

Contents

Part I Introduction: Where It All Starts

1	Basic Aspects of Growth	3
1.1	Classification.	4
1.1.1	Tip Growth	5
1.1.2	Accretive Growth	7
1.1.3	Volumetric Growth	9
1.2	The Scaling of Growth	10
1.3	Relative Growth	17
1.4	The Kinematics of Growth	23
2	Mechanics and Growth	27
2.1	Growth is Influenced by Stress	27
2.1.1	The Growth of Stems	30
2.1.2	The Growth of Axons	30
2.1.3	Thoma's Law for Arteries	32
2.1.4	Woods' Law for the Heart	32
2.1.5	Wolff's Law for Bones	34
2.1.6	Davis' Law for Soft Tissues	36
2.1.7	Tumor Spheroid Growth	38
2.2	Stress is Influenced by Growth	38
2.2.1	Tissue Tension in Plants	40
2.2.2	Residual Stress in Physiology	43
2.3	The Theory of Morphoelasticity	45
2.4	A Short History of Growth Modeling	46
2.5	A Review of Reviews	49
3	Discrete Computational Models	51
3.1	On-Lattice Models	51
3.1.1	Cellular Automata	51
3.1.2	Cellular Potts Models	52

3.2	Off-Lattice Models	54
3.2.1	Center Dynamics Models.	54
3.2.2	Vertex Dynamics Models	56
3.2.3	Advantages and Drawbacks.	58

Part II Filament Growth: A One-Dimensional Theory

4	Growing on a Line	63
4.1	Example: A Growing Rod in One Dimension.	66
4.2	Purely Elastic Deformations	67
4.3	Growth Without Elastic Deformations	69
4.3.1	Example: Tip Growth	69
4.3.2	Application: Spheroid Tumor Growth	71
4.4	Growth with Elastic Deformation	76
4.4.1	Growth of a Rod Between Two Plates	76
4.4.2	Three Different Configurations.	77
4.4.3	Homeostatic Growth	78
4.4.4	Application: The Growth of Neurons.	80
4.4.5	Is This Just Plasticity?	85
4.5	Application: The Growth of Plant Cells	88
4.5.1	Lockhart Model	89
4.5.2	Extending Lockhart's Model	92
5	Elastic Rods	97
5.1	The Kinematics of Curves and Rods.	98
5.1.1	Curves and Frenet Frames	98
5.1.2	Rods and General Frames	99
5.1.3	Inextensible, Unshearable Rods	102
5.2	The Mechanics of Elastic Rods.	103
5.2.1	Balance of Linear Momentum	104
5.2.2	Balance of Angular Momentum.	106
5.2.3	Local Mechanics of Rods	107
5.3	Constitutive Laws for Elastic Rods.	108
5.3.1	Extensible and Shearable Elastic Rods.	108
5.3.2	Inextensible and Unshearable Rods	108
5.3.3	Isotropic, Extensible, but Unshearable Rods	110
5.4	Scaling	110
5.5	Bending and Torsional Stiffnesses	111
5.6	The Kirchhoff Elastic Rod Model: A Summary	113
5.7	Example: Helical Rods	116
5.7.1	Geometry of Helices	116
5.7.2	Helical Equilibria.	117
5.7.3	Overwinding or Underwinding Helices	119

5.8	The Planar Elastica: Bernoulli–Euler Equations	122
5.8.1	Static Solutions	122
6	Morphoelastic Rods.	125
6.1	Kinematics of a Growing Rod	127
6.2	Mechanics of a Growing Rod	128
6.3	Evolution Laws for Growing Rods	129
6.4	Example: The Remodeling of Stems	130
6.5	A Buckling Criterion	132
6.5.1	Example: Michell’s Instability	133
6.5.2	A General Perturbation Expansion	136
6.5.3	Bifurcation Criterion for Elastic Buckling	138
6.5.4	Example: The Growing Ring	138
6.5.5	A Growing Ring with Remodeling	140
6.6	Growing Rods on a Rigid Foundation	142
6.6.1	Example: A Growing Ring on a Foundation	143
6.6.2	Example: A Straight Rod Growing on a Foundation.	145
6.7	Application: Growing Vines	149
6.7.1	Perversion in Tendrils	149
6.7.2	Twining Vines	162
6.7.3	Application: The Growth of <i>Bacillus subtilis</i>	168

Part III Surface Growth: A Two-Dimensional Theory

7	Accretive Growth	175
7.1	Intrinsic Accretive Growth	176
7.1.1	Simple Examples	179
7.1.2	Shape Planarity	184
7.1.3	Shape Invariance	185
7.1.4	Self-Similarity	187
7.2	Application: The Growth of Seashells.	187
7.2.1	Background	187
7.2.2	Geometric Description	189
7.2.3	Accretive Growth of Seashells.	193
7.2.4	Other Accreted Structures	194
7.2.5	The Role of Mechanics in Morphological Patterns	196
8	Membranes and Shells	207
8.1	Elastic Membranes	208
8.1.1	Kinematics.	208
8.1.2	Mechanics	211

8.1.3	Constitutive Laws	214
8.1.4	A Complete Set of Equations	214
8.1.5	Application: The Shape of Sea Urchins	216
8.2	Nonlinearly Elastic Shells	220
8.2.1	Mechanics	220
8.2.2	Scalings	223
8.2.3	Application: The Rice Blast Fungus	224
9	Growing Membranes	231
9.1	Morphoelastic Membranes	232
9.2	Application: Microbial Tip Growth	234
9.2.1	Background	234
9.2.2	Bacterial Filaments: Actinomycetes	234
9.2.3	Fungi	237
9.2.4	Root Hairs	238
9.2.5	Modeling of Tip Growth	239
9.2.6	A Model for Hyphal Growth	240
9.2.7	Tip Shapes for Filamentous Bacteria	241
9.2.8	Lysis and Beading	246
9.2.9	Shear Stress and the Normal Growth Hypothesis	247
10	Morphoelastic Plates	251
10.1	Elastic Plates	252
10.1.1	Mean and Gaussian Curvatures	252
10.1.2	Growing Elastic Plates	254
 Part IV Volumetric Growth: A Three-Dimensional Theory		
11	Nonlinear Elasticity	261
11.1	Kinematics	262
11.1.1	Scalars, Vectors, and Tensors	264
11.1.2	Spatial Derivatives of Tensors	266
11.1.3	Derivatives in Curvilinear Coordinates	268
11.1.4	Derivatives of Scalar Functions of Tensors	270
11.1.5	The Deformation Gradient	272
11.1.6	Volume, Surface, and Line Elements	274
11.1.7	Polar Decomposition Theorem	276
11.1.8	Velocity, Acceleration, and Velocity Gradient	277
11.2	Balance Laws	278
11.2.1	Balance of Mass	279
11.2.2	Balance of Linear Momentum	280
11.2.3	Balance of Angular Momentum	282
11.2.4	Many Stress Tensors	283
11.2.5	Balance of Energy for Elastic Materials	284

11.3	Constitutive Equations for Hyperelastic Materials.	285
11.3.1	Internal Material Constraints	286
11.4	Summary of Equations	287
11.5	Boundary Conditions	288
11.6	Objectivity and Material Symmetry	288
11.7	Isotropic Materials	289
11.7.1	Adscitious Inequalities.	291
11.7.2	Choice of Strain-Energy Functions	293
11.8	Examples	296
11.8.1	A Simple Homogeneous Deformation	296
11.8.2	The Half-Plane in Compression.	298
11.8.3	The Inflation–Extension of a Tube.	299
11.9	Universal Deformations for Isotropic Materials.	305
11.10	Bifurcation, Buckling, and Instability	310
11.10.1	Example: Bifurcation of the Half-Plane.	314
11.11	Anisotropic Materials	320
11.11.1	One Fiber	322
11.11.2	Two Fibers	323
11.11.3	Example: The Fiber-Reinforced Cuboid.	324
11.11.4	Example: The Fiber-Reinforced Cylinder.	328
11.11.5	Application: The Hydrostatic Skeleton.	336
11.11.6	Fiber Dispersion	339
12	The Kinematics of Growth	345
12.1	A Thought Experiment	346
12.2	Relieving Stresses.	348
12.3	The Conceptual Hypothesis of Morphoelasticity.	353
12.4	Example: The Growing Ring	355
12.5	The Problem of Incompatibility	358
12.5.1	A Differential Geometry Perspective	359
12.5.2	An Analytic Perspective	371
13	Balance Laws.	375
13.1	The Slow-Growth Assumption	376
13.2	Balance of Mass	377
13.3	Balance of Linear and Angular Momenta.	380
13.4	Energy Balance.	381
13.5	Imbalance of Entropy	382
13.6	Elastic Constitutive Laws	383
13.7	Summary of Volumetric Morphoelasticity	384
13.8	Simple Examples	385
13.8.1	A Growing Cuboid	385
13.8.2	Two Growing Cuboids	386
13.8.3	A Growing Ring	388

13.9	Mixture Models	392
13.9.1	Classical Mixture Theory	393
13.9.2	Constrained Mixture Theory	396
14	Evolution Laws and Stability	399
14.1	Symmetry of the Growth Tensor	400
14.2	Isotropic Growth and Gel Swelling.	403
14.3	Discrete Growth Steps	404
14.4	The Thermodynamics Perspective.	405
14.5	Phenomenological Laws and Homeostatic Stress	408
14.6	Dynamics of Homogeneous Deformations	410
14.6.1	Diagonal Deformations	411
14.6.2	A Two-Dimensional Example	415
14.7	Remodeling.	419
14.7.1	Fiber Remodeling of a Cuboid in Tension.	421
14.8	Growth Induced Instability	425
15	Growing Spheres	431
15.1	The Growing Shell	431
15.1.1	Kinematics of Growing Spheres	432
15.1.2	Stresses in a Growing Sphere	435
15.2	Examples	437
15.2.1	Anisotropic Growth	437
15.2.2	Differential Growth	438
15.3	Limit-point Instability and Inflation Jump.	442
15.3.1	The Effect of Growth on the Limit-point Instability	448
15.4	Singularities in Growing Solid Spheres.	449
15.5	Cavitation	450
15.5.1	Cavitation Induced by Tensile Loading	452
15.5.2	Cavitation Induced by Growth.	452
15.6	Instability Due to Anisotropic Growth	453
15.6.1	A Numerical Scheme.	460
15.6.2	Thin-Shell Limit	462
15.6.3	Thick-Shell Limit.	465
15.6.4	Bifurcation of the Growing Shell.	467
15.7	Instability Due to Differential Growth.	470
15.7.1	Instability in a Shrinking Shell	471
15.7.2	Instability of a Shell Growing Inside a Medium	473
16	Growing Cylinders	475
16.1	Kinematics of the Growing Cylinder	476
16.2	Application: Cavitation in Plants.	480
16.2.1	Background	480
16.2.2	The Model.	483

16.2.3	Analysis.	484
16.2.4	Discussion	484
16.3	Bifurcation of Growing Cylinders.	487
16.3.1	Buckling Versus Barreling.	487
16.3.2	Bifurcation and Buckling in Growing Cylinders . . .	493
16.3.3	The Effective Rigidity of a Growing Cylinder.	496
16.3.4	An Example.	498
16.4	Application: Tissue Tension in Plants.	499
16.4.1	Background	499
16.4.2	The Model.	501
16.4.3	Analysis.	503
16.4.4	Discussion	507
16.5	Application: The Buckling of Arteries	509
16.5.1	Background	509
16.5.2	The Model.	512
16.5.3	Analysis.	518
16.5.4	Discussion	523
16.6	Circumferential Buckling and Mucosal Folding	524
16.6.1	Example: Circumferential Buckling in a Cylinder	526
16.6.2	Example: A Two-Layer Cylinder.	528
16.7	Application: Asthma and Airway Remodeling	530
16.7.1	Background	530
16.7.2	The Model.	532
16.7.3	Analysis.	534
16.7.4	Discussion	538
16.8	Residual Stress Through Fiber Contraction.	539
16.8.1	Rotation of a Pressurized Anisotropic Cylinder	540
16.9	Application: The Spiral Growth of Phycomyces	542
16.9.1	Background	542
16.9.2	The Model.	544
16.9.3	Analysis.	546
16.9.4	Discussion	551

Part V Conclusion: Where It Does Not End

17	Ten Challenges	555
17.1	The Rheology of Growth	555
17.2	The Regulation of Growth and Growth Size.	557
17.3	The Elusive Growth Law	559
17.4	Multiscale: From Discrete to Continuous and Back	561
17.5	Growth Versus Diffusion	564
17.6	Multiphysics: Coupling Growth with Other Fields	567
17.7	A Theory of Accretive Growth.	570

17.8 Dynamics and Post-bifurcation Behavior 572

17.9 Active Forces, Actives Stresses, and Active Strains 574

17.10 The Mathematical Foundations 578

References 581

Index 637

The Mathematics and Mechanics of Biological Growth

Goriely, A.

2017, XXII, 646 p. 341 illus., Hardcover

ISBN: 978-0-387-87709-9