Preface

This book is designed to present recent results on some global well-posedness and asymptotic behavior of the solutions to non-classical thermo(visco)elastic models arising from physics, mechanics, and materials science such as thermoviscoelastic systems, thermoelastic systems of types II, III, and with second sound, Timoshenko-type system with past history. Some of the content of this book is based on the research carried out by the authors and their collaborators in recent years. Most of it has been previously published only in original papers, and some of the material has never been published until now. Therefore, the authors hope that the book will benefit both the interested beginner in the field and the expert.

This book is divided into nine chapters.

Chapter 1 is devoted to introducing some basic knowledge in modern analysis, some useful inequalities and basic theory of semigroups.

In Chap. 2, we investigate one-dimensional nonlinear thermoelasticity with thermal memory and second sound. Thermoelastic model is based on the continuum physics and thermodynamics, considering the distribution of the thermoelastic body deformation and temperature.

The classical model for the heat propagation turns into the well-known Fourier law

$$q + k\nabla\theta = 0, \tag{1}$$

where θ is temperature (difference to a fixed constant reference temperature), q is the heat conduction vector and k is the coefficient of thermal conductivity. The model using classic Fourier law exhibits the physical paradox of infinite propagation speed of signals. To eliminate this paradox a generalized thermoelasticity theory has been developed subsequently. The development of this theory was accelerated by the advent of the second sound effects observed experimentally in materials at a very low temperature. In heat transfer problems involving very short time intervals and/or very high heat fluxes, it has been revealed that the inclusion of the second sound effects to the original theory yields results which are realistic and very much different from those obtained with the classical Fourier's law.

The first theory, developed by Lord and Shulman [156], replaces (1) with the Cattaneo–Maxwell law

$$\tau_0 q_t + q + k\theta_x = 0. \tag{2}$$

The heat equation associated with (2) becomes hyperbolic and, hence, automatically eliminates the paradox of infinite speeds. The positive parameter τ_0 is the relaxation time describing the time lag in the response of the heat flux to a gradient in the temperature. In this chapter, we shall prove the global existence and exponential stability of solutions to nonlinear thermoelastic equations with second sound provided that the initial data are close to the equilibrium and the relaxation kernel is strongly positive definite and decays exponentially.

In Chap. 3, we consider the Timoshenko-type system with a past history. The model of a homogeneous Timoshenko beam is described by a system of second-order partial differential equations. Timoshenko [270] first proposed the transverse vibrations of a beam which are given by the following coupled partial differential equations

$$\begin{cases} \rho u_{tt} = (k(u_x - \varphi))_x, & \text{in} \quad (0, L) \times (0, +\infty), \\ I_\rho \varphi_{tt} = (EI\varphi_x)_x + k(u_x - \varphi), & \text{in} \quad (0, L) \times (0, +\infty), \end{cases}$$

where t denotes the time variable and x is the space variable along the beam, the length of which is L, in its equilibrium configuration. The function u is the transverse displacement of the beam and φ is the rotation angle of the filament of the beam. The coefficients ρ, I_ρ, E, I and k are, respectively, the mass per unit length, the polar moment of the inertia of a cross section, Young's modulus of elasticity, the moment of inertia of a cross section, and the shear modulus. The model considered here is a modified Timoshenko system, that is, a Timoshenko system in classical thermoelasticity of type I. We shall use multiplier techniques to prove the stability property for the system with a past history. For a kernel of polynomial decay, we prove the polynomial stability results for the equal wave-speed propagation, and establish a decay result for the nonequal wave-speed case under the assumption that g decays exponentially. Moreover, the existence of the global attractor is achieved.

In Chap. 4, as a continuation of Chap. 3, we consider a modified Timoshenko system, that is, a Timoshenko system in classical thermoelasticity of type III, we shall use the semigroup method to establish a polynomial stability result for Timoshenko-type system with a past history for the nonequal wave-speed case.

In Chap. 5, we study the (non)linear (non)homogeneous thermoelastic Bresse system. In their study on networks of flexible beams, Lagnese, Leugering, and Schmidt [9] derived a general model for three-dimensional nonlinear thermoelastic beams. A special case of this model is a linear planar, shearable thermoelastic beam whose motion is governed by the following system of partial differential equations

$$\begin{cases} \rho h w_{1tt} = (Eh(w_1' - kw_3) - \alpha\theta_1)' - kGh(\phi_2 + w_3' + kw_1), \\ \rho h w_{3tt} = Gh(\phi_2 + w_3' + kw_1)' + kEh(w_1' - kw_3) - k\alpha\theta_1, \\ \rho I \phi_{2tt} = EI\phi_2'' - Gh(\phi_2 + w_3' + kw_1) - \alpha\theta_3', \\ \rho c\theta_{1t} = \theta_1'' - \alpha T_0(w_{1t}' - kw_{3t}), \\ \rho c\theta_{3t} = \theta_3'' -_0 \phi_{2t}', \end{cases} \tag{3}$$

where w_1, w_2 and ϕ_2 are the longitudinal vertical and shear angle displacements; θ_1, θ_3 are the temperature deviations from the reference temperature T_0 along the longitudinal and vertical directions; $E, G, \rho, I, m, h, k, c$ are positive constants for the elastic and thermal material properties. We shall use the semigroup method and establish the global existence of solutions for the thermoelastic Bresse system.

In Chap. 6, as a continuation of Chap. 5, we shall consider the stability for thermoelastic bresse system, that is, we use multiplier techniques to prove the exponential stability result only for the equal wave-speed case.

In Chap. 7, we consider the linear thermoelastic model of type III with memory effects. In 1990s, Green and Naghdi [89–92] introduced three new types of thermoelastic theories in the aim of replacing the usual entropy production inequality with an entropy balance law. In each of these theories, the heat flux is given by a different constitutive assumption. As a result, three theories were obtained and respectively called thermoelasticity type I, type II, and type III. When the theory of type I is linearized, we obtain the classical system of thermoelasticity. The systems arising in thermoelasticity of type III are of dissipative nature whereas those of type II thermoelasticity do not sustain energy dissipation. In this chapter, we shall establish the global existence result for the higher-dimensional linear thermoviscoelastic equations of type III by using a semigroup approach. Using the multiplier techniques and Lyapunov methods, we prove that the energy for such a model decays to zero exponentially by introducing a velocity feedback on a part of the boundary of a thermoelastic body, which is clamped along the rest of its boundary to increase the loss of energy.

In Chap. 8, we study the thermoelastic model of type II. The issue of the asymptotic behavior of thermoelastic systems has attracted much attention in recent years. In the beginning, the mathematicians and engineers mainly considered the behavior of thermoelastic systems under the theory of classical thermoelasticity, in which the heat flux is given by the Fourier's law. Note that the theory of classical thermoelasticity predicts an infinite speed of heat propagation. This leads to an unrealistic property that a sudden disturbance at some point will be felt instantly everywhere else in the materials. In 1990s, Green and Naghdi [89–92] proposed three types of thermoelastic theories called thermoelasticity of types I, II, III, respectively, based on an entropy equality instead of the usual entropy inequality. The thermoelasticity of type I coincides with the classical one. In type II, known as thermoelasticity without dissipation, the heat is allowed to propagate by means of thermal waves, but without dissipating. The types I and II are limiting cases of thermoelasticity type III. Based on these three new theories of thermoelasticity, many engineers and mathematicians discussed the asymptotic behavior of several thermoelastic problems so as to describe the thermo-mechanical interactions in elastic materials. In this chapter, we shall prove the global existence for the three-dimensional thermoelastic equations of type II by means of semigroup methods.

In Chap. 9, we shall consider the thermoviscoelastic system.

For very viscous liquids, the ordinary hydrodynamic description needs to be generalized to allow for the slow relaxation processes related to the high viscosity. As far as mechanical relaxation is concerned, the theory of viscoelasticity provides such a description. However, the slow structural relaxation, which causes the high viscosity, also leads to a slow relaxation of thermal variables like temperature or entropy. Effects of such thermal relaxation are a frequency dependence of the specific heat and a coupling of heat conduction with structural relaxation. For a complete generalization of hydrodynamics for very viscous liquids, therefore, the relaxation of mechanical and thermal variables has to be treated on the same level. The result of this generalization will be a theory of linear thermoviscoelasticity. In this chapter, we shall obtain a decay result for higher-dimensional linear thermoviscoelastic equations by introducing a velocity feedback on a part of the boundary and using the multiplier technique method.

We sincerely wish that the reader will know the main ideas and essence of the basic theories and methods in deriving the global existence, uniqueness, asymptotic behavior of solutions for the models considered in this book. We also wish that the reader can be stimulated by some ideas from this book and undertake the further study after having read the related references and bibliographic comments in this book.

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