

Chapter 2

Being at the Margins $\neq$ Being Unsuccessful at Mathematics

Abstract Students at the margins are disadvantaged as they enter school, and the educational gap between disadvantaged and advantaged students widens as they progress through school. We would suggest that a primary cause for this occurring is how the envisaged mathematics curriculum is enacted in these classroom contexts. Thus this chapter is organized under the two major dimensions that are purported to influence the enactment of mathematics teaching: teachers' affective domain and their cognitive domain. This chapter critiques the literature relating to these domains and identifies particular elements that assist this educational gap being 'closed'. It also shares the theoretical constructs that underpinned the selection and development of materials for the classroom and the development of the RoleM professional learning model utilized across the four years of the project.

Affective Domain for Enacting Mathematics

Dispositions for Teaching Mathematics

The affective domain for teaching mathematics consists of three predominant multidimensional constructs: beliefs, attitudes, and emotions (Ma and Kishor 1997; McLeod 1992; Philipp 2007). These are commonly used to describe a wide spectrum of teachers' affective responses to mathematics. The three constructs represent varying levels of affective involvement, intensity and stability (Grootenboer et al. 2008). Beliefs are considered to be notions held by individuals that they consider as true (Grootenboer et al. 2008). Beliefs are more stable, cognitively related, and develop over a long period of time. Attitudes are learnt and are evident in responses to situations. For example, attitudes are responses such as confidence and anxiety, engaging and avoiding, and liking and disliking mathematics (Beswick et al. 2006; Ernest 1988; Ma and Kishor 1997). Recently there has been a greater emphasis placed on the role that emotions play in the learning of mathematics. DeBellis and

Goldin (2006) see emotions or human affect operating as an internal representational systems parallel to cognitive systems as students learn mathematics. They can be seen as two complimentary aspects of mind (Hannula 2002, p. 27). Thus states of emotion carry meaning for the individual. For example, a student's feeling of frustration may be internally represented as failure, and result in avoidance strategies occurring. Emotions can thus empower or disempower students as they learn. In a problem solving contexts an understanding of this emotional dimension can be crucial to success as disempowering emotions can hamper performance. Emotions or feelings are affective responses to particular contexts or events. As such they are transitory and unstable in nature (Schuck and Grootenboer 2004).

Teachers' beliefs about students change from context to context. This is problematic for students from marginalized areas, as many teachers believe they are incapable of learning. In a remote Australian Indigenous community, Jorgensen and colleagues (2010) found that there was a mismatch between the teachers' beliefs and what was delivered to students in these marginalized contexts. Teachers often espoused beliefs about good teaching practices, but did not incorporate them into classroom teaching practices. Commonly, teachers provided lessons based on skill and drill learning experiences (Baturu et al. 2008; Jorgensen et al. 2010). These findings align with the beliefs teachers hold in relation to Indigenous students' mathematical ability (Matthews et al. 2005). Thus it is imperative to move teachers towards a mindset of quality teaching and learning for all, regardless of context.

Teachers' beliefs and attitudes about teaching and learning affect their teaching practices (Ambrose 2004; Kagan 1992). Teachers have a profound influence on the learning environment, and this is partially determined by their self-concept (beliefs about themselves) and the confidence they have in teaching mathematics (Rohrkemper 1986). Under-confident teachers tend to create learning environments that are unsupportive of learning. While effective teachers' tend to create learning environments where students are encouraged to be enthusiastic and enjoy mathematics (McLeod 1992). In the literature, self-concept is often associated with the term self-efficacy; that is, teachers' belief about their capability to bring about desired outcomes in their students' engagement and learning (McLeod 1992; Stipek et al. 2001).

Teachers' self-efficacy impacts on how they feel, think, motivate themselves, behave and perform (Bandura 1992). Those with low self-efficacy visualize failure scenarios and labor about the many things that can go wrong. Even though they may have the knowledge and skills to create effective learning environments for their students, such teachers may find it difficult to use these under taxing conditions. As teachers gain progressive mastery over difficult contexts, their own performance is enhanced (Bandura 1991). Thus, the task of creating effective learning environments for students in disadvantaged contexts rests heavily on the self-efficacy of the teacher and beliefs in their ability to effectively teach their students (instructional efficacy).

Teachers' self-efficacy is closely related to their instructional efficacy. When teachers' instructional efficacy is high they devote more time to academic learning, assist students who are having difficulties, and praise students on their

accomplishments. If their instructional efficacy is low, they give up more easily on students, students are held accountable for their failures, and teachers generally undermine their students' abilities, efficacy and cognitive development. Within marginalized contexts, settings with higher proportions of students from low socio-economic backgrounds and higher levels of student turnover and absenteeism, teachers' instructional efficacy is predominantly low (Ashton and Webb 1986).

The confidence teachers possess and year level in which they teach also influences their instructional efficacy. For example, early years teachers are often under-confident when teaching mathematics. The literature identifies two areas where many of these teachers lack confidence: their own understandings of mathematics; and, their ability to teach mathematics. This lack of confidence has the potential to impact on (a) the types of mathematical tasks these teachers utilize, and (b) the instructional decisions they make as they teach mathematics (Chick and Beswick 2013; Sullivan et al. 2009), resulting in low levels of instructional efficacy. Clarke and colleagues (2009) hypothesize that teachers with low instructional efficacy choose an instructional pathway that consists of guiding students incrementally through a set of skills. This limits the opportunities for students to engage in high levels of mathematical thinking (Staub and Stern 2002; Stein and Lane 1996).

Thus, the affective issues that influence teachers in these marginalized contexts are (a) their own self efficacy with regard to how they feel about working in these unfamiliar contexts and how they think they will perform, (b) their beliefs about whether these students are capable of learning high levels of mathematics, and (c) their own instructional efficacy; that is, their belief in their capability to create effective learning experience for these students.

Cognitive Domain for Enacting Mathematics Teaching

Knowledge for Teaching Mathematics

In research there is shared understanding that a distinct construct, *knowledge for teaching mathematics*, underscores quality student learning. Historically, what this means has changed over the last 40 years. There has been a movement from knowledge being conceived as a static entity (e.g., Ernest 1989; Shulman 1986) to knowledge as a dynamic entity (e.g., Rowland et al. 2005), with the former categorizing knowledge into different types and the latter including the capacity to know how to act in contingent moments.

The categories developed to describe the knowledge required to teach mathematics are underscored by Shulman's (1986, 1987) work. Shulman distinguished two overarching categories of knowledge essential for teaching mathematics:

subject matter knowledge (SMK) and pedagogical content knowledge (PCK); that is, knowledge of what to teach and knowledge of how to teach it (Ernest 1989).

While PCK has been acknowledged in the literature, exactly what it is lacks clarity (Ball et al. 2008). In many instances it is not clear how it is distinguished from other forms of teacher knowledge. It has often been described as the intersection between the subject knowledge and knowledge of teaching and learning (Niess 2005). Shulman (1987) claimed that pedagogical content knowledge comprises specific knowledge that helps teachers to translate and communicate the subject matter to the students they teach. This includes knowledge of instructional strategies and representations, and knowledge of students' conceptions and misconceptions (Shulman 1987). Krauss and colleagues (2008) further clarified PCK as knowledge of mathematical tasks as instructional tools, knowledge of multiple representations and explanations of mathematics problems, and knowledge of students' thinking and explanations. Curricular knowledge encompasses the scope and sequence of teaching programs, and the resources required to implement them (Rowland et al. 2005). Thus, there is agreement that PCK includes multiple components including knowledge of student's conceptions and difficulties, knowledge of instructional strategies, knowledge of curriculum and context, and general pedagogical knowledge (Depaepe et al. 2013).

There has been much debate as to whether pedagogical content knowledge (PCK) and mathematics content knowledge (MCK) are distinct (e.g., Baumert et al. 2010) or overlapping categories (e.g., Ball et al. 2008). However, there has been agreement that strong knowledge of the subject being taught is fundamental to teacher competence (Baumert et al. 2010; Chick et al. 2006; Hill et al. 2005). Quality teachers exhibit MCK prior to and beyond the level they are teaching. Additionally, teachers' MCK together with their confidence in their MCK is strongly related to students' positive gains in their understanding of mathematics and motivation to participate in mathematics in the early years of school (Baumert et al. 2010; Hill et al. 2005). In these settings, teachers who are confident in their subject matter knowledge are more likely to recognize and maximize young children's potential to learn (Hedges and Cullen 2005). Conversely early childhood teachers who are not comfortable with their level of subject matter knowledge tend to rarely include this content in their learning environments or extend their children's thinking in this content area (Anning and Edwards 1999).

PCK and MCK are related constructs. It has been evidenced that having deep mathematics content knowledge does not necessarily result in deep pedagogical content knowledge, but obtaining deep pedagogical knowledge heavily relies on possessing deep mathematics content knowledge (Baumert et al. 2010). In addition, the repertoire of teaching strategies and pool of alternative representations and explanations teachers possess is dependent on the depth of their mathematics content knowledge (Baumert et al. 2010). Teachers who know and use an extensive repertoire of teaching strategies tend to have a deeper understanding of their subject matter knowledge.

PCK and MCK change according to the experiences teachers are involved in during their educational journey. The greatest gains in teachers' subject matter knowledge (SMK) and pedagogical content knowledge (PCK) occur during their pre-service years, and the gains made once teachers are teaching are predominantly for PCK (Kleickmann et al. 2013). There are also relationships between the PCK teachers possess, their experience, and the context in which they teach. Teachers who have taught for more than 10 years seem to have higher PCK than their less experienced counterparts (Lee 2010). In addition teachers with less MCK tend to be situated in low socio-economic status contexts (Hill et al. 2005), and traditional professional development has little impact on the development of teachers' PCK and SMK (Darling-Hammond and Richardson 2009; Kleickmann et al. 2013).

Thus, gaining MCK and associated PCK for teachers whose knowledge of both is low, is a difficult task. Mathematics is based on abstract constructs and teachers have the challenging task of making these constructs accessible for students. Providing students with a 'hands-on approach' to mathematics using appropriate concrete representations is one way of beginning to make mathematics accessible for students in marginalized contexts. This approach moves students away from reading a text that they are unfamiliar with or listening to a teacher deliver a mathematical lesson in a language that is not their first language, to a mode of engagement involving the use of high quality representations.

The Role of Representations in the Teaching and Learning of Mathematics

There are two types of representations namely, *external representations* or *internal representations*, and each has a role to play in developing our understanding of mathematics. Internal representations are mental models or images constructed in the mind. These images include verbal/syntactic, imagistic, formal notational, visual, kinaesthetic and affective (Goldin 2002). External representations are those that are physically embodied or observed such as graphs, number lines, equations, table of values (Goldin and Kaput 1996), and other concrete materials used in the teaching of mathematics. Duval (2006) claims that external representations comprise four registers of mathematics: natural language, figures/diagrams, mono-functional registers of notation systems (symbols), and graphs. He argues that mathematics comprehension results from the coordination or mapping of at least two of these registers. Figure 2.1 provides an example of some of the external representations utilized in an exploration of functional thinking.

Both teachers and students engage with external and internal representations in the teaching and learning process. As students engage with external representations presented by the teacher, they interpret and internalise these representations. Students then communicate their understanding of these representations to others

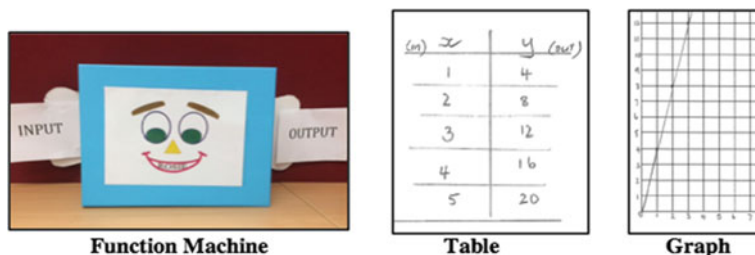


Fig. 2.1 External representations of functional thinking

through the use of additional external representations. In turn, the teacher internalises the communication from the student to inform their next point of instruction. Thus, each representation provides a new layer of understanding and this cycling continues as students work towards constructing their own understanding of mathematical concepts.

While it has been acknowledged that engaging with multiple external representations provides students with the opportunity to develop a deep understanding of mathematics (Duval 2006; Goldin and Shteingold 2001; Janvier 1987; Tall 1991), there is some debate with regard to how this best occurs. Currently, there are two approaches as to helping students explore multiple external representations for a particular concept. The first introduces the representations simultaneously, so the students engage with a number of different external representations in the one learning experience. It is conjectured that this approach does not provide students with the opportunity to develop a sound understanding of one representation before transferring this to the next representation (Ainsworth et al. 1996). The second approach encourages students to map across representations serving two purposes. It allows students to (a) access a mathematical concept that was not apparent in the previous representation, and (b) deepen their understanding of the concept itself (Duval 2006). A shortfall of the first approach is that it does not provide students with the opportunity to develop a sound understanding of one representation before transferring this to the next representation (Ainsworth et al. 1996). In fact it can lead to confusion rather than development of a deep understanding of a concept. It is the second approach that underpinned the development of the materials utilised in this project.

It is essential for teachers to explicitly map between multiple representations for students to develop deep conceptual understandings of mathematics (Cooper and Warren 2008; Dreyfus 1991; Duval 2006; Halford 1993). As students engage with a new representation they must learn how to use it, and how it links to previous representations. This mapping provides students with the opportunity to transfer their knowledge across different mathematical registers (Duval 2006; Halford 1987). Teachers connect the mathematics for students by linking representations to

particular mathematical language. Thus, as students engage with different representations, teachers need to explicitly integrate the mathematical language that links the representations to the concept being taught. This bundling of rich mathematical representations and language provides enriched learning experiences for students.

The Role of Language in Mathematics

In the early years of schooling an oral language approach is strongly related to numeracy development (e.g., Sarama et al. 2012; Warren and Miller 2013). An oral language approach is more than just communicating orally. It entails speaking and listening, and comprehending what is being said, understanding the vocabulary being used, and applying this to mathematical contexts. An oral language approach is not only foundational to literacy (Aldridge 2005), but also underpins the development of mathematical understanding (Krause et al. 2010). Mathematics has its own vocabulary that links specifically to knowledge about concepts and process. It is not enough for students to know definitions of concepts in mathematics; rather they need a deep understanding of the language, concepts and representations. While which words are presented and how they are developed is vitally important (Khisty and Chval 2002), it is not just an emphasis on vocabulary that is required. We cannot disregard other linguistic and cultural elements when helping students construct meaning in the context of mathematics (Celedon-Pattichis et al. 2010).

Mathematical language can be viewed as either a resource or as a political construct (Jorgensen 2011). The former stresses language as supporting thinking, doing, learning and teaching mathematics, whereas the latter refers to language as having the potential to position and transform by placing certain languages and speakers at a disadvantage (Planas and Civil 2013). While we acknowledge the tension between the language required to learn mathematics and the language these students use in and out of the classroom, in this book we are specifically focusing on mathematical language as a resource.

Mathematical language from a linguistic perspective is encapsulated in the literature by the term ‘mathematical register’ (Halliday 1978). While activities like counting, measuring, and ordering and comparing draw on ‘everyday’ language, this language in these contexts often serves different functions. Halliday uses the term to describe ‘a set of meanings that is appropriate to a particular function of language, together with word and structures that express their meanings’ (Halliday 1978, p. 195). This requires teachers to use mathematical language in a certain type of way. From this perspective, learning mathematics involves learning the register, its field, tenor and mode. These refer to the social activity that is occurring (what is being said), the roles and relationships between the participants (beliefs and attitudes), and how the interaction happens (speaking, writing, representing).

The language of instruction impacts on students’ access to mathematical concepts. When the language of schooling is Standard Australian English (SAE), the mathematical register used by teachers consists of words that come from two

primary sources: (i) everyday English: and (ii) mathematics. The words from everyday English may have the same meaning when used in the mathematics register (e.g. *increase*), may have a different meaning (e.g. *table*), or may have a subtly different nuance (e.g. *between*). Some words sourced from the discipline of mathematics seem to only have meaning in mathematics, including words such as *pronumeral*. In order to be positioned to engage with school mathematics, teaching and assessing using SAE, students need to possess an adequate linguistic repertoire (Meaney et al. 2008).

It is the syntactic, semantic and pragmatic features of the mathematical register that causes most problems for ESL and Indigenous students (Spanos et al. 1988; Niesche 2009). Syntactic features that prove difficult are comparisons of sizes, conditional relationships, and the use of the passive voice (Abedi and Lord 2001; Galligan 1997; Fernandes 2011). The pragmatic features refer to the use of language in particular contexts where culturally specific meanings are required to understand the problem. A prime example of this within the Queensland testing regime was the inclusion of the words *rover pass*, referring to a special ticket that allows all day travel on buses, trains or ferries. Indigenous students from a remote community, unfamiliar with metropolitan contexts, interpreted these words as something about a dog (named Rover) (Baturu et al. 2008).

As Jones et al. (1995) contest, it is not enough to acknowledge just the cultural and linguistic factors that influence the learning of Western mathematics. Such a focus can inappropriately result in a concentration on the use of language rather than the acquisition of mathematical concepts (Howard 1997).

In conclusion, it is conjectured that the main dimensions that impact on teachers' enactment of mathematics teaching in marginalized contexts, are the beliefs and attitudes they bring to these contexts and the mathematical knowledge they possess. The latter includes their knowledge of mathematics content (including its representations and language), and knowledge of how to effectively implement this content in these contexts. These dimensions underpinned the resources we created for use in classrooms. These dimensions also informed the development of the professional learning model we used across the four-year period.

Quality Resources and Activities for Teachers and Students

The principles of equitable teaching also drove the creation of RoleM resources. Equitable teaching is perceived to consist of four dimensions: access, achievement, identity and power (Gutiérrez 2012). Moschkovich (2013) argues that equitable teaching practices for students (a) support mathematical reasoning, conceptual understanding and discourse, and (b) broaden participation for students in learning mathematics. This required ensuring that the resources were: conceptually orientated; open-ended to cater for the differential that exists in students' ability; of high cognitive demand; and culturally appropriate (Boaler and Staples 2008).

Each activity exhibited the following characteristics:

- focused on particular mathematical concepts reflecting high stakes mathematics.
- delineated the specific language and representations required to fully explore the concepts.
- encompassed a degree of flexibility to cater for the learning that was context specific.
- encouraged both direct teaching and group work.
- incorporated a range of representations with a range of hands-on materials.
- allowed for easy differentiation to cater for a range of differences in prior learning and multiple entry points.
- ensured they were culturally appropriate for the different contexts.

Figure 2.2 presents a snapshot of Year 3 learning activities on teaching thirds.

The Indigenous teacher aides that were incorporated in all aspects of the projects played a vital role in ensuring that the activities were culturally appropriate.

Mathematics content knowledge

Purpose of the learning experiences

Often students draw on whole-number understanding to make sense of fractions.... Some students interpret $\frac{2}{3}$ as a number between 2 and 3. These misunderstandings occur because they have little experience in partitioning the length model or connecting the length model to the number line.

Making connections

As we move through mathematics the number system expands. ... All whole numbers are fractions.... Locating fractions on a number line helps students make this connection

Mathematical language

Vocabulary

Half, third, smaller, larger, equal parts....

Open-ended task

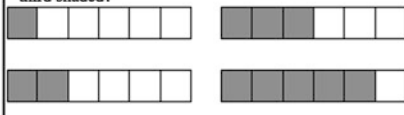
Is a quarter smaller than a third? Explain how you know

Word problem

There are 12 pens. If I was given one-third of the pens, how many pens would I have? If I was given one quarter of the pens, how many pens would I have?

NAPLAN type question

Look at the chocolate bars. Which bar shows one-third shaded?



Mathematical pedagogical knowledge

Learning experiences

Consolidating

Engaging

Extending

Independent learning

Each includes instructions to consider using, points of discussion and questions that can be asked.

Representations

Paper sheets (circles and rectangles)

Strings

Streamers

Number lines

Digital (Apps)

Teachable moments

... hands on folding activities help students 'see' how fractions are related to each other. After students have folded the string into thirds, mark one third, two thirds. Have them to fold the string in halves.... Ask: *Is two thirds closer to a half or one?*

Mathematical recording

..... Accompany this recording with appropriate language (e.g. four thirds is four out of three equal parts)

Fig. 2.2 Year 3 RoleM learning activities: *Exploring thirds*

Different representations and examples were provided by them that guaranteed the activities closely aligned with the culture and the environment in which they were situated.

In brief, the materials allowed teachers to differentiate the learning by assigning students to different types of activities (e.g., engaging, consolidating, extending) according to their level of understanding. The different types of representations and materials used within the activities also allowed teachers to cater for a wide range of learning styles (e.g. kinesthetic, auditory, visual). The activities encouraged students to explore mathematical concepts through a range of representations (e.g., number lines, number tracks, ten frames). Finally, the activities helped teachers to become aware of encouraging students to be active participators in classroom talk about mathematics (e.g., vocabulary, open-ended tasks, word problems).

Additionally, multiple representations were used to teach each concept in mathematics. As a new representation was introduced in a new learning activity, it was mapped back onto prior representations of that concept. Some representations (e.g., 10 frames) were used across all year levels to develop a deeper conceptual understanding of the multiplicative nature of our number system.

Professional Learning

One of the greatest influences on students' learning is what teachers do in the classroom. Up to 30 % of the variation in students' performance can be attributed to their teachers (Hattie 2003). Thus, improving students' performance in marginalized contexts is closely related to improving the quality of their teachers' teaching. While the relationship between teachers' mathematics content knowledge (MCK) and their teaching performance is complex, there is general consensus that good content knowledge facilitates good teaching and students' progress (Rowland et al. 2000).

Given that mentors in these contexts are few and far between, the strategy that is seen as imperative for these schools to adopt is high quality professional learning for their teachers (Desimone 2009; Hattie 2009). But efforts to bring about change via traditional professional development sessions are insufficient. Professional learning happens over a long time and is a contextualized holistic experience (Darling-Hammond et al. 2009; Vygotsky 1978). We contest that quality professional learning is underscored by theories of learning, and is thereby dependent on the interactions that occur between the learner, the context, and what is learned (Gravani 2007; Goos and Makar 2007; Jarvis and Parker 2005). Additionally, the strength of the support offered by 'experts' from within the school and wider community (Askew 2008) can make a difference to the quality of teaching. But these 'experts' need to have strong mathematical backgrounds and deep understanding of how to effectively teach mathematics.

Supporting the Professional Learning of Teachers in These Contexts

The particular theories that drove the development of this project were socio-cultural perspectives of learning (Vygotsky 1978) and communities of practice (Lave and Wenger 1991). The significance of the Vygostkian perspective is that it extinguishes the traditional boundaries between individual effort and social interactions as the individual comes to ‘know’. Learning is situated in a community of practice, a community that involves ‘ways of doing things that are shared to some significant extent among members’ (Lave and Wenger 1991). In this context, learning as an acquisition of knowledge is situated in social relationships, and involves a process of social participation and enculturation into the wider practices of society (Borko 2004). To master new knowledge and skills newcomers are required to move towards full participation in the socio-cultural practices of the community.

Integral to a socio-cultural perspective is the notion of the Zone of Proximal Development (ZPD) (Vygotsky 1978). ZPD is defined as an individual’s potential capacity for development through the assistance of a more knowing person (Vygotsky 1978). The significance of ZPD is that it determines the lower and upper bounds of the zone within which professional development (PD) instruction and teacher learning should be directed. In the lower bounds, formal PD sessions provide important information that teachers need to know about mathematical content, changes in the curriculum, innovative teaching strategies, and using resources effectively. However, instruction is only efficacious when it goes beyond the notion of simply assisting a person to acquire a particular set of skills or knowledge. Such instruction enables learners to extend themselves through active engagement, exploration and investigation of teaching and learning concepts and activities. In the upper bounds of the ZPD, the ‘more knowing person’, or ‘expert’, provides support for teachers through mentoring and scaffolding as these teachers are guided towards competent and accomplished practices (Brockbank and McGill 2006). A purported result is that the learner is better placed to independently implement innovative pedagogical practices across all curriculum areas after the ‘expert’ has withdrawn.

The depth and scope of learning is also influenced by the nature and quality of a teacher’s reflection (Phillips 2008; Wells 1999). Thus, when extensive teacher reflection is combined with action, students’ experiences are transformed into learning (Schon 1983). Teacher reflection serves both as an instrumental and a critical function (van Manen 1977). The former encourages teachers to reflect on teaching and learning problems that arise in their classrooms, and formulate practical plans that may solve the problem. Reflection, as a critical function, provides cognitive and affective insights that can challenge assumptions teachers hold about things, such as: the nature of teaching and themselves as teacher; and their students’ ability as learners in mathematics (van Manen 1977). As Dewey stated, genuine thinking only occurs ‘when there is a tendency to doubt’ (as cited in Garrison 2006

p. 3). Therefore, effective teacher learning requires multiple cyclical movements among systems of influence and teachers' worlds (Opfer and Pedder 2011). With ongoing support, teachers and 'experts' become co-constructors of knowledge moving within and beyond each other's ZPD.

The RoleM Professional Learning Model

The RoleM professional learning model is a socio-constructivist model based on the theories of Vygotsky (1978). Six principles drawn from the literature also underpin the model:

1. Meaningful learning is a slow and uncertain process for teachers, requiring time and teachers' reflection (e.g., Boyle et al. 2005).
2. Teachers' professional learning is more evident when continuing professional development (PD) includes a focus on teachers' knowledge of the subject (e.g., Borko 2004) and classroom practicalities (e.g., Porter et al. 2000).
3. PD emphasizing general teachers' knowledge and teaching competencies known to improve student learning and requires teachers to reconsider their current practices (e.g., Desimone 2009; Timperley 2008).
4. Professional development is more meaningful to teachers when it is situated within the context of their workplace (Webster-Wright 2009).
5. The most significant changes in teacher beliefs and attitudes occur when teachers have multiple opportunities to absorb new information, put it into practices and observe improved student learning outcomes (e.g., Guskey 2010; Darling-Hammond and Richardson 2009).
6. Resourcing has an impact on a teacher's capacity to effectively teach mathematics (e.g., Clements 2004).

We argue that professional development days are components of professional learning. For effective professional learning to occur teachers, together with experts, need to trial ideas in their classroom contexts and reflect on the student learning that has occurred.

The RoleM professional learning model (RPL) is cyclical in nature and involves teachers in self-reflection as they trial approaches and resources in their classrooms to improve the quality of their teaching practice. It is based on the view that teachers have the ability to improve their practice by trialing 'proven' effective learning experiences, and through continuous cycles of on-the-job reflections and discussions with experts from the field (Castle and Aichele 1994; Clarke and Hollingworth 2002). Our model builds on Guskey's (1988) three domains of practice, consequence and personal. The domain of practices entails professional experimentation with the context of the classroom. The domain of consequence refers to salient outcomes such as improved student learning. The personal domain encompasses ones knowledge, beliefs and attitudes. The model also reflects Clarke and Hollingworth's (2002) notion of effective professional learning is cyclical and

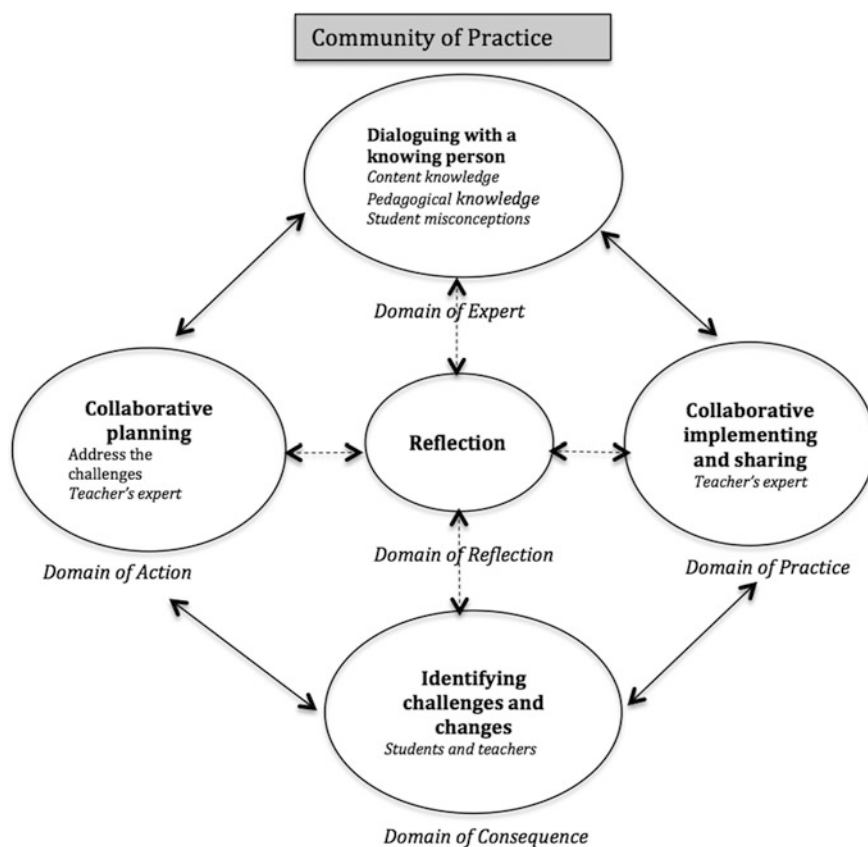


Fig. 2.3 The RoleM professional learning model

the domains are inter-connected. Figure 2.3 presents the key components together with the key focuses, of the professional learning model.

The contribution that this model makes to literature pertaining to effective professional learning is the inclusion of the domain of expert, domain of reflection and domain of action (practice).

Implementation of the Model Within These Disadvantaged Contexts

Teachers and teacher assistants (TAs) attended professional development days three times a year. The professional development days focused on a specific year level for each year (e.g., 2010 Foundation Year, 2011 Year 1). At each professional development day teachers and TAs interacted with 10 to 12 RoleM mathematical

learning experiences. The notion of active learning underpinned the professional learning that occurred over the PD day. Teachers ‘acted out’ the activities and discussed the mathematics content knowledge, pedagogical knowledge, mathematical language and representations that underpinned each. They also reflected on possible student misconceptions and how these could be addressed. At these PD days, teachers also received the RoleM resources ready for implementation in their classrooms.

Approximately three weeks after each PD day, experts conducted follow up visits (FV) in each teacher’s classroom. Before arriving at the school, teachers were contacted and asked what mathematical content they would like the expert to focus on. Each follow up day was different, as the format of each was teacher-driven. At these visits experts and teachers worked collaboratively to address any issues identified by the teacher. For example, experts may have: modeled lessons; observed and provided feedback to teachers; worked with TA’s; and/or worked with individual students to help diagnose their misconceptions or extend their learning. Finally, teachers and experts worked together to plan the next phase in students’ learning, and the cycle started again. These FV also occurred three times a year.

The literature review delineated a series of sub-questions that serve to further illuminate the overarching research question that drove the project, which was:

What is the interplay between effective teaching and effective learning in these contexts?

The sub-questions were:

How do teachers’ dispositions towards teaching mathematics influence their classroom practice in these contexts?

How does teachers’ mathematics content knowledge and pedagogical knowledge influence their classroom practices in these contexts?

How does professional learning best support the development of this interplay?

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2016, XIV, 120 p. 16 illus., Softcover

ISBN: 978-981-10-0701-9