

# Preface

The superconductivity in the  $A_3C_{60}$  systems with  $A = K, Rb, Cs$  (alkali-doped fullerenes) has been intensively studied since its discovery in 1991. The maximum superconducting transition temperature  $T_c$  of  $\sim 40$  K is the highest among the molecular superconductors. The superconductivity was seemingly well understood by the conventional phonon mechanism, in which the intramolecular  $H_g$  modes played a main role: Experimentally, the conventional  $s$ -wave pairing symmetry and the positive correlation between the superconducting transition temperature and the lattice constant supported this scenario. Theoretically, it has been claimed in the literature that the total electron phonon coupling  $\lambda \sim 0.5$ – $1$  and the high phonon frequencies on the order of  $\sim 0.1$  eV produce a high transition temperature comparable to the experimental ones.

However, the recent success in synthesizing fcc/A15  $Cs_3C_{60}$  brought to light a severe contradiction with the conventional scenario. They are both Mott insulators at ambient pressure, and the superconductivity is realized only when the lattice constant is shrunk by applying pressure. The critical temperature  $T_c$  as a function of the lattice constant shows a dome-like shape for both the A15 and fcc systems. These features cannot be explained by the conventional Migdal–Eliashberg theory. In fact, the existence of the superconducting phase in the vicinity of the Mott insulating phase indicates that the electron correlation might be essential. Furthermore, the observed low-spin state and the dynamical Jahn–Teller effect in the insulating phase revealed a substantial role for the electron–phonon interactions. Therefore, in order to understand the pairing mechanism, the Mott transition, and the low-spin state in the Mott insulating phase in a comprehensive manner, it is necessary to elucidate a nontrivial interplay between the electron correlations and the electron–phonon interactions.

In this thesis, we aim to obtain a unified description of the phase diagram. Especially, we try to answer why the  $s$ -wave superconductivity is stabilized in the vicinity of the Mott insulating phase in contrast to a naive expectation that the strong electron correlations are incompatible with the  $s$ -wave pairing. While it was proposed that a new type of phonon-mediated superconductivity distinct from the

BCS (Bardeen–Cooper–Schrieffer) superconductivity emerges near the Mott transition by cooperation of the Jahn–Teller phonons and the strong correlations, to obtain conclusive statements, we need arguments as unbiased as possible.

Another goal of the thesis is the non-empirical calculation of the transition temperatures of the alkali-doped fullerides. Historically, the  $T_c$  calculation crucially relies on empirical parameters, such as the Coulomb pseudopotential. While recent progress has enabled fully ab initio  $T_c$  calculations in high accuracy for the conventional superconductors, there is still no reliable way to predict  $T_c$  for unconventional superconductors.

To achieve the goals, we derive, from first-principles calculations, effective low-energy Hamiltonians for the fcc  $A_3C_{60}$  systems. The derived Hamiltonians consist of the electron transfer, the Coulomb interaction, the electron–phonon interaction, and the phonon one-body terms. By analyzing them accurately with a model calculation technique, we study the low-energy phenomena. This scheme requires only the chemical composition and the crystal structure, which enables quantitative studies without employing any empirical parameters.

Because a previous study has evaluated the electron transfer and the Coulomb interaction parameters, in this thesis we focus on the derivation of the phonon-related terms. To this end, we formulate a novel ab initio scheme, constrained density functional perturbation theory (cDFPT). In the cDFPT, partially renormalized phonon frequencies and electron–phonon couplings are calculated with excluding the low-energy-subspace renormalization effects, which are used as the parameters in the low-energy models. The partial renormalization allows us to take into account the effects of high-energy bands and to avoid the double counting of the low-energy-subspace renormalization effects, which are to be considered when the models are analyzed.

We apply the cDFPT to the alkali-doped fullerides. Then, the static part of the phonon-mediated negative exchange interactions  $J_{ph}(\omega = 0)$  is estimated to be  $J_{ph}(\omega = 0) \sim -51$  meV. We find that the magnitude of the negative  $J_{ph}(\omega = 0)$  is larger than that of the positive Hund’s coupling  $J_H \sim 34$  meV. It means that, effectively, negative exchange and pair-hopping interactions are realized in the  $A_3C_{60}$  systems, while the amounts of the interactions ( $\sim -17$  meV) are tiny compared to the Hubbard repulsion  $U \sim 1$  eV. Furthermore, we see that, due to the phonon-mediated attractions, the effective interorbital repulsion  $U'_{eff}$  becomes slightly larger ( $\sim 5\%$ ) than the effective intraorbital repulsion  $U_{eff}$ .

To analyze the Hamiltonians with the unusual form of the intramolecular interactions, we adopt the extended dynamical mean-field theory (extended DMFT) with employing the continuous-time quantum Monte Carlo method based on the strong coupling expansion as an impurity-model solver. The extended DMFT is one of the most powerful methods to study strongly correlated materials in three dimensions, which can accurately treat the local phonon dynamics and the dynamical screening effects originating from the long-range Coulomb interactions, on top of the local electron correlations.

We perform the extended DMFT analysis of the derived ab initio models and draw a theoretical phase diagram as a function of lattice constant and temperature.

We obtain the paramagnetic metal, the superconducting phase, and the paramagnetic Mott insulator, which will reproduce the experimentally observed phases. As a consequence of the effective negative exchange interaction, the low-spin state is realized in the insulating phase. Remarkably, the agreement is not only at a qualitative level but also at a quantitative level. In particular, the calculations indicate the maximum  $T_c$  of  $\sim 28$  K, in good agreement with the experimental result ( $\sim 35$  K).

As for the pairing mechanism, we identify two crucial factors. One is the singlet pair generation by  $U'_{\text{eff}} > U_{\text{eff}}$ , and the other is the tunneling of the pairs by the negative pair-hopping interactions (the Suhl–Kondo mechanism). The inequality  $U'_{\text{eff}} > U_{\text{eff}}$  and the negative pair-hopping term originate from the phonon-mediated attractions, thus the superconductivity essentially relies on the phonons. However, this superconductivity differs from the conventional ones in that the strong electron correlations also play an important role: The pair formation is originally inefficient because the difference between  $U'_{\text{eff}}$  and  $U_{\text{eff}}$  is very small ( $\sim 0.03$ – $0.04$  eV) compared to the typical kinetic energy  $\sim 0.5$  eV, and becomes efficient only when the electronic kinetic energy is suppressed by the correlations. As a result, we see the increase of  $T_c$  with the increase of the correlation strength. These considerations lead to a conclusion that the alkali-doped fullerenes are unconventional superconductors, whose essence is the unusual synergy between the strong electron correlations and phonons.

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