

## Chapter 2

# Background of Heavy Quarkonium Physics

**Abstract** Since the discovery of  $J/\psi$  in 1974 [1, 2], heavy quarkonium physics has played an important role in revealing and in investigating the QCD at the interplay between the perturbative regime and the non-perturbative regime. However, till now, we are still unable to understand the heavy quarkonium production mechanism very well. In particular, we do not know which theory can describe its production at various colliders. In this chapter, we review the main theoretical background, recent progress, and challenges in heavy quarkonium production physics. The organization of this chapter is: in Sect. 2.1, we will give an introduction of some basic theoretical ideas and establish the notations and nomenclature; in Sect. 2.2, we will present the challenges of theories in confront of experiments.

### 2.1 Theoretical Framework

Heavy quarkonium is a kind of color-singlet (CS) bound state  $H_{Q\bar{Q}}$ , which is composed of a pair of heavy-flavor quarks  $Q\bar{Q}$ . For example,  $J/\psi(\Upsilon)$  is composed of a charm (bottom) quark and a charm (bottom) antiquark, while  $B_c^+$  meson is composed of a charm quark and a bottom antiquark. The mass of a heavy quarkonium is almost the sum of the masses in its constituent quarks, which indicates that heavy-flavor quarks in a heavy quarkonium meson are nonrelativistic and their relative velocity  $v$  in the rest frame of the meson should be a small parameter. For charmonium,<sup>1</sup> the typical  $v^2 \simeq 0.3$ , whereas for bottomonium,  $v^2 \simeq 0.1$ . Hence, there are three intrinsic scales in a heavy quarkonium meson: the heavy-flavor quark mass  $m$ , the relative momentum of the heavy quark pair  $mv$ , and the binding energy of the heavy quark pair  $mv^2$ .

Let us consider a process with a heavy quarkonium production at hadron colliders or  $e^-e^+$  colliders. Since the heavy-flavor quark pair  $Q\bar{Q}$  is produced in a hard-scattering process, a hard scale  $Q_{hard}$  should enter into the description of the production process. In a hadroproduction process, the hard-scattering scale  $Q_{hard}$  is

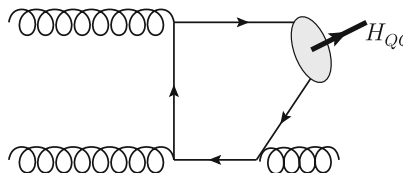
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<sup>1</sup>Charmonium is composed of a pair of charm quark and antiquark, whereas bottomonium is composed of a pair of bottom quark and antiquark.

the order of the transverse momentum  $p_T$  of the heavy quarkonium, while in  $e^-e^+$  annihilation process,  $Q_{hard}$  is the order of the three-dimensional momentum  $p^*$  of the heavy quarkonium in the rest frame of the initial colliding particles. It would be imagined that the production could be understood in terms of two district steps. The first step is producing a quasi-collinear heavy quark pair  $Q\bar{Q}$  at hard scale  $Q_{hard} \gtrsim m$ . If the quark pair is moving too far away from each other, it would be difficult to bind together afterward. This scattering process should happen at “short” distance  $1/Q_{hard}$ . After the hard-scattering process,  $Q\bar{Q}$  will evolve into a color-singlet hadron state (i.e., a heavy quarkonium  $H_{Q\bar{Q}}$ ) with a probability smaller than unity. The dynamical scales involved in the second evolution process are  $mv$  and  $mv^2$ , and it happens at a relatively “long” distance.

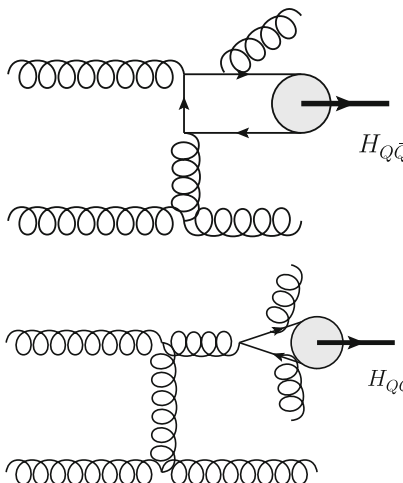
The quantitative description of this physical picture relies on the validation of a factorization theorem; i.e., the short-distance physics at scale  $Q_{hard}$  can be completely separated from the long-distance physics at  $mv$ ,  $mv^2$  and at the QCD scale  $\Lambda_{QCD}$ . No important interference terms will contribute into the physical observables. To prove such factorization theorem, one must be able to express the heavy quarkonium amplitude into a sum of products of IR safe short-distance coefficients (SDCs) with well-defined long-distance matrix elements (LDMEs). Due to  $Q_{hard} \gg \Lambda_{QCD}$ , the short-distance coefficients can be calculated in perturbative QCD (pQCD), while the long-distance terms are the only non-perturbative stuff. A better situation is the long-distance part has the less independent freedom than experiments and it can be determined once from some experiments and predicts the others. There are several approaches in describing the evolution of a heavy-flavor quark pair into a heavy quarkonium meson (see reviews in [3–5]): the color-singlet model (CSM)(see, e.g., Refs. [6–9]), the color-evaporation model (CEM) [10–15], the non-relativistic QCD (NRQCD) effective theory [16] and the fragmentation-function (FF) approach [17–24].

In the CSM, the heavy quark pair  $Q\bar{Q}$  is produced in CS states at the hard-scattering process with scale  $Q_{hard}$ . The quantum numbers of the quark pair are the same with those of the heavy quarkonium. The LDME of the quarkonium  $H_{Q\bar{Q}}$  in the CSM can be estimated from its decay rates measurements or in a potential model [25]. The CSM was successful in describing the heavy quarkonium production rates at the relatively low-momentum transfer  $Q_{hard} \sim m$  [26–28] regime. However, it was found that the CSM underestimated the production rate of  $J/\psi$  and  $\psi(2S)$  at Tevatron [29–31] at larger  $p_T$ . The discrepancy can be reduced a lot if one includes higher-order contributions [32, 33]. Moreover, the polarization of  $J/\psi$  and  $\psi(2S)$  in the CSM will be changed from being transverse at leading order (LO) to be longitudinal at NLO [34]. This behavior is understood as new topologies only appear at higher orders. In specific, at LO, the parton-level distribution  $d\hat{\sigma}/dp_T^2$  is scaling as



$$p_T \gg m \sim \alpha_S^3 \frac{(2m)^4}{p_T^8}, \quad (2.1)$$

whereas at NLO and next-to-next-to-leading order (NNLO), the corresponding scalings are



$$p_T \gg m \sim \alpha_S^4 \frac{(2m)^2}{p_T^6},$$

$$p_T \gg m \sim \alpha_S^5 \frac{1}{p_T^4}.$$

$$(2.2)$$

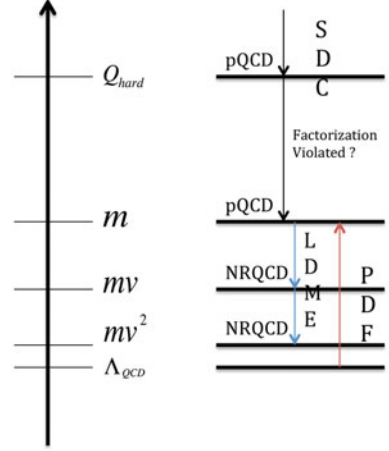
Because of the  $p_T$ -enhancement in the new topologies, the yields and polarization patterns of the heavy quarkonium will completely change in the CSM. After considering up to  $\mathcal{O}(\alpha_S^5)^2$  contributions [33, 35], the yields and polarizations of  $\psi$  are still in contradicted with the experimental data measured at the Tevatron and the LHC (e.g., see Ref. [36]). Moreover, in the production or the decay processes of the P-wave mesons, for example  $\chi_c$ , the CSM is known to be lacking the ability to cancel IR divergences.

The CEM is based on the principle of quark-hadron duality, in which the heavy-flavor quark pair evolves into a heavy quarkonium only when the invariant mass of the quark pair is less than the threshold for producing a pair of open-flavor heavy mesons. The probability of the quark pair evolving into a quarkonium is a universal and scale-independent constant. It also explores the large predictive power of the theory. However, the drawback of this model is that it predicts a fixed cross-sectional ratios of various heavy quarkonium productions, which is apparently in contradiction with the current experiments.

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<sup>2</sup>Only double real contribution is included at  $\mathcal{O}(\alpha_S^5)$ .

**Fig. 2.1** Various scales for the effective theory NRQCD setup



The FF approach is derived from the collinear factorization theorem that a heavy quarkonium production cross section can be factorized in terms of convolutions of parton-level production cross sections with light-cone like FFs [17–21] in the large  $p_T$  limit  $p_T \gg m$ . The leading power in  $m/p_T$  is coming from the terms of convolutions of a single parton production cross section with a single parton FF [17]. The factorization theorem of the subleading power of  $m/p_T$  contribution was introduced and proven in Refs. [18, 20]. Later, in Ref. [21], it was also proven in the Soft-Collinear Effective Theory (SCET) [37, 38]. The subleading contribution is given by  $Q\bar{Q}$  production cross sections convoluted by double parton FFs. The good thing in FF approach is that the collinear factorization theorem has been proven rigorously to subleading power of  $m/p_T$  and the large logarithms of  $\log \frac{p_T^2}{(2m)^2}$  can be systematically resummed via renormalization group running of FFs. However, it only applies in the large  $p_T$  regime and the predictive power relies heavily on the knowing of the non-perturbative FFs.

The state-of-the-art theory in describing the evolution of  $Q\bar{Q}$  in heavy quarkonium  $H_{Q\bar{Q}}$  is the NRQCD effective theory [16]. Its applicability to quarkonium production processes is based on the validity of NRQCD factorization theorem. NRQCD reproduces the complete QCD dynamics at the scales of  $mv$  and  $mv^2$ . The typical scales in NRQCD are shown in Fig. 2.1. In the regime of the dynamic scales at the order of  $mv$  and  $mv^2$ , the full QCD Lagrangian can be reorganized in the expansion of operators in different powers of  $v$ . The vacuum expectation values of some operators can be interpreted as the probability for a  $Q\bar{Q}$  pair evolving into a heavy quarkonium  $H_{Q\bar{Q}}$  and are the LDMEs in NRQCD. Hence, the inclusive cross section for a heavy quarkonium production at large momentum transfer can be written as a sum of SDCs times the NRQCD LDMEs. In addition to CS states, the hard-scattering production of  $Q\bar{Q}$  can also be in color-octet (CO) states. At the evolution stage, the CO  $Q\bar{Q}$  radiates  $mv$  gluon(s) to form CS mesons. We call such mechanism, which does not happen in CSM, as CO mechanism (COM). Although in general there are infinite

intermediate states<sup>3</sup> contributing to a heavy quarkonium  $H_{Q\bar{Q}}$  production process, according to the velocity scaling rule [16], its number is always finite when one truncates it at some specific accuracy. For example, in Table 2.1, we show the  $Q\bar{Q}$  Fock states contribute to some quarkonia up to  $\mathcal{O}(v^7)$ . Because the only unknown pieces in NRQCD are the vacuum expectation values of the NRQCD operators, the predictive power of NRQCD is much larger than that of the FF approach. Actually, the practical applications of the FF approach in heavy quarkonium production are based on NRQCD factorization [17, 22–24, 39–44]. The non-perturbative LDMEs play the similar role as the parton-distribution functions (PDFs) in QCD. Moreover, the non-cancellation of the IR divergences in the production of a P-wave quarkonium in the CSM can be naturally absorbed into the renormalization of the CO S-wave Fock state in NRQCD. Like the CSM, we have the similar  $p_T$  scaling behavior in producing CO intermediate states, for example

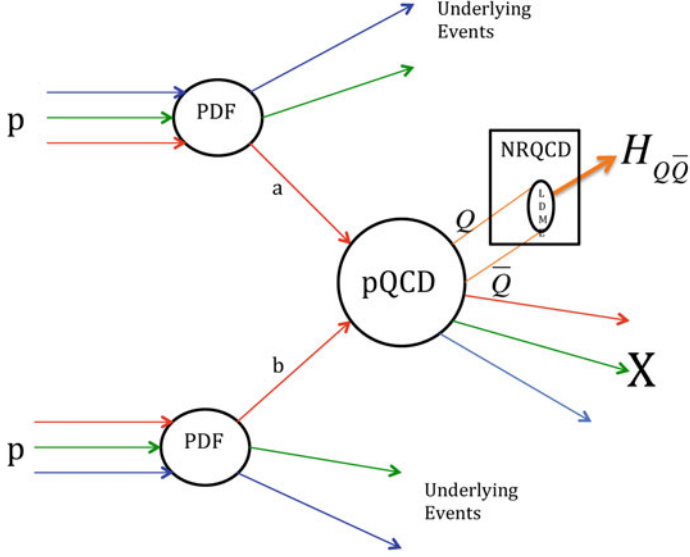
$$\begin{aligned}
 p_T \gg m & \sim \alpha_S^3 \frac{1}{p_T^4}, \\
 p_T \gg m & \sim \alpha_S^3 \frac{(2m)^2}{p_T^6}, \\
 p_T \gg m & \sim \alpha_S^4 \frac{1}{p_T^4}.
 \end{aligned} \tag{2.3}$$

However, the statement that NRQCD can describe the quarkonium production heavily relies on the validity of NRQCD factorization theorem, which is now only a conjecture and is still lacking a compelling proof. Schematically, based on QCD factorization and NRQCD factorization, a heavy quarkonium hadroproduction cross section  $d\sigma(pp \rightarrow H_{Q\bar{Q}} + X)$  can be written as

<sup>3</sup>We call the CS and CO intermediate states as Fock states.

**Table 2.1** Power counting of Fock states in NRQCD velocity scaling rule [16] for some quarkonia

Power counting	$\eta_c, \eta_b$	$J/\psi, \psi(2S), \Upsilon$	$h_c, h_b$	$\chi_{cJ}, \chi_{bJ}$
$v^3$	$1S_0^{[1]}$	$3S_0^{[1]}$	—	—
$v^5$	—	—	$1P_1^{[1]}, 1S_0^{[8]}$	$3P_J^{[1]}, 3S_1^{[8]}$
$v^7$	$1S_0^{[8]}, 3S_1^{[8]}, 1P_1^{[8]}$	$1S_0^{[8]}, 3S_1^{[8]}, 3P_J^{[8]}$	—	—

**Fig. 2.2** An illustrative graph for the QCD factorization and the NRQCD factorization in describing a heavy quarkonium  $H_{Q\bar{Q}}$  production at a proton–proton collider

$$d\sigma(pp \rightarrow H_{Q\bar{Q}} + X) = \sum_n d\sigma(pp \rightarrow Q\bar{Q}[n] + X) \times \langle \mathcal{O}^{H_{Q\bar{Q}}}(n) \rangle, \quad (2.4)$$

where

$$d\sigma(pp \rightarrow Q\bar{Q}[n] + X) = \sum_{a,b} f_{a/p}(x_1) f_{b/p}(x_2) |\mathcal{A}(ab \rightarrow Q\bar{Q}[n] + X)|^2. \quad (2.5)$$

Its graphic representation is displayed in Fig. 2.2.

## 2.2 Challenges

### 2.2.1 Challenges in Proving NRQCD Factorization

It is a difficult task to prove the validity of NRQCD factorization theorem. One difficulty is the NRQCD factorization formula [16] is only a perturbative expansion of  $\alpha_S$  and  $v$  in SDCs, which makes the NRQCD results usually suffer from large higher-order corrections. In other words, we need an optimal reorganization of the heavy quarkonium production cross sections into simultaneously  $\alpha_S$  and  $m/p_T$ . The FF approach provides such a reorganization. As proposed in Refs. [18, 19, 45], the proof can be done into two steps:

1. One should first prove that the inclusive heavy quarkonium production cross section can be written as convolutions of the heavy quarkonium FFs with the short-distance parton production cross sections. The factorization theorem in leading power of  $m/Q_{hard}$  was proven at electron–positron annihilation [46] and at hadronic collisions [18], whereas up to subleading power in hadroproduction, it was proven recently in Ref. [20].
2. One then tries to prove that the FFs can be written as a sum of SDCs times NRQCD LDMEs, which is the only obstacle to proving the NRQCD factorization theorem up to subleading power. A proof of the second step requires that all soft singularities can cancel or can be absorbed into the renormalization of NRQCD LDMEs and all collinear singularities and spectator interactions can be absorbed into PDFs to all orders in  $\alpha_S$ .

Two possible situations might violate the factorization at step 2. One difficulty is when there is a gluon with the momentum of order  $m$  in the heavy quarkonium rest frame, and it has nonvanishing soft exchanges in the FF [5, 47]. Another situation is when there are comoving heavy quark(s) with the  $Q\bar{Q}$  pair that forms  $H_{Q\bar{Q}}$ , the heavy quark(s) can have a soft-color exchange with the  $Q\bar{Q}$  pair [48, 49]. This process was not considered in the NRQCD factorization picture and would break its factorization.

If NRQCD factorization was proven to be valid to all orders in  $\alpha_S$  and up to subleading in  $m/Q_{hard}$ , one might expect that the non-factorizable corrections would be at least  $m^3/Q_{hard}^3$ . It indicates that NRQCD will fail to describe the heavy quarkonium production physics when  $Q_{hard}$  is not large enough compared to  $m$ .

### 2.2.2 NRQCD Versus Experiments

It is apparent that NRQCD is a more rigorous theory than CSM and CEM. In particular, when one takes  $v \rightarrow 0$ , NRQCD will collapse into CSM. However, till now, it is still a debate how significant part will CO states contribute to the heavy quarkonium production physics. Since it is really a long story, we will try to recall it briefly.

More than a decade ago, the first measurements [29–31] by the CDF collaboration of the yields of  $J/\psi$  and  $\psi(2S)$  production at Tevatron Run I revealed an unexpected large discrepancy with the theoretical calculations. The observed production rates were more than an order of magnitude greater than the LO CSM theoretical result, which motivates a lot of theoretical studies on NRQCD since the unknown CO contributions can compensate the large gap. At LO in  $\alpha_S$ , the dominant contribution at sufficiently large  $p_T$  is coming from a gluon fragmenting into a S-wave CO state. Consequently, LO NRQCD predicts a transverse polarization in its helicity frame [50–52]. It is in contradiction with the CDF measurements [53, 54], which was called “polarization puzzle” in NRQCD. On the other side, NLO QCD corrections to  $J/\psi$  and  $\Upsilon$  hadroproduction enhance the production rates substantially [32] due to the  $p_T$ -enhancement topologies. Later on, part of  $\mathcal{O}(\alpha_S^5)$  contribution was also taken into account [33]. All these efforts significantly reduce the discrepancy between CSM theoretical predictions and experiments [29–31]. After that, more and more theoretical advances are dedicated to higher-order computations in  $\alpha_S$  [24, 34, 55–69] and in  $v^2$  [70, 71]. It explores that CSM cannot explain the polarization data as well as large  $p_T$  yields data [35, 57]. However, as far as the  $p_T$ -integrated yield is concerned, CSM contribution agree relatively well with the existing data [28, 72], which is dominated by  $p_T \sim m$  phase space. We guide the reader to a recent review [73] on heavy quarkonium polarization in hadronic collisions.

Based on our original works [66, 68, 74, 75], in the next few chapters, we will show that the NRQCD yields and polarizations of  $J/\psi$  and  $\psi(2S)$  in hadronic collisions are compatible with the Tevatron and the LHC data when  $p_T > 11\text{GeV}$ . It is compatible with the recent observations by two theoretical groups [24, 69], which implies that NRQCD factorization may be only valid at large enough  $p_T \gg m$  regime. However, it is really tough to say that all of the available data outside this  $p_T$  constraints are lacking a theory to describe them. Given the large uncertainties, the data in small momentum transfer regime (including the data at B factories [76–83], in photoproduction [84–86], in fixed-target production [27, 87] and in hadroproduction [28, 72]) are described relatively well by CSM. A more severe thing is we do not have a satisfactory theory to describe the data between the small and large momentum transfer regions. Moreover, in exclusive heavy quarkonium production processes, only CS state can be taken into account.

More measurements on various heavy quarkonium production processes and on various observables are necessary in the future to assess the universality of NRQCD LDMEs. Theoretically, many interesting quarkonium-associated production processes have been promoted to NLO level in  $\alpha_S$ . They are  $J/\psi + \gamma$  [88, 89],  $J/\psi + Z$  [90, 91],  $J/\psi + W^\pm$  [92, 93] and  $J/\psi + J/\psi$  [94]. Finally, other interesting processes deserved mentioning here are  $J/\psi + c\bar{c}$  and  $\Upsilon + b\bar{b}$  [55], which may be important in understanding the factorization breakdown effects via the color-transfer mechanisms as proposed in Refs. [48, 49].



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