

Chapter 2

Static Bionanosensor Networks for Target Detection

Abstract This chapter considers static bionanosensor networks for detecting target signals that may appear in the monitoring environment. The key problem considered in this chapter is to determine the number of bionanosensors that need to be distributed in the monitoring environment in order to meet application-dependent goals (e.g., in terms of the probability of detecting target signals). In this chapter, we first formulate the target detection problem and introduce two bionanosensor placement schemes: random and proportional placement schemes. We then show how the number of bionanosensors impacts the target detection performance of static bionanosensor networks that are formed according to the two placement schemes.

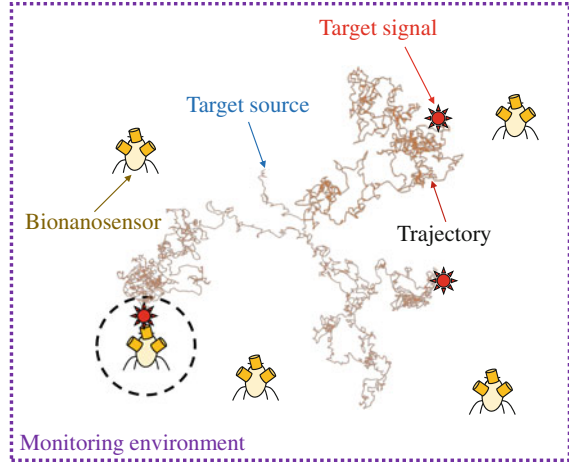
Keywords Static bionanosensor network · Target detection · Sensor placement · Residence time

2.1 Overview

Target detection is a fundamental functionality of bionanosensor networks that enables wide varieties of applications. In nanomedical applications, bionanosensors may be placed deep inside the human body, monitor the physiological environment, and perform required operations upon detecting targets (e.g., release drug molecules upon detecting disease-indicating signals). To enable precise and timely detection of targets, bionanosensors may be massively distributed; however, the number of deployable bionanosensors may be limited when the cost of bionanosensors is expensive or when possible side effects with the environment are concerned.

Figure 2.1 illustrates a bionanosensor network for target detection. Here a static bionanosensor network is considered where bionanosensors form a certain spatial distribution in the environment for detecting targets. Bionanosensors may be spatially organized in a distributed manner [4] or in a centralized manner (e.g., using a magnetic [1] or an electric field [3]). The environment contains target sources where target signals (i.e., molecules) may be generated. An example of a target source is a cell that requires nanoscale diagnosis, while that of a target signal is a disease-indicating molecule that may be released from the cell. A target source is distributed in a very

Fig. 2.1 A static bionanosensor network



small area (e.g., consisting of a small number of cells) in the environment, and target signals generated from a target source are extremely weak, meaning that target signals degrade and disappear in a relatively short amount of time (e.g., within several tens of seconds). Bionanosensors distributed in the environment are capable of chemically reacting with target signals; since these are weak, the goal of bionanosensor networks is to detect the target signals immediately after they are generated.

In the target detection problem described in this chapter, the residence time (RT) is defined as the amount of time from when a target signal is generated to when it is detected by a bionanosensor [5]. When three target signals are generated at time $t = 0$ in a monitoring environment containing multiple bionanosensors, their residence times are given as, for instance, $RT_1 = 14$ (s), $RT_2 = 41$ (s), and $RT_3 = 112$ (s), respectively. Their mean, namely the mean residence time (MRT), is calculated as the average; i.e., $MRT = 1/3 \sum_{i=1}^3 RT_i = 55.7$ (s). Note that target signals in the considered scale may be too weak and disappear before detected by bionanosensors. If these signals disappear at time $t = 100$ (s), only two signals can be detected. In this case, the probability of successfully detecting the signals, referred to as the probability of successful detection (PSD), is $2/3$.

The remainder of this chapter is organized as follows. In Sect. 2.2, we formulate the target detection problem to maximize PSD. In Sect. 2.3, we introduce two placement schemes for distributing bionanosensors for target detection: random and proportional placement schemes. In Sect. 2.4, we conduct numerical experiments to examine the impact of the number of bionanosensors on the PSD. Finally, we give a summary of this chapter in Sect. 2.5.

2.2 Problem Formulation

In this section, we consider the target detection problem in two-dimensional space using the notation shown in Table 2.1. We first derive the concentration of target signal in one-dimension and extend it to two-dimensions. Assume that a target signal performs discrete-space and -time random walk in one-dimensional space; namely, a target signal, starting at location $x = 0$ at time $t = 0$, moves by a step length δ in the forward or backward direction equiprobably at time $t = i \cdot \tau$ ($i = 1, 2, \dots$) where τ is the time step interval.

Let $p(m, n)$ be the probability of a target signal found at location $x = m\delta$ at time $t = n\tau$ after it moves a steps forward and b steps backward: i.e., $n = a + b$ and $m = a - b$. The probability $p(m, n)$ is given as the probability that a successes are obtained from n independent Bernoulli trials with the success probability of $1/2$:

$$p(m, n) = \frac{1}{2^n} \cdot \frac{n!}{a!(n-a)!} \quad (2.1)$$

The continuous version of $p(m, n)$, denoted by $c(x, t)$, is derived as follows. With Stirling's formula $k! = \sqrt{2\pi k} (k/e)^k$ for $k \gg 1$, (2.1) is approximated as

$$p(m, n) = \sqrt{\frac{2}{n\pi}} \exp\left(-\frac{m^2}{2n}\right). \quad (2.2)$$

By dividing (2.2) by 2δ and substituting $m = x/\delta$ and $n = t/\tau$, we have

Table 2.1 Notation

Notation	Description
D	Diffusion coefficient of the target signal
t_d	Average life-time of the target signal
K	Dissociation coefficient on the chemical reaction between a bionanosensor and the target signal
M	Number of target signals that are generated
N	Number of bionanosensors deployed
$f(\cdot)$	Sensor placement function
$p_{\text{detect}}(t)$	Signal detection probability by a single bionanosensor
$p_{\text{detect}}^{(N)}(t)$	Signal detection probability by N bionanosensors
R	Radius of the monitoring environment
Δt	Sensing interval
t_i	i -th time separator
p_i	The probability that the target signal is detected at time t_i
q_k	The probability that the target signal is detected for the first time at time t_k

$$\begin{aligned}
c(x, t) &= \lim_{\substack{\delta \rightarrow 0 \\ \tau \rightarrow 0}} \frac{p\left(\frac{x}{\delta}, \frac{t}{\tau}\right)}{2\delta} \\
&= \sqrt{\frac{1}{4\pi Dt}} \exp\left(-\frac{x^2}{4Dt}\right),
\end{aligned} \tag{2.3}$$

with

$$\lim_{\substack{\delta \rightarrow 0 \\ \tau \rightarrow 0}} \frac{\delta^2}{2\tau} = D \neq 0, \tag{2.4}$$

where D is the diffusion coefficient of the target signal. Note that $c(x, t) \cdot 2\delta$ is the probability that a target signal is found in the interval $[x, x + \delta]$ at time t , and $c(x, t)$ is interpreted as a concentration of the target signal at location x and at time t .

The two-dimensional version $c(x, y, t)$ is given as a product of $c(x, t)$ and $c(y, t)$ since the random walks along x and y axes are independent processes. Therefore, $c(x, y, t)$ is given as

$$c(r, t) = \frac{1}{4\pi Dt} \exp\left(-\frac{x^2 + y^2}{4Dt}\right) = \frac{1}{4\pi Dt} \exp\left(-\frac{r^2}{4Dt}\right). \tag{2.5}$$

We now formulate the target detection problem in bionanosensor networks. We consider a two-dimensional area \mathcal{R}^2 , where a single target source is located at the origin of the area. A total of M target signal molecules are secreted from the target source at time $t = 0$, and these molecules diffuse in the environment independently from each other. The concentration $c_M(r, t)$ of the target signal at location r at time t is given as

$$c_M(r, t) = M \cdot c(r, t). \tag{2.6}$$

Without loss of generality, target signals degrade and disappear according to the exponential distribution with the average life-time of t_d . The concentration $c_M(r, t)$ is then rewritten as

$$\begin{aligned}
C_M(r, t) &= c_M(r, t) \int_t^{+\infty} \frac{1}{t_d} \exp\left(-\frac{u}{t_d}\right) du \\
&= \exp\left(-\frac{t}{t_d}\right) \cdot c_M(r, t).
\end{aligned} \tag{2.7}$$

Figure 2.2 shows the concentration of the target signal observed by a bionanosensor located at distance $r = 10$ away from the origin. The average life-time t_d of the target

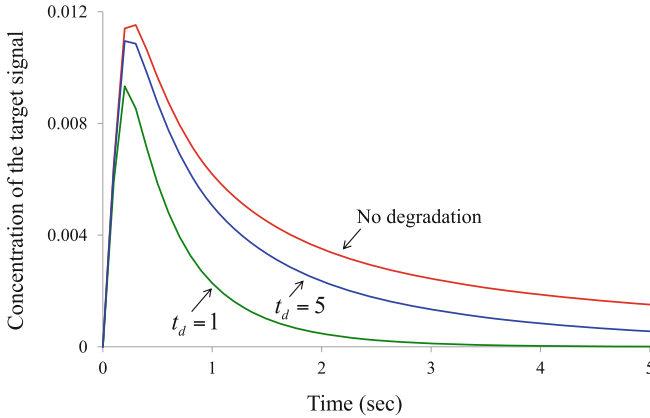


Fig. 2.2 Concentration of the target signal. $r = 10$

signal is varied in the range $\{1, 5, \infty\}$ where $t_d = \infty$ is labeled as “No degradation” in the figure.

Depending on the concentration of the target signal, a bionanosensor detects the target signal based on the following probability:

$$p(r, t) = \frac{C_M(r, t)}{K + C_M(r, t)}, \quad (2.8)$$

where r is the distance from the target source to the bionanosensor and K is a parameter. Note that the probability becomes 0.5 when $C_M(r, t) = K$.

Bionanosensors are distributed based on the *sensor placement function* $f(r)$ where r is the distance from the target source. Note that $f(r)$ is a probability density function for a single bionanosensor located at distance r from the target source. Therefore, the probability that a single bionanosensor, placed based on $f(r)$, detects the target signal at time t , referred to as the *signal detection probability*, is given by averaging $p(r, t)$ with respect to bionanosensor’s location, i.e.,

$$P_{detect}(t) = \int_0^R p(r, t) \cdot f(r) dr. \quad (2.9)$$

The probability that at least one of N bionanosensors independently placed based on $f(r)$ detects the target signal at time t is

$$P_{detect}^{(N)}(t) = 1 - \{1 - P_{detect}(t)\}^N. \quad (2.10)$$

The objective of sensor placement for target detection is to maximize the probability of successful detection (PSD). The PSD is defined as the probability that the

target signal is detected by a group of bionanosensors before the signal degrades and disappears from the environment.

Here bionanosensors are assumed to be measurement devices [2] that sense the concentration of the target signal every Δt seconds. The probability that bionanosensors detect the target signal for the first time at time t_k is the joint probability that bionanosensors fail to detect the signal at t_i for $i \in \{0, 1, 2, \dots, k-1\}$, multiplied with the probability that bionanosensors successfully detect the signal at t_k ; hence the probability mass function of RT is given by

$$q_k = \Pr[RT = t_k] = p_k \prod_{i=0}^{k-1} (1 - p_i), \quad (2.11)$$

where RT represents a random variable for the residence time. With the probability mass function of RT in (2.11), PSD is defined as a cumulative sum of the probability of the residence time up to an application-dependent cut-off time T , i.e.,

$$\begin{aligned} PSD(T) &= \Pr[RT \leq T] \\ &= \sum_{k=0}^{\lfloor T/\Delta t \rfloor} q_k, \end{aligned} \quad (2.12)$$

and the mean residence time (MRT) is described as the mean of RT normalized by $PSD(T)$, i.e.,

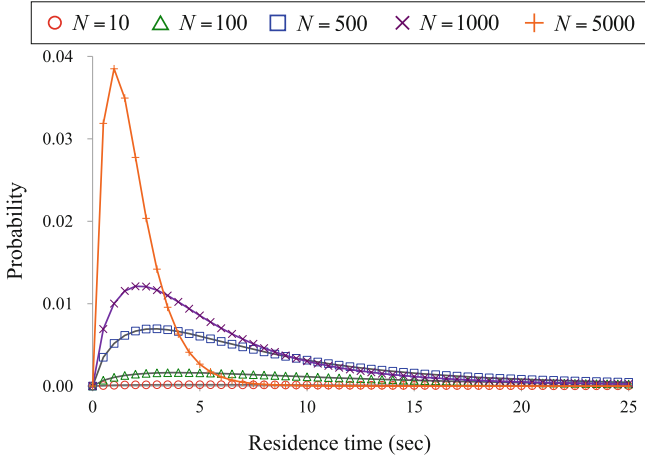


Fig. 2.3 Residence time distribution

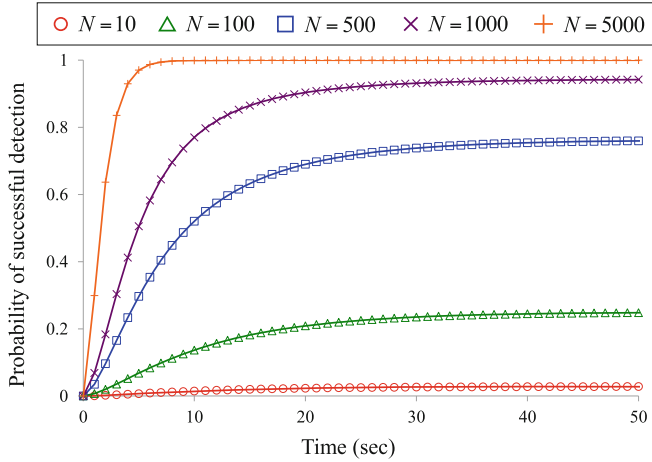


Fig. 2.4 Probability of successful detection

$$MRT(T) = \frac{\sum_{k=0}^{\lfloor T/\Delta t \rfloor} q_k t_k}{PSD(T)}. \quad (2.13)$$

Figure 2.3 shows the distribution of the residence time defined by (2.11) with the number N of bionanosensors varied in $\{10, 100, 500, 1000, 5000\}$. The figure shows that, as N increases, target signals are detected in a shorter period of time with a higher probability, which may result in a smaller MRT.

Figure 2.4 shows the PSD as a function of cut-off time T in (2.12) with the number N of bionanosensors varied. The figure shows that the PSD converges if sufficiently large T is used for all cases. Note that the PSD converges to the value smaller than 1 even when a large T is used since target signals that degraded are never detected.

In the following sections, we introduce two sensor placement schemes as solutions to the target detection problem defined in this section, and then conduct numerical experiments to evaluate the detection performance.

2.3 Sensor Placement Schemes

We consider two sensor placement schemes: *random* and *proportional*. In the random placement, bionanosensors are distributed uniformly in the environment. The random placement is the simplest scheme and it may be the only one option for many applications, as in wireless sensor networks [6]. In the proportional placement, more bionanosensors are placed near the origin where the target source exists. The proportional placement may be achieved using external control [1, 3]. The proportional

placement may also be implemented by utilizing bionanosensors capable of moving close to the target source and maintaining the distribution by adhering to the surface of the environment.

In the following, we consider a polar coordinate in which the monitoring environment is modeled as an area enclosed by a circle with radius R , and define the sensor placement function $f(r)$ for the random placement and proportional placement, respectively.

In the random placement, the sensor density in the environment is uniform. Therefore, the probability that a single bionanosensor exists within a circle centered at the origin with radius r increases in proportion to the area size of the circle; the cumulative distribution function (CDF) for the random placement with respect to r is

$$F_{rand}(r) = \frac{\pi r^2}{\pi R^2} = \frac{r^2}{R^2}. \quad (2.14)$$

The corresponding probability density function (PDF) is obtained by differentiating the CDF with respect to r , i.e.,

$$f_{rand}(r) = \frac{\partial F_{rand}(r)}{\partial r} = \frac{2r}{R^2}. \quad (2.15)$$

In the proportional placement, more bionanosensors are placed near the origin. In the orthogonal coordinate system, the sensor placement function may be described as a product of two normal distributions with mean 0 and standard deviation σ :

$$f_{X,Y}(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right). \quad (2.16)$$

where σ represents the “precision” of the proportional placement; when σ is close to 0, bionanosensors are placed near the origin with a high probability. This sensor placement function (2.16) in the orthogonal coordinate system is converted to that $f_{R,\Theta}(r, \theta)$ in the polar coordinate system as follows.

$$f_{R,\Theta}(r, \theta) = |J|f_{X,Y}(x, y), \quad (2.17)$$

where $|J|$ is a Jacobian defined by

$$|J| = \det \begin{bmatrix} \frac{\partial r \cdot \cos \theta}{\partial r} & \frac{\partial r \cdot \cos \theta}{\partial \theta} \\ \frac{\partial r \cdot \sin \theta}{\partial r} & \frac{\partial r \cdot \sin \theta}{\partial \theta} \end{bmatrix} = r. \quad (2.18)$$

Now, we assume that there is no orientation in terms of direction:

$$f_R(r) = \int_0^{2\pi} d\theta f_{R,\theta}(r, \theta). \quad (2.19)$$

This leads to

$$f_R(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), & r \geq 0 \\ 0, & r < 0 \end{cases}. \quad (2.20)$$

Note that the derived distribution in (2.20) with respect to r is known as *Rayleigh distribution*. To obtain the PDF for the proportional placement, we normalize the Rayleigh distribution (2.20) over the interval $[0, R]$:

$$f_{prop}(r; \sigma) = \begin{cases} \frac{\frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)}{1 - \exp\left(-\frac{R^2}{2\sigma^2}\right)}, & r \in [0, R] \\ 0, & \text{otherwise} \end{cases}. \quad (2.21)$$

Note that when $\sigma \rightarrow \infty$, the proportional placement function in (2.21) coincides with the random placement function in (2.15), i.e.,

$$\lim_{\sigma \rightarrow \infty} f_{prop}(r; \sigma) = f_{rand}(r). \quad (2.22)$$

Figure 2.5 shows example placements of bionanosensors; monitoring environment without bionanosensors, bionanosensors placed based on the random placement and the proportional placement with σ varied in a range $[1e3, 8e3]$.

2.4 Numerical Experiments

In numerical experiments, we consider the random and proportional placement schemes described in Sect. 2.3, and examine the impact of the number of bionanosensors on the target detection performance, namely, MRT (2.13) and PSD (2.12).

2.4.1 Parameter Configurations

By default, $N = 1000$ bionanosensors are placed within a circle centered at the origin with radius $R = 1.0e4 \text{ } (\mu\text{m}) = 1.0 \text{ (cm)}$. In the proportional placement, $\sigma = 1e4 \text{ } (\mu\text{m})$ is used. For the detection of the target signal, the dissociation constant

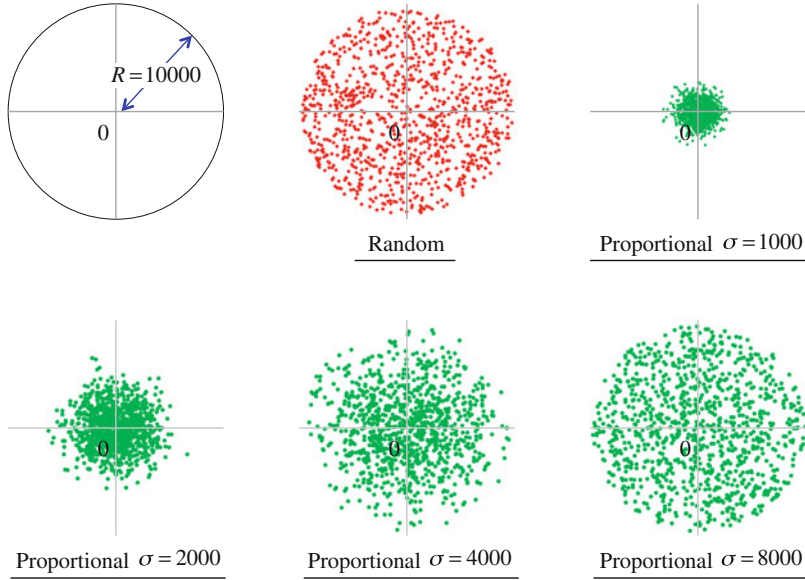


Fig. 2.5 Distributions of bionanosensors in random and proportional placement schemes

$K = 1e - 3$ is used. Target signals diffuse in the environment with the diffusion coefficient $D = 100 \text{ } (\mu\text{m}^2/\text{s})$ and target signals degrade and disappear from the environment with the average lifetime of $t_d = 10 \text{ (s)}$ assuming that target signals are protein molecules in mammalian cells.¹ The number of target signals generated is $M = 10$. For the time T of calculating PSD, a sufficiently large value is used.

2.4.2 Results

Figures 2.6 and 2.7 show the PSD and MRT when the random placement scheme is employed. In the random placement scheme, the number N of bionanosensors becomes the control parameter. The two figures show the impact of N on PSD and MRT, respectively; these also show the impact of average life-time t_d of the target signal. As shown in these figures, PSD increases and MRT decreases as N increases. In practice, N may be selected based on the target detection performance (PSD and MRT) as well as the cost of bionanosensors and potential side effects of bionanosensors with the environment.

¹The lifetime of proteins ranges from several minutes to years, and 1-2 days on average. Abnormal molecules can be unstable and degrade rapidly, for example, in several tens of seconds.

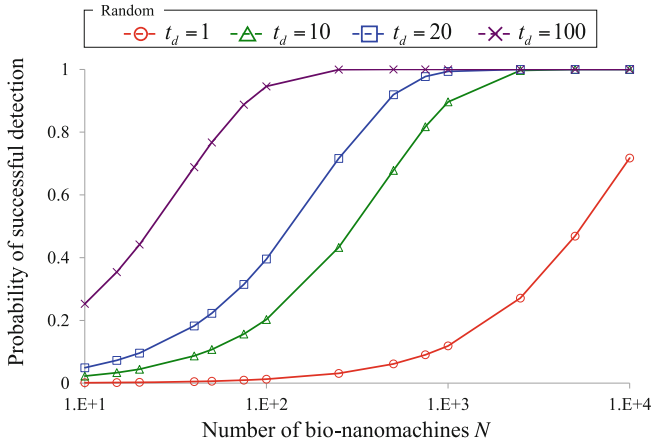


Fig. 2.6 Impact of the number of bionanosensors on probability of successful detection

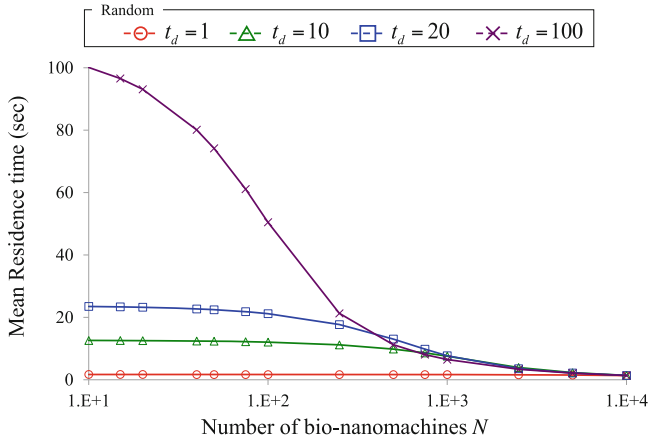


Fig. 2.7 Impact of the number of bionanosensors on mean residence time

Figures 2.8 and 2.9 show the PSD and MRT when the proportional placement scheme is employed. In the proportional placement scheme, PSD and MRT depend on the placement precision σ whose impact is shown in these two figures. These two figures also show the impact of average life-time t_d of the target signal. These figures show that there exists an effective range in σ to improve PSD and MRT. In the case of $t_d = 1$, for instance, PSD and MRT increase quickly in $[1e3, 1e4]$. Remember that when σ is sufficiently large, the proportional placement coincides with the random placement; in these figures, PSD and MRT in the random placement scheme can be approximated where $\sigma > 1e5$.

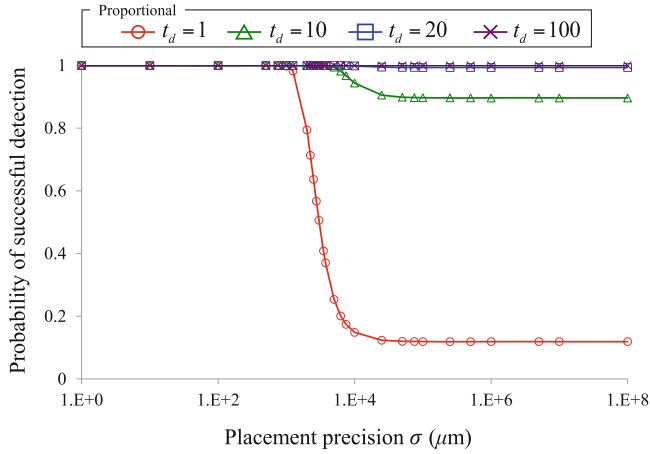


Fig. 2.8 Impact of placement precision σ of proportional placement on the probability of successful detection

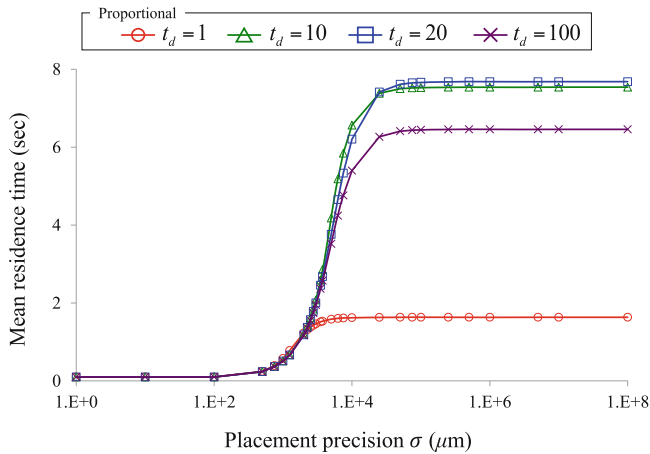


Fig. 2.9 Impact of placement precision σ of proportional placement on mean residence time

2.5 Summary

In this chapter, we designed, modeled and evaluated the performance of static bionanosensor networks for target detection. In the static bionanosensor networks considered in this chapter, bionanosensors are placed in the environment to detect target signals that diffuse in the environment. We formulated the target detection problem using the mean residence time (MRT) of target signals (i.e., the mean amount of time from when the target signal appears in the environment to when it is detected by a bionanosensor) and the probability of successful detection (PSD) (i.e., the prob-

ability that bionanosensors successfully detect the target signal before it degrades and disappears). We examined the random and proportional placement schemes as solutions to the target detection problem. Further, we evaluated the target detection performance of bionanosensor networks in terms of the probability of successful detection and the mean residence time.

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