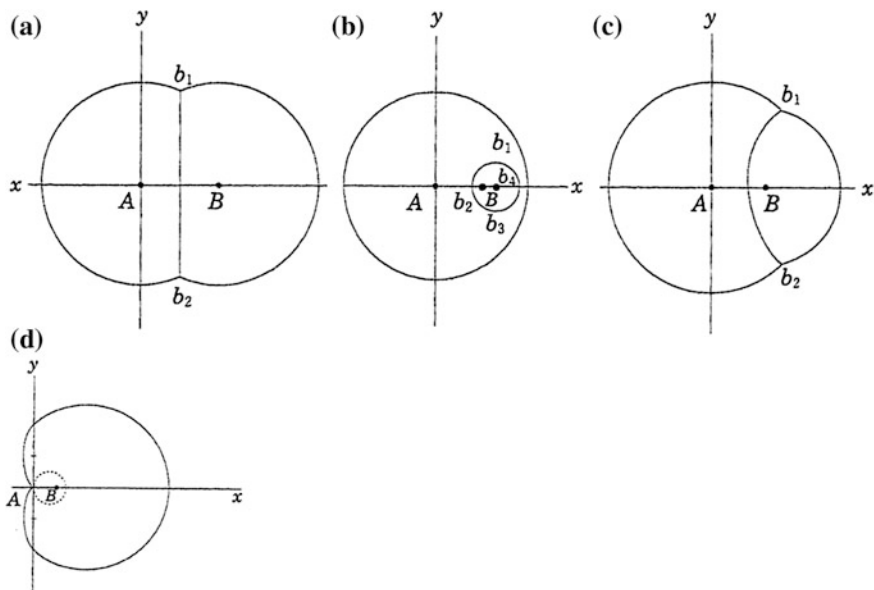


Fig. 2.2 Four shapes of the market boundary of two market areas. **a** Straight line, **b** Circle, **c** Hyperbola, **d** Limacon

2.1.3 *Boundary Shape in a Case of the Same Prices and Different Transport Costs*

In the second case, it is assumed that the prices at the two retailers' stores are the same, while the transportation cost per mile of retailer A is lower than that of retailer B, $\theta < \theta^*$, and $S = 0$. Solving the simultaneous Eq. (2.2a) and (2.3) gives Eq. (2.5):

$$(\tan^2 \theta^* - \tan^2 \theta)x^2 + (\tan^2 \theta^* - \tan^2 \theta)y^2 - 2L \tan^2 \theta^* x + L^2 \tan^2 \theta^* = 0 \quad (2.5)$$

Let us clarify the shape indicated by the Eq. (2.5). Dividing Eq. (2.5) by $(\tan^2 \theta^* - \tan^2 \theta)$ gives Eq. (2.6):

$$x^2 + y^2 - 2L \tan^2 \theta^* x / (\tan^2 \theta^* - \tan^2 \theta) + L^2 \tan^2 \theta^* / (\tan^2 \theta^* - \tan^2 \theta) = 0 \quad (2.6)$$

Equation (2.6) describes the circle of origin located at point $(L \tan^2 \theta^* / (\tan^2 \theta^* - \tan^2 \theta), 0)$; the radius of the circle is $(L \tan^2 \theta^* / (\tan^2 \theta^* - \tan^2 \theta))^{0.5}$. This circular market boundary is illustrated by the small circle shown by point b_1 – b_2 – b_3 in Fig. 2.2b, and its origin is indicated by point b_4 in Fig. 2.2b.

2.1.4 *Boundary Shape in a Case of Different Prices and the Same Transportation Costs*

In the third case, it is supposed that the price of the goods at retailer B's store is higher than that of retailer A by S . The transportation cost per mile is the same between the two retailers. Because the transportation cost per mile is the same, the transportation costs of the two retailers are noted by the same notation, θ . The market boundary in this case is obtained as follows. The righthand side of Eq. (2.3) is developed to Eq. (2.7):

$$\left((x - L)^2 + y^2 \right) \tan^2 \theta = z^2 - 2zS + S^2 \quad (2.7)$$

Substituting z in Eq. (2.7) with Eq. (2.2a) gives Eq. (2.8):

$$\left((x - L)^2 + y^2 - (x^2 + y^2) \right) \tan^2 \theta - S^2 = -2zS \quad (2.8)$$

As both sides of Eq. (2.8) are squared, Eq. (2.9) is obtained:

$$\left((x - L)^2 + y^2 - (x^2 + y^2) \right) \tan^2 \theta - S^2)^2 = 4z^2 S^2 \quad (2.9)$$

Substituting z^2 in Eq. (2.7) with Eq. (2.2a), then dividing it by $(1/\cot^2 \theta)^2$, gives Eq. (2.10):

$$(L^2 - 2Lx - S^2 \cot^2 \theta)^2 = 4S^2(x^2 + y^2) \cot^2 \theta \quad (2.10)$$

Equation (2.10) is transformed to Eq. (2.11):

$$(L^2 - S^2 \cot^2 \theta)x^2 - S^2 \cot^2 \theta y^2 - L(L^2 - S^2 \cot^2 \theta)x + (1/4)(L^2 - S^2 \cot^2 \theta)^2 = 0 \quad (2.11)$$

Transforming the x - y coordination, Eq. (2.11) is shown by Eq. (2.12), which implies a hyperbola:

$$x^2 / (S \cot \theta / 2)^2 - y^2 / \left((L^2 - S^2 \cot^2 \theta)^{0.5} / 2 \right)^2 = 1 \quad (2.12)$$

The asymptotic lines of the hyperbola are given by Eqs. (2.13) and (2.14):

$$y = + \left((L^2 - S^2 \cot^2 \theta)^{0.5} / S \cot \theta \right) x \quad (2.13)$$

$$y = - \left((L^2 - S^2 \cot^2 \theta)^{0.5} / S \cot \theta \right) x \quad (2.14)$$

The boundary of the retailers' market areas in this case is shown by b_1 - b_2 in Fig. 2.2c.

2.1.5 *Boundary Shape in a Case of Different Prices and Different Transportation Costs*

The fourth case deals with a rather complicated situation that the transportation cost per mile of retailer B is higher than that of retailer A, and the price of retailer B is lower than that of retailer A by S . Then, retailer B's delivered price at the location of retailer A is just equal to the price at retailer A. Retailer B's regular cone surface is indicated by Eq. (2.15):

$$\left((x - L)^2 + y^2 \right) \tan^2 \theta^* = (z + S)^2 \quad (2.15)$$

The market boundary can be obtained by solving the simultaneous Eqs. (2.2a) and (2.15) with respect to x and y . By developing the right-hand side and then substituting z^2 with Eqs. (2.2a) and (2.16) is derived:

$$\left((x-L)^2 + y^2\right) \tan^2 \theta^* = (x^2 + y^2) \tan^2 \theta + 2zS + S^2 \quad (2.16)$$

Equation (2.16) is transformed to Eq. (2.17):

$$(\tan^2 \theta^* - \tan^2 \theta) x^2 + (\tan^2 \theta^* - \tan^2 \theta) y^2 - 2L \tan^2 \theta^* x + L^2 \tan^2 \theta^* - S^2 = 2zS \quad (2.17)$$

By squaring the both sides of Eq. (2.17) and substituting z with Eq. (2.2a), Eq. (2.18) is obtained:

$$\left(W(x^2 + y^2)^2 \tan^2 \theta^* Lx + \tan^2 \theta L^2 - S^2\right)^2 = 4S^2 \tan^2 \theta (x^2 + y^2) \quad (2.18)$$

where $W = \tan^2 \theta^* - \tan^2 \theta$. Because $\tan \theta^* = S/L$, Eq. (2.18) is transformed as Eq. (2.19):

$$(x^2 + y^2 - 2 \tan^2 \theta^* Lx/W)^2 = (2S \tan \theta / W)^2 (x^2 + y^2) \quad (2.19)$$

Equation (2.19) describes the limaçon, which is shown in Fig. 2.2d.

2.1.6 Boundary in a Case of the Same Retailers' Locations and Different Transportation Costs

The fifth case examines the market boundary when the prices at the stores of the two retailers are different and the transportation costs per mile also are different between the two retailers, while the location of the stores of the retailers are coincidence. In this case, the regular cone surface of retailer B is given by Eq. (2.20):

$$(x^2 + y^2) \tan^2 \theta^* = (z + S)^2 \quad (2.20)$$

The shape of the market boundary in this situation is derived from Eq. (2.21), which is derived by using Eqs. (2.2a) and (2.20):

$$(\tan^2 \theta^* - \tan^2 \theta) (x^2 + y^2)^2 + 2S^2 (\tan^2 \theta^* + \tan^2 \theta) (x^2 + y^2) + S^4 = 0 \quad (2.21)$$

By transforming Eqs. (2.21) and (2.22) is obtained:

$$x^2 + y^2 = (S(\tan^2 \theta^* + \tan^2 \theta) / (\tan^2 \theta^* - \tan^2 \theta))^2 \quad (2.22)$$

Equation (2.22) indicates a circle with an origin located at the origin $(0, 0)$; the radius of the circle is $S(\tan^2\theta^* + \tan^2\theta)/(\tan^2\theta^* - \tan^2\theta)$.

2.1.7 Market Boundary Shapes Derived by a Method of Numerical Calculation

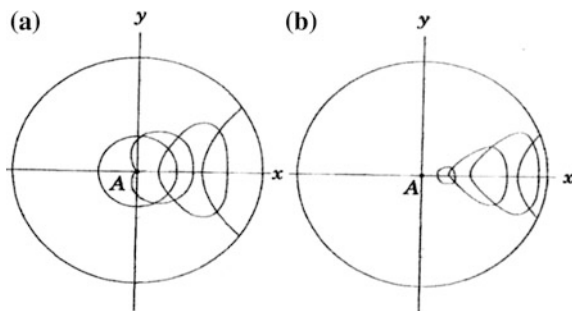
When the two retailers' locations, prices, and transportation costs per mile are different, the shape of the market area boundary can be usually derived by using a numerical calculation. In this subsection, the following two typical market conditions are selected and the resulting market area boundaries are shown by assigning numerical values to the transport costs per mile and the retailers' prices:

- (I) Retailer B's price is lower than that of retailer A and the transportation cost per mile of retailer B is higher than retailer A. Retailer B's location is moved from the origin point to the end-point of the market area of retailer A along the x axis.
- (II) Retailer B's price is higher than that of retailer A and the transportation cost per mile of retailer B is higher than retailer A. Retailer B's location is moved from a point at which retailer B starts to sell its goods in the market space to the end-point of the market area of the retailer A along the x axis.

First, let us examine the market area boundary of case (I), where $\theta = 1$, $\theta^* = 1.5$ and $p_A = 5$, $p_B = 2$ are assumed. In this case, as retailer B's location is moved from the origin point of the plane to the end-point of retailer A's market area, the market area boundary is changed as shown by the curves in Fig. 2.3a; from a circle to the limaçon, and then one of the vertically stretched curves of the ellipse finally.

In case (II) where $\theta = 1$, $\theta^* = 1.5$ and $p_A = 5$, $p_B = 6$ are assumed. The location of retailer B is assumed to be moved from a point at which retailer B start to sell its goods in the market space to the end-point of the market area of the retailer A along the x axis. In this case, as retailer B's location is moved toward the end point, the market area boundary is illustrated by the curves in Fig. 2.3b. Each market boundary belongs to one of the horizontally stretched ellipse families.

Fig. 2.3 Market boundary described by the fourth degree functions. **a** Vertically stretched ellipse curves, **b** Horizontally stretched curves



2.1.8 *A Comment on the Market Boundary Analysis*

Although the history of the analysis of the retailers' market boundary is rather long, the achievements of the analysis have not been effectively utilized. One of the reasons why the shape analysis of the market area boundary has not been successfully developed in economics is that the analysis relies on only a single method of the equation treatment: There are many complicated shapes among the market area borders so that the shapes of the market areas of the retailers are diversified. It is thus difficult to derive the sales volumes of the retailers in their market areas. If it is difficult to derive the sales volume, which is indispensable to obtain the retailers' profits and costs, the motive and incentive to analyze the market area boundaries are so weak that the significance of the market area boundary analysis is not noticed by many researchers and expert regional policy makers. It may be important to use a numerical calculation method, such as Monte Carlo, as well as equation treatment in order to grasp the sales quantity in the complicated market areas of the retailers.² If it would be easy to derive the sale quantities in many complicated market areas, it would stimulate the analysis of the regional economics, especially the analysis of the city systems in regions, which are the basic spatial unit of the regional economy.

2.2 Market Area in Spatial Free-Entry Equilibrium

Assuming that a plane market space is opened to allow for a new retailer to enter and the market space is under free-entry competition, this section analyses the retailers' market area sizes and market area shapes in a free-entry competitive equilibrium. Also, the prices and the delivered prices of goods established in the competitive market space are examined.

2.2.1 *Feasible Market Shapes in a Free-Entry Equilibrium*

Let us derive the potential market area shapes that can emerge in a free-entry competitive equilibrium in a market space. Assume that the same regular polynomial markets of the retailers cover a plane space and the regular polynomial markets have n apexes, and m apexes gather at an apex of the polynomial market. In this case, the vertical angle of the regular market area with n apexes is shown by $\pi(n - 2)/n$. Therefore, the condition that the vertical angle multiplied by m times makes 2π is given by $m(1 - 2/n) = 2$. Hence, the following relationship is obtained:

²The derivation of sales volumes using Monte Carlo has been conducted by Ishikawa and Toda (2002). It is applied to a retailer's location strategy in a plane space.

$$mn = 2m + 2n \quad (2.23)$$

Adding 4 to the both sides of Eq. (2.23) gives Eq. (2.24).

$$(m - 2)(n - 2) = 4 \quad (2.24)$$

The integer solutions of Eq. (2.24) are confined to the following the three sets:

$$(m, n) = (4, 4), (3, 6), (6, 3) \quad (2.24a)$$

The above integer solution sets mean that the shapes of market areas in a free-entry equilibrium are limited to the three shapes; square, regular triangle, and regular hexagon.

2.2.2 Market Shapes of Retailers in Free-Entry Competitive Equilibria

This subsection deals with the retailers' market area shape, which appears in a free-entry competitive equilibrium.³ In this analysis, the same framework is used as in the preceding analysis in the linear market space. Thus, the demand function and cost function are the same those used in the analysis in the linear space. There are two conditions that must be satisfied in a free-entry competitive equilibrium: (1) each retailer's profit is just zero and (2) each retailer sets the price to maximize its profit. The two equilibrium conditions are represented by Eqs. (2.25) and (2.26), respectively:

$$Y = (p - k)Q - F = 0 \quad (2.25)$$

$$dY/dp = (p - k)(dQ/dp + dQ/dU \cdot dU/dp) + Q = 0 \quad (2.26)$$

The term dU/dp in Eq. (2.26) is dependent on the conjectural price valuation dp'/dp , as shown by Eq. (2.27):

$$\begin{aligned} dU/dp &= (dp'/dp - 1)/2tR_i \quad i = t, s, h \\ R_t &= 2, R_s = 2^{0.5}, R_h = 2/3^{0.5} \end{aligned} \quad (2.27)$$

where p' is the price of the retailer, which is located at farthest between the retailers surrounding the retailer in question. As explained in Chap. 1, when the value of dp'/dp is 1, 0, and -1 , the competition style in the retailing space is called Lösch (L),

³An analysis of the market area shapes under free-entry competition is carefully conducted by Ishikawa and Toda (1990).

Table 2.1a Equilibrium price and market size established in a free-entry market

F	Style	Triangle		Square		Hexagon	
		p	U	p	U	p	U
$0.01a^4$	L	$0.4557a$	$0.0963a$	$0.4583a$	$0.1091a$	$0.4589a$	$0.1171a$
	S-H	$0.1902a$	$0.1203a$	$0.1683a$	$0.1434a$	$0.1562a$	$0.1589a$
	G-O	$0.1287a$	$0.1421a$	$0.1128a$	$0.1712a$	$0.1042a$	$0.1907a$
$0.06a^4$	L	$0.3643a$	$0.2950a$	$0.3751a$	$0.3265a$	$0.3776a$	$0.3485a$
	S-H	$0.2909a$	$0.3041a$	$0.2750a$	$0.3436a$	$0.2636a$	$0.3718a$
	G-O	$0.2356a$	$0.3242a$	$0.2133a$	$0.3752a$	$0.2009a$	$0.4110a$

Hotelling-Smithies (S-H), and Greenhut-Ohta (G-O) competition, respectively. In this analysis, these three kinds of competition styles are selected as the competition styles prevailing in the plane market space.

Let us suppose that the marginal cost is zero and the two levels of the fixed cost, F , are assumed to be $0.01a^4$ and $0.06a^4$. Solving simultaneous Eq. (2.26) and (2.27) gives the competitive equilibrium price and market area size for the three different market area shapes against the two levels of fixed cost. The equilibrium values obtained by calculations are shown in Table 2.1a.

From the values of the radius of the inscribed circle of the market area, the size of the market area is derived for the three kinds of market shapes. Table 2.1b shows the equilibrium market area size for the three different market area shapes against each fixed cost level. When the market space is under Lösch competition style (L), the market shape that makes the market area size the smallest is a hexagon against each fixed cost level. When the market space is under the Greenhut-Ohta competition style (G-O), the market shape that makes the market area size the smallest is the triangle to each fixed cost level. When the market space is under the Hotelling-Smithies competition style (S-H), the market shape that makes the market area size the smallest is changed according to the fixed cost level. When the fixed cost is low, the triangular market shape makes the market area size the smallest; when the fixed cost is high, the square market shape makes the market area size the smallest.

Based on the criterion that the market shape which makes the market area size the smallest appears in a free-entry competitive equilibrium, it can be concluded

Table 2.1b Equilibrium market size established in a free-entry market

F	Style	Triangle area	Square area	Hexagon area
$0.01a^4$	L	$0.0482a^2$	$0.0476a^2$	$0.0475a^4$
	H-S	$0.075a^2$	$0.082a^2$	$0.089a^4$
	G-O	$0.011a^4$	$0.117a^2$	$0.126a^4$
$0.06a^4$	L	$0.452a^2$	$0.426a^2$	$0.421a^2$
	H-S	$0.481a^2$	$0.472a^4$	$0.479a^2$
	G-O	$0.546a^2$	$0.563a^2$	$0.585a^4$

that the three kinds of shapes have opportunity to appear in the free-entry competitive equilibrium. Which shape appears is dependent on the competition style and the fixed cost level of the retailers in the market space.

In addition, it is noteworthy that although Lösch competition style minimizes the market absolute size, the price of the retailers in Lösch competition becomes the highest irrelative market shapes. Greenhut-Ohta competition style makes the absolute market area the largest, whereas the price of the goods for retailers in Greenhut-Ohta competition becomes the lowest irrelative market shapes. Because the equilibrium profit of each retailer must be zero, the market shape problem is not so serious to each retailer. However, for every consumer, the problem of competition style among the retailers is important because the consumer surplus differs by the market shape in equilibrium.⁴

2.2.3 Shapes of Demand Curves in Retail Market Space

The retailers' prices and market areas in a free-entry competitive equilibrium vary according to the competition style among retailers. The differences in the retailers' equilibrium situations can be explained by the shapes of the demand curve in a plane market space. In this section, assuming a hexagonal market area for simplicity of the analysis, the shapes of the demand curves in plane market space are analyzed.

2.2.3.1 Demand Curved in a Greenhut-Ohta Competition Model

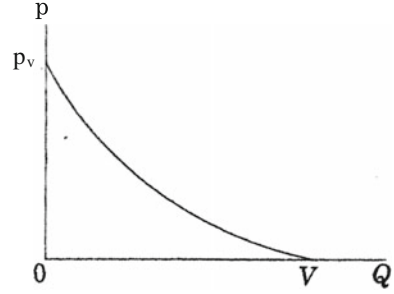
First, the demand curve in a Greenhut-Ohta model is examined. As shown in the above analysis, the retailers' equilibrium price and market area size in a Greenhut-Ohta model are derived by solving the simultaneous equation system of (2.25) and (2.26), assigning the value of -1 to the conjectural price valuation. In the original framework of the analysis, however, it is assumed that the delivered price at an apex of the retailer's market area is parametrically given to the retailer. Then, the competitive equilibrium price and market area are analyzed.

According to the original framework, the shape of the demand curve is examined. The quantity demanded in a retailer's market area is given by Eq. (2.28).

$$Q = 12 \int_0^{30^\circ} \int_0^{U/\cos\theta} (a - p - tu) u du d\theta \quad (2.28)$$

⁴See Ishikawa and Toda (1990). They shows the changes of the consumer surplus and the social surplus according the market area shape and the level of the retailers' fixed cost.

Fig. 2.4 Demand curve in Greenhut-Ohta competition model



If the delivered price at an apex is denoted by p_v , the radius of the inscribed circle of the market area U is shown by $U = 3^{0.5}/2(p_v - p)$. Thus, by integrating Eq. (2.28), the sales quantity of a retailer is given as Eq. (2.29).

$$Q = 9(p_v - p)^2 \left(\left(\frac{1}{2} \cdot 3^{0.5} \right) a - \left(\frac{1}{2} \cdot 3^{0.5} \right) p - 0.2027(p_v - p)3^{0.5}/2 \right) \quad (2.29)$$

Equation (2.29) is described by the curve p_v - V in Fig. 2.4. This curve shows the demand curve in a Greenhut-Ohta competition model.

2.2.3.2 Demand Curve in a Lösch Competition Model

Second, the demand curve in a Lösch model is examined. Here, the equilibrium price and market area size are derived by assigning the value of 1 to the conjectural price valuation. In the original framework of the analysis, it is assumed that the size of the hexagonal market area is parametrically given to a retailer. The retailers' equilibrium price and market area are derived. Thus, according to the original framework, the shape of the demand curve is examined.

Suppose that a hexagonal market area is described as points $H-I-J-L-M-N$ in Fig. 2.5 and its radius of the inscribed circle of the market area is denoted by U . In this market area, the demand curve is shown by curve $a-K'-J'-R$ in Fig. 2.6. This curve is explained step by step as follows.

When the retailer lowers its price at its store from the maximum reservation price a to P_k , at which the delivered price at point K in Fig. 2.5 reaches the value of a , the number of consumers increases and the quantity demanded by individual consumers is raised. The shape of the demand curve, therefore, is shown by $a-K'$ in Fig. 2.6. This shape is the same as in a case when the retailer is a monopoly.

As the retailer's price decreases toward P_j at which the delivered price at point J in Fig. 2.5 reaches just a , the demand curve is shown by $a-K'-J'$. Because that as the price reduces, the increase rate of the number of the consumers is gradually decreased by the market area border, the slope of the demand curve of $K'-J'$ is steeper than that of $a-K'$. Then, the retailer's price is reduced less than P_j ; the

Fig. 2.5 Hexagonal marker area

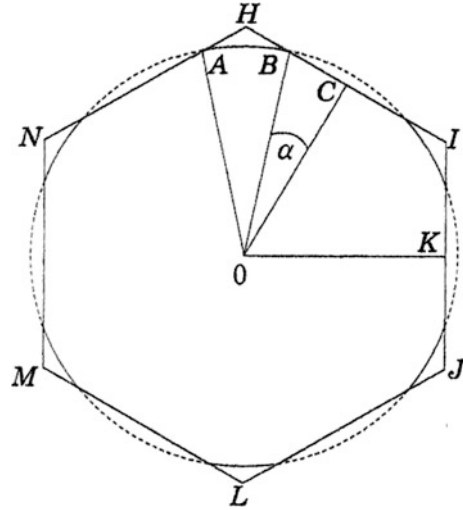
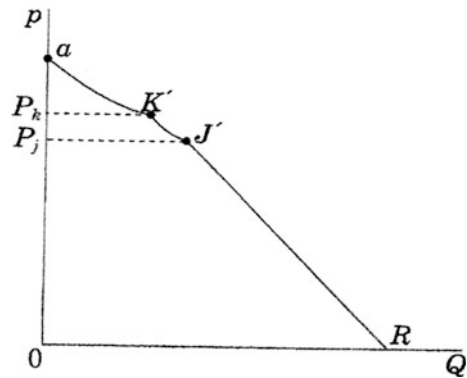


Fig. 2.6 Demand curve in a Lössch competition model



demand curve is shown by $J'-R$. Because the reduction of the price increases the quantity demanded by individual consumers and it does not increase the number of customers, the corresponding part of the demand curve becomes a straight line with a steeper slope. Consequently, the demand curve in a Lössch model is represented by the curve of $a-K'-J'-R$.

Let us formulate each part of the demand curve of $a-K'-J'-R$. As shown by Fig. 2.6, the demand curve of a Lössch competition model is formed by three parts. The part $a-K'$ of the demand curve is the same as that of the case when the retailer is a monopoly. Thus, the part is formulated by Eq. (2.28). In Eq. (2.28), the price p of the goods must not be higher than the price level at which the delivered price at an apex of its market is equal to the maximum demand price a .

The part $K'-J'$ of the demand curve is formulated by Eq. (2.30), which is derived by obtaining the demand quantity Q_2 in a pseudo-hexagonal market area in Fig. 2.5 as a function of price p that is in the range of $P_k > p > P_j$. The demand quantity is obtained by the summing up the quantities that are derived in the two areas of two pseudo-triangles OAB and OBC in Fig. 2.5.

$$Q_2 = 12 \int_0^{\alpha} \int_0^{U/\cos\theta} (a - p - tu) u du d\theta + 6 \int_0^{\pi/3-2\alpha} \int_0^{U/\cos\theta} (a - p - tu) u du d\theta \quad (2.30)$$

Here, α is the angle of BOC and $\alpha < 30^\circ$. Equation (2.30) is integrated as Eq. (2.31):

$$\begin{aligned} Q_2 = & 12U^2((\alpha/2)\tan\alpha - (p/2)\tan\alpha \\ & - (U/3)(\sin\alpha/(2\cos^2\alpha) + (1/2)L_N|\tan(\alpha/2 + \pi/4)|)) \\ & + 6U^2((\alpha/2)\tan(\pi/3 - 2\alpha) - (p/2)\tan(\pi/3 - 2\alpha) \\ & - (U/3)(\sin(\pi/3 - 2\alpha)/2\cos^2(\pi/3 - 2\alpha)) + (1/2)L_N|\tan((1/2)(\pi/3 - 2\alpha) + \pi/4)|)) \end{aligned} \quad (2.31)$$

Here, α is represented as a function of price as Eq. (2.32).

$$\alpha = \cos^{-1}(U/(a - p)) \quad (2.32)$$

Eventually, the part $K'-J$ of the demand curve is expressed as a function of price p .

Lastly, the part $J'-R$ of the demand curve is formulated as Eq. (2.33) by deriving the quantity demanded in a hexagonal market area where there is no new customer.

$$Q_3 = 12U^2((1/2 \cdot 3^{0.5})a - (1/2 \cdot 3^{0.5})p - 0.2027U) \quad (2.33)$$

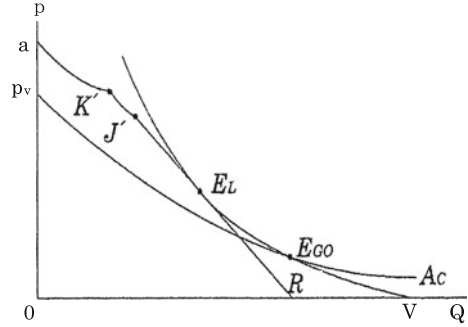
The part of $J'-R$ of the demand curve is a function of price p , which is in the range of $p < P_j$. It is expressed as a straight line.

2.2.3.3 Equilibrium Situations in Lösch and Greenhut-Ohta Competition Models

Using the two demand curves derived in the above analyses, the equilibria in Lösch and Greenhut-Ohta competition models are explained. The demand curve of Lösch model is steeper than that of a Greenhut-Ohta model. Figure 2.7 depicts these two demand curves introducing an average cost curve AC .

In Fig. 2.7, the two different equilibria are established at points E_L and E_{GO} . They show the equilibrium situations in Lösch and Greenhut-Ohta equilibria,

Fig. 2.7 The relationship between equilibrium values and the shape of the demand curve



respectively. As shown in Fig. 2.7, the equilibrium price of the Lösch model is always higher than that of the Greenhut-Ohta model, and the quantity demanded in the Greenhut-Ohta model is larger than that of the Lösch model.

2.2.3.4 Change of Market Area Due to Decrease of Transportation Costs

This section examines the effects of a reduction of the transportation cost per mile on the competitive equilibria derived from the three kinds of the competition models.⁵ In this examination, it is assumed that the retailers' market area shape is hexagon, the marginal cost is zero, the fixed cost is $0.05a^4$, and the transportation cost per mile is reduced from 1.47 to 0.2. In the same manner, the equilibrium price and inscribed radius of the market area are derived for the three kinds of the competition models. The loci of the three equilibrium situations are shown in Fig. 2.8. Table 2.2 indicates the numerical equilibrium values for the Lösch and Greenhut-Ohta competition model.

Figure 2.8 and Table 2.2 show that if the retailers are under the Greenhut-Ohta competition style, as the transportation cost per mile decreases from 1.47 to 0.2, the equilibrium price decreases and the size of the retailers' market area expands. The market area when the transportation cost per mile is 0.2 becomes larger than that of the original size. The size of the market area in the Lösch model simply squeezes as the transportation cost per mile decreases and the retail price increases. The reason for the difference in the direction of the change is the same as that shown in the analysis in a linear market space in Chap. 1.

⁵Ishikawa and Toda (1998) start to analysis the correspondence between the changes of the equilibrium situation and the competition styles.

Fig. 2.8 Changes of equilibrium situations due to reducing transportation costs

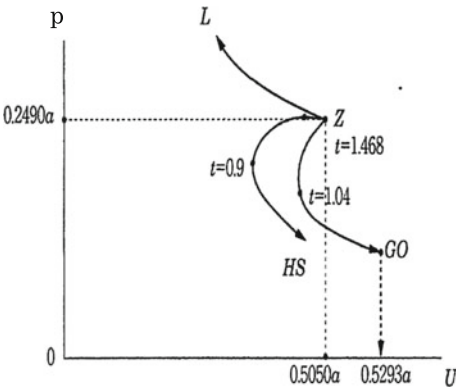


Table 2.2 Changes of equilibrium values of Lösch and Greenhut-Ohta models

<i>t</i>	Lösch		G-O	
	<i>U</i>	<i>p</i>	<i>U</i>	<i>p</i>
1.47	0.505 <i>a</i>	0.25 <i>a</i>	0.505 <i>a</i>	0.25 <i>a</i>
1.2	0.3346 <i>a</i>	0.3590 <i>a</i>	0.3798 <i>a</i>	0.2153 <i>a</i>
1.0	0.3060 <i>a</i>	0.3929 <i>a</i>	0.3744 <i>a</i>	0.1873 <i>a</i>
0.8	0.2864 <i>a</i>	0.4196 <i>a</i>	0.3809 <i>a</i>	0.1585 <i>a</i>
0.5	0.2649 <i>a</i>	0.4535 <i>a</i>	0.4155 <i>a</i>	0.1128 <i>a</i>
0.4	0.2591 <i>a</i>	0.4636 <i>a</i>	0.4382 <i>a</i>	0.0963 <i>a</i>
0.2	0.2490 <i>a</i>	0.4825 <i>a</i>	0.5293 <i>a</i>	0.060 <i>a</i>

As the transportation cost per mile decreases, while the retailers’ market area size becomes smaller than that of the original market size in a Lösch competition style, the retailers’ market size becomes larger than that of the original size in a Greenhut-Ohta competition style. These facts are very important when examining the structural change of city systems that are constructed on the basis of retailers’ market areas.

2.3 Effects of a Variety of Items on Retailers’ Market Situations

This section discussed the variety of items in retailers’ stores, analyzing the relationships between the varieties of goods at retailers’ stores and market areas. Incorporating the competition styles used in the above analysis into the framework developed by Baumol and Ide (1956), this section examines the effects of the variety of items on the market boundary and the retailers’ market areas in a competitive plane market space.

2.3.1 *Effects of the Variety of Items on the Boundary of Market Areas*

2.3.1.1 Assumptions and Derivation of Retailer's Profit Function

According to Baumol and Ide (1956), who systematically analyzed the relationships between the variety of items and the retailers' market areas, the following assumptions are set:

- (i) Consumers evenly reside in a plane space with density K .
- (ii) Retailers provide a number of items at their stores to consumers.
- (iii) The probability that a consumer visits a retailer becomes higher as the number of the items increases. This probability, Z , is expressed by Eq. (2.34)⁶:

$$z(N) = (N/A_Z)^\alpha \quad (2.34)$$

where N is the number of the items and A_Z and α are positive parameters, respectively. The value of $Z(N)$ belongs to the range $0 \leq Z(N) \leq 1$.

- (iv) By going to a retailer, a customer incurs some costs. The difficulty of shopping increases with the congestion of a store and with the transportation costs to the store. The congestion level, J , increases with the number of the items, and the transportation costs, S , increase with the distance to the retailer. The congestion is represented by Eq. (2.35),

$$J(N) = C_n N^\beta \quad (2.35)$$

where C_n is the marginal cost of congestion and β is a positive parameter. The transportation cost is shown by Eq. (2.36):

$$S = tu \quad (2.36)$$

where t is the transportation cost per mile and u is the distance from a customer to a retailer's store.

- (v) The willingness of a customer to visit a retailer is influenced negatively by the average price of the items, p . This influence, Pa , is expressed by Eq. (2.37)⁷:

$$Pa(p) = \phi p^\phi \quad (2.37)$$

where ϕ and ϕ are positive parameters.

⁶Baumol-Ide does not use a concrete function like Eq. (2.34).

⁷The average price of items is not incorporated into the analysis by Baumol and Ide (1956).

Based on these assumptions, a retailer's capacity of pulling customers, f , can be determined by the three factors: the number of the items N , the average price p , and the distance u . A retailer's capacity of pulling customers (i.e., its traction power) can be represented by Eq. (2.38):

$$f(N, p, u) = \omega(N/A_Z)^\alpha - v(C_n N^\beta + tu + \phi p^\phi) \quad (2.38)$$

where ω and v are positive parameters.

- (vi) The quantity demanded in a retailer's market area is proportional to the traction power of attracting customers to the retailer's store. When a retailer's market area is a circle, the quantity demanded, Q , in its market area is expressed by Eq. (2.39):

$$Q = 2\pi \int_0^U ((N/A_Z)^\alpha - v(tu + C_n N^\beta + \phi p^\phi)) u du \quad (2.39)$$

where U is the radius of the circular market area. Because the average price of the items is represented by p , the retailer's revenue, Rv , is derived by Eq. (2.40):

$$Rv = pQ. \quad (2.40)$$

- (vii) Inventory costs of the retailer are shown by Eq. (2.41):

$$IC = (E/I)r + (I/2 + R)T, \quad (2.41)$$

where E is the expected sales volume of all commodities per period, r is the handling costs of reordering, I is the quantity ordered for inventory each time stocks are replaced, T is the warehousing cost per item per period, and R is the stock level at which inventory is replaced (i.e., the stocks on hand fall to R and the retailer replaces inventory). From Eq. (2.41), the optimal inventory (ICI) per item is derived by Eq. (2.42):

$$ICI = (2rTQ/N)^{0.5} + RT. \quad (2.42)$$

- (viii) The fixed costs of the warehouse and the retail facility are shown by F . The dealing costs of goods are proportional to the number of the items, and the costs are expressed as $eN^{0.5}$, where e is the parameter. The total costs of a retailer are given by Eq. (2.43):

$$TC = N(2rTQ/N)^{0.5} + NRT + eN^{0.5} + F. \quad (2.43)$$

Finally, the retailer's profit, Y , is given by Eq. (2.44):

$$\begin{aligned}
 Y = & p(2\pi KU^2(0.5(\omega(N/A_Z)^\alpha - vC_n N^\beta - v\phi p^\varnothing) - vtU/3)) \\
 & - N^{0.5}((2rT(2\pi K U^2(0.5(\omega(N/A_Z)^\alpha - vC_n N^\beta - v\phi p^\varnothing) \\
 & - vtU/3)))^{0.5} + e) - NRT - F
 \end{aligned} \tag{2.44}$$

2.3.1.2 Effect of a Variety of Items on the Market Area Boundary

Before conducting the analysis of the market area boundary, let us examine the retailer's capacity of pulling customers when the variety of the items is considered.

We assigned the following numerical values to the parameters in Eq. (2.38): $A_Z = 20$, $\omega = 45$, $v = 5.6667$, $t = 0.2$, $C_n = 1.2$, $\alpha = 0.26$, $\beta = 0.5$, $\varphi = 0.2$, and $\varnothing = 1.05$. The retailer's capacity of pulling customers is then given by Eq. (2.45):

$$f(N, p, u) = 45(N/20)^{0.26} - 5.6667(0.2u + 1.2N^{0.5} + 0.2p^{1.05}) \tag{2.45}$$

When one retailer is located in a plane space, the retailer's market area is established by points at which the retailer's traction power of customers is zero; that is, Eq. (2.45) is zero at these points. For example, assume that the retailer is located at the origin of the coordinates of x and y , the number of the items N is 0.5, and the average price p is 4.0. The points at which the retailer's power to pull customers is zero are derived by solving Eq. (2.46) with respect to x and y .

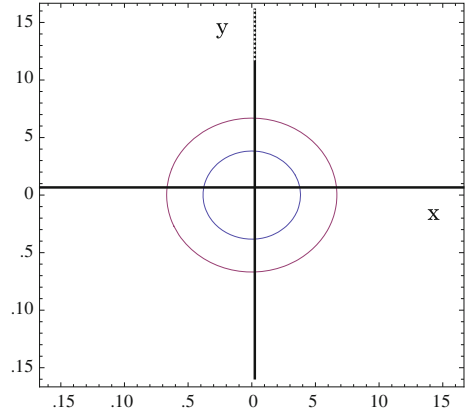
$$f(x, y) = 45(0.5/20)^{0.26} - 5.6667(0.2(x^2 + y^2)^{0.5} + 1.2 \cdot 0.5^{0.5} + 0.2 \cdot 4^{1.05}) = 0 \tag{2.46}$$

In this case, the retailer's market area is indicated by a circle, which is depicted by the larger circle in Fig. 2.9. The radius U of the circle is 6.69. If the number of the items N decreases to 0.1, the circular market area shrinks to the small circle with the radius of 3.83, which is shown by the smaller circle in Fig. 2.9. Keeping the average price at the same level, the reduction in the number of the items shrinks the retailer's market area by 95.53. This shrunken area reveals the effect of the variety of the items on the size of the retailer's market area.

Now, let us suppose that two rival retailers A and B are in a plane market space. Retailer A and B are located at points $(-2, 0)$ and $(2, 0)$, respectively. It is also assumed that the retailers' varieties of the items are denoted by N_A and N_B ; their transportation costs per mile and the average prices are represented by t_A , t_B and p_A , p_B , respectively.

The market area boundary of the two retailers is formed by points where the two retailers' powers to attract customers are the same. Equation (2.47) must be satisfied at the market area boundary:

Fig. 2.9 Formation of a retailer's market area



$$\begin{aligned}
 & 45(N_A/20)^{0.26} - 5.6667 \left(t_A \left((x+2)^2 + y^2 \right)^{0.5} + 1.2N_A^{0.5} + 0.2p_A^{1.05} \right) \\
 & = 45(N_B/20)^{0.26} - 5.6667 \left(t_B \left((x-2)^2 + y^2 \right)^{0.5} + 1.2N_B^{0.5} + 0.2p_B^{1.05} \right)
 \end{aligned}
 \tag{2.47}$$

Supposing that $N_A = N_B = 0.98$, $t_A = t_B = 0.2$, $p_A = p_B = 1.1$, the market area boundary is obtained by Eq. (2.47). In this case, the market area boundary is a straight line, as shown by the line in Fig. 2.10. Then, when the number of the items stocked at the store of retailer A increases from 0.98 to 1.27, keeping the average prices of both retailers at the same level, the boundary becomes a hyperbola, which is represented in Fig. 2.10. Retailer A expands its market area toward retailer B.

Fig. 2.10 Market area boundaries indicated by a straight line and a hyperbola

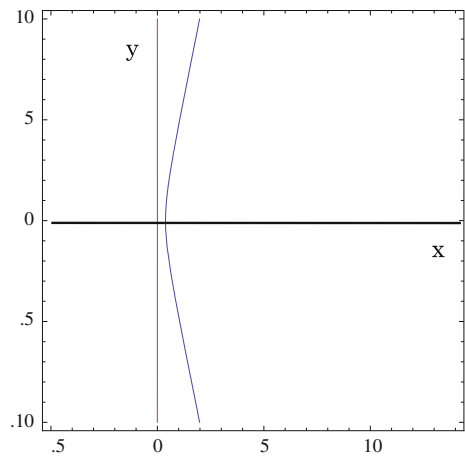
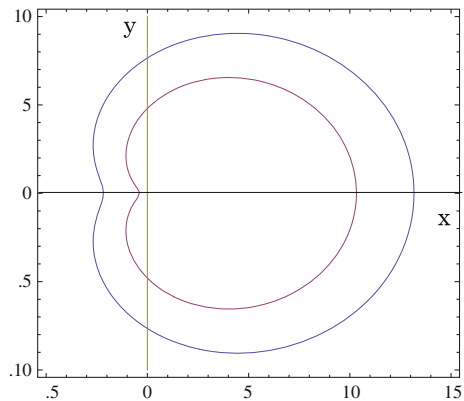


Fig. 2.11 Market area boundary indicated by ellipses



On this boundary, the delivered average price level of retailer A is higher than that of retailer B. Because retailer A provides more variety of items than its rival retailer B, retailer A occupies a larger market area than retailer B.

Then, assume that retailers A and B are located at $(-0.5, 0)$ and $(0.5, 0)$, respectively, and that the number of items are the same between the retailers, $N_A = N_B = 0.98$. The average price and transportation cost per mile are different between the retailers: $p_A = 2$, $p_B = 1.1$, and $t_A = 0.2$, $t_B = 0.24$. In this case, the market boundary is depicted by the small closed ellipse in Fig. 2.11. The area enclosed by the small ellipse is the market area of retailer B. When the number of the items of retailer B increases to $N_B = 1.37$, the market area boundary expands to the large ellipse in Fig. 2.11. The increase of the variety of the items expands the market area of retailer B. It can be said that incorporating the variety of the items stocked at the retailers' stores into consideration gives the flexibility of the market area analyses and provides the ability to explain the locations and the market areas of the retailers in the real world.

2.3.2 *Effect of the Variety of Items on Retailers' Market Area Size*

2.3.2.1 **Influence of Competition Styles on the Equilibrium for the Variety of Items**

Assuming that free-entry competition among the retailers prevails in a plane market space, this section derives the number of the items, the average price of the items, and the market area size in a spatial free-entry equilibrium.

There are three conditions for a free-entry equilibrium to be established in a spatial free-entry market space when the variety of the items in the retailers' stores is included in the preceding framework of the analysis. Every retailer determines the number of the items and the average price to maximize its profit. A new retailer comes into the market space until every retailer's profit is just zero. These conditions are shown by Eqs. (2.48a, b, c):

$$\partial Y/\partial N = \partial Y/\partial N + \partial Y/\partial U \cdot \partial U/\partial N = 0 \quad (2.48a)$$

$$\partial Y/\partial p = \partial Y/\partial p + \partial Y/\partial U \cdot \partial U/\partial p = 0 \quad (2.48b)$$

$$Y = 0 \quad (2.48c)$$

In Eq. (2.48a), $\partial U/\partial N$ represents the variation of the radius of the market area when a retailer changes the number of the items by one unit; $\partial U/\partial N$ and $\partial U/\partial p$ are given by Eqs. (2.49a), (b), respectively:

$$\partial U/\partial N = (((\omega/\nu)(\alpha/A^\alpha)N^{\alpha-1} - \beta C_n N^{\beta-1})/2t)(1 - dN'/dN) \quad (2.49a)$$

$$\partial U/\partial p = ((1/2t)\phi p^{\phi-1})(1 - dp'/dp) \quad (2.49b)$$

where N' in Eq. (2.49a) is the rival retailer's number of the items and dN'/dN is the conjectural variation of the variety.

When the value of dN'/dN equals 1, the Lösch competition style is indicated in terms of the items. In this case, when a retailer increases an item at the store, the retailer conjectures that the rival also increases one item at its store. Assuming that the values of dN'/dN and dp'/dp are 1 and solving the simultaneous Eqs. (2.48a, b, c) with respect to N , p , and U , the number of the items, the average price, and the market area size in the Lösch equilibrium are obtained. When the values of dN'/dN and dp'/dp are assumed to be 0, it indicates Hotelling-Smithies competition style; in the same way, the values can be obtained for the Hotelling-Smithies equilibrium.

For the values of the Lösch and Hotelling-Smithies equilibria, the following numerical values were assigned to parameters; $A_Z = 20$, $F = 20$, $K = 1.75$, $R = 5$, $T = 2$, $e = 0.15$, $r = 1.6$, $\alpha = 0.26$, $\omega = 45$, $\nu = 5.6667$, $t = 1.2$, $\phi = 0.2$, $\varnothing = 1.05$, $\beta = 0.5$, and $C_n = 1.2$. First, the retailer's profit function Y is shown by Eq. (2.50).

$$\begin{aligned} Y = & p(2 \cdot 3.14 \cdot 1.75U^2(0.5(45(N/20)^{0.26} - (5.6667 \cdot 1.2)N^{0.5} - (5.6667 \cdot 2p^{1.05})) \\ & - (5.6667 \cdot 1.2/3)U)) - 20 - N^{0.5}((2 \cdot 3.14 \cdot 1.75U^2(0.5(45(N/20)^{0.26} - 6.8N^{0.5} \\ & - (5.6667 \cdot 0.2p^{1.05})) - (5.6667 \cdot 1.2/3)U)))^{0.5} + 0.15) - N \cdot 5 \cdot 2 \end{aligned} \quad (2.50)$$

Then, $\partial Y/\partial N$ and $\partial Y/\partial p$ are given by Eqs. (2.51a), (b):

$$\begin{aligned} \partial Y/\partial N = & -10 + 5.49778(5.3693/N^{0.74} - 3.40002/N^{0.5})pU^2 \\ & - (2.09719(5.3693/N^{0.74} - 3.4/N^{0.5})N^{0.5}U^2)/((0.5(20.6512N^{0.26} \\ & - 6.8N^{0.5} - 1.13334p^{1.05}) - 2.26668U)U^2)^{0.5} - (0.5(0.15 + 8.38878 \\ & ((0.5(20.6512N^{0.26} - 6.8N^{0.5} - 1.13334p^{1.05}) - 2.26668U)U^2)^{0.5}))/N^{0.5} \end{aligned} \quad (2.51a)$$

$$\begin{aligned} Y/\partial p = & -6.5424p^{1.05}U^2 + 10.9956(0.5(20.6512N^{0.26} \\ & - 6.80004N^{0.5} - 1.13334p^{1.05}) - 2.26668U)U^2 \\ & + (2.49568N^{0.5}p^{0.05}U^2)/((0.5(20.6512N^{0.26} - 6.8N^{0.5} \\ & - 1.13334p^{1.05}) - 2.26668U)U^2)^{0.5} \end{aligned} \quad (2.51b)$$

Lastly, the term $\partial Y/\partial U$ in Eqs. (2.48a), (b) is derived as Eq. (2.52):

$$\begin{aligned} \partial Y/\partial U = & 21.9911p(0.5(20.6512N^{0.26} - 6.80004N^{0.5} - 1.13334p^{1.05}) \\ & - 2.26668U)U - 24.9234pU^2 - (4.19439N^{0.5}(2(0.5(20.6512N^{0.26} \\ & - 6.8N^{0.5} - 1.13334p^{1.05}) - 2.26668U)U - 2.26668U^2))/((0.5(20.6512N^{0.26} - 6.8N^{0.5} \\ & - 1.13334p^{1.05}) - 2.26668U))(U^2)^{0.5} \end{aligned} \quad (2.52)$$

Now, using the above four equations, Eqs. (2.48a, b, c) can be rewritten to be calculable. Solving the simultaneous equations system (2.48a, b, c) with respect to N , p , and U gives the equilibrium number of the items, the average price, and the market area size.⁸ The equilibrium values are shown in Table 2.3.

When the transportation costs per mile decreases from 1.2 to 0.2, the equilibrium values of the retailer are changed. In this case, the equilibrium values are shown in Table 2.4.

By comparing the figures shown in Tables 2.3 and 2.4, it is found that the number of the items in the Hotelling-Smithies equilibrium is higher than that of Lösch equilibrium, the average price in Hotelling-Smithies is lower than Lösch, and the market area size in Hotelling-Smithies is larger than Lösch. In addition, the changes of the equilibrium values due to the decrease of the transportation cost per mile are different according to the competition style between the retailers. When Lösch competition prevails among the retailers, because the transportation costs are lower, the number of the items decreases and the market area size shrinks; in

⁸In this analysis, the Greenhut-Ota competition style is not selected because the main purpose of this subsection is to show that the difference of the competition styles influences the retailers' market situations.

Table 2.3 The equilibrium number of items, average price, and market area size

Competition style	Item N	Average price p	Market area size U
Lösch	1.081	4.894	0.506
H-S	2.822	7.011	0.659

H-S: Hotelling-Smithies competition style

Table 2.4 The equilibrium values when the transportation costs is low

Competition style	Item N	Average price p	Market area size U
Lösch	0.881	5.491	0.422
H-S	5.188	10.941	1.035

H-S: Hotelling-Smithies competition style

Hotelling-Smithies competition, the number of the items increases and the market area size expands.

2.3.2.2 Changes of the Number of Items and Market Size by Decreasing Transportation Costs

Now, by observing the retail types in the real world, the retailer's types could be roughly classified into three categories: convenience store, supermarket store, and department store. It is possible to assign conjectural variations of the item and of the price to the competition style prevailing in each category. The distances between convenience stores are short and they deal with small goods with low prices. Thus, the conjectural variation of item and price would be assumed as $dp'/dp = 1$ and $dN'/dN = 1$ —that is, a Lösch competition style. The distances between supermarkets are relatively long and they deal with daily goods with low and medium prices. Thus, it is possible to assume that dp'/dp is 1, and the conjectural variation of the item is $dN'/dN = 0.75$. A supermarket store assumes that when it changes the number of items by one unit, the rival store changes the number of items by 0.75 because the rival cannot immediately change the number of the items to the same level. It is assumed in the case of department stores that the conjectural variations are $dp'/dp = -0.5$ and $dN'/dN = -0.25$. Customers visit a department store to purchase goods with high prices and they attach great importance to their taste. The department store attempts to provide many varieties of goods and plans to sell the goods by differentiating from their rivals. Thus, the department store assumes that against the change of the number of the items and the average price of its store, the rival takes the opposite response. Hence, in this analysis, it is assumed that the department store sets the conjectural variations of the items and price as $dp'/dp = -0.5$ and $dN'/dN = -0.25$ in order to maximize its profits.⁹

⁹Schöler (1993) shows the cases that the conjectural variation of price takes a negative value.

Then, because the fixed costs, the number of the variety, and some parameters' values seems to be different between the three retailing types, this section assumes the following values:

- For convenience stores, $F = 5$, $A_Z = 20$, $R = 5$, $r = 1.5$, and $\omega = 40$
- For supermarkets, $F = 27$, $A_Z = 20$, $R = 20$, $r = 1.5$, and $\omega = 42$
- For department stores, $F = 30$, $A_Z = 16$, $R = 10$, $r = 1.7$, and $\omega = 46$.

Other values of parameters are assumed to be the same for simplicity of the analysis. By lowering the transportation cost per mile from 0.6 to 0.2, the equilibrium market values are derived for the three retail types at each level of the transportation costs. The results are shown in Tables 2.5a, b, c, respectively.

The comparison of the figures shown in these three tables gives the following interesting facts: the market area of the convenience stores becomes smaller and the number of the variety stocked in the stores decreases as the transportation cost per mile lowers. The market area of the supermarket stores becomes smaller until transportation costs per mile lower to 0.4 and it begins to expand by a reduction in

Table 2.5a Changes of the equilibrium values of a convenience store

t	N	P	U
0.6	0.140	1.883	0.391
0.5	0.136	1.914	0.382
0.4	0.132	1.945	0.374
0.3	0.129	1.974	0.367
0.2	0.126	2.002	0.360

Table 2.5b Changes of the equilibrium values of a supermarket

t	N	p	U
0.6	0.689	4.211	0.738
0.5	0.822	4.55	0.731
0.4	0.847	4.585	0.712
0.3	1.056	4.831	0.720
0.2	1.389	5.097	0.745

Table 2.5c Changes of the equilibrium values of a department store

t	N	p	U
0.6	5.085	10.763	0.979
0.5	5.471	11.229	1.036
0.4	5.912	11.724	1.113
0.3	6.424	12.262	1.224
0.2	7.042	12.864	1.402

the transportation costs; the number of the items stocked in the stores increases by decreasing transportation costs. The number of the items stocked in department stores increases and the market areas expand as the transportation cost per mile lowers.

It is interesting to know that changes of the number of the items and the market area size due to the reduction of the transportation cost are greatly different between the retail store types.

2.3.3 The Difference in Traction Powers at a Store and the Market Boundary

This section examines the retailer’s capacity to pull customers (i.e., its traction power) at its store and at the market area boundary. Figure 2.12 illustrates the traction powers at the store’s location for the three kinds of the retailers against the six different transport cost levels from 0.6 to 0.2. The supermarket store’s traction power, which is denoted by the square mark, is the highest between the three kinds of stores. Its traction power changes in a U-shape as the transport cost decreases. The department store’s traction power, which is denoted by the triangle mark, is the lowest between them. The convenience store’s power, which is denoted by the diamond mark, is between the supermarket and the department store.

Figure 2.13 illustrates the traction powers at the market area boundary for the three kinds of retailers against the six transport cost levels. The supermarket store’s traction power at the market area is the highest between the three kinds of stores. Its traction power increases as the transport cost decreases. The department store’s traction power is the lowest between them. The traction powers of both the supermarket store and the convenience store increase as the transport cost decreases. The department store’s power decreases as the transport cost is reduced.

Figure 2.14 shows the difference between the traction powers at the store and at the market area boundary for the three kinds of retailers against the six transport

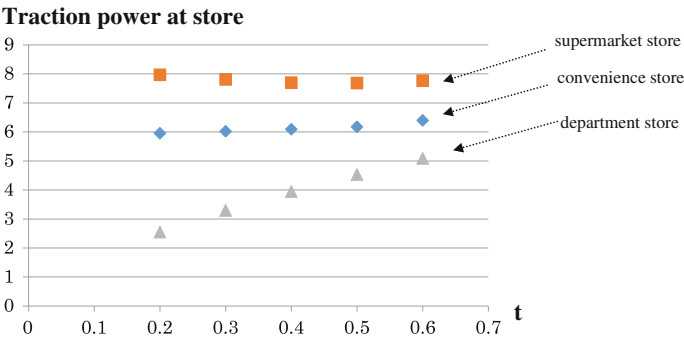


Fig. 2.12 Retailers’ capacity to pull customers at the store

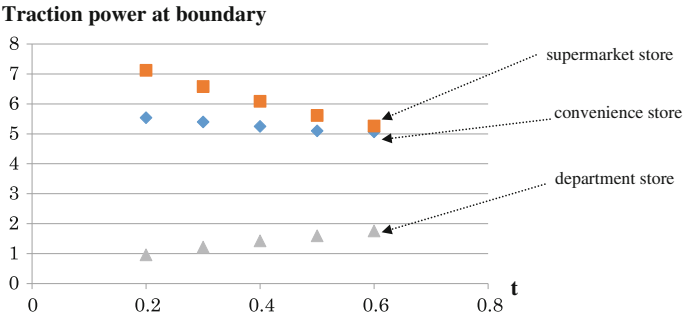


Fig. 2.13 Retailers' capacity to pull customers at the boundary

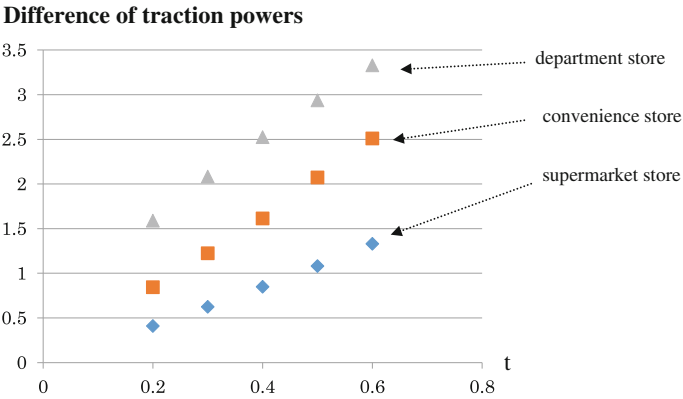


Fig. 2.14 Difference of the traction powers at the store and the boundary

cost levels from 0.6 to 0.2. The difference of the traction powers of every kind of retailer becomes small as the transport cost reduces. Although the directions of the change of the number of the items, the market size, and the average price of the items are different between the three kinds of the stores, there is a common trend between them: the difference of the traction powers at the stores' location and at the market area boundary becomes smaller as the transportation cost decreases.

2.4 Effects of Freight Rate on Items and Market of a Shopping Center

Since Dixit-Stiglitz's work (1977) on production and utility functions, a variety of items has been incorporated into the analysis of the retail industry. By using the utility function developed by Dixit-Stiglitz, many studies have provided useful insights about the economic effects of the variety of items on the locations of the

retailers inducing the operations of shopping centers in a plane market space. Henkel-Stahl-Walz (2000) incorporated the diversification of items into the utility function of consumers and examined the influence of the number of the items on the market area of the shopping center. This section incorporates the concept of the interaction between the shopping centers and analyzes, utilizing the framework that Henkel-Stahl-Walz built, the relationship between the variety of the items and the market areas of shopping centers.

2.4.1 Assumptions and Framework

This section sets the following assumptions. Consumers are equally distributed over a plane space with a density of 1. Each consumer has a utility function G , which is shown by Eq. (2.53):

$$G = \left[\int_0^n q(i, u)^{\sigma - 1/\sigma} di \right]^{\sigma/\sigma - 1} \quad (2.53)$$

where $q(i, u)$ is the quantity demanded for goods i , which depends on the distance u from a consumer to the retailer at which the consumer purchases the goods' n denotes the number of the items of the retailer; σ is a parameter; and $\sigma > 1$. The constraint of the budget is shown by Eq. (2.54):

$$y = \int_0^n p(i)q(i, u)di + tu \quad (2.54)$$

where y is the consumer's income, $p(i)$ is the price of goods i , and t denotes the transportation cost per mile. Each consumer purchases goods while aiming at maximization of their utility under the income restraint shown by Eq. (2.54). The quantity demanded for goods i is given by Eq. (2.55):

$$q(i) = \frac{p_i^{-\sigma}}{\int_0^n p(i)^{1-\sigma} di} (y - tu) \quad (2.55)$$

Because a kind of good is supplied by a retailer, the number of kinds of goods, n , implies the number of retailers. Each retailer is located at a shopping center, which occupies the central sales place of a city. A retailer sets a price to maximize its profit at the shopping center. There is not a manager who dictates the shopping center. Hence, retailers can enter into the shopping center until a retailer's profit becomes just zero. The market area of the shopping center is expanded to points where

consumers give up visiting the shopping center. The shape of the market area is given by a circle with the radius, U .

The retailer that sells the goods, i , at the shopping center obtains the profit $Y(i)$, which is shown by Eq. (2.56):

$$Y(i) = 2\pi \int_0^U (p(i) - c)q(i, u)du - F \quad (2.56)$$

where c and F are the marginal and fixed costs, respectively.

Because each retailer sets the price to maximize its profit, the price of goods, i , is given by Eq. (2.57):

$$p = c\sigma/(\sigma - 1) \quad (2.57)$$

where p is the *representative* price of all kinds of goods at the shopping center. Because the quantity demanded by a consumer is given by Eq. (2.55), the retailer's profit is rewritten by Eq. (2.57a):

$$Y(i) = (2\pi/n\sigma) \int_0^U (y - tu)udu - F \quad (2.57a)$$

The retailer's profit is eventually shown by a function of the radius of the market area, as shown by Eq. (2.57b):

$$Y(i) = (2\pi/n\sigma)U^2(y - tU/2) - F \quad (2.57b)$$

Because a retailer can enter into the shopping center until its profit become to zero, the equilibrium number of the retailers at the shopping center, n^* , is derived by Eq. (2.57b) as a function of the radius of the market area as Eq. (2.58):

$$n^* = 2\pi U^2(y - tU/2)/\sigma F \quad (2.58)$$

The utility of the consumer who comes to the shopping center must not be negative. Thus, the inverse utility of the consumer who comes to the shopping center is derived as Eq. (2.59):

$$V = \left(\frac{\sigma-1}{c\sigma} \right) (2\pi u^2(y - tu/2)/\sigma F)^{\frac{1}{\sigma-1}}(y - tu) \quad (2.59)$$

The market area cannot be expanded beyond the site at which the retailer's profit becomes zero. In addition, it is clarified that the market area is not expanded to cross the point where the utility of a consumer becomes just zero.

2.4.2 The Number of Retailers and Market Area of a Shopping Center

Let us derive the number of the retailers, the market area size, and the level of utility of a consumer by numerical calculation. The following numerical values were assigned to the parameters: the transport cost per mile, $t = 1.2$; parameter, $\sigma = 5$; fixed cost, $F = 25$, and consumer's income, $y = 50$. Under this situation, from Eq. (2.58), the relationship between the number of the retailers and the market area size is illustrated in Fig. 2.15. When the radius of the market area of the shopping center is 55.55, the number of the retailers is maximized at $n = 2585.7$. The utility of a consumer who resides at an end-point of the market area is shown in Fig. 2.16.

As shown in Fig. 2.16, when the radius of the shopping center's market area is 41.67, the utility of the consumers who reside at the boundary of the market area becomes just zero. Thus, the radius of the market area is determined by that size. The number of retailers is 2181.7 when the size of the market area of the shopping center is $U = 41.67$. This market area size is less than that of the market area that makes the retailer's profit just zero, $U = 55.55$. When the radius of the market area is determined at 41.67, the level of utility of each consumer in the market area is shown by Fig. 2.17 as a function of the distance from the shopping center to each consumer. As shown by Fig. 2.17, the consumer residing next to the shopping center enjoys the highest utility of 195.3, while the utility of the consumer who lives at the boundary of the market area becomes zero.

Fig. 2.15 The change of the number of retailers due to the expansion of the market area

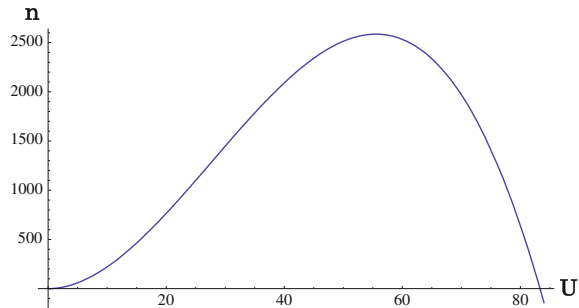


Fig. 2.16 The utility level of the consumer at the boundary of the market area

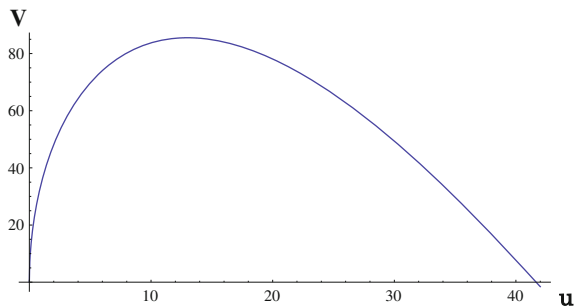


Fig. 2.17 The relationship between a consumer's utility and distance from the shopping center

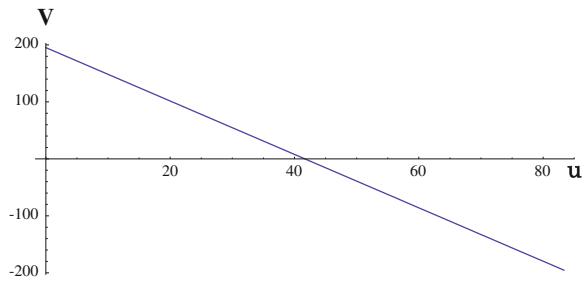
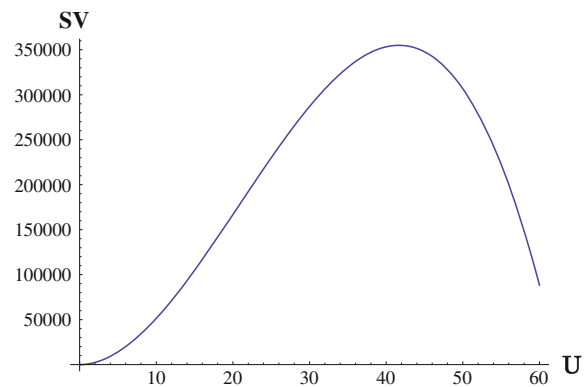


Fig. 2.18 The relationship between total utility and the market area size



Finally, let us examine the variation of the total utility of all consumers, SV , as the market area of the shopping center expands. The variation is illustrated in Fig. 2.18. As illustrated by Fig. 2.18, the total utility of all consumers is maximized, $SV = 355,006$, at the market size of $U = 41.67$. Thus, the boundary of the market area becomes settled at point where the utility of the consumers becomes just zero; such a market area size maximizes the total utility of all consumers in the market area.

2.4.3 *Change of the Number of Retailers at Centers Due to Decreasing Freight Rates*

2.4.3.1 Change of the Framework of the Analysis

In the following analysis, additional assumptions are set as follows:

1. Many shopping centers are in a plane market space. Their market area shape is a circle.
2. Each shopping center is dictated by a manager who can adjust the number of retailers located at the center to maximize every retailer's profit.

3. The competition between the shopping centers arises; then, the market areas of the shopping centers are forced to be reduced to the size that makes the profit of each retailer becomes just zero.
4. Every shopping center moves instantly without any costs. The location of every center adjusts to establish an equilibrium situation.

Under these assumptions, the number of the retailers at the shopping center and the center's market area size and consumer's utility are analyzed.

2.4.3.2 Analysis of a Shopping Center in a Monopolistic Competition

Because the manager of a shopping center can adjust the number of the retailers to maximize every retailer's profit, the manager determines the number of the retailers to satisfy Eq. (2.60):

$$\partial Y_i / \partial n_A = \left(2\pi U / (\sigma n_A^2) \right) \left(-yU + tU^2/2 + n_A(2y - 3tU/2)dU/dn_A \right) = 0 \quad (2.60)$$

Here, dU/dn_A is determined by Eq. (2.61):

$$dU/dn_A = (1/(\sigma - 1))n_A^{(2-\sigma)/(\sigma-1)}(y - tU_A) / \left(2\pi n_A^{1/(\sigma-1)} \right) \times (1 - dn_B/dn_A) \quad (2.61)$$

where n_A is the number of the retailers at shopping center A and n_B is the number of the retailers at the rival shopping center B, which exists nearest to shopping center A.

dn_B/dn_A is the value forecasted by the manager of shopping center A. The conjectural variation of number (CVN) of the rival shopping center B is when shopping center A changes the number of the retailers at the center A. As mentioned in the preceding analysis, the value of CVN is usually assumed to be $-1 \leq \text{CVN} \leq 1$. The typically assumed values are 1, 0, and -1 . When the value of dn_B/dn_A equals 1, the Lösch competition style is indicated. The values 0 and -1 give the Hotelling-Smithies and the Greenhut-Ohta competition styles, respectively. These three typical competition styles are assumed in this analysis.

Because a monopolistic competition arises between the shopping centers, the profit of every retailer at each shopping center must be zero. Thus, the market area size of each shopping center is forced to have a market area size that makes the retailer's profit zero. This equilibrium condition is given by Eq. (2.62):

$$Y(i) = (2\pi/n_A\sigma)U^2(y - tU/2) - F = 0 \quad (2.62)$$

The number of retailers and the market area size of the shopping center under a monopolistic competition equilibrium is derived by solving Eqs. (2.60) and (2.62)

with respect to n_A and U . Assigning the same values used in the above analysis to the parameters and lowering the freight rates from 1.2 to 0.8 by the interval 0.2, the equilibrium number of the retailers and the radius of the market area are obtained. The equilibrium values for the three competition styles are shown in Table 2.6. The following interesting facts are obtained from Table 2.6. Due to a decline of the transportation cost per mile, the number of the retailers increases and the market area of the shopping center expands in Hotelling-Smithies and Greenhut-Ohta competition styles. While in Lösch competition style, the number of retailers decreases and the market area shrinks.

When the transportation cost per mile is high and the competition style between the shopping centers is the Lösch type, the number of the retailers is the highest and the market area is the largest. On the other hand, when the competition style is the Hotelling-Smithies type, the number of the retailers is lowest and the market area is the smallest. When the transportation cost per mile is low, in the Greenhut-Ohta competition style, the number of the retailers is the highest and the market area becomes the largest. It is interesting that as the transportation cost per mile declines, whether the number of the retailers at the shopping center increases or decreases is dependent on the competition style between the shopping centers. Similarly, the market area size is also influenced by the competition style.

Then, the total utility level of all the consumers, SV , and the utility level per person, vp , in a market area of a shopping center can be derived against each freight rate. Table 2.7 shows the interesting results: As the transportation cost per mile falls, the total utility of all consumers increases in the Hotelling-Smithies and Greenhut-Ohta competition styles, while it decreases in the case of Lösch style. The utility level per person goes up for the three competition styles. When the transportation cost per mile is high, the competition style that maximizes the utility level per person is the Lösch style. Conversely, when the transportation cost per mile is low, the utility is maximized in the Greenhut-Ohta competition style.

Table 2.6 Change of the number of retailers and market due to decreasing freight rates

t	Lösch		Hotelling-Smithies		Greenhut-Ohta	
	n	U	n	U	n	U
1.2	402.2	13.9	114.6	8.0	358.8	13.0
1.0	345.8	12.6	208.2	9.6	516.6	15.6
0.8	311.8	11.8	325.4	12.0	807.2	19.5

Table 2.7 Change of utility of consumers due to decreasing transportation costs per mile

t	Lösch		Hotelling-Smithies		Greenhut-Ohta	
	SV	vp	SV	vp	SV	vp
1.2	60,243	99.5	16,314	81.5	52,287	98.5
1.0	50,981	102.5	27,243	94.7	82,571	107.9
0.8	45,692	105.0	47,647	105.9	144,086	120.6

2.4.3.3 The Importance of an Expanding Market Area in an Equilibrium

In the real world, the locations of shopping centers cannot move easily. If the transportation cost per mile becomes lower, the equilibrium market area size of the shopping centers becomes larger. This fact has an important economic meaning: As innovation in transportation progresses, the freight rate gradually falls. When the competition style between the shopping centers is the Hotelling-Smithies type, the number of the retailers at the shopping centers increases and the necessitated spatial size of their market areas expands. This implies that some shopping centers are eventually forced to leave from the market space. Therefore, the structure of the retail systems established in the early stages of economic development is altered as economic development progresses to a certain stage: The decreasing transportation cost collapses the spatial organization of the existing competitive equilibrium of the retailers and the shopping centers. This alters the existing city systems in a market space and provides the surviving retailers with positive profits.

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