Framework of Compressive Sampling with Its Applications to One- and Two-Dimensional Signals

Rachit Patel, Prabhat Thakur and Sapna Katiyar

Abstract Compressive sampling emerged as a very useful random protocol and has become an active research area for almost a decade. Compressive sampling allows us to sample a signal below Shannon Nyquist rate and assures its successful reconstruction with some limitations on signal, that is, signal should be sparse in some domain. In this paper, we have used compressive sampling for an arbitrary one-dimensional signal and two-dimensional image signal compression and successfully reconstructed them by solving L1-norm optimization problems. We also have showed that compressive sampling can be implemented if a signal is sparse and incoherent through simulations. Further, we have analyzed the effect of noise on the recovery.

Keywords Basis function · Compressive sampling · Incoherent signal · L1-norm · Sparse signal

1 Introduction

Today we are moving towards digital domains, but origination of the signal most of the times be analog. Therefore, analog-to-digital converting systems are required but these systems bounded by criteria that sampling frequency should be greater than twice of the analog signal frequency (Shannon Nyquist criteria) [1]. But if frequency of signal is very high then it is very tiresome to use Nyquist criteria

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because the number of samples will be very large. It becomes costly and sometimes infeasible to store and process such large number of samples.

Nevertheless, if somehow we can overcome Shannon Nyquist criteria that we can reconstruct original signal by using a very less number of samples as compare to Nyquist criteria, then problem of storage and processing of large data can be solved. This problem may be solved by compressive sampling [2–4] a random approach if signal is sparse in some domain.

Compressive sampling uses a very less number of samples as compared to Shannon Nyquist rate which reduces the hardware and software loads and then signal is recovered by using various recovery mechanisms [5–7]. Compressive sampling uses a random matrix to form out linear random projections of signals with most of the desired information. It is possible due to two properties of signal, i.e., sparsity and incoherence [8]. Sparsity refers to the property of signal according to which information present in signal is very less as compared to the bandwidth occupied by the signal. Incoherence is a property of sparse signal to get transformed into desired domain. Desired domain is the domain in which signal is sparse. If signal is more sparse, i.e., low sparsity level, its reconstruction will be better as compared to less sparse signal. Basically, incoherence refers to not coherent, i.e., the dictionary (domain) elements should be independent to the sampling matrix.

The paper is organized as follows. Section 2 provides some background on compressive sampling, mathematical model, and signal reconstruction by solving optimization problems. In Sect. 3 we use compressive sampling for one-dimensional and two-dimensional signals compression and its successful reconstruction. Section 4 presents simulation results on its performance and Sect. 5 presents conclusion and future scope.

2 Background

2.1 Compressive Sampling

Compressive sampling (CS) is an emerging theory which allows us to project random measurements of signal of interest so that we can sample the signal at information rate and not at its ambient data rate. This reduces the number of samples to represent a signal. These less number of samples can be stored easily and processing of such small number of samples can be performed efficiently. But to apply compressive sampling on the signal, signal should be sparse and incoherent.

Sparse Signal: For a signal to be sparse, only some of the components should have considerable magnitude and all other components should have very less magnitude, i.e., closer to zero.

Incoherent Signal: For two signals to be coherent, they should be independent of each other.

2.2 Mathematical Approach for Compression

Consider a signal r which is sparse. r is said to be sparse if it can be represented as a linear combination of basic functions where some of the coefficient's magnitude are significant and all others have zero magnitude

$$r = \psi c$$

ψ—basis functions and c—basis coefficients

For compressive sampling, a random matrix $\emptyset$ needs to project random projections or random measurements

$b = \emptyset r$, here b is random measurement vector

2.3 Reconstruction

Now we need to reconstruct back r form b

$$b = \emptyset \psi c \tag{1}$$

By solving equation we can find basis coefficient c.

Information of c leads us towards recovery solution

$$r = \psi c$$

2.4 Optimization Problem Formulation

The equation we have to solve, i.e., (1) is an underdetermined system as number of equations is less than number of unknowns. So we need to use norm minimization techniques to solve above problem.

Mathematically norm provides the total size or positive lengths of all vectors in a vector space or matrices. Generally, Norm n of vectors x is defined as

$$\|x\|_n = \sqrt[n]{\sum_i \|x\|^n} \text{ Where}, n \in R$$

Frequently using norms are L0, L1, L2 but here we use L1-norm.

L1-norm: L1-norm is defined as $\|x\|_1 = \sum_i |x|$

L1 optimization problem is formulated as

$$\min \|x\|_1 \quad \text{subject to } |Ax = b|$$

Above problem can be solved using least square optimization

$$x = A^{+}b,$$

where A^{+} –Psuedoinverse of A

Even though this method is easy to compute it is not necessary that it provides best solution. That is why we use L1-norm optimization.

So our optimization problem can be formulated as

$$\min \|c\|_1 \quad \text{subject to} \quad |(\varnothing\psi)c = b|$$

2.5 L1 Optimization Solution

L1 optimization problems can be solved by using linear or nonlinear programming algorithms such as greedy-type orthogonal matching pursuit, basic pursuit [9].

3 Applications in One- and Two-Dimensional Signal

Reduced load on hardware and software leads us to use compressive sampling in all possible fields such as compression, image compression, speech compression, audio and video compression, wireless sensor networks, etc. But here we apply compressive sampling on one-dimensional and two-dimensional image signals.

3.1 One-Dimensional Signal Compression and Recovery

In our daily life, number of times we deal with one-dimensional signal such as audio signals speech signals and we need to sample these signal for performing some digital operation on these signal. Less number of samples can be processed easily with a short processing time. So we go for compressive sampling of such signals if they are sparse. Complete recovery of signal depends on sparsity level (SL) and compression ratio (CR). Sparsity level is a number of components having significant magnitude. Compression ratio is the ratio that up to what level we have compressed the signal, e.g., $N/10$, where N is the total number of samples present in the signal.

If sparsity level is low, recovery will be better. If compression ratio is more, recovery will be better. Consider one-dimensional signal r having length n.

r_{n*1} can be represented with the help of basic functions and its coefficients

$$r_{n*1} = \psi_{n*n}\, c_{n*1} \tag{2}$$

$$\begin{aligned} \psi_{n*n} - n*n &\quad \text{Matrix of basis function} \\ c_{n*1} - n*1 &\quad \text{Vector of basis coefficients} \end{aligned}$$

For random measurements after random sampling we use measurement matrix $\varnothing_{m*n}$

$$b_{n*1} = \varnothing_{m*n}\, r_{n*1}$$

$$\varnothing_{m*n} \text{–Measurement Matrix.}$$

$$b_{n*1} = (\varnothing_{m*n}\, \psi_{n*n}) c_{n*1} \quad \text{using (2)}$$

Above equation needs to be solved using L1-norm optimization. L1-norm optimization problem is formulated as

$$\min\|c_{n*1}\| \quad \text{subject to } (\varnothing_{m*n}\, \psi_{n*n}) c_{n*1} = b_{n*1} \tag{3}$$

Reconstruction using above solution

$$\widehat{r_{n*1}} = \psi_{n*n}\, c_{n*1}$$

3.2 Recovery Error (Rerr)

Recovery error is a very important parameter that gives us the error for successful recovery of the signal and is defined as

$$\text{Rerr} = \text{norm2}(r - \hat{r})$$

3.3 Two-Dimensional Signal Compression and Recovery

Two-dimensional signals like image can also be compressed using its Fourier or wavelet domain where image shows some sparse nature. A mathematical approach for image remains same as for signals but we choose basis functions either on Fourier or wavelet domain. Instead of a one-dimensional vector we deal with a two-dimensional matrix.

3.4 Effect of Noise on Recovery Error

Noise is an undesired signal that may affect the performance of the system. So we analyzed the effects of noise on our system of compressive sampling, i.e., how recovery error is going to vary with respect to noise.

For compressive sampling, noise may affect the sampled values and mathematical equation for sampled signal will be defined using Eq. (2)

$$b_{m*1} = \varnothing_{m*n}\, r_{n*1} + n_{m*1}$$

where n_{m*1} —noise vector

Recovery procedure will be same as in Eq. (3), i.e., we need to solve L1-norm minimization problem

$$\min \|\mathrm{c}_{\mathrm{n}*1}\|_1$$

$$\text{such that } \|\mathrm{b}_{\mathrm{m}*1} - (\varnothing_{\mathrm{m}*\mathrm{n}}\, \psi_{\mathrm{n}*\mathrm{n}})\mathrm{c}_{\mathrm{n}*1}\|_2 \leq \varepsilon$$

But due to addition of noise, values of vector *b* change. Thus, above problem can be solved with affected value of *b*. So value of coefficient vector *c* also varies from the desired values. By this way reconstruction or recovery gets affected.

4 Simulations and Results

For implementation of all algorithms, we used Matrix Laboratory (Matlab) on a standard computer. The L1-magic toolbox is used to achieve the solution of L1-norm optimization problems.

4.1 Signal Reconstruction

We considered a signal in time domain and made it sparse in frequency domain by taking all frequency domain coefficients zero which are below some threshold value. Here threshold value is assumed to be one-fifth of the maximum amplitude of the coefficients. We used same procedure used for signal compression and recovery of original signal in Sect. 3.1. We sampled the signal using sampling rate which is ten times lesser than Nyquist rate and successfully reconstructed the original signal as shown in Fig. 1. Here we varied the sparsity level of signal and analyzed its results on recovery error. In addition to this, we also analyzed the effect of variation of compression ratio on recovery error.

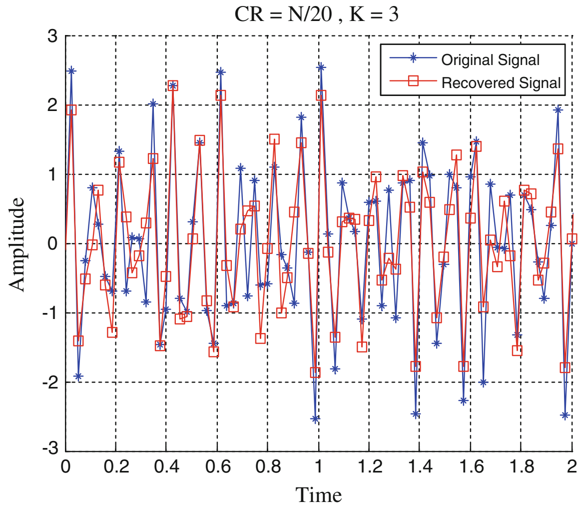

Fig. 1 Original signal versus recovered signal

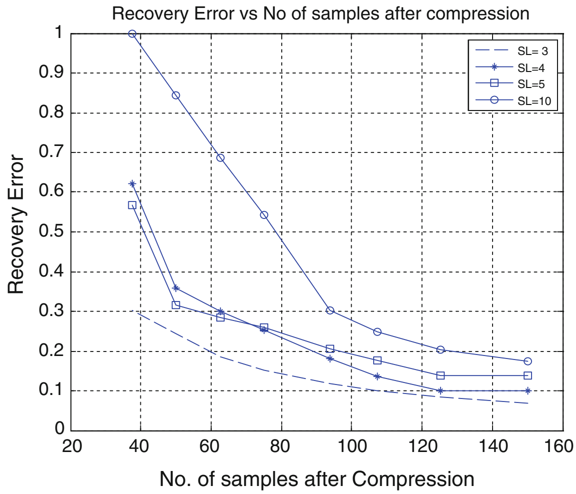

Fig. 2 Recovery error versus number of samples after compression with different sparsity level

Figure 2 explains the behavior of recovery error with variation in compression ratio for different sparsity levels. It is clear that with the increase in compression ratio, recovery error increases. More the sparsity level is more the recovery error. So it confirms that the compressive sampling can be implemented on the signals having sparse nature otherwise recovery or reconstruction cannot be done successfully. Recovery error relies on sparsity level and compression ratio. If signal is sparser, it can be compressed more and can be reconstructed successfully.

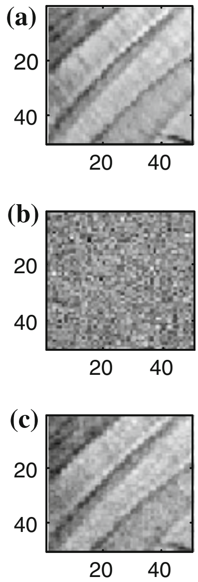

Fig. 3 Original image versus recovered image. **a** Original signal. **b** Recovered image using least square method. **c** Recovered image using basic pursuit

4.2 Two-Dimensional Signal's Reconstruction

Further, we compress the image by taking random measurements and recover the image using least square method (LSM) and basis pursuit (BP, an algorithm to solve L1-norm optimization problem).

Figure 3 contains three images, namely original image, recovered image using least square method, and recovered image using basis pursuit. From here it is clear that image recovered using least square method is much distorted but image recovered using basic pursuit is almost similar to original image.

4.3 Effect of Noise on Recovery Error

We have seen in Sect. 2.4 how noise affects our recovery error. Here we analyzed the effect of noise on the recovery error for different values of sparsity level (Fig. 4).

5 Conclusion and Future Scope

Compressive sampling appears to be a revolutionary technique for data acquisition and successful reconstruction. We have implemented this technique for one-dimensional signal as well as two-dimensional image signal and successfully

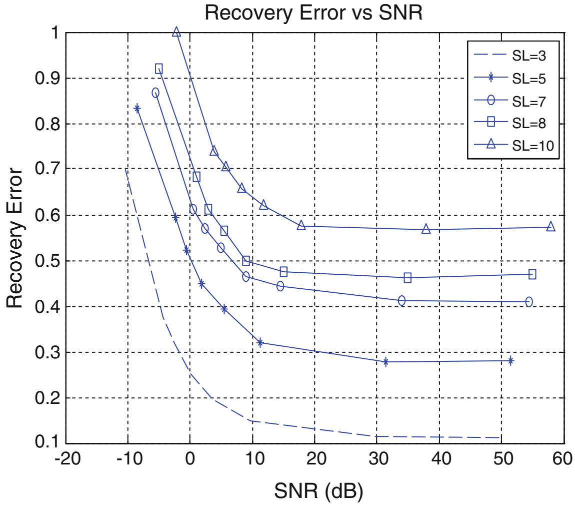

Fig. 4 Effect of SNR on recovery error

recover them from compressive random measurements. We have analyzed recovery error due to variations in sparsity level and compression ratio and assured that successful reconstruction of signal relies on sparsity level and compression ratio. Effect of noise is also considered in compressive sampling and we verified that with increase in SNR, recovery error decreases, that is, according to our system expectations.

In future scope, we can use this technique for compression of another kind of one-dimensional or multidimensional signal if they are sparse in some domain. This technique can play important role in microwave applications where sampling rate is very high due to high frequencies.

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