Preface

Random matrix theory (RMT) has a long history, starting with the statistics of nuclear energy levels by Wigner, and it has found applications in wide areas of all sciences from mathematics to biology. The universality of properties derived from random matrices has led to a large number of applications such as network statistics, big data analysis, and biological information. Useful surveys maybe found in [61, 97, 111, 117, 124].

This book deals with Gaussian random matrix models with an external deterministic matrix source. There are also numerous reasons to consider such extensions of standard RMT theory. For instance for a system in the presence of impurities with Hamiltonian $H = H_0 + V$, a Gaussian distribution for the impurity potential V yields a Gaussian distribution for H with a coupling term between H and H_0. However, we have restricted ourselves to a systematic exposition of the main subject that we have studied in earlier publications, namely the geometric properties of surfaces of arbitrary genera deduced from RMT with a nonrandom matrix source.

Indeed for a number of years random matrix theory has been known to be a powerful tool for characterizing geometric properties of surfaces, leading in particular to explicit solution to 2D quantum gravity, and after Kontsevich's resolution of Witten conjectures, to the computation of intersection numbers for curves drawn on Riemann surfaces of given genus with fixed marked points. Many techniques have been used in such problems such as loop equations, Virasoro constraints, and integrable hierarchies. However our work, over a long period, has consisted in showing that simple Gaussian models with an appropriately tuned external source provide an elementary alternative approach to these topics. A systematic exposition of the main tools that underlie this method is the aim of this book. Two properties are basic: (i) the n-point functions are known explicitly for a given arbitrary matrix source. This is true for finite $N \times N$ matrices. (ii) A remarkable duality holds for this probability distribution: the expectation value of the product of K characteristic polynomials over $N \times N$ random matrices is equal to the expectation value of the product of N characteristic polynomials over $K \times K$ random matrices.

Then, an appropriate tuning of the external source generates, in a “double scaling” limit, well-known models such as the Kontsevich Airy matrix model, the Penner model and various generalizations. The duality makes it possible to relate those models to Gaussian models whose correlations functions are known explicitly, providing thereby a simple tool to compute intersection numbers.

Among the mathematical techniques underlying this approach, the Harish Chandra [77], Itzykson-Zuber [80] integral over the unitary group plays a central role. The random matrices there are Hermitian, and the associated random surfaces are orientable. However the general Harish Chandra formula holds for integration over Lie algebras, such as antisymmetric or symplectic matrices for $so(N)$ or $sp(N)$. The corresponding random surfaces are nonorientable. Therefore one can generalize to these Gaussian models in a source the same strategy, namely finding explicit expressions for the correlation functions plus a duality. In the double scaling limit, this provides explicit results for finding the geometric intersection numbers for nonorientable surfaces.

Another extension concerns a supersymmetric duality, whose geometric content is not known to us.

Although the contents mostly consist of a systematic exposition of results scattered through earlier publications, a few new results are presented in the last chapters.

Paris, France Edouard Brézin
Kunigami-gun, Japan Shinobu Hikami

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Brézin, E.; Hikami, S.

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