

Chapter 2

Background Guide to Random Walk Analysis

Abstract This chapter consists of three major subsections. Following a discussion of the ability to model a large class of environmental processes governing natural disasters as random walks, this chapter provides the theoretical background for this book which will be useful throughout the rest of the chapters. The first subsection will summarise the main notion used throughout the book and the adaptation of the theoretical background for natural disaster prediction in an engineering applications perspective. The second subsection will review recent research advancements in *MFPT* prediction methods. The final subsection will briefly describe some mathematical tools used in the proceeding chapters.

Random walks and their properties have been studied widely in literature due to the important role played by them in a range of disciplines. These methods can be adapted and improved for predicting natural disasters. This chapter discusses the literature background for the remaining chapters.

2.1 Introducing Some Basic Notions

Some common concepts that are used in the next chapters are presented in this section in the perspective of this book. Since the main focus of this book is on natural disasters, these definitions might be slightly different from traditional definitions in different disciplines. The more common definitions referred throughout this book are presented in the Glossary at the end of the book.

2.1.1 Random Processes

This book will use the term ‘random’ with the meaning of being unpredictable or non-deterministic. When one tosses a coin, it is commonly understood that one of the two sides would turn up with almost equal likelihood. Although it might be theoretically possible to calculate exactly which side turns up using the initial conditions such as coin weight distribution, initial velocity and point of contact, the complexity

of the dynamics and the interaction with the surrounding environment makes this calculation practically very difficult. For problems such as these, it is common to use probability distributions rather than absolute values to describe and analyse the system. An experiment tossing a coin a large N number of times and showing the ‘head’ up n_h number of times would describe the system with a probability of n_h/N of getting a ‘head’ and a $(1 - n_h/N)$ of getting a ‘tail’.

This book suggests that considering complex dynamic systems like the motion of tropical cyclones as random processes is a feasible approach of getting better understanding of such systems.

- A **random process** is a probabilistic process characterised by a probability distribution(s) of one or more *random variables* [1].
- A **random variable** is one which can take any value out of a continuous or discrete range but the exact value is not deterministic.

There is a large class of complex real world random processes that govern the dynamics of natural disasters and of which the states map into random walks in their respective state spaces.

2.1.2 Random Walks

A *random walk* is a fundamental dynamic process [2] which consists of a *path made up of a succession of random steps*. At each step the walker takes a decision on the next step either from static or dynamically connected nodes. This book defines a random walk as *a stochastic process which results in a non-deterministic array of state changes*. The state space is application dependent. An example of a one dimensional random walk is the monthly mean temperature profiles. Figure 2.1 shows the monthly mean temperature profiles of Melbourne, Australia between 1971 and 2016.

A few diverse example random walks which result from real world dynamic systems are temporal variation of share prices and exchange rates [1], disaster propagation [4–6], spread of epidemics and pandemics [7], path of gossip spread [8], routing of information packets in large-scale infrastructures such as the Internet [9], and many other state transitions of physical non-perpetual systems.

Analysing random walks have also gained a lot of popularity due to them being one of the simplest mechanisms of exploring of and search on networks [10]. Random search processes are widespread in nature ranging from target searching molecules in living cells [11] to animals searching for food [12]. Search processes have been shown to be optimal if the walker follows the shortest path (i.e. one with the smallest number of links) between the two nodes under consideration [2]. But the shortest

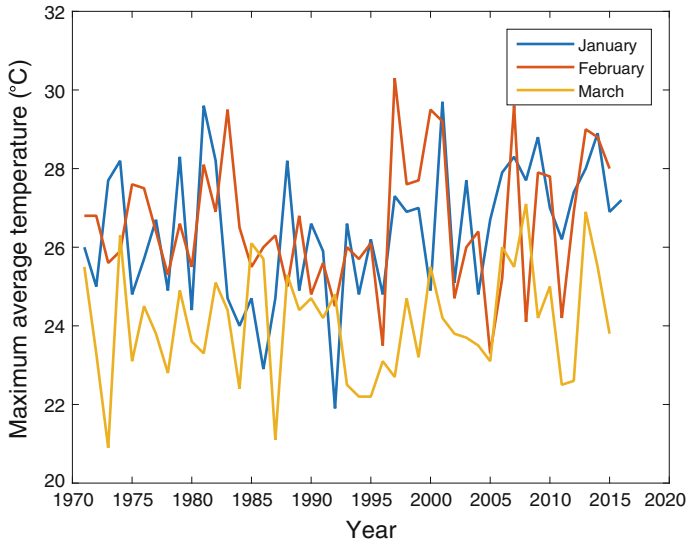


Fig. 2.1 One dimensional random walk of the monthly mean temperature of Melbourne, Australia between 1971 and 2016 (data from [3])

path can be only found theoretically when the global connectivity of the network is known, which is not always possible. Therefore, random walks have been used extensively as a mechanism to explore a network where only local connectivity is known at each node.

It has been observed that a large class of random walks resulting from natural processes are not purely Brownian [1]. This is true for many complex systems in natural disaster dynamics and therefore the methods described in the later chapters do not require the processes to be Brownian.

2.1.3 Networks

In the context of random walks of interest in this book, a network is defined as follows:

A *network* is the complete set of states which the random walker can occupy.

Networks can either be *discrete* or *continuous*. A discrete network for a random walker is one where there are a limited number of nodes which are reachable by the walker on that network. A continuous network is one where the states which are achievable by the walker are continuous in nature and therefore *infinite*. For such

network, the probability of a random walker being at a location is expressed in terms of a subset on state space as opposed to a single point and a ‘probability density distribution’ (P_{DD}) is used to calculate this.

For a d dimensional state space, the probability that a walker is inside the subspace $\forall_{i=1:d} a_i < x_i < b_i$ is given as $\int P_{DD}(x_i, \dots, x_d)$.

Another differentiation between continuous networks is that when the state space that is reachable by the walker under any circumstance is limited, the network is said to be *bounded* where boundaries can be *reflecting* or *absorbing*.

Random walks on a continuous state space or network space can be either continuous or discrete in nature. For engineering solutions, even continuous walks are usually discretised by measurement and therefore a random walk consists of a succession of *steps* in the time domain. Selecting the next step of the walk could either be solely dependent on the current step, where such random walk is said to possess the *Markovian property*, or might be shaped by the history of preceding steps (non-Markovian). Either way, the sum of the probabilities of the immediate neighbours being selected is equal to one.

Connected networks are ones where there would be *edges* that would connect the *nodes* such that there would be at least one path the walker can take to reach any target node from its current node. The *chemical distance* between two connected nodes is the shortest number of nodes between them [13]. The *degree* of a node is the number of edges connected to that node. There is another class of *dynamically connected* or *temporal* networks [14] where the connections or the edges of the network are dynamic and dependent on the current state of the network and walker. These types of networks, although have been given relatively less attention, are quite common in real world dynamic systems. For example, if there is a bushfire of which the spread relies on the fire at a single point reaching discrete fuel nodes (e.g. trees, bushes), the reachability of nodes depends on the spread rate which is defined by the wind and topography in the area. Therefore nodes that would otherwise be isolated could be connected given stronger winds. For a discrete *connected* or *dynamically connected* network, the probability of the walker being at any given node is a value dependent on the current state of the network and would always add up to one for the total node set. Most parts of this book would consider network connections to be temporal.

2.1.3.1 Random Walks on Complex Networks

A discrete network is said to be *complex* when the node structure becomes complicated and different from a normal lattice structure.

Table 2.1 Common features of complex networks

Scale-free networks	The degree distribution of these networks follow the power-law (at least asymptotically) where the probability that a given node has k neighbours is $P(k)/k$ [1]. Some example scale-free networks [24] are social networks [14], protein interaction networks [17] and metabolic network
Scale-invariant networks	The network properties would remain almost unchanged irrespective of the scale on which the network is looked at [18]. For such networks the number of nodes within a shell of size r is $N(r) \propto r^{df}$, where df is the fractal dimension [1] common at any node. Among these networks, deterministic fractal networks are when the same pattern is observed at all scales
Self-similar networks	The main properties (e.g. degree distribution) of these networks remain unchanged under length scale renormalisation. In [13] renormalisation is when a fractal network is covered with N_B number of non-overlapping boxes and when each box is replaced by a super-node and the links between boxes are transferred to boxes
Small-world networks	The network diameter D or the longer chemical distance between any two nodes, is proportional as $D \propto \ln(N)$ for a network with N nodes [1]. A phenomenon observed as early as 1929 and is well known as <i>the six degrees of separation</i> [15]

A considerable portion of current literature on *MFPT* estimations looks at complex networks. The study on complex network structures began in the 1950s with Erdős-Renyi random graph which assumes that complex networks are wired randomly together [15]. This hypothesis was the basis of most initial work on complex networks in sociology, biology and computer sciences. Scientists have now identified several topological properties common in complex networks that are very different from usual Euclidean lattices [1]. Some common *complex features* are presented in Table 2.1.

Many real world networks have been shown to possess one or more of these features [2, 19]. An example such network that is commonly used to describe these properties is the yeast protein interaction network (PIN) which is scale-free and also has the small-world structure [1]. Some other such networks are the world wide web [20], social networks [14] and electric power transmission grids [21]. This book discusses the applicability of some of these features for networks on which random walks that describe the processes governing natural disasters occur.

Many theoretical methods have been researched on to predict the motion of a random walker in networks having a deterministic node structure (i.e. for networks which would fall under a predefined network classification whether simple or complex). An observation worth noting here is that there are many random walks describing processes that govern natural disaster dynamics that do not fall into such deterministic structures or are composed as a compound of several such structures having ideal temporal, directional, and spatial conditions.

2.1.4 *Random Spread Versus Diffusion*

Up to this point the considered networks consisted of only a single propagator in the network. Extending this concept, the cumulative behaviour of a large number of independent random walkers in the same network is defined as *random diffusion*. Diffusion is the continuum limit of a random-walk process [22]. As common as diffusions are in real world systems (e.g. Brownian motion), a contrasting phenomenon of *random spreading* exists where it is also a large collection of random walkers but which are interdependent. A similar concept is a *branching random walk* where at each step, several steps are branched out. The network is changed by each passing walker and therefore will have to be considered in calculations. An example random spread is the spread of bushfire over a network of fuel distribution biased by wind and topography (see Sect. 6.1 of Chap. 6). *Anomalous diffusion* is when the mean square displacement r^2 is no longer proportional to the time t [1].

2.1.5 *Transport Characteristics Describing Random Walks*

Understanding the properties of transport for random walks describing natural processes is crucial for a large class of applications [13]. Depending on the application and the network type, different transport properties are used to evaluate the performance of a system [19]. Some of the main such transport characteristics are summarised in Table 2.2.

In addition to the characteristics described in Table 2.2, there are several others which have been used to describe random walks such as ‘transience’, ‘recurrence’, ‘number of distinct sites visited’, ‘average recurrence time’, ‘number of returns to origin’, and ‘occupancy of set by a random walk’ [26, 27]. Other transport properties such as the ‘walk dimension’ (dw), ‘fractal dimension’ (df), ‘spectral dimension’ (ds), ‘random walk centrality’ (C) [2], etc. which are commonly used to measure certain characteristics of the network are also used in literature to derive methods to calculate the properties such as the *MFPT*. These will be discussed in detail in Chap. 4.

For networks of regular lattices, most of these transport properties can be determined analytically. This is however not the case for many real-world networks and therefore have been an objective of active research [10]. This book mainly focusses on calculating the *MFPT* for real world dynamic systems behind natural disasters of which state transitions form random walks on Euclidean state spaces.

Table 2.2 Common transport properties

Cover time	The time for a random walk to visit all sites of the network [23]
Mean exit time from a sphere (t_{exit})	This is the first time a random walker reaches any point at a distance r from its starting point. This quantity is commonly used in analysing Brownian motion in Euclidean spaces [16]. In many cases, the length-scale-invariant properties of the random walk leads to the scaling form t_{exit}/r^{dw} , which defines the walk dimension dw [24]. This will be discussed in detail in Chap. 4. As opposed to <i>MFPT</i> , t_{exit} is not sensitive to the confining environment as only a sphere of r radius is explored
Mean first passage time (<i>MFPT</i>)	This is the probability that a random walker or a diffusing particle hits a specified target point T for the first time starting from a source point S . As discussed in the introduction, this metric is the most common transport feature used in applications where first encounters trigger important events. It has been shown in [24] that the <i>MFPT</i> crucially depends on the confining environment and the distance between S and T [23]
Mean return time	The average time the walker takes to come back to the starting node [10]
Mixing time	The time scale that determines how the probability distribution approaches its limiting behaviour [23]
Number of nodes visited	This is the number of nodes visited during a given time period t
Occupation probability	The probability that a diffusing particle is located at a known location T at time t [22]
Relaxation time	Asymptotic time of convergence to the <i>stationary distribution</i> [25]
Splitting probability	In the existence of two absorbing boundaries (B_1, B_2), this is the probability of absorbing in B_1 and B_2 as a function of S [22]
Stationary distribution	The probability density distribution of the random walker's position as $t \rightarrow \infty$
Survival probability	For an absorbing domain, this equals the integral of the first passage probability over all time and spatial extent of the boundary [22]

2.2 MFPT: A Literature Survey

Estimating *MFPT* for random walks has gained a lot of attention in various fields in the past decade due to the significant role played by first encounters in a large range of application fields. This book looks at two of the main approaches that have been used in literature to address this problem. One approach is through analytical expressions using transport properties of joint characteristics of the network and the random walker. The next is to use Eigen value analysis on transition probability matrices (TPM) for the state transitions modelled as random walks.

2.2.1 *Estimating MFPT Using Random Walks' Transport Characteristics*

Analysing transport properties of random walks on networks with known characteristics can be found in numerous studies. Among them are Brownian motion [16, 28], Lévy flights [24, 29], random walks on scale-invariant media [24], fractals [30], scale-free networks [2, 31] and many others. Studies have also been conducted on random walks which combines several types of motion [7]. Until recently, most methods of determining first passage properties in bounded domains have been mainly limited to one-dimensional geometries with higher spatial dimensions considered only in homogeneous media [24]. This book will discuss some of these methods in Chap. 4 with special consideration to methods presented in [24] that uses the pseudo Green function, that is referred throughout several chapters. However one limitation in most of the methods proposed in literature is that the results are generally applicable to deterministic networks with homogeneous node distributions [32, 33].

2.2.2 *Analysing the Transition Probability Matrix to Estimate MFPT*

The other method commonly discussed in many *MFPT* prediction methods is by Eigen value analysis on transition probability matrices (TPM) for dynamic systems modelled as random walks on their respective state spaces. For random walks possessing the 'Markovian' property (discussed in the next subsection), TPMs are a useful tool for reducing the complexity of *MFPT* computation.

Transition probability matrix (TPM)

For a system with n possible discrete states (network nodes), an $n \times n$ TPM gives the transition probabilities from a state i to j in the (i, j) th cell.

If the state transitions are Markovian, TPM for M state changes is simply obtained by the Markov chain with the result being the current state vector multiplied by $[TPM]^M$. The probability distribution of a regular Markov chain will converge to a unique stationary distribution regardless of the starting position [19].

Eigen value analysis gives the relaxation time of a Markov Chain in a global result as $1/(1 - \varepsilon_2)$, where ε_2 is the second largest eigenvalue [19].

Therefore if the state space can be digitised, TPM is a useful tool.

This is a more popular approach for many engineering applications such as that discussed in [34]. One advantage of this approach is that it is possible to derive the transition probability matrix from prior knowledge of the system either in the form of system modelling or past datasets. These will be discussed in detail in Chap. 3. One major drawback of this approach is that the global connectivity of the network generally needs to be known for generating the TPM and the analysis is only applicable to connected networks [19]. And since the computation involves multiple matrix multiplication, when applied to large scale networks with millions of nodes, the computational complexity makes the method infeasible for many applications [19]. Moreover, the solutions obtained from this fundamental matrix approach are known to be too generic to interpret when it is unclear on which factors (eigenvalue, node degree, connectivity, etc.) govern the *MFPT* from the solution expression [19]. Therefore, although the TPM method provides a very versatile approach for *MFPT* calculations, there still exists room for further improvements.

2.2.3 Random Walks in Natural Disaster Dynamics

Most *MFPT* prediction methods described in literature assume that the random walker has no memory. This is called the ‘Markov property’ (discussed under mathematical tools). However, most random walks governing natural disaster dynamics do not follow the strict Markov conditions. It has also been observed that a large class of natural processes are not always Brownian [1]. This book looks at systems which deviate from such ideal conditions especially in the existence of directional bias and network inhomogeneity. Current literature on these aspects are discussed in the following sections.

2.2.3.1 Bias on Natural Systems

An isotropic random walk that starts at the origin [22] is the basis on which many prediction methods depend on. However, in the past few years more literature has addressed the question of bias in *MFPT* calculations [2, 12, 19, 22]. In [22] it is shown that for a biased random walk, *MFPT* is dominated by the bias.

For a walk on a d dimensional Euclidean space, where $d > 1$, biased random walks are mainly within a narrow cone of t length along the bias direction and having a width of $t^{1/2}$ where t is the time.

The most popular approach in current literature on the definition of bias in random walks is in the form of bias resulting from the node degree distribution [35]. Zhuo et al. [19] have shown that there are cases where the nodes share the same node

degree while having greatly different local topologies. Here they look at bias as being controlled by arbitrarily controlled node weights and show that the *MFPT* is a function of both node degree distribution and node weight distribution. The work in [36] shows that directed self-avoiding walks show anisotropic behaviour by decomposing the random walk as a forward walk along the preferred direction and a random walk perpendicular to that direction.

In one example, animal motion's central foraging behaviour is described as biased random walks in [12] describing *MFPT* through the effect of landscape on animal movement and search time. Directional bias has been introduced into the model with an advection term which describes the directed motion component. The relation between the magnitudes of the directed and random components of movement is shown to determine the shape of the *MFPT* curve.

When the random walks are biased

$$MFPT(S \rightarrow T) \neq MFPT(T \rightarrow S). \quad (2.1)$$

The asymmetry is characterised with the difference in the *MFPT*'s [22, 23]. In [2] it is shown that the difference can be determined by the *random walk centrality*.

The directional bias has also been addressed using TPM based methods in order to calculate *MFPT* for random walks [19] but is still computationally infeasible when networks topologies are not 100 % known or when networks have millions of nodes. This book discusses both methods of addressing directional bias on random walks in Chaps. 3, and 4.

Effect of Bias in Anomalous Diffusion

Biased anomalous diffusion has been shown in [37] as constant external fields giving rise to constant currents which increase with the strength of the biasing field. The drift velocity is shown to respond non-monotonically to the biasing field.

Correlated Walks and Biased Random Walks

Initial work analysing random walks considered the walkers to be 'uncorrelated'.

Uncorrelated walks are where each step taken by the walker is essentially independent from the previous step. Such motion sums up the concept of 'Brownian' motion.

It has long been realised that a large class of random walks are in fact correlated in the form of either *auto-correlation* where the walkers current step is shaped by its previous steps or *cross-correlation* where there is a relationship between two walkers

in that network. The effect of problem specific bias, is another factor shaping random walks in that network. The effect of bias may or may not lead to correlated random walks as will be explained with the following examples. Animal migrating paths which are shaped by biasing factors such as wind, temperature or water [12] as well as life history and morphological traits [38] often lead to correlated random walks as the vector field biasing the network remains more or less unchanged. But in the example of fire spread, the dynamicity of the vector field created by winds make the random walks biased but not correlated [39]. This book concentrates on the effect of bias on random walks irrespective of correlation.

A network without loops is generally called a ‘tree’ and it is impossible for a walker with the memory of the last step to go in a loop in a network with a tree structure. Loops in the network can lead to loops in the random walks. The vector field resulting from biasing factors in a network could be a deciding factor for the existence of loops in a random walk. This book considers random walks without loops for simplicity and limitation of computational resources. Moreover, the effect of biasing vector fields often shape the probability density distribution of selecting the next step at each step of a random walk. The probability of reaching nodes in the network is also shaped by the biasing vector field. Furthermore, biased networks lead to ‘directed’ links where the motion from one node to another is not the same when the nodes are reversed.

2.2.3.2 Inhomogeneous Networks

Network inhomogeneity is defined when a random walk has transition probabilities dependant on its position.

In most *MFPT* prediction methods found in literature, transport properties are assumed to be scale-invariant. Most use network renormalisation schemes such as those found in [13, 15, 24], that use self-similarity and scale-free properties. However, in recent work, network inhomogeneity has been addressed in several approaches. Structural inhomogeneity effects of nature of the diffusive and relaxation dynamics of a random walk has been studied in [2]. The majority of literature on network inhomogeneity looks at the inhomogeneity in node degree distribution [2, 13, 40] with some also looking at local weight distributions [19]. For animal motion, network inhomogeneity has been introduced in terms of landscape features in [12] where landscape heterogeneity on animal movement is considered in two parts: movement rate and movement direction. Seasonal hot-spots have been identified in motion of some predatory animals in [41]. In [2] it is shown that for homogeneous networks, random walks’ centrality is the same for all nodes and any difference determines the difference of *MFPT*.

2.3 Some Mathematical Tools

In this section, a summary of some of the mathematical tools used throughout this book is included. The reader is encouraged to refer to this section for clarifications regarding the mathematical tools in the context of the methods discussed in the next chapters. The tools which are used only once are discussed within the chapter itself.

2.3.1 Bayesian Probability Theory

Contrasting to the Frequentist view of probability being a result of infinitely many attempts, in the Bayesian view, probability is a measure of belief of predicted outcome [42]. It uses prior data to refine prediction and hypothesis.

The main equation governing Bayesian probability is

$$P(h|D) = \frac{P(D|h)P(h)}{P(D)} \quad (2.2)$$

where h is the hypothesis (prior probability) and D denotes the prior data (evidence). $P(h|D)$ gives the posterior probability and $P(D|h)$ gives the generative model or the likelihood.

In making an inference, several tools are available depending on the problem.

2.3.1.1 Maximum Likelihood (ML) Hypothesis

This is considered a good prediction option when no past data is available but not the ideal choice when there is available prior knowledge. The equation for this is

$$h_{ML} = \operatorname{argmax}_{h \in H} P(D|h). \quad (2.3)$$

2.3.1.2 Maximum A Posteriori (MAP) Estimate

The MAP estimate gives the hypothesis with the highest probability given observed data. It is still a point estimate and obtained using

$$h_{MAP} = \operatorname{argmax}_{h \in H} P(D|h)P(h). \quad (2.4)$$

2.3.1.3 The Bayesian Estimate

This method uses the full probability density distribution to infer on the system.

2.3.1.4 Bayes' Filtering Method

The method of Bayes' filtering is a practical approach which uses the posterior probability as the prior probability in the next step. This is used for rainfall prediction in Chap. 3. This method is used in many practical applications through many variations such as in *Kalman filter* which assumes linear dynamics and Gaussian noise, and a more recent version known as the *particle filter*. This gives the probability of being at a state X_t at time t given a sequence of observations $y = (y_1, y_2, \dots, y_t)$ per [42] as

$$P(X_t|y_1, y_2, \dots, y_t) \propto P(y_t|X_t)P(X_t|X_{t-1})P(X_{t-1}|y_1, y_2, \dots, y_t). \quad (2.5)$$

2.3.2 Markov Matrices

A Markov Process is one where when given the current state of a process at time t , any information regarding the states of the process until time t does not effect on the prognosis of the system state after time t [43, 44].

In the context of a random walk, this is when the walker is at the i th node, the probability $P_{i,j}$ to jump to j th node does not depend on previous steps [1]. This memoryless property is called the *Markovian property*.

The Markovian property for a random walks is

$$P(s_{n+1} = x | s_1, s_2, \dots, s_n) = P(s_{n+1} = x | s_n). \quad (2.6)$$

This probability condition defines the *Discrete Markov Chain*. A *Markov matrix*, also known as a stochastic matrix, describes the transitions of a Markov chain. The j th element of the i th row consists of the non-negative value $P_{i,j}$. A process with n possible states produces an $n \times n$ Markov matrix ($A_{n \times n}$). Two major properties of this matrix are,

- $\sum_{i=1}^n p_{ij} = 1, \forall j \in \{1, 2, \dots, n\}$ and
- $0 \leq p_{ij} \leq 1, \forall i, j \in \{1, 2, \dots, n\}$.

The largest Eigen-value of a Markov matrix is 1. Finally, because when a random walk has no memory, it is “renewed” every time a specific node is reached. Thus recurrence also implies that every site is visited infinitely often for an isotropic random walk with the Markovian Property.

2.3.3 Linear Least Square Fitting Technique

When a data model is developed to explain a given system, and when such model is linear in the model coefficients, *linear regression* is used to fit the model to the dataset. The most common linear regression type is the *linear least square fit*. This method is used throughout this book to fit linear models.

If a linear model is given as

$$y = f(x) \quad (2.7)$$

the least squares method minimises the sum of the square of residuals. A residual for the k th data point (r_k), is the difference between the response observed (y_k) and the model response ($\hat{y}_k$) given as

$$r_k = y_k - \hat{y}_k \quad (2.8)$$

and the sum of square of residuals (S) is

$$S = \sum_{k=1}^n r_k^2 = \sum_{k=1}^n (y_k - \hat{y}_k)^2 \quad (2.9)$$

for n number of data points included in the fit.

The individual regression coefficients are found by solving the differentiation of S with respect to each parameter as

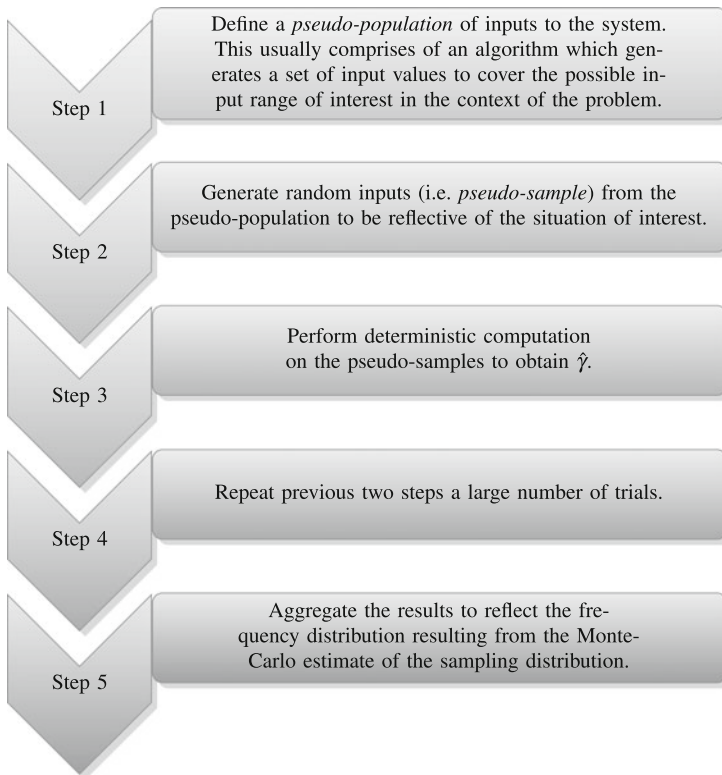
$$\forall_{(i=1,2,\dots,m)} \frac{\partial[S]}{\partial p_i} = 0 \quad (2.10)$$

where p_i ; $i = 1, 2, \dots, m$ are the parameters.

2.3.4 Monte-Carlo Simulations

Monte Carlo simulations serve the objective of estimating a stochastic characteristic γ with an estimator $\hat{\gamma}$ computed from observed data [45]. The roots of modern Monte Carlo methods run back to the 1940s with the use of random numbers to examine problems from a stochastic perspective [46]. Monte Carlo methods have proven to be extremely useful in solving problems where analytical solutions do not exist or when conditions required for mathematical theory to be valid do not hold. Monte-Carlo simulations offer an alternative to analytical mathematics for understanding systems behaviour using random samples of known simulated data populations.

The basic steps of a Monte-Carlo simulation are as follows [45]:



The basic Monte-Carlo simulations are therefore very simple. The accuracy of a Monte-Carlo estimate depends upon creation of the pseudo-population in the thoroughness with which it is explored through pseudo-samples. Improvement of the result could be possible simply by increasing the number of trials. Therefore, a major limitation of Monte-Carlo simulations is the limitation of computational resources. This method is used extensively in the next several chapters of this book.

2.4 Summary

This chapter outlined the basic concepts and notions, a literature review of *MFPT* prediction methods and a few mathematical tools used throughout the rest of this book. The main objective of the rest of this book is on discussing new techniques to calculate the transport property introduced as *MFPT*, for random walks under different conditions and situations commonly imposed by processes governing natural disaster activity. The concepts and definitions introduced in this chapter will be used throughout the book. It is encouraged to revisit this chapter and the literature cited herein whenever clarifications are needed.

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