

Preface

Mathematicians continue to make major contributions to the classical but continually evolving discipline of Fluid Mechanics, which is often included in advanced undergraduate and postgraduate applied mathematics or engineering programmes. On the other hand, Magnetohydrodynamics (MHD) is more often left to undergraduate and postgraduate physics programmes, and relatively few research mathematicians proceed to work on MHD or plasma theory. However, Fluid Mechanics and MHD are both branches of Continuum Mechanics, often drawing upon much the same mathematics and yielding many closely related results. Moreover, both are fundamental to many fascinating areas in geophysics and astrophysics, and have a wide variety of human and laboratory or engineering applications (from blood flow to power generation).

This book is primarily intended to enable graduate research students in science and engineering to enhance their understanding and expertise in Fluid Mechanics and MHD, which are no longer treated in isolation as in other texts. A previous background or preliminary reading in either could be an advantage, and prior knowledge of multivariate calculus and differential equations is expected. However, we write in a didactic way that reflects our scientific inclination and experience (not least as past research student supervisors), and the mathematical development is largely self-contained. Our presentation could be supplemented by historical or more descriptive material (including relevant basic physics) readily obtained from elsewhere if desired—in particular, research students are now generally well aware that the Internet can be a source of quite accessible ideas and information. Incidentally, the mathematical theory and applications of Fluid Mechanics and MHD are usually presented in the classical Newtonian context as in this book, but there are relativistic formulations in the literature for motion nearer the speed of light.

Common fluids (such as air or water) flow so readily that early theory in Fluid Mechanics, sometimes called “classical” but usually ideal in this book, entirely neglected viscosity. Wave motion is one area where the ideal (inviscid) model is often appropriate, although an observer of water waves on the surface of a lake might notice that some insects exploit the force of surface tension rather well there!

Ideal MHD is the corresponding inviscid model that Hannes Alfvén invented last century, to predict the famous new wave named after him (in a plasma permeated by a magnetic field). Air flow past a streamlined object such as an aerofoil (a cross-section of an aircraft wing) may appear to be well described by the ideal fluid model, but the air does not slip past freely close to its surface as envisaged in that theory—and even more obviously, it does not predict flow past a blunter object, typically associated with trailing swirls and drag. As is now well known, there is a boundary layer near the surface of an obstacle where even quite small fluid viscosity cannot be neglected, and in MHD (including stability theory) non-ideal properties may similarly be expected to modify an otherwise largely acceptable ideal MHD model in particular regions. Mathematically, one may proceed to invoke singular perturbation theory, where an ideal “outer” solution is matched to a non-ideal “inner” solution at the relevant transition layer boundaries. Variational formulations and energy principles also provide a common and often powerful mathematical approach, particularly for ideal theory but sometimes to investigate non-ideal effects.

As indicated above, we have deliberately correlated essential theory in Fluid Mechanics and MHD, and we also proceed to some notable applications with which we are most familiar. Three sections indicated by a star (*) are not referenced later, but introduce aspects that some readers may find interesting. Certain fundamental topics are dealt with more briefly than elsewhere, for there are several well written basic textbooks on either fluid mechanics or MHD, such as Acheson (“Elementary Fluid Mechanics”, Clarendon Press, Oxford) and Davidson (“An Introduction to Magnetohydrodynamics”, Cambridge University Press). We do provide more detail in some areas—e.g. where we have personally contributed or where we believe future research work may prove fruitful, trusting our readers will enjoy that personal touch without in any way losing an appreciation of the essential component. Our orientation in the application of MHD is not to fluids but to plasmas, mainly in the context of magnetic confinement in controlled thermonuclear fusion research, although we also discuss some aspects in astrophysics and solar physics that caught our interest in the past. Incidentally, we recommend Prof. Kulsrud’s book on “Plasma Physics for Astrophysics” (Princeton University Press) as an excellent point of departure for research students interested in astrophysical applications.

The mathematical development from Newton’s basic description of the motion of a single particle to modern mathematical modelling of complex dynamical systems is a remarkable scientific achievement, and the evolution of Fluid Mechanics and MHD is no small part of that tapestry to which a new generation of young scientists can be expected to contribute. While our mathematical development here is largely self-contained, there are inevitably quite important areas that cannot be included in a book of this size—e.g. nonlinear modelling underlying the description of turbulence or chaos, phenomena that often follow or precede coherent structures in fluids and plasmas. Indeed, given the rapid increase in computing power, numerical simulation has become increasingly important in Fluid Mechanics and MHD, especially in nonlinear calculations. However, the

fundamental theory and analytical procedures represented in this book are essentially indispensable for any emerging researcher in Fluid Mechanics and MHD, and we hope that more established members of the research community may also enjoy our correlated presentation. We list various books in the bibliography to cover further material in many areas, ranging from introductory to classical and more specialist treatments. Further, we occasionally include additional comments in footnotes to the text, often with reference to relevant original sources.

Each chapter in this book begins with a brief preamble, but it may be helpful to comment on our overall presentation here. Chapter 1 on Vectors and Tensors incorporates notable material on dyadics and vector and dyadic representations, not only in familiar orthogonal coordinate systems but in general as non-orthogonal coordinates are needed later. The additional topics that follow in this chapter then complete a deliberately chosen near-sufficient “mathematical toolkit” for the rest of the book. After briefly introducing the concept of conservation equations and discussing several basic topics often found elsewhere, in Chap. 2 on the Fundamental Equations we proceed to a unified derivation that in particular clearly distinguishes the viscosity of a neutral fluid from the viscosity of a plasma in a magnetic field (to be subsequently described by MHD). Thus although this detailed derivation of the macroscopic equations from a Boltzmann-like equation may seem quite complicated at first sight, it applies to either a collection of molecules in a fluid or of charged particles in a plasma, and we encourage our readers to follow it (with the help of some associated analysis in related Exercises) as a suitable theoretical foundation at the outset with results for reference later. The adopted closure at thirteen moments also allows for a generalisation of the classical law of heat conduction, but this section is starred as that aspect is not pursued in this book. Chapter 2 concludes with a brief summary of forms of the magnetic induction equation needed to complete any MHD model, either the ideal or some non-ideal model to be discussed.

Only then do we proceed to discuss Basic Fluid Mechanics in Chap. 3 and Waves in Fluids in Chap. 4, before returning again to MHD in Chap. 5 followed by MHD Stability Theory in Chap. 6. Given the diversity of formal background in Fluid Mechanics or MHD previously mentioned, we anticipate that certain topics will already be familiar to our readers but others less so—some in Chaps. 3 and 4 with an entertaining history in fluid mechanics but others more recent, often serving as precursors to our discussion of various topics in Chaps. 5 and 6. The emphasis on waves rather than instabilities in Chap. 4, but on stability theory rather than waves in Chap. 6, is quite deliberate as there are already some excellent books on hydrodynamic stability. Moreover, we consider important aspects of wave propagation in appropriate places in Chap. 5; and in Chap. 6 we emphasise further developments in both ideal and non-ideal MHD stability from the perspective of our personal research experience, given the extensive classical treatise by Chandrasekhar on “Hydrodynamic and Hydromagnetic Stability” (Dover).

The Exercises throughout the book often serve to provide additional quite significant knowledge or develop mathematical skill, and may also fill in certain details or enhance understanding of some essential concept—or even to extend the

discussion in the text in some interesting way. We strongly recommend that any student first attempt to answer each Exercise personally and independently, before participating in a group discussion or referring to the Answers we have provided for help (available online). Over many years, we have often found that the independent effort involved significantly assists student learning.

One of us (RJH) acknowledges the hospitality of Prof. Ian Craig and his group at the University of Waikato, and that of Prof. John Howard and Dr. Matthew Hole and his group at the Australian National University, during two important stages in completing this book. Michael Howard and several others provided perceptive and helpful comments that led to a clearer exposition at various places in the discussion, and we dedicate it more generally to the continual stimulation and enthusiasm of so many of our students and colleagues over the years.

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