

# Regression-Based Neural Network Simulation for Vibration Frequencies of the Rotating Blade

Atma Sahu and S. Chakravarty

**Abstract** The aim of this paper is to demonstrate the use of regression-based neural network (RBNN) method to study the problem of the natural frequencies of the rotor blade for micro-unmanned helicopter [3]. The training of the traditional artificial neural network (ANN) model and proposed RBNN model has been implemented in the MATLAB environment using neural network tools (NNT) built-in functions. The graphs for angular velocity ( $\Omega$ ) of the micro-unmanned helicopter are plotted for estimation of the natural frequencies ( $f_1$ ,  $f_2$ ,  $f_3$ ) of transverse vibrations. The results obtained in this research show that the RBNN model, when trained, can give the vibration frequency parameters directly without going through traditional and lengthy numerical solutions procedures. Succeeding this, the numerical results, when plotted, show that with the increase in  $\Omega$ , there is increase in lagging motion frequencies. Additionally, it is found that the increase in the lower mode natural frequencies is smaller than that of the higher modes. This finding is in agreement with the results reported in earlier research [3–5] carried out by employing Rayleigh–Ritz and FEM, respectively.

**Keywords** Transverse vibrations · Artificial neural network · Harmonic motion · Mean square error · Micro-unmanned helicopters · Rotor blade vibrations

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## 1 Introduction

The micro-unmanned helicopters are quite different from the conventional manned helicopters in their design scheme. Therefore, in the case of micro-unmanned helicopter, the rotor mechanism is altered in order to optimize the manufacturing costs [3] without compromising on its needed functionality. In this paper, however, for the prototype engineering design requirements, the vibrations of helicopter rotor blades, whether manned or unmanned, are of a major concern. The purpose of this paper is to use a regression-based neural network (RBNN) method [1] to solve the problem of studying the natural frequencies of the rotor blade for micro-unmanned helicopter [3]. With this R&D effort resulting in appropriate mathematical calculations, the design engineers are able to overcome blade resonance problems (maybe by putting damper on the blade or any other vibrations correction method). The authors choose not to go into the fluid (air) resistance motion problem of blades' airfoil system (Appendix Fig. 5).

## 2 Transverse Vibrations Analysis

In this paper, the rotor manipulation mechanism is based on the use of the inertia characteristic of the rotor and its elastic features as considered by J Lu [3]. Also, an equally important characteristic in rotor parametric manipulation is the blade shape change that can be affected by the leading and trailing edges of the entire airfoil system (Appendix Fig. 8). However, the authors in this research paper will limit the scope to RBNN-based analysis of the transverse vibrations of the rotor blade. Also, it is reasonable to assume that the blade length is very large compared to its width. For this reason, Euler–Bernoulli beam theory is adequate for our purposes. The scheme of the blade and notations (see Appendix Fig. 8) is adopted in this paper from Lü [3] to make comparisons of this work easier and comprehensible. In this paper, ANN model for blade vibrations has been undertaken. The training of network is performed using the pattern calculated with the help of Boundary Characteristic Orthogonal Polynomials (BCOPs) in the Rayleigh–Ritz method.

We adopt below the kinetic energy (KE) and potential energy (PE) equations as derived by Lü et al. [3]. Considering the conditions of small deflections, the KE of the rotor blade of length  $L$  is given by  $T$  as follows:

$$T = \frac{1}{2} \int_0^L \rho \left( \dot{u}^2 + [u^2 + (a+x)^2] \dot{\theta} + 2\dot{\theta}u(a+x) \right) dx;$$

and PE is given by  $U$  as follows:

$$U = \frac{1}{2} \int_0^L EI u_x^2 dx + \frac{1}{2} \int_0^L \left( \frac{1}{2} \rho \dot{\theta}^2 (L-x)^2 + \rho \dot{\theta}^2 a(L-x) \right) (u_x)^2 dx$$

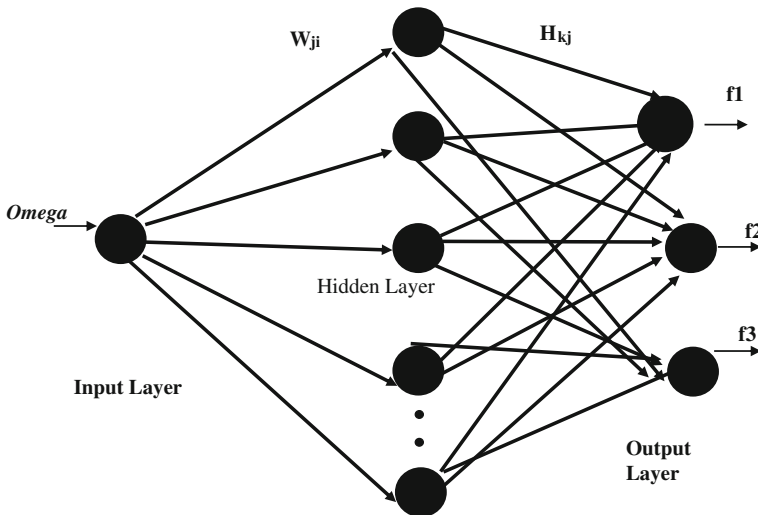
For harmonic motion, the blade deflection is given by  $u(x, t) = Y(x) \sin(\omega t + \phi)$ ; using  $u(x)$  in  $T$  and  $U$  above, Lagrangian is obtained. Following notations are used:

$L$  = Length of the blade (m),  $\rho$  = mass in unit length of the blade (kg/m),  $\omega = \dot{\theta}$  = angular velocity of rotor,  $EI$  = flexural rigidity of the blade ( $Nm^2$ ),  $XO'Y$  = Inertia reference frame,  $xOu$  = Flying reference frame,  $V_{ji}$  = Hidden layer weights, and  $W_{kj}$  = Output layer weights.

Substituting the linear combination of BCOPs in the Rayleigh–Ritz method for  $T$  and  $U$ , we may turn it to a standard eigenvalue problem. The solution of the standard eigenvalue problem then gives the natural frequencies at various rotational speeds [2]. The computations have been carried out by taking  $EI = 1.392 \text{ Nm}$ ,  $L = 0.15 \text{ m}$ ,  $\rho = 0.1260 \text{ Kg/m}$ . [2]. As such natural frequencies have been computed for the blade at various rotational speeds for the simulation in RBNN model. In the following paragraphs, ANN architecture is described for the estimation of natural frequencies for given values of  $\Omega$  which is the angular velocity parameter.

### 3 Identification of the RBNN Model: Solution Technique

Three-layer architecture for regression-based artificial neural network approach is considered here to understand the proposed model for solving the present problem. Figure 1 show the neural network used in the process. The input layer consists of single input as  $\Omega$  and the output layer consists of three outputs in the form of the corresponding frequency parameters  $f_1$ ,  $f_2$ , and  $f_3$ . Three cases of the number of nodes depending upon the proposed parameter of the methodology have been considered in the hidden layer to facilitate a comparative study on the architecture of the network. The output of the network is computed by regression analysis combined



**Fig. 1** ANN architecture used for estimation of frequencies for given values of  $\Omega$

with neural activation function performed at two stages, i.e., the stage of hidden layer and the stage of output layer. Number of neurons in the hidden layer depends upon the degree of the regression polynomial that is used to fit the data between input and output. If we consider a polynomial of degree  $n$ , then number of nodes in hidden layer will be  $(n + 1)$  and the corresponding  $(n + 1)$  coefficients of this polynomial (say,  $a_i, i = 0, 1, \dots, n$ ) are taken as the initial weights from input layer to the hidden layer ( $H_{kj}$ ). Architecture of the network for a polynomial of  $n$ th degree is shown in Fig. 1.

## 4 Numerical Results

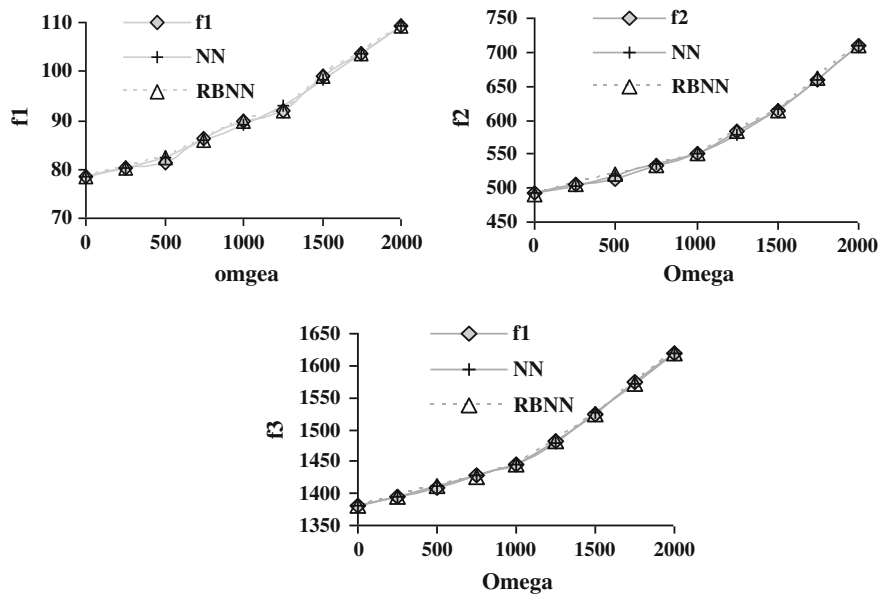
The training of the traditional artificial neural network (ANN) model and proposed RBNN model has been implemented in the MATLAB environment using neural network tools (NNT) built-in functions. Also, in the following paragraphs, the graphs for angular velocities ( $\Omega$ ) of the micro-unmanned helicopter are plotted for estimation of the natural frequencies ( $f_1, f_2, f_3$ ).

### 4.1 The Experiment 1

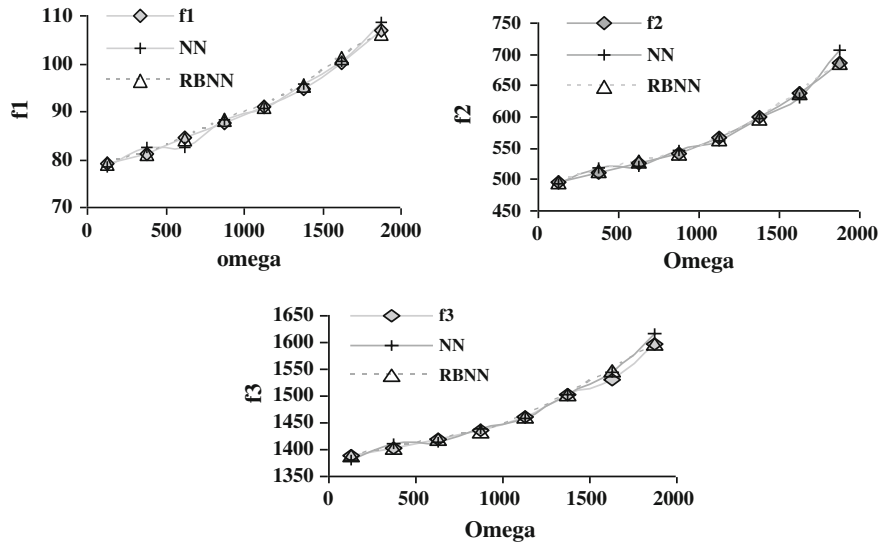
The training of the traditional ANN model and proposed RBNN model has been implemented for estimation of the frequencies with respect to  $\omega$  values. In the traditional model, the output of the network is computed by built-in transfer functions, namely, tan-sigmoid (`tansig`) and linear (`purelin`) of the neural network tool (NNT) performed at two stages, i.e., the stage of hidden layer and the stage of output layer. The connection weights interconnecting the neurons between different layers are taken through a random number generator built-in function in the NNT. The neural network based on this feedforward back propagation algorithm has been trained with Levenberg–Marquardt training function of the NNT.

### 4.2 The Experiment 2

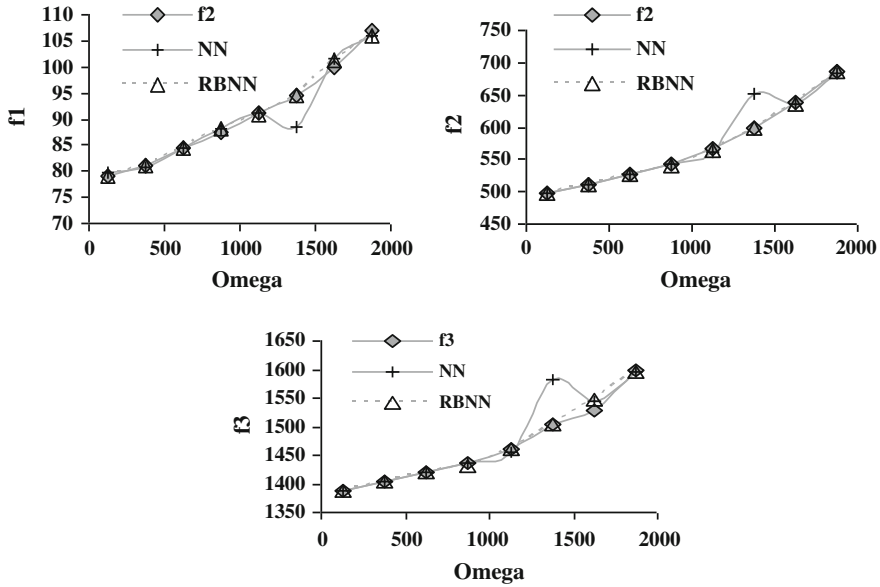
In proposed RBNN model, regression polynomials of degree three are fitted to the training patterns. The coefficients of this polynomial are taken as the connecting weights for the hidden layer, as described earlier. The output of the neurons in the hidden layer is calculated using activation function. At this stage, the error of the RBNN model is calculated and a decision is taken as to whether the network has been trained or not. If the tolerance level of the error is not achieved, the procedure is repeated; otherwise, we say that the network has got trained. In this case, the network has been converged with the desired accuracy as shown in the Fig. 1 for



**Fig. 2** Results of training of *RBNN* and *ANN* models for  $\Omega$  versus Frequency  $F_1$ ,  $F_2$ , and  $F_3$  (D-3)



**Fig. 3** Performance of the proposed regression-based neural network for  $\Omega$  versus Frequency  $F_1$ ,  $F_2$ , and  $F_3$  (D-3)



**Fig. 4** Performance of the proposed regression-based neural network for Omega versus Frequency F1, F2, and F3 (D-4)

the problems under consideration. The output of the network  $f_1$ ,  $f_2$ , and  $f_3$  and the mean square error (MSE) between neural and desired output are calculated. In this figure,  $f_1$ ,  $f_2$ , and  $f_3$  represent the desired output values, NN represents these values obtained by the traditional ANN models, and RBNN represents the values of these parameters obtained from the proposed model with four nodes in the hidden layer. The performance of the proposed model is given in the Fig. 2. The pattern characteristics of the traditional ANN model and RBNN model for degree four are incorporated in Fig. 3. The performance of the proposed model for degree four is given in the Fig. 4.

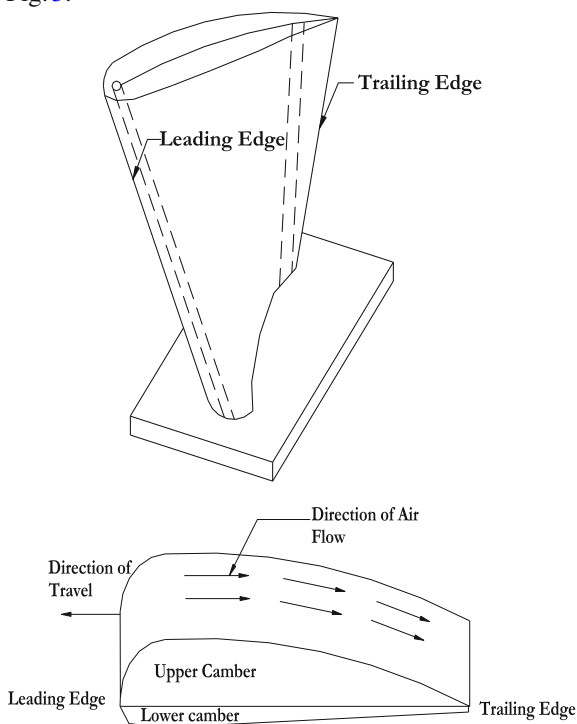
## 5 Conclusion

The RBNN method employed to solve fourth order partial differential equation for rotor blade in this paper gives a direct estimation of frequencies without going through traditional and lengthy numerical solutions procedures. The numerical results, when plotted, show that with the increase in Omega (angular velocity), there is increase in lagging motion frequencies. The increase in the lower mode natural frequencies is smaller than that of the higher modes. This finding is in agreement with the results reported in earlier researches [3–5] that have been carried out by employing Rayleigh–Ritz and FEM, respectively. Furthermore, RBNN soft computing method

used in this research is useful to solve other beam, plates, and shell vibration problems and guide engineers immensely in their structures design needs. Last of all, NN methods in general [2, 6] have attracted extensive attention in recent past as NN approaches have led many efficient algorithms help in exploring the intrinsic structure of data set.

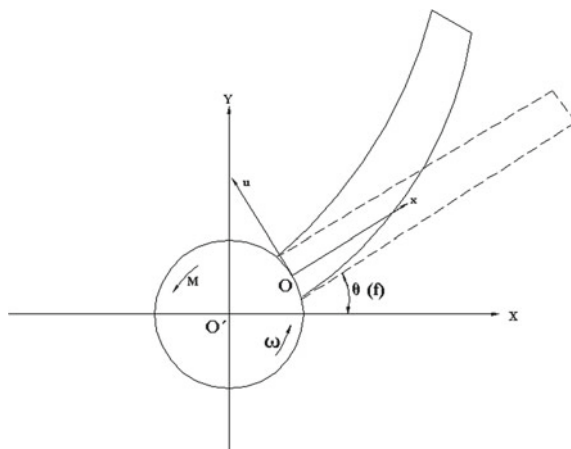
## Appendix

See Appendix Fig. 5.



Leading and trailing edges of a blade

**Fig. 5** Helicopter blade scheme



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Modern Mathematical Methods and High Performance  
Computing in Science and Technology

M3HPCST, Ghaziabad, India, December 2015

Singh, V.K.; Srivastava, H.M.; Venturino, E.; Resch, M.;  
Gupta, V. (Eds.)

2016, XXI, 309 p. 50 illus., 37 illus. in color., Hardcover

ISBN: 978-981-10-1453-6