

Contents

1	Crystals	1
1.1	Periodicity of Crystal Structure, Crystal Cell	1
1.2	Three-Dimensional Lattice Types	2
1.3	Symmetry and Point Groups	2
1.4	Reciprocal Lattice	5
1.5	Appendix of Chapter 1: Some Basic Concepts	6
1.5.1	Concept of Phonon	6
1.5.2	Incommensurate Crystals	10
1.5.3	Glassy Structure	11
1.5.4	Mathematical Aspect of Group	11
	References	12
2	Framework of Crystal Elasticity	13
2.1	Review on Some Basic Concepts	13
2.1.1	Vector	13
2.1.2	Coordinate Frame	14
2.1.3	Coordinate Transformation	14
2.1.4	Tensor	16
2.1.5	Algebraic Operation of Tensor	16
2.2	Basic Assumptions of Theory of Elasticity	17
2.3	Displacement and Deformation	17
2.4	Stress Analysis	19
2.5	Generalized Hooke's Law	20
2.6	Elastodynamics, Wave Motion	24
2.7	Summary	25
	References	26
3	Quasicrystal and Its Properties	27
3.1	Discovery of Quasicrystal	27
3.2	Structure and Symmetry of Quasicrystals	29
3.3	A Brief Introduction on Physical Properties of Quasicrystals	31

3.4	One-, Two- and Three-Dimensional Quasicrystals	32
3.5	Two-Dimensional Quasicrystals and Planar Quasicrystals	32
	References	33
4	The Physical Basis of Elasticity of Solid Quasicrystals	37
4.1	Physical Basis of Elasticity of Quasicrystals	37
4.2	Deformation Tensors	38
4.3	Stress Tensors and Equations of Motion	40
4.4	Free Energy Density and Elastic Constants	42
4.5	Generalized Hooke's Law	44
4.6	Boundary Conditions and Initial Conditions	44
4.7	A Brief Introduction on Relevant Material Constants of Solid Quasicrystals.	46
4.8	Summary and Mathematical Solvability of Boundary Value or Initial-Boundary Value Problem	47
4.9	Appendix of Chapter 4: Description on Physical Basis of Elasticity of Quasicrystals Based on the Landau Density Wave Theory	48
	References	53
5	Elasticity Theory of One-Dimensional Quasicrystals and Simplification	55
5.1	Elasticity of Hexagonal Quasicrystals	55
5.2	Decomposition of the Elasticity into a Superposition of Plane and Anti-plane Elasticity	58
5.3	Elasticity of Monoclinic Quasicrystals	61
5.4	Elasticity of Orthorhombic Quasicrystals.	64
5.5	Tetragonal Quasicrystals	65
5.6	The Space Elasticity of Hexagonal Quasicrystals	66
5.7	Other Results of Elasticity of One-Dimensional Quasicrystals	68
	References	68
6	Elasticity of Two-Dimensional Quasicrystals and Simplification . . .	71
6.1	Basic Equations of Plane Elasticity of Two-Dimensional Quasicrystals: Point Groups $5m$ and $10mm$ in Five- and Tenfold Symmetries	75
6.2	Simplification of the Basic Equation Set: Displacement Potential Function Method	81
6.3	Simplification of Basic Equations Set: Stress Potential Function Method	83
6.4	Plane Elasticity of Point Group $5, \bar{5}$ and $10, \bar{10}$ Pentagonal and Decagonal Quasicrystals	85
6.5	Plane Elasticity of Point Group $12mm$ of Dodecagonal Quasicrystals	89

6.6	Plane Elasticity of Point Group $8mm$ of Octagonal Quasicrystals, Displacement Potential	93
6.7	Stress Potential of Point Group $5, \bar{5}$ Pentagonal and Point Group $10, \bar{10}$ Decagonal Quasicrystals	98
6.8	Stress Potential of Point Group $8mm$ Octagonal Quasicrystals . . .	100
6.9	Engineering and Mathematical Elasticity of Quasicrystals	103
	References	106
7	Application I—Some Dislocation and Interface Problems and Solutions in One- and Two-Dimensional Quasicrystals	109
7.1	Dislocations in One-Dimensional Hexagonal Quasicrystals	110
7.2	Dislocations in Quasicrystals with Point Groups $5m$ and $10mm$ Symmetries	112
7.3	Dislocations in Quasicrystals with Point Groups $5, \bar{5}$ Fivefold and $10, \bar{10}$ Tenfold Symmetries	119
7.4	Dislocations in Quasicrystals with Eightfold Symmetry	124
	7.4.1 Fourier Transform Method	125
	7.4.2 Complex Variable Function Method	127
7.5	Dislocations in Dodecagonal Quasicrystals	128
7.6	Interface Between Quasicrystal and Crystal	129
7.7	Dislocation Pile up, Dislocation Group and Plastic Zone	133
7.8	Discussions and Conclusions	134
	References	134
8	Application II—Solutions of Notch and Crack Problems of One- and Two-Dimensional Quasicrystals	137
8.1	Crack Problem and Solution of One-Dimensional Quasicrystals	138
	8.1.1 Griffith Crack	138
	8.1.2 Brittle Fracture Theory	143
8.2	Crack Problem in Finite-Sized One-Dimensional Quasicrystals	145
	8.2.1 Cracked Quasicrystal Strip with Finite Height	145
	8.2.2 Finite Strip with Two Cracks	149
8.3	Griffith Crack Problems in Point Groups $5m$ and $10mm$ Quasicrystal Based on Displacement Potential Function Method	150
8.4	Stress Potential Function Formulation and Complex Analysis Method for Solving Notch/Crack Problem of Quasicrystals of Point Groups $5, \bar{5}$ and $10, \bar{10}$	155
	8.4.1 Complex Analysis Method	156
	8.4.2 The Complex Representation of Stresses and Displacements	156
	8.4.3 Elliptic Notch Problem	158
	8.4.4 Elastic Field Caused by a Griffith Crack	162

8.5	Solutions of Crack/Notch Problems of Two-Dimensional Octagonal Quasicrystals	163
8.6	Approximate Analytic Solutions of Notch/Crack of Two-Dimensional Quasicrystals with 5- and 10-Fold Symmetries	165
8.7	Cracked Strip with Finite Height of Two-Dimensional Quasicrystals with 5- and 10-Fold Symmetries and Exact Analytic Solution	168
8.8	Exact Analytic Solution of Single Edge Crack in a Finite Width Specimen of a Two-Dimensional Quasicrystal of 10-Fold Symmetry	172
8.9	Perturbation Solution of Three-Dimensional Elliptic Disk Crack in One-Dimensional Hexagonal Quasicrystals	175
8.10	Other Crack Problems in One- and Two-Dimensional Quasicrystals	179
8.11	Plastic Zone Around Crack Tip	179
8.12	Appendix 1 of Chapter 8: Some Derivations in Sect. 8.1	179
8.13	Appendix 2 of Chapter 8: Some Further Derivation of Solution in Sect. 8.9.	181
	References	186
9	Theory of Elasticity of Three-Dimensional Quasicrystals and Its Applications	189
9.1	Basic Equations of Elasticity of Icosahedral Quasicrystals	190
9.2	Anti-plane Elasticity of Icosahedral Quasicrystals and Problem of Interface of Quasicrystal–Crystal	194
9.3	Phonon-Phason Decoupled Plane Elasticity of Icosahedral Quasicrystals	200
9.4	Phonon-Phason Coupled Plane Elasticity of Icosahedral Quasicrystals—Displacement Potential Formulation	202
9.5	Phonon-Phason Coupled Plane Elasticity of Icosahedral Quasicrystals—Stress Potential Formulation	205
9.6	A Straight Dislocation in an Icosahedral Quasicrystal	207
9.7	Application of Displacement Potential to Crack Problem of Icosahedral Quasicrystal	212
9.8	An Elliptic Notch/Griffith Crack in an Icosahedral Quasicrystal	220
9.8.1	The Complex Representation of Stresses and Displacements	220
9.8.2	Elliptic Notch Problem	222
9.8.3	Brief Summary	226
9.9	Elasticity of Cubic Quasicrystals—The Anti-plane and Axisymmetric Deformation	226
	References	231

10 Phonon-Phason Dynamics and Defect Dynamics of Solid Quasicrystals	233
10.1 Elastodynamics of Quasicrystals Followed Bak's Argument	234
10.2 Elastodynamics of Anti-plane Elasticity for Some Quasicrystals	235
10.3 Moving Screw Dislocation in Anti-plane Elasticity	236
10.4 Mode III Moving Griffith Crack in Anti-plane Elasticity	240
10.5 Two-Dimensional Phonon-Phason Dynamics, Fundamental Solution	243
10.6 Phonon-Phason Dynamics and Solutions of Two-Dimensional Decagonal Quasicrystals	249
10.6.1 The Mathematical Formalism of Dynamic Crack Problems of Decagonal Quasicrystals	249
10.6.2 Examination on the Physical Model	252
10.6.3 Testing the Scheme and the Computer Programme	254
10.6.4 Results of Dynamic Initiation of Crack Growth	256
10.6.5 Results of the Fast Crack Propagation	257
10.7 Phonon-Phason Dynamics and Applications to Fracture Dynamics of Icosahedral Quasicrystals	259
10.7.1 Basic Equations, Boundary and Initial Conditions	259
10.7.2 Some Results	261
10.7.3 Conclusion and Discussion	263
10.8 Appendix of Chapter 10: The Detail of Finite Difference Scheme	264
References	268
11 Complex Analysis Method for Elasticity of Quasicrystals	271
11.1 Harmonic and Biharmonic in Anti-Plane Elasticity of One-Dimensional Quasicrystals	272
11.2 Biharmonic Equations in Plane Elasticity of Point Group $12mm$ Two-Dimensional Quasicrystals	272
11.3 The Complex Analysis of Quadruple Harmonic Equations and Applications in Two-Dimensional Quasicrystals	273
11.3.1 Complex Representation of Solution of the Governing Equation	273
11.3.2 Complex Representation of the Stresses and Displacements	274
11.3.3 The Complex Representation of Boundary Conditions	275
11.3.4 Structure of Complex Potentials	276
11.3.5 Conformal Mapping	281
11.3.6 Reduction in the Boundary Value Problem to Function Equations	282
11.3.7 Solution of the Function Equations	283

11.3.8	Example 1 Elliptic Notch/Crack Problem and Solution	284
11.3.9	Example 2 Infinite Plane with an Elliptic Hole Subjected to a Tension at Infinity	286
11.3.10	Example 3 Infinite Plane with an Elliptic Hole Subjected to a Distributed Pressure at a Part of Surface of the Hole	286
11.4	Complex Analysis for Sextuple Harmonic Equation and Applications to Three-Dimensional Icosahedral Quasicrystals	287
11.4.1	The Complex Representation of Stresses and Displacements	288
11.4.2	The Complex Representation of Boundary Conditions	290
11.4.3	Structure of Complex Potentials	291
11.4.4	Case of Infinite Regions	294
11.4.5	Conformal Mapping and Function Equations at ζ -Plane	295
11.4.6	Example: Elliptic Notch Problem and Solution	297
11.5	Complex Analysis of Generalized Quadruple Harmonic Equation	300
11.6	Conclusion and Discussion	301
11.7	Appendix of Chapter 11: Basic Formulas of Complex Analysis	302
11.7.1	Complex Functions, Analytic Functions	302
11.7.2	Cauchy's formula	303
11.7.3	Poles	306
11.7.4	Residual Theorem	306
11.7.5	Analytic Extension	309
11.7.6	Conformal Mapping	309
	References	311
12	Variational Principle of Elasticity of Quasicrystals, Numerical Analysis and Applications	313
12.1	Review of Basic Relations of Elasticity of Icosahedral Quasicrystals	314
12.2	General Variational Principle for Static Elasticity of Quasicrystals	315
12.3	Finite Element Method for Elasticity of Icosahedral Quasicrystals	319
12.4	Numerical Results	323
12.5	Conclusion	332
	References	332

13	Some Mathematical Principles on Solutions of Elasticity of Quasicrystals	333
13.1	Uniqueness of Solution of Elasticity of Quasicrystals	333
13.2	Generalized Lax–Milgram Theorem	335
13.3	Matrix Expression of Elasticity of Three-Dimensional Quasicrystals	339
13.4	The Weak Solution of Boundary Value Problem of Elasticity of Quasicrystals	343
13.5	The Uniqueness of Weak Solution	344
13.6	Conclusion and Discussion	347
	References	347
14	Nonlinear Behaviour of Quasicrystals	349
14.1	Macroscopic Behaviour of Plastic Deformation of Quasicrystals	350
14.2	Possible Scheme of Plastic Constitutive Equations	352
14.3	Nonlinear Elasticity and Its Formulation	355
14.4	Nonlinear Solutions Based on Some Simple Models	356
14.4.1	Generalized Dugdale–Barenblatt Model for Anti-plane Elasticity for Some Quasicrystals	356
14.4.2	Generalized Dugdale–Barenblatt Model for Plane Elasticity of Two-Dimensional Point Groups 5 m, 10 mm and $5, \bar{5}, 10, \bar{10}$ Quasicrystals	359
14.4.3	Generalized Dugdale–Barenblatt Model for Plane Elasticity of Three-Dimensional Icosahedral Quasicrystals	361
14.5	Nonlinear Analysis Based on the Generalized Eshelby Theory	362
14.5.1	Generalized Eshelby Energy-Momentum Tensor and Generalized Eshelby Integral	362
14.5.2	Relation Between Crack Tip Opening Displacement and the Generalized Eshelby Integral	364
14.5.3	Some Further Interpretation on Application of E-Integral to the Nonlinear Fracture Analysis of Quasicrystals	365
14.6	Nonlinear Analysis Based on the Dislocation Model	366
14.6.1	Screw Dislocation Pile-Up for Hexagonal or Icosahedral or Cubic Quasicrystals	366
14.6.2	Edge Dislocation Pile-Up for Pentagonal or Decagonal Two-Dimensional Quasicrystals	369
14.6.3	Edge Dislocation Pile-Up for Three-Dimensional Icosahedral Quasicrystals	370
14.7	Conclusion and Discussion	371

14.8	Appendix of Chapter 14: Some Mathematical Details	371
14.8.1	Proof on Path-Independency of E-Integral	371
14.8.2	Proof on the Equivalency of E-Integral to Energy Release Rate Under Linear Elastic Case for Quasicrystals	373
14.8.3	On the Evaluation of the Critic Value of E-Integral	376
	References	377
15	Fracture Theory of Solid Quasicrystals	379
15.1	Linear Fracture Theory of Quasicrystals	379
15.2	Crack Extension Force Expressions of Standard Quasicrystal Samples and Related Testing Strategy for Determining Critical Value G_{IC}	383
15.2.1	Characterization of G_I and G_{IC} of Three-Point Bending Quasicrystal Samples	383
15.2.2	Characterization of G_I and G_{IC} of Compact Tension Quasicrystal Sample	384
15.3	Nonlinear Fracture Mechanics	385
15.4	Dynamic Fracture	387
15.5	Measurement of Fracture Toughness and Relevant Mechanical Parameters of Quasicrystalline Material	388
15.5.1	Fracture Toughness.	389
15.5.2	Tension Strength	389
	References	391
16	Hydrodynamics of Solid Quasicrystals	393
16.1	Viscosity of Solid	393
16.2	Generalized Hydrodynamics of Solid Quasicrystals	394
16.3	Simplification of Plane Field Equations in Two-Dimensional 5- and 10-Fold Symmetrical Solid Quasicrystals	396
16.4	Numerical Solution	396
16.5	Conclusion and Discussion	405
	References	405
17	Remarkable Conclusion	407
	References	408
	Major Appendix: On Some Mathematical Additional Materials	411

Mathematical Theory of Elasticity of Quasicrystals and
Its Applications

Fan, T.-Y.

2016, XVI, 452 p. 103 illus., 40 illus. in color., Hardcover

ISBN: 978-981-10-1982-1