Chapter 2
Class-Based Storage with a Finite Number of Items in AS/RS

Abstract Class-based storage is widely studied in the literature and applied in practice. It divides all stored items into a number of classes according to their turnover. A class of items with a high turnover is allocated to a region close to the warehouse depot. Studies have shown that the use of more storage classes leads to a shorter travel time for storing and retrieving items. A basic assumption in this literature is that the required storage space for all the items is equal to their average inventory level, which is valid only if an infinite number of items are stored in each storage region. Therefore, this chapter revisits class-based storage by considering a finite number of items and by relaxing the assumption that the “required storage space of all the items equals their average inventory level”. We develop a travel-time model and algorithm that can be used for determining the optimal number and the boundaries of storage classes in warehouses. Different from the results of conventional research, our findings illustrate that a small number of classes is generally optimal. In addition, we find that travel time is fairly insensitive to the number of storage classes in a wide range around the optimum. This finding suggests that managers can select a near-optimal number of storage classes in an easy way, and they should not be worried about the effect of storage-class reconfigurations. We validate our findings for various cases, including different ABC demand curves, space-sharing factors, number of items, storage rack shapes, discrete storage locations, and stochastic item demand.

2.1 Research Background

Class-based storage is the most commonly used storage policy in practice and is widely discussed in many operations management textbooks (Tompkins et al. 2010; Heragu 2006; Adams 1996) and scientific papers (Rosenblatt and Eynan 1989; Kouvelis and Papanicolaou 1995; Johnson and Brandeau 1996; Gu et al. 2007; Eynan and Rosenblatt 1994; De Koster et al. 2007). It divides stored items into different classes (using three classes is common in practice) according to the ABC demand curve (see Fig. 2.1). In case of ABC class-based storage, a relatively small number of highly demanded items are grouped as A-class items and are then stored in a

X. Guo, *Storage Policies and Maintenance Support Strategies in Warehousing Systems*, Springer Theses, DOI 10.1007/978-981-10-1448-2_2

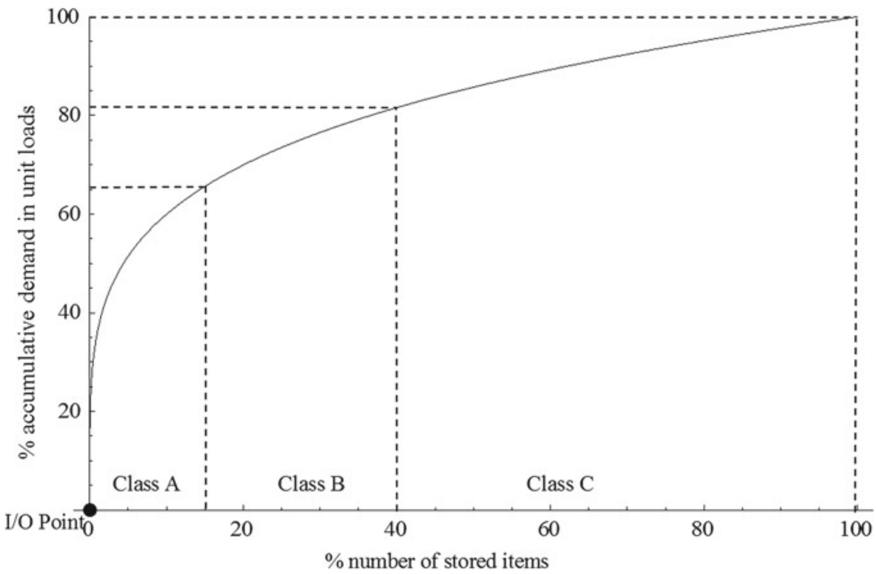

Fig. 2.1 An example of an ABC demand curve

warehouse region closest to the depot (the entrance and exit position). Grouped as C-class items, rarely demanded items are stored in the region farthest from the depot. Within each class, items are stored randomly.

Figure 2.2 illustrates the side view of a SIT storage rack with an example of an ABC class-based storage as used in automated storage and retrieval (AS/R) warehousing systems. In such a system, the optimal boundary of each region is square because the storage/retrieval (S/R) machines can drive and lift simultaneously. This capability leads to a Chebyshev distance metric (Bozer et al. 1990) used to measure the distance between a storage location and the depot.

Hausman et al. (1976) modeled and analyzed the two- and three-class-based storage policies; Rosenblatt and Eynan (1989) and Eynan and Rosenblatt (1994) formulated a travel-time model for n-class-based storage and addressed the benefits of class-based storage by increasing the number of classes. Following these studies and the earlier paper, most studies on class-based storage (Eynan and Rosenblatt 1994; Larson et al. 1997; Gu et al. 2007; Yu and Koster 2009) have implicitly or explicitly assumed that the total required storage space does not depend on the number of classes in modeling. This assumption is valid when the number of items in each class is sufficiently large (infinite). Within each storage class, multiple items are stored randomly and share a common storage space. They are replenished in the system at different points in time. When an item is replenished, any available empty storage location in its class can be used for storing it. As a result, if the number of

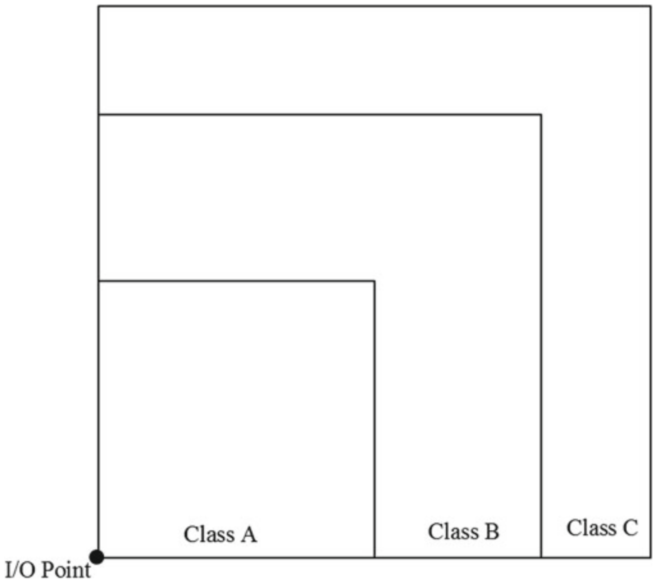

Fig. 2.2 Side view of a rack with ABC class-based storage regions

items in a class is infinite, then the required storage space of the class is approximately equal to the average total inventory level of all items in the class. With this assumption, conventional research has indicated that an increase in the number of classes reduces the average travel time for storing or retrieving items (see the curve indicating conventional research in Fig. 2.3).

However, the abovementioned widely cited finding is inconsistent with the practice in which only a few (three to five) classes are usually implemented (Roodbergen and Vis 2009). To the best of our knowledge, no study has theoretically demonstrated that an excessive number of classes degrade system performance. This deficiency motivates us to investigate all assumptions made in the literature. The cause of the inconsistency appears to be the assumption that regardless of the number of storage classes, the space needed for each storage class is equal to the sum of the average inventory levels of the items in the class. This assumption can be justified in the case of an infinite number of items stored for each product class. However, every time an item is received, sufficient space should be available for storing the entire batch. Therefore, items sharing a storage class need more space than just their average stock level. If the number of storage classes increases, then the number of items in each class decreases, and more space is needed for each item as the opportunity for space-sharing decreases. This condition increases the average travel time for storing and retrieving items and finally offsets the travel-time reduction resulting from dividing items over a large number of classes according to their turnover. This tradeoff has

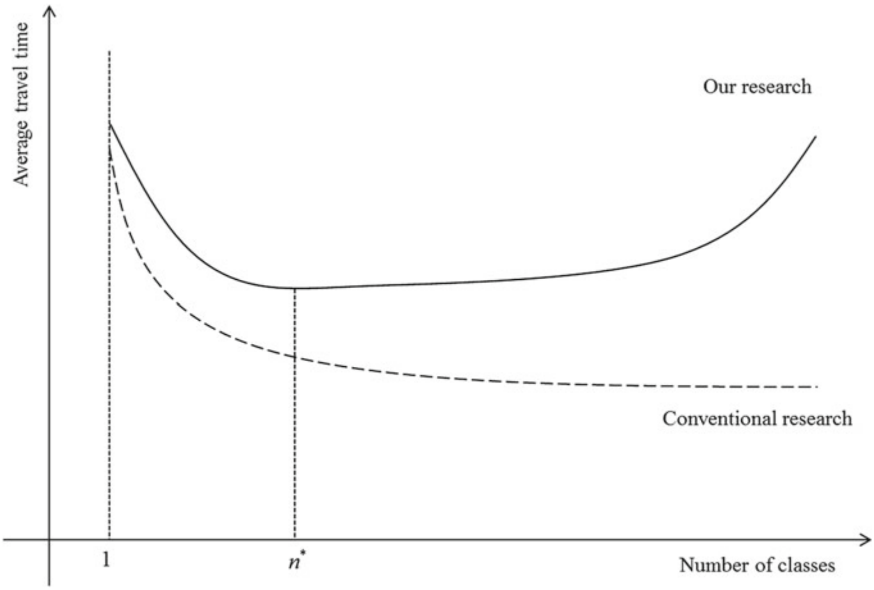

Fig. 2.3 Travel times in two different lines of research

not been investigated in the literature. Relaxing other common assumptions, such as deterministic demand, continuous and SIT racks, and use of the classic EOQ replenishment policy, does not affect the existence of this tradeoff. Therefore, we first relax the (implicit) finite number of items assumption and then validate the robustness of our findings by relaxing other assumptions made in the literature.

To investigate this tradeoff overlooked in the existing literature and to deduce managerial insights for warehouse managers, this chapter develops a new expression for estimating the required storage space as a function of the number of storage classes. Based on this expression, a travel-time model is developed from which the optimal number of classes and their boundaries can be determined for a warehouse with a finite number of items. The model is solved using dynamic programming with time complexity $O(N^3)$, where N is the number of stored items in the system. The results demonstrate that travel time is commonly a bowl-shaped function of the number of classes as shown in Fig. 2.3. This flat curve shape yields some important findings. First, beyond a small number of classes, an increase in the number of classes cannot reduce travel time. Second, a small number of classes is usually enough to yield an optimal solution. Third, travel time appears to be insensitive to the number of classes in a wide range around the optimum (corresponding to the bottom of the bowl-shaped curve in Fig. 2.3). This finding implies that warehouse managers should not hesitate to change their number of classes if necessary because "they cannot go wrong".

2.2 Problem Description and Conventional Travel-Time Model

This section describes the system studied and develops the mathematical model. The traditional model in the literature is also revisited.

2.2.1 Problem Description

Without loss of generality, the basic idea of n-class-based storage is usually discussed in the abovementioned literature with a basic automated warehousing system: the AS/RS. This system consists of an S/R machine, a continuous storage rack, and one depot where all items enter and leave the system. Items can be finished goods, work-in-process, or raw materials, which are stored on standardized unit loads (e.g., pallets or totes) in the AS/RS. The system works as follows: when a storage unit load arrives at the depot of the system, the machine retrieves and transports it to any given storage location in the rack. When a stored unit load is requested, the machine picks it up and moves it to the depot. The system has the following properties:

1. All storage locations are the same size for storing standardized unit loads.
2. The depot is located on the lower-left side of the storage rack.
3. The continuous-space storage rack is SIT; the time for the machine to move from the depot to the most distant column is equal to the time for the machine to move from the depot to the most distant tier. The machine travels in horizontal and vertical directions simultaneously, thus resulting in a Chebyshev distance metric used to measure the distance between a storage location and the depot Roodbergen and Vis (2009). An extended model incorporating NSIT racks is given in Sect. 2.4.1. A discrete space rack is modeled in Sect. 2.4.3.
4. The capacity of the machine is one unit load. The machine operates in a single-command mode, and it stores or retrieves one unit load each time.
5. The pick-up or deposit time for the machine to load or unload a unit load is constant and ignored.
6. The turnover of each item is measured as the number of unit loads requested in a unit-time period, such as a week, a month, or a year, and is determined by the ABC demand curve given in Eq. (2.1). All the items are ranked according to their marginal contribution to the total turnover; an item that has a smaller contribution is indexed with a larger number. We extend this method to the stochastic demand in Sect. 2.4.3.
7. Item inventories are replenished according to the classic EOQ model.

As used in conventional research, the ABC demand curve is a plot of ranked cumulative percentage expected demand per unit time, $G(i)$, and is modeled by the following:

$$G(i) = i^s = \int_0^i D(j)dj \Big/ \int_0^1 D(j)dj, \qquad 0 < s \leq 1, \tag{2.1}$$

where i is the item at the ith percentile in the ranked sequence of all items, $D(i)$ is the demand of item i per unit time, and s is the shape factor of the ABC demand curve. Given $s = 0.222$, we know that 20 % of the total items (i.e., $i = 20\,\%$) contribute $G(i) = i^s = (20\,\%)^{0.222} = 70\,\%$ of the total demand. A lower s means a more skewed ABC demand curve. For example, $s = 0.222$ represents a 20 %/70 % demand curve that is more skewed than a 20 %/50 % ABC demand curve for $s = 0.431$. We relax the ABC curve function with another demand curve function in Sect. 2.4.2. By normalizing the total demand $\int_0^1 D(j)dj = 1$, we obtain the following without loss of generality:

$$D(i) = dG(i)/di = si^{s-1}, \qquad 0 < s \leq 1, \tag{2.2}$$

according to Hausman et al. (1976) and Rosenblatt and Eynan (1989).

Given the abovementioned system properties and the item demands determined by Eq. (2.2), we intend to find the average one-way travel time for storing or retrieving a unit load in a class-based storage system. The one-way travel time is the travel time from the depot to a unit-load storage location.

The class-based storage policy divides the storage space into n regions. Region k is dedicated to storing items of class k, $k = 1, 2, \ldots, n$. As shown in Fig. 2.1, a region with items of high demand is located close to the depot. Items are randomly stored in each region. Furthermore, the regions are L-shaped in a SIT storage system.

Based on the description given so far, the notations used in this chapter are given and defined in Table 2.1.

With the abovementioned notations and according to Rosenblatt and Eynan (1989), the average one-way travel time (called "travel time" hereafter) in an n-class system, T_n, can be formulated as the follows:

$$T_n = \frac{\sum_{k=1}^n t_k \Lambda(k)}{\sum_{k=1}^n \Lambda(k)} = \sum_{k=1}^n t_k \left(\frac{\Lambda(k)}{\sum_{k=1}^n \Lambda(k)} \right), \tag{2.3}$$

where $\Lambda(k) \Big/ \sum_{k=1}^n \Lambda(k)$ is the weighted retrieval rate of class k, and $\sum_{k=1}^n \Lambda(k)$ is the total turnover in the whole system. Rosenblatt and Eynan (1989) simplified the calculation using Eq. (2.2) to obtain the following:

Table 2.1 Notations used in this chapter

Notations	Definitions
i	Index of the ith item. An item with a lower demand has a larger index
j	Index of the jth storage location (or unit load). A location closer to the depot has a smaller index
n	Number of classes in the storage system
k	Index of the kth class, $k = 1, 2, \ldots, n$
i_k	Index of the item with the lowest turnover in class k
j_k	Storage location (a corresponding unit load) farthest from the depot in class k. It also corresponds to the total required storage space of items 1 to k
t_k	Average one-way travel time for storing/retrieving a unit load of class k
R_k	One-way travel time for storing/retrieving a unit load at the further boundary of class k
G_k	100×(cumulative demand for the first k classes)/(the total demand of all items in a unit-time period)
$\Lambda(k)$	Total turnover, in number of unit loads per unit-time period of all items stored in class k
T_n	Average one-way travel time of a unit load for an n-class storage system

$$\Lambda(k) \Big/ \sum_{k=1}^{n} \Lambda(k) = G_k - G_{k-1} = i_k^s - i_{k-1}^s, \qquad k = 1, 2, \ldots, n. \tag{2.4}$$

Furthermore, in case of a SIT system, according to Hausman et al. (1976) and Rosenblatt and Eynan (1989), t_k can be obtained as the follows:

$$t_k = \frac{2\left(R_k^3 - R_{k-1}^3\right)}{3\left(R_k^2 - R_{k-1}^2\right)}, \qquad k = 1, 2, \ldots, n. \tag{2.5}$$

Consequently, by substituting Eqs. (2.4) and (2.5) into Eq. (2.3), the average travel time for the system can be rewritten as follows:

$$T_n = \sum_{k=1}^{n} \frac{2\left(R_k^3 - R_{k-1}^3\right)}{3\left(R_k^2 - R_{k-1}^2\right)} \left(i_k^s - i_{k-1}^s\right). \tag{2.6}$$

To minimize T_n in Eq. (2.6), we must derive the relationship between R_k and i_k (or G_k), $k = 1, 2, \ldots, n$. When the relationship is obtained, travel time can be minimized through the optimization of either R_k or i_k, for $k = 1, 2, \ldots, n$.

In the next subsection, the conventional travel-time model based on the assumption that "the required storage space of all the items equals their average inventory level" is revisited.

2.2.2 Conventional Travel-Time Model

Conventional research (Rosenblatt and Eynan 1989; Hausman et al. 1976; Eynan and Rosenblatt 1994) assumes that the total required storage space for storing all items is equal to the total average inventory level of the items regardless of the number of classes of the system

$$A = L = \int_0^1 Q(i)\big/2di = \sqrt{2Ks}\Big/(s+1), \tag{2.7}$$

where A is the total required storage space (in number of unit load locations) for storing all items, L is the total average inventory level of the items, $Q(i)$ is the economic order quantity of item i, and K is the ratio of order cost to holding cost and is assumed to be equal for all items. On the basis of the abovementioned assumptions, Rosenblatt and Eynan (1989) provided the relationship between R_k and G_k as $G_k = R_k^{4s/(s+1)}$ with $G_k = i_k^s$. As a result, the conventional model (hereafter called as Model CM) can be defined as follows:

$$\begin{aligned}
&\textbf{Model CM}: \\
&Min \quad T_n = \sum_{k=1}^{n} \frac{2\left(R_k^3 - R_{k-1}^3\right)}{3\left(R_k^2 - R_{k-1}^2\right)} \left(R_k^{4s/(s+1)} - R_{k-1}^{4s/(s+1)}\right), \\
&s.t. \quad 0 = R_0 < R_{k-1} < R_k < R_n = 1, \\
&d.v. \quad R_k, \quad k = 1, 2, \ldots, n-1.
\end{aligned} \tag{2.8}$$

Model CM can be solved using the following recursive equation proposed by Rosenblatt and Eynan (1989):

$$T_n = \frac{2\left(1 - R_{n-1}^3\right)}{3\left(1 - R_{n-1}^2\right)} \left(1 - R_{n-1}^{4s/(s+1)}\right) + R_{n-1}^{(5s+1)/(s+1)} T_{n-1}. \tag{2.9}$$

2.3 Travel-Time Model with a Finite Number of Items

This section relaxes the assumption of conventional research by adopting a finite number of items in the system. The total required storage space is not simply equal to the average inventory level similar to the Model CM but becomes a function of the number of classes and the number of items in each class. In Sect. 2.3.1, the required storage space for each storage class and the relationship between R_k and i_k are derived. Section 2.3.2 presents our basic model that considers the required storage space. A solution methodology for the model is provided in Sect. 2.3.3.

2.3.1 Required Storage Space Function and Relationship Between R_k and i_k

If the number of items sharing a common storage space is finite, then the required storage space of an item depends on several factors such as the number of items sharing the space, the skewness of the ABC demand curve (s), the inventory replenishment policies, and the ratio of order cost to holding cost (K). We first determine the required storage space of an item as a function of the number of items in the same shared space by considering the replenishment quantity $Q(i)$ that incorporates the other factors.

The function $a_i(N_k)$ denotes the space required (average over time) to store item i in class k together with $(N_k - 1)$ other items, where N_k represents the number of items sharing a common storage space within class k. A large number of storages and retrievals are simulated for obtaining the presentation of $a_i(N_k)$, and the general shape is sketched in Fig. 2.4. In particular, when $N_k = 1$, the storage method turns into a dedicated storage, and the required storage space for item i is equal to its order quantity $Q(i)$. When $N_k = +\infty$, the method turns into the situation with an infinite number of items, and the required storage space for item i is now equal to its average inventory level, $Q(i)\big/2$ (Hausman et al. 1976; Rosenblatt and Eynan 1989). For $1 < N_k < +\infty$, the value of $a_i(N_k)$ is between $Q(i)$ and $Q(i)\big/2$, and it decreases in N_k convexly as shown in Fig. 2.4. The relationship can be represented by the following mathematical presentation:

$$a_i(N_k) = 0.5\left(1 + N_k^{-\varepsilon}\right) Q(i) = 0.5\left(1 + N_k^{-\varepsilon}\right)\sqrt{2KD(i)}, \tag{2.10}$$

where ε is the space-sharing factor.

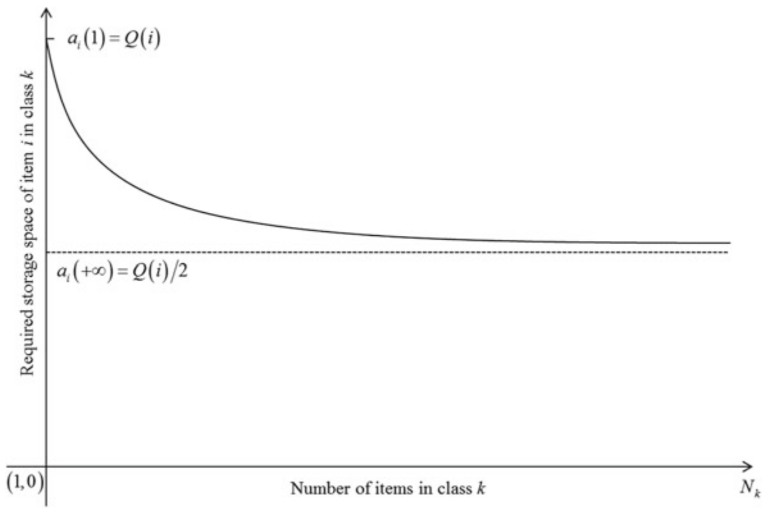

Fig. 2.4 Required storage space of item i as a function of N_k

To provide a better understanding of the above mentioned relationship, we develop an analytical model for a special case by assuming that N_k identical items exist in class k. If the replenishments for the N_k items are fully coordinated, that is, item 1 arrives at time T, item 2 at time $T + T/N_k$, ..., item i at time $T + (i - 1)T/N_k$, and so on, then the inventory patterns of the items are according to those shown in Fig. 2.5. Therefore, the total required storage space of this class, k, can be obtained as the total inventory level of the N_k items when an order arrives. In other words, $\sum_{i=1}^{N_k} iQ(i)/N_k = (1 + N_k)Q(i)/2$. Consequently, the required storage space for item i can be obtained as follows:

$$a_i(N_k) = ((1 + N_k)Q(i)/2) \big/ N_k = 0.5\left(1 + N_k^{-1}\right) Q(i). \tag{2.11}$$

In this special case, the space-sharing factor is $\varepsilon = 1$, which yields the best space-sharing because of the item symmetry and synchronization of the item ordering. However, in practice, the value of ε is significantly smaller than 1 because of the heterogeneity of the items in demand volumes, reorder points, order quantities, delivery lead times, holding costs, and others. Therefore, we use simulation to determine the

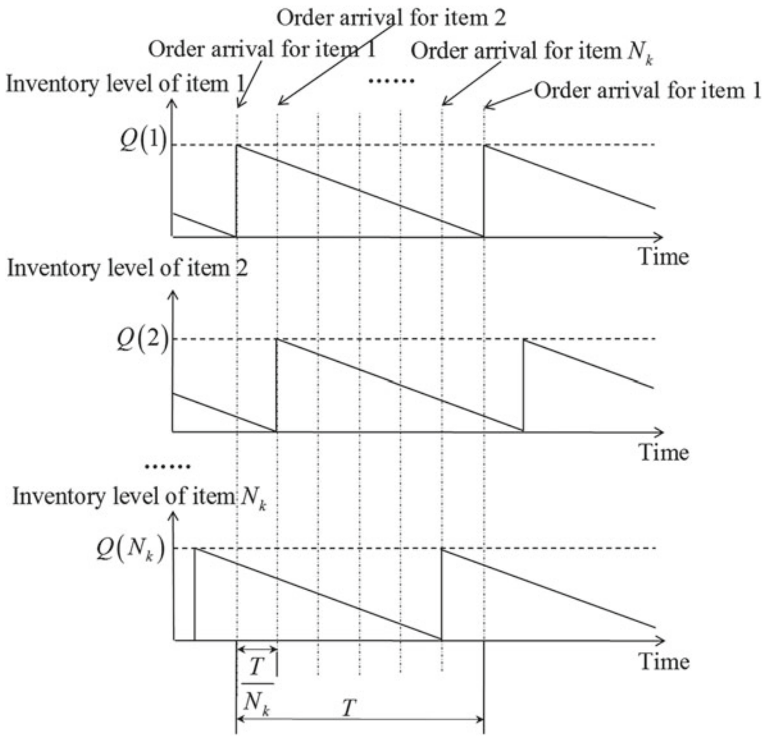

Fig. 2.5 Coordinated replenishment cycles for N_k identical items in a class

Table 2.2 Algorithm used to simulate ε

Steps	Job description
Step 1	Obtain the initial inventory level for the N_k items randomly from their possible values
Step 2	Obtain the inventory level as a function of time for all the N_k items
Step 3	Obtain the total inventory level of this class as a function of time based on the results of *Step 2*
Step 4	Obtain the maximum value of the function obtained from *Step 3* as the required storage space of this class
Step 5	Obtain the simulated required storage space for each item according to their weighted contribution to the total required storage space of the class
Step 6	Repeat *Steps 1–5* for m times, and obtain an average value as the required storage space for the items to estimate ε
Step 7	Repeat *Steps 1–6* for every possible N_k
Step 8	Estimate ε for Eq. (2.10) using least squares method based on the simulated values

average value of ε. The algorithm steps shown in Table 2.2 are used in the simulation, and the details are given in Appendix A.

The method of simulation does not depend on the demand curves or replenishment policies. In the details presented in Appendix A, the continuous review (r,S) policy, classic EOQ policy, and the different kinds of ABC demand curves are verified. The estimated average value of ε appears to be mostly in the range of 0.15–0.25 and is quite insensitive to the system parameters.

Consequently, using Eq. (2.10), we can obtain the required storage space for class k with a finite number of items

$$j_k - j_{k-1} = \int_{i_{k-1}}^{i_k} a_i(N_k)di = \int_{i_{k-1}}^{i_k} \left(1 + N_k^{-\varepsilon}\right)\sqrt{0.5KD(i)}di, \tag{2.12}$$

where j_k is the storage location farthest from the depot in class k, which also corresponds to the total required storage space of items 1 to i_k, and $j_0 = 0$.

To make our result comparable with those of the conventional model, we also rescale j by $j * L$, where $L = \sqrt{2Ks}\big/(s+1)$ (see Eq. (2.7)) according to Hausman et al. (1976) and Rosenblatt and Eynan (1989). Thereafter, using Eqs. (2.2) and (2.12), we obtain the total required storage space for the first k classes as follows:

$$j_k = i_k^{(s+1)/2} + \sum_{l=1}^{k} N_l^{-\varepsilon}\left(i_l^{(s+1)/2} - i_{l-1}^{(s+1)/2}\right). \tag{2.13}$$

Furthermore, the relationship between R_k and j_k in a SIT continuous storage system is $R_k = \sqrt{j_k}$ (Hausman et al. 1976; Rosenblatt and Eynan 1989). Therefore, the boundary of the kth class, R_k, can now be rewritten as follows:

$$R_k = \sqrt{i_k^{(s+1)/2} + \sum_{l=1}^{k} N_l^{-\varepsilon}\left(i_l^{(s+1)/2} - i_{l-1}^{(s+1)/2}\right)}, \tag{2.14}$$

where N_l is the number of items in class l, which is obtained with $N_l = N(i_l - i_{l-1})$, and N is the total number of items in the system.

2.3.2 Basic Travel-Time Model with a Finite Number of Items

Considering the relationship between R_k and i_k shown in Eq. (2.14), we obtain the new model (hereafter called "Model BM" to differentiate the basic model from the extensions in the following sections) that enables us to determine the optimal class boundaries of the continuous AS/RS.

Model BM:

$$\begin{aligned}
&\textit{Min} \quad && T_n = \sum_{k=1}^{n} \frac{2\left(R_k^3 - R_{k-1}^3\right)}{3\left(R_k^2 - R_{k-1}^2\right)}\left(i_k^s - i_{k-1}^s\right), && (2.15)\\
&s.t. && N(i_k - i_{k-1}) \geq 1, && (2.16)\\
& && R_k = \sqrt{i_k^{(s+1)/2} + \sum_{l=1}^{k} N_l^{-\varepsilon}\left(i_l^{(s+1)/2} - i_{l-1}^{(s+1)/2}\right)}, &&\\
&d.v. && R_k,\ k = 1, 2, \ldots, n; \ \text{ or } \ i_k, \quad k = 1, 2, \ldots, n-1, &&
\end{aligned}$$

where $R_0 = i_0 = 0$ and $i_n = 1$ are known.

Model BM differs from Model CM in three respects. (i) Eq. (2.14) indicates that the class boundary, R_k, is not only related to the last item and the items' demand but also to the number of items in class k and all its preceding classes $1, \ldots, k-1$. (ii) Constraints (2.16) are required to ensure that at least one item is stored in each class because the items and the total required storage space are finite. (iii) The total required storage space of the system, R_n, is an unknown value in Model BM because R_n relates to R_k, $k = 1, 2, \ldots, n-1$, but $R_n = 1$ is known in Model CM. To determine an efficient manner of solving this problem, we introduce a solution methodology based on dynamic programming.

2.3.3 Solution Methodology

Unfortunately, the methodology for solving Model CM used by Rosenblatt and Eynan (1989) cannot be applied because an iterative relation similar to Eq. (2.9) does not hold in this case. In addition, the objective function (2.15) is nonlinear, and we do not know if it is a convex function of R_k (or i_k), $k = 1, 2, \ldots, n$. In addition, constraints (2.16) are nonlinear functions of R_k, $k = 1, 2, \ldots, n$, in considering Eq. (2.14). For a small number of classes, a grid search can be applied to identify approximate solutions. We use a different solution method because we are interested in a fast algorithm that determines the optimal solution for a larger number of classes. Rewriting expressions (2.14) and (2.15) is possible to enable the use of a dynamic programming solution approach. We denote $Y_k = \sum_{l=1}^{k} N_l$ as the cumulative number of items of the first k classes, where N_l is the number of items in the lth class. The relationship between i_k and Y_k is as follows:

$$i_k = Y_k/N. \tag{2.17}$$

By substituting Eq. (2.17) into Eqs. (2.14) and (2.15), we can rewrite Model BM in a Solution Model (SM) as follows:

Model SM:

$$Min\ \ T_n = \sum_{k=1}^{n} \frac{2\left(R_k^3 - R_{k-1}^3\right)}{3\left(R_k^2 - R_{k-1}^2\right)} \left(\left(\frac{Y_k}{N}\right)^s - \left(\frac{Y_{k-1}}{N}\right)^s\right), \tag{2.18}$$

$$s.t.\ \ Y_k = \sum_{l=1}^{k} N_l,$$

$$R_k = \sqrt{\left(\frac{Y_k}{N}\right)^{(s+1)/2} + \sum_{l=1}^{k} N_l^{-\varepsilon} \left(\left(\frac{Y_l}{N}\right)^{(s+1)/2} - \left(\frac{Y_{l-1}}{N}\right)^{(s+1)/2}\right)}, \tag{2.19}$$

$$d.v.\ \ N_k > 0,\ k = 1, 2, \ldots, n.$$

The details of the dynamic programming solution methodology of this model are described in Appendix B. The complexity of the algorithm appears to be $O(N^3)$.

2.4 Model Extensions

This section extends the basic model in Sect. 2.3 to consider an NSIT storage rack, a different ABC demand curve, a discrete rack, and a stochastic item demand. In particular, Sect. 2.4.1 considers the NSIT rack model, Sect. 2.4.2 examines the model based on Bender's ABC demand curve, and Sect. 2.4.3 extends the model to a discrete storage rack with stochastic item demand.

2.4.1 NSIT Storage Racks

In practice, storage racks are usually NSIT. Following Eynan and Rosenblatt (1994), we discuss an NSIT case with a rack face with a fixed height, $\sqrt{b}$, in the vertical direction, where b is a shape factor with $0 < b \leq 1$.

When class-based storage is applied, three types of class regions exist for the NSIT storage rack: square regions, rectangular regions, and a transient region as shown in Fig. 2.6. Square regions like class 1 or L-shaped regions with square outer boundaries (surrounding a square class 1). The boundary of such a region is $R_k \leq \sqrt{b}$, and the total area containing the previous k classes is $R_k \times R_k$. Rectangular regions like class 3 with $R_{k-1} \geq \sqrt{b}$. The total area containing the previous k classes is $R_k \times \sqrt{b}$. A transient region is like class 2. It is a region between the square area and the rectangular area. For this class, $R_{k-1} \leq \sqrt{b}$ and $R_k \geq \sqrt{b}$. The total area containing the previous k classes is $R_k \times \sqrt{b}$. We note that this region may not exist if the boundary of the $(k-1)$th region is exactly at $R_{k-1} = \sqrt{b}$. According to Eynan and Rosenblatt (1994) and with class $\hat{k}$ as the transient region, the following are true:

$$R_k = \begin{cases} \sqrt{j_k}, & \text{if } 1 \leq k < \hat{k}, \\ j_k/\sqrt{b}, & \text{if } k \geq \hat{k}. \end{cases}$$

$$t_k = \begin{cases} 2\left(R_k^3 - R_{k-1}^3\right)/\left(3R_k^2 - 3R_{k-1}^2\right), & \text{if } 1 \leq k < \hat{k}, \\ \left(b^{3/2} + 3\sqrt{b}R_k^2 - 4R_{k-1}^3\right)/\left(6\sqrt{b}R_k - 6R_{k-1}^2\right), & \text{if } k = \hat{k}, \\ \left(R_k + R_{k-1}\right)/2, & \text{if } k > \hat{k}. \end{cases}$$

Therefore, the travel-time model with a finite number of items in an NSIT system can be obtained as follows:

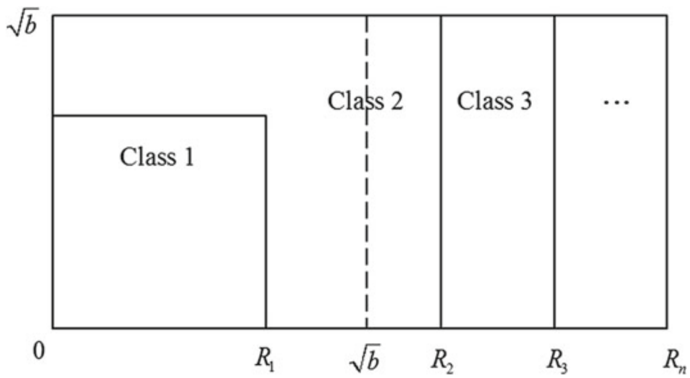

Fig. 2.6 Different types of classes in a NSIT warehouse

Model NSIT:

$$Min \qquad T_n = \sum_{k=1}^{\hat{k}-1} \frac{2\left(R_k^3 - R_{k-1}^3\right)}{3\left(R_k^2 - R_{k-1}^2\right)} \left(i_k^s - i_{k-1}^s\right) + \sum_{k=\hat{k}+1}^{n} \frac{\left(R_k + R_{k-1}\right)}{2} \left(i_k^s - i_{k-1}^s\right)$$
$$+ \frac{\left(b^{3/2} + 3\sqrt{b}R_k^2 - 4R_{k-1}^3\right)}{6\left(\sqrt{b}R_k - R_{k-1}^2\right)} \left(i_k^s - i_{k-1}^s\right), \tag{2.20}$$

$$s.t. \qquad Y_k = \sum_{l=1}^{k} N_l, \;\; i_k = Y_k \big/ N,$$
$$j_k = i_k^{(s+1)/2} + \sum_{l=1}^{k} N_l^{-\varepsilon} \left(i_l^{(s+1)/2} - i_{l-1}^{(s+1)/2}\right),$$
$$R_k = \sqrt{j_k}, \quad \text{for } 1 \le k < \hat{k},$$
$$R_k = j_k \big/ \sqrt{b}, \quad \text{for } k \ge \hat{k},$$
$$d.v. \qquad N_k > 0, \; k = 1, 2, \ldots, n.$$

The optimal T_n and $N_k > 0$, $k = 1, 2, \ldots, n$ can be found through the solution methodology presented in Sect. 2.3.3.

2.4.2 Bender's ABC Demand Curve

Although the ABC demand curve in Eq. (2.1) is widely used in the literature, Bender (1981) empirically showed that the following equation well represents the ABC demand curve in reality (Pohl et al. 2011).

$$G(i) = (1 + B)i \big/ (B + i). \tag{2.21}$$

where B is the shape factor of the ABC demand curve. The difference of these two demand curves can be clearly observed in Fig. 2.7 by choosing the 20 %/70 % curve as an example. To examine the effect of these different ABC demand curves on the optimal number of classes, we revise Model BM described in Sect. 2.3 as follows:

With regard to the difference between the two curves, the corresponding item demand, $D(i)$, and the cumulative required storage space for the first k classes, j_k, can be realized as the following:

$$D(i) = B(1 + B) \big/ (B + i)^2, \tag{2.22}$$

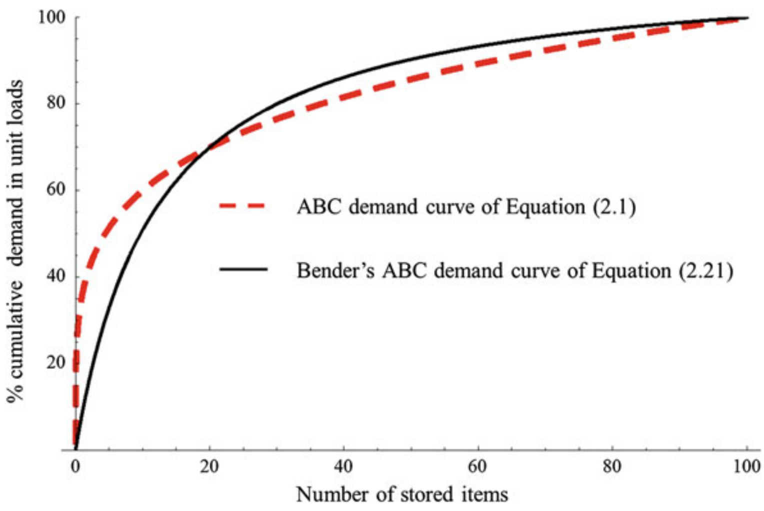

Fig. 2.7 Two different 20 %/70 % ABC demand curves

$$j_k = \left(\ln \frac{B + i_k}{B} + \sum_{l=1}^{k} \ln \frac{B + i_l}{B + i_{l-1}} \right) \Big/ \ln \frac{B+1}{B}. \tag{2.23}$$

Therefore, by replacing i_k with Y_k/N and with respect to $R_k = \sqrt{j_k}$, we obtain the model for Bender's ABC demand curve Model SMB ("SMB" represents "Solution Methodology with Bender's ABC curve") as follows:

Model SMB:

$$\text{Min} \quad T_n = \sum_{k=1}^{n} \frac{2\left(R_k^3 - R_{k-1}^3\right)}{3\left(R_k^2 - R_{k-1}^2\right)} \left(\frac{(1+B)Y_k}{NB + Y_k} - \frac{(1+B)Y_{k-1}}{NB + Y_{k-1}} \right), \tag{2.24}$$

$$s.t. \quad Y_k = \sum_{l=1}^{k} N_l,$$

$$R_k = \sqrt{\left(\ln \frac{B + i_k}{B} + \sum_{l=1}^{k} \ln \frac{B + i_l}{B + i_{l-1}} \right) \Big/ \ln \frac{B+1}{B}}, \tag{2.25}$$

$$d.v. \quad N_k > 0, \; k = 1, 2, \ldots, n.$$

Through Model NSIT, the optimal T_n and $N_k > 0, k = 1, 2, \ldots, n$ can be obtained similarly through the same solution methodology shown in Sect. 2.3.3.

2.4.3 Discrete Racks and Stochastic Demand

In this section, our basic model (Model BM) for continuous racks and deterministic demand in Sect. 2.3 is extended to a more realistic model with discrete storage locations and stochastic item demand. Different from the problem described in Sect. 2.2, the demand of each item follows a stochastic distribution over a unit-time period. The expected demand is determined according to the ABC demand curve. Item inventories are replenished according to a continuous review (r,S) policy with a replenishment lead time. Here, r is the reorder point, and S is the order-up-to level.

Shortages may occur and lead to backorders because item demand is stochastic and a delivery lead time exists for replenishing orders. Therefore, a maximum stockout probability is set. The stockout probability of item i in the warehouse must be less than α_i, with $(1-\alpha_i)$ defined as the service level (or fill rate). As a result, a safety stock ss_i for item i is needed to achieve this service level.

Correspondingly, in addition to the notations given in Sect. 2.2, the notations shown in Table 2.3 are defined for this model.

To obtain the travel-time model for an n-class-based storage system shown as Eq. (2.6), we need to get the average travel time for each storage class and the weighted expected turnover frequency of each class.

First, the cumulative fraction of the total expected demand of the i items can be expressed as follows according to Hausman et al. (1976):

$$G(i) = (i/N)^s = \sum_{x=1}^{i} \lambda(x) \Big/ \sum_{x=1}^{N} \lambda(x), \quad 0 < s \leq 1. \tag{2.26}$$

Second, we define τ_j as the one-way travel time from location j to the depot. For a discrete SIT system, τ_j can be expressed as follows:

$$\tau_j = \left\lceil \sqrt{j} \right\rceil, \tag{2.27}$$

Table 2.3 Notations used in Sect. 2.4.3

Notations	Definitions
r_i	Reorder point of item i
S_i	Order-up-to level of item i
l_i	Delivery lead time for the orders of item i, which is constant for each item
$\lambda(i)$	Expected demand (i.e., expected number of retrievals) of item i in a unit-time period with $\lambda(i) \geq \lambda(i+1)$ for all i. The expected demand of the item over the delivery lead time is $l_i\lambda(i)$
$f_i(\cdot)$	Demand probability density function of item i during the delivery lead time l_i
$F_i(\cdot)$	Cumulative demand distribution function of item i during the delivery lead time l_i

where $\lceil\sqrt{j}\rceil$ represents the smallest integer number not less than $\sqrt{j}$, and the travel speed is supposed to be one location per second. As a result, the average one-way travel time for storing (or retrieving) a unit load of class k can be obtained as follows:

$$t_k = \frac{1}{j_k - j_{k-1}} \sum_{j=j_{k-1}+1}^{j_k} \tau_j. \tag{2.28}$$

Therefore, similar to Eq. (2.6), the average travel time for the system can be obtained as follows:

$$T_n = \sum_{k=1}^{n} \frac{\sum_{j=j_{k-1}+1}^{j_k} \tau_j}{j_k - j_{k-1}} \left(\left(\frac{Y_k}{N}\right)^s - \left(\frac{Y_{k-1}}{N}\right)^s \right). \tag{2.29}$$

With the description given so far, the problem now is to determine the relationship between j_k and Y_k. First, the function of the required storage space of item i, $a_i(N_k)$ for determining $j_k - j_{k-1}$ can be obtained through simulations similar to Model BM as follows:

$$a_i(N_k) = 0.5\left(1 + N_k^{-\varepsilon}\right)(S_i - r_i) + ss_i. \tag{2.30}$$

Thereafter, the required storage space for class k can be obtained as follows:

$$\begin{aligned} j_k - j_{k-1} &= \left\lceil \sum_{i=Y_{k-1}+1}^{Y_k} a_i(N_k) \right\rceil \\ &= \left\lceil \sum_{i=Y_{k-1}+1}^{Y_k} \left(0.5\left(1 + N_k^{-\varepsilon}\right)(S_i - r_i) + ss_i\right) \right\rceil, \end{aligned} \tag{2.31}$$

where $j_0 = 0$.

Consequently, the total required storage space of the first k classes is equal to the following:

$$j_k = \left\lceil \sum_{l=1}^{k} \sum_{i=Y_{l-1}+1}^{Y_l} \left(0.5\left(1 + N_l^{-\varepsilon}\right)(S_i - r_i) + ss_i\right) \right\rceil, \tag{2.32}$$

where $N_l = Y_l - Y_{l-1}$ and $Y_0 = 0$.

Considering the previous analysis, the relationship between j_k and Y_k expressed in Eq. (2.32), and the objective function given in Eq. (2.29), we obtain the following Model DSM (i.e., discrete-stochastic model for discrete racks and stochastic demand) to determine the optimal class allocations of the AS/RS in the discrete space scenario.

Model DSM:

$$
\begin{aligned}
&Min \quad && T_n = \sum_{k=1}^{n} \frac{\sum_{j=j_{k-1}+1}^{j_k} \tau_j}{j_k - j_{k-1}} \left(\left(\frac{Y_k}{N}\right)^s - \left(\frac{Y_{k-1}}{N}\right)^s \right), \qquad (2.33)\\
&s.t. && \tau_j = \left\lceil \sqrt{j} \right\rceil, \;\; Y_k > Y_{k-1} \text{ and Eq. (2.32)},\\
&d.v. && Y_k > 0, \;\; k = 1, 2, \ldots, n,
\end{aligned}
$$

where $j_0 = Y_0 = 0$ is known.

Similarly, this model can be solved by the methodology provided in Sect. 2.3.3.

2.5 Numerical Illustrations

This section provides detailed numerical illustrations to present in detail the results of the models deduced in this chapter and to show managerial insights to help warehouse managers make useful decisions. In Sect. 2.5.1, the results of Model CM from Sect. 2.2 and those of our new Model BM from Sect. 2.3 are compared under different ABC demand curves. Section 2.5.2 presents the results of the extended models with respect to NSIT storage racks, Bender's ABC demand curve, and discrete rack and stochastic item demand.

2.5.1 Base Examples: Results for Basic Model

In our base example, the total number of items in the system is $N = 100$, and the space-sharing factor is $\varepsilon = 0.22$ (the average value, obtained through simulation, see Appendix A). This section shows the comparison between Model BM and Model CM for different numbers of classes. We solve both models for 1 class to 100 classes. The results for the optimal travel time T_n as a function of the number of classes n are shown in Fig. 2.8 under four different ABC demand curves, with $s = 1(20\,\%/20\,\%)$, $0.431(20\,\%/50\,\%)$, $0.222(20\,\%/70\,\%)$, and $0.065(20\,\%/90\,\%)$. The corresponding required storage space of the system is shown in Fig. 2.9.

The results shown in Figs. 2.8 and 2.9 imply the following:

(i) The optimal number of classes n^* is small, and $n = 3$ provides near-optimal solutions in all cases tested. In Fig. 2.8, $n^* \leq 5$ for all our examples, which cover all practical values of the ABC demand curves. A small number of classes is very close to the warehousing practice, in which only three storage classes are often used (Roodbergen and Vis 2009).

(ii) Travel time is insensitive to the number of classes in a wide range beyond the optimum. In Fig. 2.8, the differences in travel time between $n = 3$ and $n = 8$ are basically negligible. However, the range of the number of classes yielding the

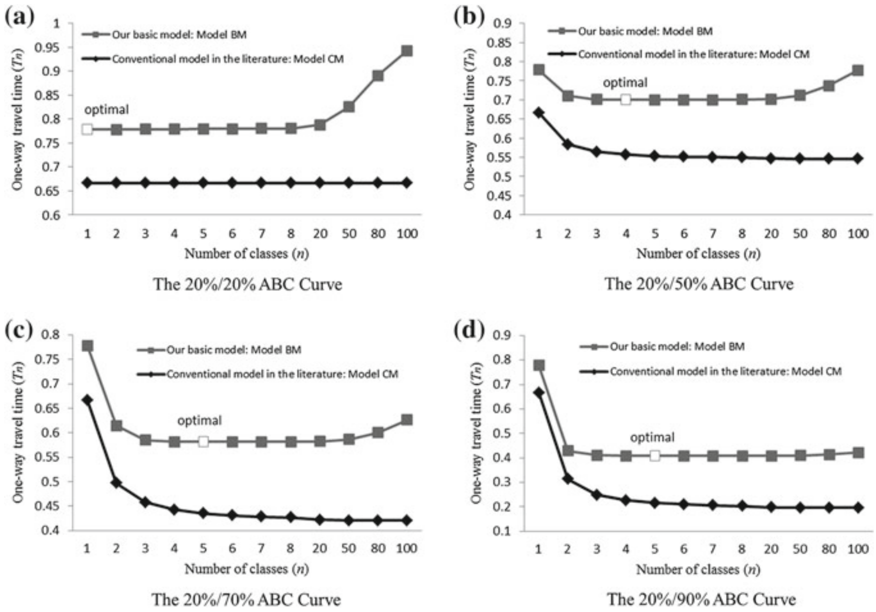

Fig. 2.8 Travel times in our basic model compared to those in the literature. **a** The 20 %/20 % ABC curve. **b** The 20 %/50 % ABC curve. **c** The 20 %/70 % ABC curve. **d** The 20 %/90 % ABC curve.

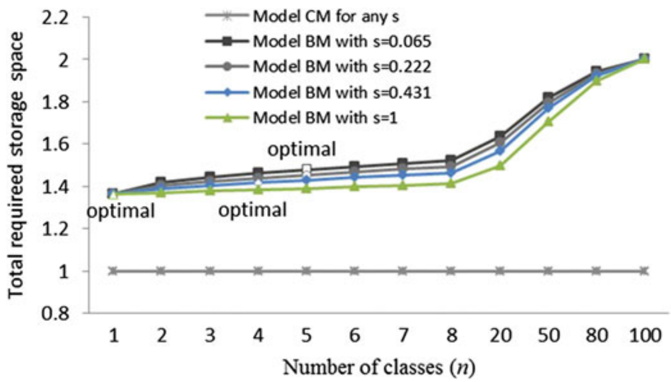

Fig. 2.9 Total required storage space as a function of the number of classes

near-optimal travel time depends on the ABC demand curves. For instance, if we define that the acceptable gap is 1 % for the 20 %/20 % curve, then the range of 1 class to 17 classes is acceptable, whereas for the 20 %/90 % curve, the acceptable range is from 3 classes to 75 classes. If the acceptable gap is 5 %, then the range of 1 class to 44 classes is acceptable for the 20 %/20 % curve and that of 3 classes to 100 classes is all acceptable for the 20 %/90 % curve. That is, a full turnover-based policy is acceptable for this curve. Therefore, managers should not hesitate to select any reasonable small number of classes because the result is essentially near optimal. This result is also true even when the required storage space is considered because the required storage space does not increase significantly from $n = 3$ to 8 as shown in Fig. 2.9.

(iii) The relative gaps between the travel times of our Model BM and those of Model CM increase with n in all examples. Even at $n = 1$, the gaps are still quite large (almost 15 % for all examples in Fig. 2.8). Therefore, warehouse managers should not only simply adopt the average inventory level as the required storage space for the warehousing system. The underestimation of required storage space may lead to managers' incorrect decisions, such as response time estimation and cost budgeting.

(iv) Compared with the required storage space adopted in conventional research, the required storage space of Model BM and that of Model CM have a significant gap that is usually more than 30 % and can be as large as 100 % when dedicated storage is adopted. Consequently, warehouse managers should consider the difference among the required storage spaces when the storage policy changes, and this result provides warehouse managers a direction about warehouse capacity design.

2.5.2 *Results for Extended Models*

To verify the robustness of the results obtained in Sect. 2.5.1, this section presents the numerical results for the extended models given in Sect. 2.4. Noting that the values of parameters are different section between Model DSM and the others, we first show the results for the NSIT storage rack and Bender's ABC curve in Sect. 2.5.2.1 and then provide the results of Model DSM in Sect. 2.5.2.2.

2.5.2.1 Results for the NSIT Storage Racks and Bender's ABC Demand Curve

This section presents the results for the NSIT storage racks and Bender's ABC demand curve. The results shown in Fig. 2.10 are based on a 20 %/70 % curve, which is a common case in practice. The results based on other curves with different skewness are omitted here because they have the same trend.

The results show that the main findings of Sect. 2.5.1 still hold for different storage racks and different kinds of ABC demand curves: a small number of classes yield the minimum travel time of the system, and any number of classes around the optimal one is a near-optimal solution.

In particular, an NSIT storage rack leads to longer travel times than a SIT rack because the optimal rack configuration is not NSIT but SIT (as shown in Fig. 2.10a). In Fig. 2.10b, the travel time of the conventional ABC curve (Eq. (2.1)) is shorter than

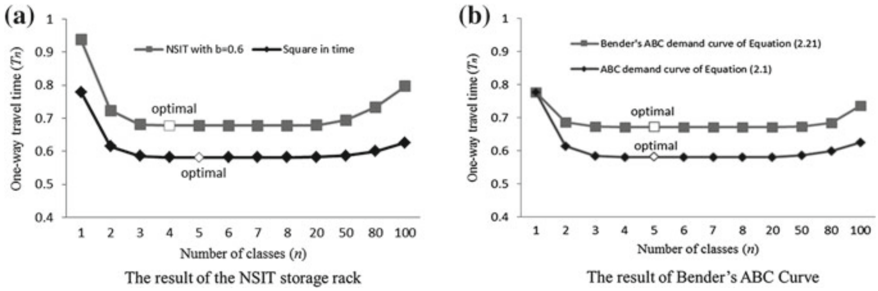

Fig. 2.10 Travel time for Model NSIT and Model SMB. **a** The result of the NSIT storage rack. **b** The result of Bender's ABC curve

that of Bender's ABC curve (Eq. (2.21)) because the conventional ABC demand curve is more skewed (as shown in Fig. 2.7).

2.5.2.2 Results for Discrete Racks and Stochastic Demand

The basic parameters in this section are as follows: $N = 100$ items are stored in the warehousing system; the total annual demand of all the items is $\sum_{i=1}^{N} \lambda(i) = 10,000$; and $K = 2$ and $l_i = 1/50$ year for $i = 1, 2, \ldots, N$. A continuous review order-up-to level replenishment policy is adopted for item i, $i = 1, 2, \ldots, N$ with the required service level $1 - \alpha_i = 95\,\%$ for a 20 %/70 % demand curve. The demand of each item follows a normal distribution with $\sigma_i/\mu_i = 0.2$ and $\mu_i = \lambda(i)$ for all $i = 1, 2, \ldots, N$. The reorder point r_i is chosen according to the service level as $r_i = F_i^{-1}(1 - \alpha_i)$. Thereafter, the safety stock level can be obtained as $ss_i = r_i - l_i\lambda(i)$. Finally, according to Kapalka et al. (1999), the order-up-to level is $S_i = r_i + \sqrt{2K\lambda(i)}$.

The results of Model DSM for discrete racks and stochastic demand are given in Figs. 2.11, 2.12 and 2.13. The comparison of the results with those of Model BM can be found in Fig. 2.11, in which the results of Model DSM are normalized through the replacement of R_j with $R_j \Big/ \sqrt{\sum_{i=1}^{N} Q(i)/2}$, comparable with the normalized results of Model BM. We only present the result of $s = 0.222$ for an example in Fig. 2.11 because the results of different ABC demand curves are similar. Figures 2.12 and 2.13 show the sensitivity results of the optimal number of classes and the corresponding required storage space with varying parameters.

The results indicate that our major findings in Sect. 2.5.1 hold in the case of discrete rack and stochastic demand; a small number of classes lead to a minimum travel time (as shown in Figs. 2.11a and 2.12) through varying number of items, service level, and demand variability. The minimum travel time is insensitive to the optimal number of classes around the optimum.

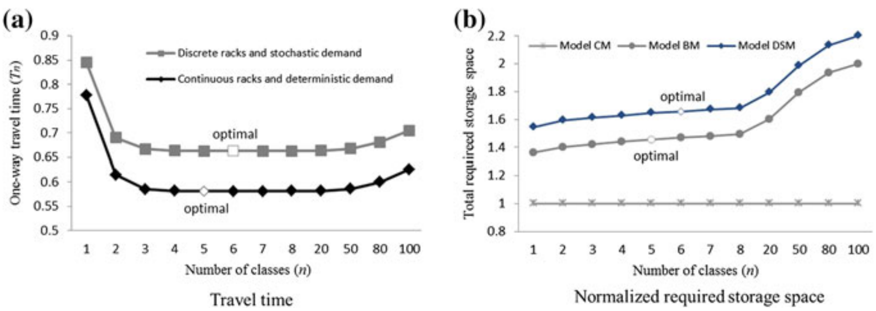

Fig. 2.11 Comparison of Model BM and Model DSM, $s = 0.222$. **a** Travel time. **b** Normalized required storage space

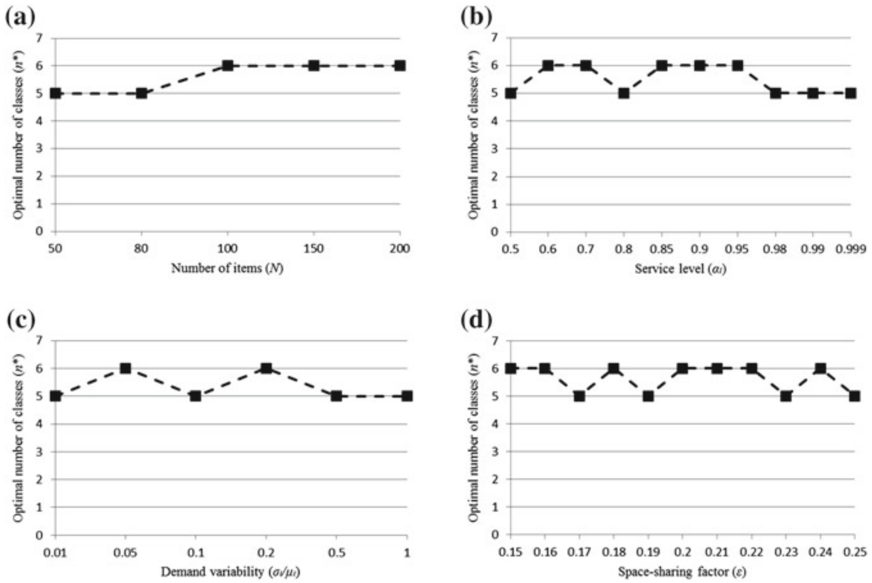

Fig. 2.12 Optimal number of classes, depending on N, α_i, σ_i/μ_i and ε. **a** Number of items. **b** Service level. **c** Demand variability. **d** Space-sharing factor

In particular, as shown in Fig. 2.10a, the travel time of Model DSM is longer than that of Model BM. The difference mainly comes from the extra space needed for the safety stock of the items in Model DSM. Figure 2.10b shows that more required storage spaces for Model DSM are needed, that is, at least 60 % larger than those of Model CM for eight storage classes and even 100 % for a large number of classes.

Figure 2.13 illustrates that the required storage space increases with the required service level, demand variability, and number of items in the system but decreases with the space-sharing factor. The required storage space convexly increases because

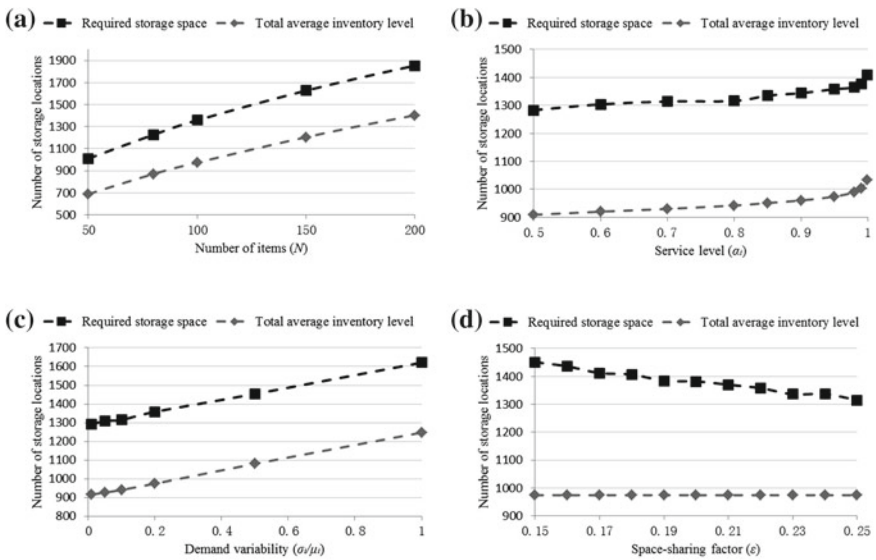

Fig. 2.13 Total required storage space at the optimal number of classes, depending on N, α_i, σ_i/μ_i and ε. **a** Number of items. **b** Service level. **c** Demand variability. **d** Space-sharing factor

the safety stock increases with the service level convexly. The required space is largely linear in the demand variability σ_i/μ_i.

2.6 Chapter Summary

This chapter extends the research on class-based storage by considering a finite number of items to be stored in an AS/RS. Our results reveal that the optimal number of classes is relatively small, and three classes give the near-shortest travel time for the ABC demand curves between 20 %/50 % and 20 %/90 %. This finding contradicts the common idea in the literature that more storage classes are better in view of travel time.

The results also show a flat range of the number of classes yielding the near-optimal solution, which is dependent on the skewness of the demand curves. For example, for the 20 %/20 % curve, 17 classes lead to a 1 % increase in travel time compared with the minimum value, and 45 classes lead to more than 5 %. For the 20 %/90 % curve, a 1 % gap enables managers to adopt up to 75 classes, and the gap is less than 3.5 % for the 100-class system. Although the flat range is influenced by the demand curves, a common range provides a near-optimal solution for all the cases. This finding is important for warehouse managers because it suggests that any reasonable number of classes such as between 3 and 8 is near optimal. As a result, warehouse managers can freely change the number of classes when necessary (e.g., for space use purposes) because travel time is not sensitive to the number of classes.

Furthermore, by analyzing the extended models, we show that the results are robust for different storage rack shapes, different kinds of ABC demand curves, discrete storage locations, and stochastic item demand. These findings provide important managerial directions about item classification when class-based storage is adopted in a warehousing system.

Moreover, we reveal that the space needed for a warehouse with an optimal number of storage classes should be at least 30–50 % of the average inventory level. This result is in accordance with the practical knowledge that every warehouse needs a slack space to warrant a smooth operation. An important conclusion for the research is that the space-sharing effect cannot be ignored in the class-based storage system. Models assuming perfect space-sharing (i.e., by implicitly assuming an infinite number of items per class) underestimate the space requirements, the travel time needed, and investments in racks and equipment. This finding provides a clearer guidance for capacity design, instead of simply adopting the average inventory level, in a class-based storage system.

Therefore, further research on storage policies based on a finite number of items is called for because many studies use class-based storage and implicitly assuming an infinite number of items. The results of these studies should be revisited to address the consequence of assuming a finite number of items to be stored in the system. The results for a finite number of items may substantially differ from those for an infinite number of items. First, travel time results under the assumption of an infinite number of items per class are usually overly optimistic because a division in storage classes requires more storage space than that accounted for. Second, an increasing number of storage classes increases the response times rather than reducing them. Similar results will hold for parallel-aisle or fishbone-layout warehouses (Gue and Meller 2009). Our contribution to identify the tradeoff between travel time reduction by item ranking and increase through increased storage space leading to an optimal product and storage number of classes may also be applied to other areas where products are classified by some criterion.

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Storage Policies and Maintenance Support Strategies
in Warehousing Systems
Guo, X.
2016, XXIII, 104 p. 29 illus., 12 illus. in color., Hardcover
ISBN: 978-981-10-1447-5