Dynamical Flocking of Multi-agent Systems with Multiple Leaders and Uncertain Parameters

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Abstract. Dynamical flocking of multi-agent systems with multiple leaders is studied. Supposing topologies are dynamically changed with jointly-connected, flocking algorithm of multi-agent systems with time-varying delays is proposed. Multi-agent systems with uncertain parameters and time-varying delays is investigated, and sufficient conditions are given for flocking control of multi-agent systems with dynamical topologies. Finally, simulations are provided to prove the effectiveness of the conclusion.

Keywords: Flocking control · Multi-agent systems · Uncertain parameters

1 Introduction

Distributed cooperative control of multi-agent systems has attracted great attention in the fields of control theory, mathematics, computer science, etc. Consensus is an important research problem of distributed cooperative control of multi-agent systems, which has been studies deeply [1–10].

Consensus of multi-agent systems means to reach agreement state with the evolution of time. Average consensus of continuous-time agents with delayed information and jointly-connected topologies is investigated, and a sufficient condition for average consensus of multi-agent system by employing Barbalat's Lemma in [1]. Multi-agent systems with time delays is studied in [2, 3], finite-time consensus algorithm for multi-agent with disturbance is researched in [4, 5]. Dynamical flocking is said as containment control with multiple leaders to drive followers moving into a target area (convex hull formed by the leaders). In [6], necessary and sufficient containment criteria with time delay are established for continuous-time and sampled-data sysems. In [7], containment control problem for second-order systems with time-varying delays is considered. In [8, 9], containment control of multi-agent systems with directed topology and communication time-delays and communication noises is investigated. In [10], containment control for multiple Lagrangian systems with multiple dynamic

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leaders in the presence of parametric uncertainties and external disturbances is studied. In [11], containment control of uncertain nonlinear multi-agent systems with multiple dynamic leaders under switching directed topologies is considered.

In this paper, dynamical flocking containment control of second-order multi-agent systems with multiple stationary leaders and time-varying delays is studied. The innovation of this paper is that distributed containment control algorithm for uncertain multi-agent systems with jointly-connected topologies is presented. By applying linear matrix inequality method, the convergence of the algorithm for the multi-agent systems with disconnected topologies is studied on Lyapunov-Krasovskii method.

2 Preliminaries

Let $G = \{V, E\}$ be an undirected graph of order n, where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of nodes, $E = \{(v_i, v_j) : v_i, v_j \in V\}$ is the set of edges. If $(v_j, v_i) \in E$, then v_j is the neighbor of v_i. The set of the neighbors of node v_i is denoted by $N_i = \{v_j \in V | (v_i, v_j) \in E, j \neq i\}$. The union of a collection of graphs $G_1, G_2, \ldots, G_m$ with the same note set V, is defined as the graph G_{1-m} with the note set V and edge set equaling to the union of the edge sets of all of the graphs in the collection. Moreover, $G_1, G_2, \ldots, G_m$ is jointly-connected if its union graph G_{1-m} is connected [12].

The weighted adjacency matrix $A = [a_{ij}]$ of undirected graph G satisfying $a_{\mathrm{ij}} > 0$ if $(i,j) \in E$, $a_{\mathrm{ij}} = 0$, otherwise. The Laplacian corresponding to the undirected graph G is defined as $L = [l_{ij}]$, where l_{ij} is defined as follows:

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j \\ \sum\limits_{j \in N_i} a_{ij}, & i = j \end{cases}$$

Consider an infinite sequence of nonempty, bounded and contiguous time-intervals $[t_r, t_{r+1})$, $r = 1, 2, \ldots,$ with $t_1 = 0$ and for some constant $T_a > 0$. In each interval $[t_r, t_{r+1})$ there is a sequence of subintervals $[t_{r,j}, t_{r,j+1})$, $j = 1, 2, \ldots, m_r$ with $t_{r,1} = t_r$ and $t_{r,m_r+1} = t_{r+1}$ satisfying $t_{r,j+1} - t_{r,j} \geq T_b$, $J = 1, \ldots, m_r$ for some integer $m_r \geq 1$. Such that the communication topology switches at $t_{r,j}$ and it does not change during each subinterval $[t_{r,j}, t_{r,j+1})$. Let $\sigma(t) : [0, +\infty) \to \Gamma$, $\Gamma = \{1, 2, \ldots, N\}$ be a piecewise constant switching function, where N denotes the total number of all possible topologies. Suppose the multi-agent system consisting of n followers and m leaders in this paper. The communication topology of multi-agent systems at time is denoted by $G_{\sigma(t)}$ and the corresponding Laplacian matrix is denoted by $L_{\sigma(t)}$.

3 Flocking Control of Uncertain Multi-agent Systems

Consider second-order multi-agent systems of n followers and m leaders, and using $F = \{1, 2, \ldots, n\}$ and $L = \{n+1, n+2, \ldots, n+m\}$ to denote, respectively, the followers' set and the leaders' set. Suppose that the dynamics of agent i is described by the following equation

$$\begin{aligned} \dot{q}_i(t) &= p_i(t-\tau(t)), \\ \dot{p}_i(t) &= u_i(t-\tau(t)), \quad i = 1,\ldots, n, n+1,\ldots, n+m. \end{aligned} \tag{1}$$

where $p_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ are the position vector, the velocity vector, and the control input vector, respectively. $\tau(t)$ is the time-varying communication delay.

Assumption 1. The time-varying communication delay $\tau(t)$ in the multi-agent system (1) is bounded, i.e., there exists $h > 0$ satisfying: $0 \le \tau(t) < h$, $t \ge 0$.

Assumption 2. The communication topologies generated by n followers and m leaders, in each interval $[t_r, t_{r+1}), r = 1, 2, \ldots,$ are jointly connected.

Assumption 3. There exists a connectivity subset for multi-agent systems in each non-overlapping time intervals $[t_{r,j}, t_{r,j+1}) \subset [t_r, t_{r+1}), j = 1, 2, \ldots, m_r$. For each follower, there exists at least one leader that has a path to the follower in the connectivity subset.

Considering the case of stationary leaders, and suppose the control protocol of second-order multi-agent systems is

$$\begin{aligned} & u_i(t) = -k_1 p_i(t) - \sum_{j \in N_i} (a_{ij} + \Delta a_{ij}(t))(q_i(t) - q_j(t)), \quad i \in F; \\ & p_i(t) = 0, \quad i \in L. \end{aligned} \tag{2}$$

where $k_1 > 0$, and $\Delta a_{ij}(t)$ is the uncertain parameter with $\Delta a_{ii}(t) = 0$.

Definition 1. L is the Laplacian matrix of graph G, and $\Delta L(t) = [\Delta l_{ij}(t)]$ is the uncertain matrix of graph G defined by $\Delta l_{ij}(t) = \begin{cases} -\Delta a_{ij}(t), & i \ne j \\ \sum\limits_{j \in N_i} \Delta a_{ij}(t), & i = j \end{cases}$, where $L = \begin{bmatrix} L_F & L_{FL} \\ 0_{m\times n} & 0_{m\times m} \end{bmatrix}$, $\Delta L(t) = \begin{bmatrix} \Delta L_F(t) & \Delta L_{FL}(t) \\ 0_{m\times n} & 0_{m\times m} \end{bmatrix}$, $L_F \in \mathbb{R}^{n\times n}$, $\Delta L_F(t) \in \mathbb{R}^{n\times n}$, $L_{FL} \in \mathbb{R}^{n\times m}$, $\Delta L_{FL}(t) \in \mathbb{R}^{n\times m}$.

Definition 2. Norm bounded parameter uncertainty $\Delta L(t)$ and $\Delta L_F(t)$ satisfying

$$\Delta L^{\mathrm{T}}(t)\Delta L(t) \le \alpha^2 I_{n+m} \tag{3}$$

$$\Delta L_F^{\mathrm{T}}(t)\Delta L_F(t) \le \alpha^2 I_n \tag{4}$$

According to Definition 1, the system (1) and (2) can be written as

$$\dot{x}_F(t) = -H_1 x_F(t-\tau(t)) - H_2 x_L(t-\tau(t)) \tag{5}$$

$$\dot{x}_L(t) = H_3 x_L(t-\tau(t)) \tag{6}$$

where $H_1 = \begin{bmatrix} 0_{n\times n} & -I_n \\ E_F & k_1 I_n \end{bmatrix}$, $H_2 = \begin{bmatrix} 0_{n\times m} & 0_{n\times m} \\ E_{FL} & 0_{n\times m} \end{bmatrix}$, $H_3 = \begin{bmatrix} 0_{m\times m} & I_{m\times m} \\ 0_{m\times m} & 0_{m\times m} \end{bmatrix}$, $E_F = L_F + \Delta L_F(t)$, $E_{FL} = L_{FL} + \Delta L_{FL}(t)$, $x_F(t) = [q_F^{\mathrm{T}}(t), p_F^{\mathrm{T}}(t)]^{\mathrm{T}}$, $q_F(t)$ denotes the position vector of followers, $p_F(t)$ denotes the velocity vector of followers.

Let $\tilde{x}_F(t) = x_F(t) + H_1^{-1} H_2 x_L(t)$, Eq. (5) can be written as

$$\dot{\tilde{x}}_F(t) = -H_1 \tilde{x}_F(t - \tau(t)) \tag{7}$$

where $H_1^{-1} = \begin{bmatrix} k_1 E_F^{-1} & E_F^{-1} \\ -I_n & 0_{n\times n} \end{bmatrix}$.

Lemma 1 [13]. For any real differentiable vector function $x(t) \in \mathbb{R}^n$ and any $n \times n$ dimensional constant matrix $W = W^{\mathrm{T}} > 0$, we have the following inequality

$$h^{-1}[x(t) - x(t-\tau(t))]^{\mathrm{T}} W [x(t) - x(t-\tau(t))] \le \int_{t-\tau(t)}^{t} \dot{x}^{\mathrm{T}}(s) W \dot{x}(s) ds.$$

where $t \ge 0$, $0 \le \tau(t) \le h$.

Lemma 2 [14]. Let $\Xi = \Xi^{\mathrm{T}}$, F_1, F_2, $H(t)$ be a matrices with appropriate dimensions, and matrix $H(t)$ satisfied $H^{\mathrm{T}}(t)H(t) \le I$, then

$$\Xi + F_1 H(t) F_2 + F_2^{\mathrm{T}} H^{\mathrm{T}}(t) F_1^{\mathrm{T}} < 0,$$

satisfied if and only if there exists a positive constant $\varepsilon > 0$ satisfied

$$\Xi + \varepsilon^{-1} F_1 F_1^{\mathrm{T}} + \varepsilon F_2^{\mathrm{T}} F_2 < 0.$$

Lemma 3 [15]. For undirected graph G_F, if Assumption 3 holds, then E_F is positive definite, $-E_F^{-1} E_{FL}$ is a non-negative matrix and the sum of the entries in every row equals 1.

Lemma 4. Let $x_L = [x_{n+1}, \ldots, x_{n+m}]^{\mathrm{T}}$, $x_F = [x_1, \ldots, x_n]^{\mathrm{T}}$, if $x_F \to -H_1^{-1} H_2 x_L$, then the containment control of the multi-agent system can be achieved.

Proof. From Lemma 3, we can get the proof.

Theorem 1. Consider a second-order dynamic system (1) of n followers and m leaders with the switching topologies. When Assumptions 1, 2 and 3 hold, for each subinterval $[t_{r,j}, t_{r,j+1})$, if there exists a constant $\varepsilon > 0$, such that

$$\Pi_\sigma^i = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0, \quad i = 1, 2, \ldots, n_\sigma. \tag{8}$$

where $\Pi_{11} = -\frac{1}{h}I_{2\times d^i_{\sigma F}} + \varepsilon I_{2\times d^i_{\sigma F}}$, $\Pi_{22} = (\frac{1}{\varepsilon} - \frac{1}{h})I_{2\times d^i_{\sigma F}} + (h+\varepsilon h^2)\bar{H}^i_{\sigma 1}{}^{\mathrm{T}}\bar{H}^i_{\sigma 1} + h\alpha^2 J^i_\sigma$, $\Pi_{12} = \frac{1}{h}I_{2\times d^i_{\sigma F}} - \bar{H}^i_{\sigma 1} + \varepsilon h\bar{H}^i_{\sigma 1}$, $J^i_\sigma = \begin{bmatrix} 0_{d^i_{\sigma F}} & 0_{d^i_{\sigma F}} \\ I_{d^i_{\sigma F}} & 0_{d^i_{\sigma F}} \end{bmatrix}$. Then, the control protocol (2) solves the containment control problem of second-order systems with time-varying delays and uncertain topologies.

Proof. Define a common Lyapunov-Krasovskii function for system (7) as follows

$$V(t) = \tilde{x}_F^{\mathrm{T}}(t)\tilde{x}_F(t) + \int_{t-h}^{t}(s-t+h)\dot{\tilde{x}}_F^{\mathrm{T}}(s)\dot{\tilde{x}}_F(s)ds \tag{9}$$

Taking the derivative of $V(t)$ along the trajectories of (9) yields

$$\begin{aligned}\dot{V}(t) = \sum_{i=1}^{n_\sigma}\Big\{&\left[-H^i_{\sigma 1}\tilde{x}^i_{\sigma F}(t-\tau(t))\right]^{\mathrm{T}}\tilde{x}^i_{\sigma F}(t) + \tilde{x}^i_{\sigma F}{}^{\mathrm{T}}(t)\left[-H^i_{\sigma 1}\tilde{x}^i_{\sigma F}(t-\tau(t))\right] \\ &+ h\dot{\tilde{x}}^i_{\sigma F}{}^{\mathrm{T}}(t)\dot{\tilde{x}}^i_{\sigma F}(t) - \textstyle\int_{t-h}^{t}\tilde{x}^i_{\sigma F}{}^{\mathrm{T}}(s)\dot{\tilde{x}}^i_{\sigma F}(s)ds\Big\}\end{aligned} \tag{10}$$

From Assumption 1, since $\tau(t) < h$, $h > 0$, we get

$$-\int_{t-h}^{t}\dot{\tilde{x}}^i_{\sigma F}{}^{\mathrm{T}}(s)\dot{\tilde{x}}^i_{\sigma F}(s)ds \le -\int_{t-\tau(t)}^{t}\dot{\tilde{x}}^i_{\sigma F}{}^{\mathrm{T}}(s)\dot{\tilde{x}}^i_{\sigma F}(s)ds \tag{11}$$

According to Lemma 1, we can get

$$-\int_{t-\tau(t)}^{t}\dot{\tilde{x}}^i_{\sigma F}{}^{\mathrm{T}}(s)\dot{\tilde{x}}^i_{\sigma F}(s)ds \le -\frac{1}{h}[\tilde{x}^i_{\sigma F}(t) - \tilde{x}^i_{\sigma F}(t-\tau(t))]^{\mathrm{T}}[\tilde{x}^i_{\sigma F}(t) - \tilde{x}^i_{\sigma F}(t-\tau(t))] \tag{12}$$

we have

$$\begin{aligned}\dot{V}(t) \le \sum_{i=1}^{n_\sigma}\{&-\tilde{x}^i_{\sigma_F}{}^{\mathrm{T}}(t)H^i_{\sigma 1}\tilde{x}^i_{\sigma_F}(t-\tau(t)) - \tilde{x}^i_{\sigma_F}{}^{\mathrm{T}}(t-\tau(t))\,H^i_{\sigma 1}{}^{\mathrm{T}}\tilde{x}^i_{\sigma_F}(t) \\ &+ h\,\tilde{x}^i_{\sigma_F}(t)^{\mathrm{T}}(t-\tau(t))\,H^i_{\sigma 1}{}^{\mathrm{T}}H^i_{\sigma 1}\tilde{x}^i_{\sigma_F}(t-\tau(t)) \\ &-\tfrac{1}{h}\left[\tilde{x}^i_{\sigma_F}(t) - \tilde{x}^i_{\sigma_F}(t-\tau(t))\right]^{\mathrm{T}}\left[\tilde{x}^i_{\sigma_F}(t) - \tilde{x}^i_{\sigma_F}(t-\tau(t))\right]\} \\ = \sum_{i=1}^{n_\sigma} & y^i_\sigma{}^{\mathrm{T}}\Omega^i_\sigma y^i_\sigma\end{aligned}$$

where $y^i_\sigma = \left[\tilde{x}^i_{\sigma F}{}^{\mathrm{T}}(t), \tilde{x}^i_{\sigma F}{}^{\mathrm{T}}(t-\tau(t))\right]^{\mathrm{T}}$, and

$$\Omega^i_\sigma = \begin{bmatrix} -\frac{1}{h}I_{2\times d^i_{\sigma F}} & \frac{1}{h}I_{2\times d^i_{\sigma F}} - H^i_{\sigma 1} \\ \frac{1}{h}I_{2\times d^i_{\sigma F}} - H^i_{\sigma 1}{}^{\mathrm{T}} & -\frac{1}{h}I_{2\times d^i_{\sigma F}} + hH^i_{\sigma 1}{}^{\mathrm{T}}H^i_{\sigma 1}\end{bmatrix}.$$

According to Lemma 2,

$$\Psi_{\sigma}^{i}=\begin{bmatrix}0\\ I_{2\times d_{\sigma F}^{i}}\end{bmatrix}{B_{\sigma}^{i}}^{\mathrm{T}}\begin{bmatrix}I_{2\times d_{\sigma F}^{i}} & h\bar{H}_{\sigma 1}^{i}\end{bmatrix}+\begin{bmatrix}I_{2\times d_{\sigma F}^{i}} & h\bar{H}_{\sigma 1}^{i}\end{bmatrix}^{\mathrm{T}}B_{\sigma}^{i}\begin{bmatrix}0\\ I_{2\times d_{\sigma F}^{i}}\end{bmatrix}^{\mathrm{T}},\ \Omega_{\sigma}^{i}<0$$

they equivalent to

$$\Phi_{\sigma}^{i}+\varepsilon^{-1}\begin{bmatrix}0\\ I_{2\times d_{\sigma F}^{i}}\end{bmatrix}\begin{bmatrix}0\\ I_{2\times d_{\sigma F}^{i}}\end{bmatrix}^{\mathrm{T}}+\varepsilon\begin{bmatrix}I_{2\times d_{\sigma F}^{i}} & h\bar{H}_{\sigma 1}^{i}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}I_{2\times d_{\sigma F}^{i}} & h\bar{H}_{\sigma 1}^{i}\end{bmatrix}=\Phi_{\sigma}^{i}+\Xi_{\sigma}^{i}<0,$$

where $\Xi_{\sigma}^{i}=\begin{bmatrix}\varepsilon I_{2\times d_{\sigma F}^{i}} & \varepsilon h\bar{H}_{\sigma 1}^{i}\\ \varepsilon h{\bar{H}_{\sigma 1}^{i}}^{\mathrm{T}} & \varepsilon h^{2}{\bar{H}_{\sigma 1}^{i}}^{\mathrm{T}}\bar{H}_{\sigma 1}^{i}+\frac{1}{\varepsilon}I_{2\times d_{\sigma F}^{i}}\end{bmatrix}$, $\varepsilon>0$. Definition 2, ${\Delta L_{\sigma F}^{i}}^{\mathrm{T}}(t)\Delta L_{\sigma F}^{i}(t)\le\alpha^{2}I_{d_{\sigma F}^{i}}$, ${B_{\sigma}^{i}}^{\mathrm{T}}B_{\sigma}^{i}\le\begin{bmatrix}0_{d_{\sigma F}^{i}\times d_{\sigma F}^{i}} & 0_{d_{\sigma F}^{i}\times d_{\sigma F}^{i}}\\ \alpha^{2}I_{d_{\sigma F}^{i}} & 0_{d_{\sigma F}^{i}\times d_{\sigma F}^{i}}\end{bmatrix}$, then $\Phi_{\sigma}^{i}\le\begin{bmatrix}-\frac{1}{h}I_{2\times d_{\sigma F}^{i}} & \frac{1}{h}I_{2\times d_{\sigma F}^{i}}-\bar{H}_{\sigma 1}^{i}\\ \frac{1}{h}I_{2\times d_{\sigma F}^{i}}-{\bar{H}_{\sigma 1}^{i}}^{\mathrm{T}} & -\frac{1}{h}I_{2\times d_{\sigma F}^{i}}+h{\bar{H}_{\sigma 1}^{i}}^{\mathrm{T}}\bar{H}_{\sigma 1}^{i}+h\alpha^{2}J_{\sigma}^{i}\end{bmatrix}$, where $J_{\sigma}^{i}=\begin{bmatrix}0_{d_{\sigma F}^{i}} & 0_{d_{\sigma F}^{i}}\\ I_{d_{\sigma F}^{i}} & 0_{d_{\sigma F}^{i}}\end{bmatrix}$. Therefore, if Eq. (8) holds, then $\Omega_{\sigma}^{i}<0$, i.e., $\dot{V}(t)<0$. The system (7) is asymptotically stable. Since $V(t)$ is nonincreasing and bounded below by 0, then $V(t)$ must approach a limit as $t\to\infty$. Note that

$$0\ge\int_{0}^{t}\sum_{i=1}^{n_{\sigma}}y_{\sigma}^{i}(s)^{\mathrm{T}}\Omega_{\sigma}^{i}y_{\sigma}^{i}(s)ds\ge\int_{0}^{t}\dot{V}(s)ds=V(t)-V(0)$$

and $\sum_{i=1}^{n_{\sigma}}{y_{\sigma}^{i}}^{\mathrm{T}}\Omega_{\sigma}^{i}y_{\sigma}^{i}<0$. It follows that $\int_{0}^{+\infty}(\sum_{i=1}^{n_{\sigma}}y_{\sigma}^{i}(s)^{\mathrm{T}}\Omega_{\sigma}^{i}y_{\sigma}^{i}(s))ds$ exists and finite. We have $y_{\sigma}^{i}=0$, i.e., $\tilde{x}_{\sigma F}^{i}(t)=\tilde{x}_{\sigma F}^{i}(t-\tau(t))=0,\ t\to+\infty$. Then by induction, it has $\lim\limits_{t\to+\infty}x_{F}(t)=\lim\limits_{t\to+\infty}x_{F}(t-\tau(t))=-H_{1}^{-1}H_{2}x_{L}(t)$, $\lim\limits_{t\to+\infty}q_{F}(t)=-E_{F}^{-1}E_{FL}q_{L}(t)$.

Therefore, containment control of the second-order multi-agent systems with time-varying delays and uncertain topologies can be achieved.

4 Simulations

Considering the dynamic switching topology with 5 followers and 3 leaders shown in Fig. 1, and using $F=\{1,2,3,4,5\}$ and $L=\{6,7,8\}$ to denote, respectively, the followers' set and the leaders' set, where the connection weights of each edge is 1. Suppose the communication topology of the multi-agent system randomly switches in G1 to G3 at $t=kT$, $k=0,1,\ldots$, where T=0.5s.

From the communication topology of the union of a collection of simple graphs $G1\sim G3$, the system matrix $L_{\sigma F}$ can be obtained. Using the Matlab's LMI toolbox to solve (8) we get $k_1>4.3278$, $h<0.4621$. Let $k_1=4.4$, and let $\tau(t)=0.2+0.2\sin t$ is the time-varying delays of multi-agent systems in the experiments. The initial position

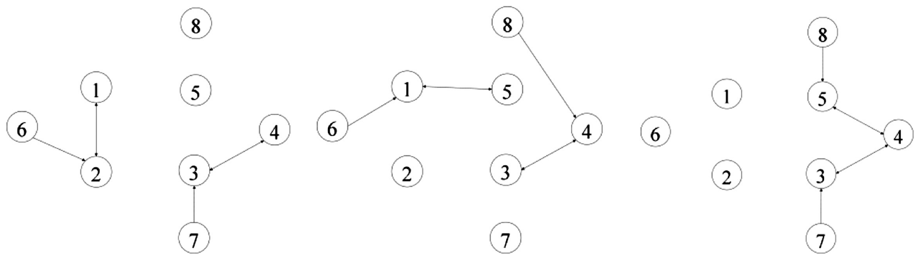

Fig. 1. Communication topology of the multi-agent systems

of followers and leaders are taken $q_1(0) = (1,1)$, $q_2(0) = (1,3)$, $q_3(0) = (3,1)$, $q_5(0) = (5,3)$, $q_6(0) = (7,10)$, $q_7(0) = (10,7)$, $q_8(0) = (10,10)$, respectively. The initial velocity of followers and leaders are taken $p_1(0) = (1,1)$, $p_3(0) = (1,1)$, $p_4(0) = (2,2)$, $p_5(0) = (3,3)$, $p_6(0) = p_7(0) = p_8(0) = (0,0)$, respectively.

For $\Delta L_{\sigma F} = \alpha \sin(t) * L_{\sigma F}$ with $\alpha = 0.1$, state trajectories of all agents are shown in Fig. 2, which shows that those followers can asymptotically convergence to the triangle formed by three leaders, i.e., the containment control of multi-agent systems can be achieved.

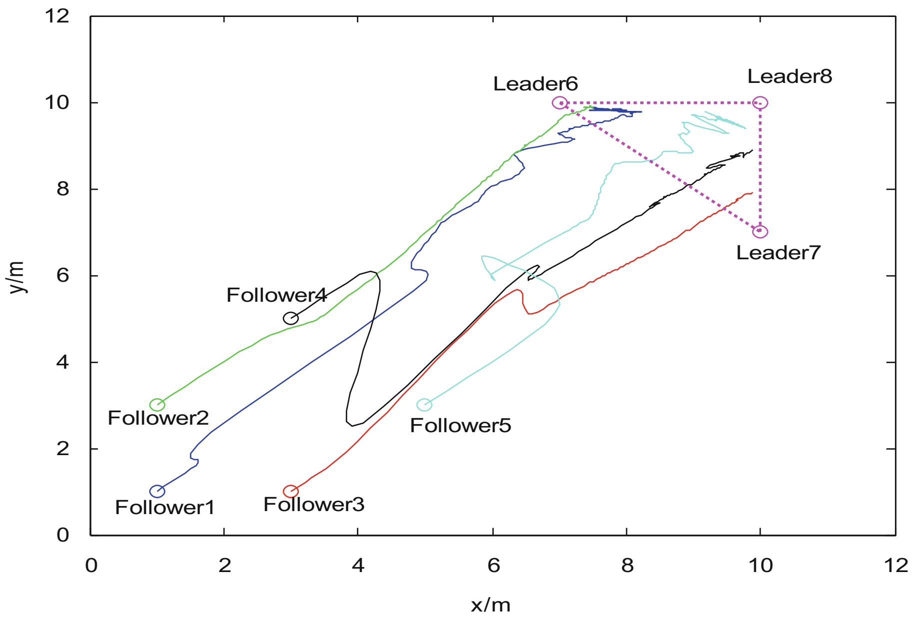

Fig. 2. Moving trajectories of multi-agent systems with uncertain topologies

5 Conclusion

In this paper, dynamical flocking containment control for second-order multi-agent systems with multiple leaders and jointly-connected topologies is studied. Flocking control algorithm of multi-agent systems with time-varying delays and uncertain parameters is proposed. The convergence of multi-agent systems is analyzed on Lyapunov-Krasovskii method, and some sufficient conditions in terms of linear matrix inequalities(LMIs) are obtained for flocking containment control of multi-agent systems.

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