2

Callability Features

2.1 Introduction and Objectives

In this chapter, we introduce callability which gives one party in a transaction the right (but not the obligation) to terminate the transaction early. The valuation of such optional rights requires more than a static curve as in Chap. 1. A starting point is the dynamic term structure model as presented in Chap. 10. We give valuation examples in terms of this model in the following sections.

2.2 Callability

In order to pay a higher coupon, thus making the bond look more attractive to the investor, one can add features that have a positive value from the issuer perspective. This excess value can then be distributed via the coupon payments such that in effect a higher coupon can be paid without any economical disadvantage taken by the issuer. One of the most common of such features is callability. This gives the issuer the right to pay back the notional of the bond before its scheduled maturity, usually at par and without any compensation for the outstanding coupon payments. There are variations to this repayment style, which we look at in Chap. 3. The call right is a one-sided right owned by the issuer that he may, but is not obliged, exercise, thus a long option held by the issuer.

There are also bonds that are puttable, meaning that the investor instead of the issuer has the right to early terminate the bond and receive the notional before reaching its maturity. Clearly, this constitutes a long option for the investor. Finally, there are bonds which are both callable and puttable.

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We focus on callable bonds, that is bonds with a call right owned by the issuer. On the one hand, they are more common than puttable bonds and on the other hand, a put right is similar to a call in terms of valuation methodology.

A general remark about the exercise of an early redemption option is in order. While for the derivative side, that is the hedge swaps the exercise is usually purely driven by what is captured in (an appropriate) valuation model, the early redemption decision of a bond goes usually beyond the exercise criterion given by a pure valuation model: Firstly, the valuation model itself is harder to calibrate to the current market, since variables like the dynamic of the credit spread and its correlation to the interest rate dynamics are required as an input which are much harder to derive from market observables than the pure interest rate swap curves and vanilla options that are required for a callable swap valuation. This is, however, not entirely true, since the inter-temporal correlation of rate movements is essential for multi-callable swap valuation as well, which is, for example, represented by a parameter mean reversion in, for example, a LGM called model, see the exposition in later chapters, in particular Chap. 9 or Sect. 10.3. Furthermore, liquidity considerations may influence an early redemption decision to a great extent, which are rarely part of the valuation model at all or if so, very hard to calibrate to any objective reference. We take this model as a placeholder for any other term structure model in the sequel.

In the fixed rate bond example from Chap. 1 suppose that the issuer has a call right after 5 year of the bond's lifetime (say exactly on the start of the 6th coupon accrual period). Whenever the NPV of the remaining 5 year bond is above par (that is the notional amount), the issuer should buy back the bond and issue a new 5 year bond at the same time at par. Since the NPV of the original bond was above par, the coupon of the new bond is necessarily below the original coupon, such that the issuer then has a gain from this switch. If on the other hand the NPV is below par on the option date, the issuer does not exercise the option. It is this one-sided option right which constitutes a positive value of the call option to the issuer.

Note that both interest rate and credit spread movements influence the NPV of the bond, such that the option value is driven by the common dynamics in particular their volatilities and mutual correlation of the two quantities in general.

The early exercise right can be the right to pay back the notional early at only one predefined date in the future. In this case, it is known as an European exercise right. If the right can be exercised at one of the several predefined future dates, it is known as a Bermudan exercise right. It is understood that a Bermudan right is at least as valuable as the most valuable European right corresponding to its exercise dates. Actually, in general the value is not equal to the maximum of the corresponding European rights, but greater than this maximum. This is because the decision which exercise date will potentially be chosen to pay back the notional early has not to be taken until a certain exercise date is reached. Only then the single decision if this particular exercise opportunity is taken has to be made or if one rather hopes that a later exercise may be more favourable. There is also the notion of an American Exercise right, which means that the notional can be paid back on each day during the life time of the bond. Technically, this is nothing else than a Bermudan exercise right with each day being an exercise date.

While for a plain bond without any early redemption rights a swap exchanging the bond coupons against a floating rate like Euribor or Libor plus a spread is the natural hedging instrument, for a callable bond it is a swap that can be terminated early at the same dates as the bonds. Such swaps are known as callable swaps, again European, Bermudan or American callable swaps depending on how many exercise opportunities exist.

A certain period at the beginning of the lifetime of a bond where it can not be called is common, for example 2 years in a 10-year bond, and only after this period a Bermudan call right may start with yearly call rights. In this case, the bond would be callable after 3, 4, 5, 6, 7, 8 and 9 years. If it is not called then the notional is paid back at maturity.

As explained in Sect. 1.5, callable swaps are imperfect hedges unless they are made credit-linked. The situation is, in a sense, worse than for the non-callable case though, since even without a default event the swap may be called on an exercise date while the bond does not. This happens, for example, if a low interest rate level suggests a call of the swap, but an overcompensating high credit spread level lets the issuer keep the bond as is (because in economical terms he would have to pay a higher coupon for a new issue despite the low interest rates because their credit spread is high enough).

We note that a call right may not be exercised purely by interest rate level and credit spread level considerations, but that in practice the liquidity situation of the issuer and the actual ability to place a new issuance may play a role whether an exercise right is executed or not. It is important to add that the credit spread is in general a quantity that is not directly observable in the market, but has to be implied from other instruments like other bonds (with similar characteristics) of the same issuer or even CDS. Since between the bond and CDS market there is a basis, the latter approach is even more difficult to apply.

2.3 Callable Fixed Bonds, Callable Swaps and Swaptions

We continue to give a valuation example. As above, we assume a flat yield term structure @2% as the benchmark discounting curve and an OAS spread of 300 basis points. The bond is a 10-y bond with fixed, yearly coupons @5%, as above, but now the bond is callable by the issuer @100% at the start of each coupon accrual period. In practice, the issuer has to decide whether or not to exercise a few days before the start of such a period. In our example we assume 5 business days. Table 2.1 lists the exercise dates and corresponding accrual start dates on which the redemption amount (in our case the notional) has to be paid.

We assume the model's interest rate volatility to be 0.0050 and the reversion speed to be 0.0020. Likewise, we assume the credit spread volatility to be 0.0080 and its reversion speed to be 0.0040. The joint dynamics of interest rates and credit spreads is specified by the correlation between their driving Brownian motions. We assume this to be 50% for the moment. When we consider short rate models, you notice that the above numbers are quite small, especially for the mean reversion.

In terms of numerical procedures, PDEs, numerical integration or Monte Carlo Simulation are all possible, whereas PDEs are probably most common for the product in question here. For the example calculation, however we use a general Monte Carlo Engine for structured bonds valuation which was designed to handle the possibility of a larger number of price factors (like

Table 2.1 Exercise and settlement dates for a callable fix rate bond, the exercise decision has to be taken on the exercise date, while in case of an exercise the redemption amount is to be paid on the settlement date. Here, the exercise date is chosen to have a 5 business days notice period before the settlement date, which is the accrual start date of the first coupon period that is part of the exercise

Exercise date	Settlement date
2017-05-18	2017-05-25
2018-05-18	2018-05-25
2019-05-20	2019-05-25
2020-05-18	2020-05-25
2021-05-18	2021-05-25
2022-05-18	2022-05-25
2023-05-18	2023-05-25
2024-05-20	2024-05-25
2025-05-19	2025-05-25

Table 2.2 NPV of a callable fix rate bond and its Asset Swap (Euribor 6M + 315.6bp) for OAS = 3%, IR volatility 0.0050, CR volatility 0.0080, IR-CR correlation 50%

	NPV (EUR)	NPV (relative) (%)
Bond underlying	989,763.11	98.9763
Bond call option	−49,783.90	−4.9783
Callable bond total	939,979.21	93.9979
Swap underlying NPV	21,441.42	2.1441
Swap call option	38,218.36	3.8218
Callable swap total	59,659.78	5.9660

Inflation, FX) as well as path dependency. Numerical methods are outlined in Appendix A.

Table 2.2 lists the NPV of the bond and its asset swap under these assumptions. The margin on the asset swap's float leg evaluates to 315.6 basis points to make the package of the callable bond and callable asset swap worth the notional on the settlement date 25-May-2016. Note that, this is not exactly the same as the asset swap spread for the non-callable bond (which was 303.33 basis points, cf. Table 1.6), which once again illustrates that the asset swap spread is not a pure measure of credit risk but reflects other price drivers as well.

The small difference of the bond's underlying price compared to the calculation in Table 1.2, which is 27.62 EUR or 0.2 basis points relative to the nominal, comes from the fact that we use a Monte Carlo valuation model here which introduces a simulation error.

We already noted that the asset swap belonging to the callable fixed rate bond is a callable fix versus float vanilla swap, that is a swap with schedule as in Table 1.4 plus an early termination right given by an exercise schedule identical to Table 2.1, meaning that all payments with respect to accrual periods with accrual start date greater or equal to the exercise date are cancelled in case of an exercise. This is the prototype of a Bermudan callable fix versus float swap. Note that the call right is held by the investor to offset the bond's call right held by the bond's issuer.

The option to enter into the tail of the underlying swap on one of the exercise dates is called a Bermudan swaption. If from the option holder perspective the underlying is a payer swap, the option is a payer swaption and otherwise a receiver swaption. The call option of a Bermudan callable swap is actually a Bermudan swaption: For a vanilla payer swap, a Bermudan receiver swaption represents the right to enter into a swap whose cashflows exactly offset the original swap's flows. In other words, the portfolio consisting of a vanilla payer swap and a Bermudan receiver swaption is equivalent to a Bermudan callable payer swap.

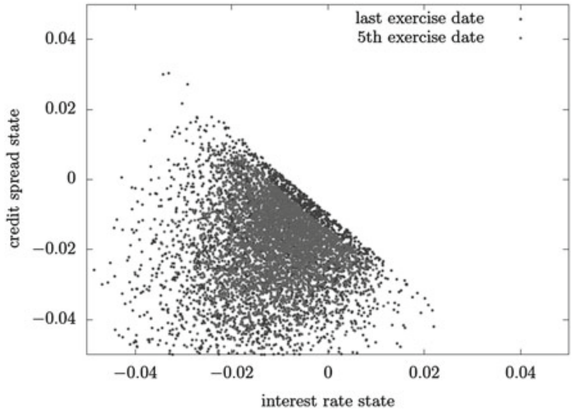

Fig. 2.1 Exercise region for 9th (the last, 2025-05-19) and the 5th (2021-05-18) exercise dates of a fixed rate bond, states in which the bond is exercised are taken from the Monte Carlo simulation used for pricing

Figure 2.1 shows points in the (x, y) plane for which the fix bond is called at the 9th and the 5th exercise dates.

Firstly, the shape of the exercise region (for both exercise dates) indicates that the bond is exercised when either the interest rate level or the credit spread level is low enough. More specifically, the picture confirms the intuition that the exercise boundary is given by the condition $ax + by < 1$ for some constants a, b. x is the level of interest rate and y that of credit spread.

In addition to this, one sees that the exercise region moves to the lower left for earlier exercise dates. This is because the exercise decision depends on two criteria:

1. the exercise as such must make sense economically, that is the bond's fair value at exercise must be greater or equal to the redemption price (usually the nominal);
2. the pay-off from the exercise (that is the difference of the bond's fair value and the redemption value) must exceed the present value of future early redemption rights.

The second point is exactly the reason why the pay-off at earlier exercise dates must increase.

Next, we look at how the callable bond price reacts to changes in

1. credit and interest rate volatility levels,
2. correlation of credit and interest rates,
3. credit and interest rates reversion.

Table 2.3 NPV of a callable fix rate bond under different model parameter scenarios

	NPV (relative) (%)	NPV change (%)
Original parameters	93.9979	0.0000
IR volatility $+0.0010$	93.6350	−0.3628
CR volatility $+0.0010$	93.5664	−0.4315
IR reversion $+0.0010$	94.0095	+0.0116
CR reversion $+0.0010$	94.0229	+0.0250
IR-CR correlation $+10\%$	93.8504	−0.1475
IR-CR correlation -10%	94.1653	+0.16740

Table 2.3 summarizes the results. The qualitative behaviour of the NPV under the different scenarios for the model parameters can be explained as follows: an increase both in IR or CS volatility increases the bond's price volatility at each exercise date; therefore, the value of the call option increases as well and, thus, the callable bond prices decrease then, since there is a higher chance of a price increase then.

For a term structure model that we consider later, a parameter governing the behaviour that control the tendency towards a long-term average, the mean reversion, for instance in a LGM model controls two things:

1. the inter-temporal correlations between rates (see above) and a higher reversion produces a lower inter-temporal correlation, which in turn increases the call option value;
2. the effective volatility for rates which is lower for higher reversion rates (given the same model volatility parameter) and, therefore, decrease the call option value.

Here, the second effect outweighs the first which effectively decreases the call option value then (and increases the callable bond's price). It should be noted that if the model volatility is calibrated to a market instrument that is discussed below, the second effect is washed away, since after increasing the model's reversion the recalibration of the model volatility will lead to an increased model volatility compensating for the change in the reversion. Then, the role of the reversion parameter is solely to control the inter-temporal correlations and an increase in reversion would lead to an increase in option value opposite to the result here. We give an example for this in Sect. 2.4.

Finally, a higher correlation between interest rates and credit spreads increases the effective price volatility of the bond at the exercise dates again and, therefore, increases the call option value and decreases the callable bond's value.

2.4 Hedging and Model Calibration

Up to now, we have looked at valuations in a model with arbitrary volatility, reversion and correlation parameters. To get market consistent prices, one has to calibrate the model's parameters in such a way that relevant market prices of reference instruments are matched. Here, "reference instruments" refer to instruments that are liquidly traded and for which a tradable market premium is known. "Relevant" means that the reference instruments whose value is reproduced by the model are related to the features of the exotic structure to be priced.

In case of a Bermudan call right, the usual choice is the underlying single European call rights, which in turn are given by European swaptions, at least in the case of a vanilla Bermudan callable swap. In some sense, the Bermudan option can be seen as a (dynamically) weighted sum of the single European rights, since as seen from today the exercise of the Bermudan option coincides with the exercise of each of the European rights with a certain probability (which changes over time). Indeed, one approximation for the Bermudan option value is simply the maximum of the values of the European options. See also below for the influence of inter-temporal correlations on the Bermudan option value.

As soon as the underlying instrument of the exotic call deviates from the market swaption's standard underlying, one has to perform some matching procedures to find a basket of market swaptions that appropriately describes the European call rights on the non-standard underlying of interest.

In our case, the callable bond's underlying deviates from the standard market swaption's underlying (with equal fixed rate as the bond, notional, underlying start date and underlying maturity date) in a number of points:

1. most importantly, the market swaption is not driven by the bond's credit spread, while for the bond it is one of the main price drivers;
2. the valuation of the underlying on the bond's benchmark discounting curve plus OAS spread versus the valuation of the swaption's underlying on the O/S curve;
3. the exchange of the redemption amount against the remaining bond cash-flows in case of the bond being called versus the exchange of the fixed and the Euribor 6M floating leg on exercise of the swaption;
4. the expiry date of the call, which is 5 business days in case of the bond and 2 in case of the market swaption;
5. the accrual schedule, which is unadjusted in the case of the bond and adjusted modified following in case of the market swaption's underlying swap.

Considering the first point, the only way out would be to use a swaption with a credit-linked swap underlying for hedging. However, such a product is not a liquidly quoted instrument in the market, so it does not help to follow this route in terms of model calibration. Also, one usually relies on standard market swaptions for hedging, since those are the instruments liquidly available and therefore cheap to trade.

There are several possible ways of accounting for the remaining differences, such as,

1. roughly adjusting parameters, like setting the swaption's strike to the bond fixed rate minus the Z-spread (appropriately converted to take the different rate conventions into account), or (more usual) adding an appropriate margin to the floating leg of the swaption's underlying swap;
2. ignoring other parameters such as the different notification periods of the call, the different exchanged financial instruments in case of the bond's and swaption's exercise (see above), or the different accrual schedules;
3. applying some numerical optimization to find the best-matching market instrument, for example by matching the NPV, Delta and Gamma of the respective underlying instruments in the model on the relevant expiry date, see Andersen and Piterbarg (2010b), 19.4 (representative basket approach), and Caspers (2013) for a specific application of the method to callable fixed bonds and other situations;
4. applying other matching methods, see Andersen and Piterbarg (2010b), 19.4.4, 19.4.5.

In the following, we illustrate the representative basket method on our examples.

So far we have identified market instruments to calibrate the model's interest rate volatility. To calibrate the model's volatility parameter to more than one instrument, it is usual to use a piecewise constant or piecewise linear volatility function with step times equal to the expiry times of the instrument one calibrates to. This gives the model's volatility exactly the same number of degrees of freedom as there are number of calibration instruments.

Likewise, one has to calibrate the credit volatility to market instruments like CDS options, if available. If no such appropriate instruments are liquidly quoted in the market (which is likely to happen), a historical estimation of the volatility can be used as a second best solution. Lichters et al. (2015), 12.3.3 provides a matching procedure to translate a historical estimate to a model parameter.

The reversion parameters of the model fine-tune the difference of the value of the Bermudan exercise right compared to the underlying European rights. See Andersen and Piterbarg (2010a), 19.2 for several possibilities to determine this parameter (in the case of interest rates at least):

1. calibrated to inter-temporal correlation from a global model such as a Libor Market Model;
2. used as an external, free parameter to match market prices for Bermudan swaptions directly;
3. calibrated to additional vanilla options such as caps and floors.

Finally, the correlation between interest rates and credit has to be specified. Since there are most probably no liquid market instruments quoted which are sensitive to this correlation, one again has to rely on a historical estimate. Again, see Lichters et al. (2015), 12.3.3 for a procedure to match the historical estimate with the model correlation.

We continue the valuation example from above by calibrating the interest rate component of our model to a basket of market-quoted swaptions. As indicated above, we ignore the credit spread dynamics, but adjust for the remaining differences listed in Sect. 2.4 by looking for a best-matching basket of market swaptions in the sense of Andersen and Piterbarg (2010b), 19.4.

Table 2.4 lists the resulting calibration basket. The expiry dates are exactly the exercise dates of the bond. The start dates of the underlying swaps however correspond to a standard market swaption's start delay of 2 business days opposed to the notice period of the bond's exercise of 5 business days. This causes the accrual periods and coupon payment dates of the swap are shifted by a few days compared to the bond. The nominal is a bit lower than the bond's nominal, which accounts for the default possibility of the bond leading

Table 2.4 Calibration basket for a callable fixed rate bond, following the representative basket approach

Expiry	Maturity	Nominal	Strike
2017-05-18	2025-12-22	929,972.53	0.019965
2018-05-18	2026-01-22	936,966.66	0.019968
2019-05-20	2026-02-23	942,693.56	0.020010
2020-05-18	2026-03-20	950,322.17	0.019965
2021-05-18	2026-04-20	957,875.24	0.019967
2022-05-18	2026-04-20	965,760.47	0.019960
2023-05-18	2026-05-22	976,226.73	0.019949
2024-05-20	2026-05-22	976,006.14	0.020122
2025-05-19	2026-05-21	958,570.07	0.020133

to a reduced hedge nominal for the swaptions. Note that, the nominal for the calibrating swaptions is not really relevant for the model calibration. For the same reason, the maturity dates lie before the bond's maturity, by 6 months for the first swaption up to only the few days we already observed for the last swaption. Finally, the strike of the calibrating swaptions is close to 2%, which is expected since the bond's coupon is 5% and the valuation spread (OAS) is 3%.

In summary, this basket is the result of a numerical optimization which accounts for the before-mentioned differences between the actual bond's call right and the standard market swaptions available for hedging.

We now calibrate the interest rate component of our LGM model to this basket assuming a (normal) market volatility of 0.0050. Table 2.5 shows the calibrated interest rate volatility. Note that, we keep the reversion fixed at the level we assumed before. Note also that the model volatility is close to the market volatility. This is, however, due to the facts that we use a normal market volatility (which corresponds to the nature of the model volatility) and that the reversion of the model is close to zero. Otherwise, the model volatility—in numeric terms—might significantly deviate from the market volatility.

Table 2.7 shows the effect of a reversion shift like in Table 2.3 before, but now when re-calibrating the model volatility to the market swaption basket. Now, the effect on the NPV is indeed negative because of the lower inter-temporal correlations increasing the call option's value. Table 2.6 shows the calibrated model volatility under the shifted reversion 0.0030. Although the calibration instruments have exactly the same value, the model volatility is slightly different because of the different reversion, which shows that reversion and volatility are interconnected.

Table 2.5 Calibrated model interest rate volatility, reversion is fixed at 0.0020

Swaption expiry	Market premium	Model premium	Model volatility
2017-05-18	13,321.19	13,321.19	0.00495
2018-05-18	17,143.09	17,143.09	0.00496
2019-05-20	18,694.90	18,694.90	0.00497
2020-05-18	18,638.26	18,638.26	0.00501
2021-05-18	17,561.88	17,561.88	0.00499
2022-05-18	15,326.62	15,326.62	0.00500
2023-05-18	12,691.53	12,691.53	0.00501
2024-05-20	9,131.95	9,131.95	0.00506
2025-05-19	4,716.95	4,716.95	0.00504

Table 2.6 Calibrated model interest rate volatility, with different reversion, now fixed at 0.0030

Swaption expiry	Market premium	Model premium	Model volatility
2017-05-18	13,321.19	13,321.19	0.00498
2018-05-18	17,143.09	17,143.09	0.00499
2019-05-20	18,694.90	18,694.90	0.00500
2020-05-18	18,638.26	18,638.26	0.00505
2021-05-18	17,561.88	17,561.88	0.00504
2022-05-18	15,326.62	15,326.62	0.00505
2023-05-18	12,691.53	12,691.53	0.00507
2024-05-20	9,131.95	9,131.95	0.00512
2025-05-19	4,716.95	4,716.95	0.00511

Table 2.7 NPV of a callable fix rate bond with calibrated interest rate volatility, both with and without recalibration under the reversion shift

	NPV (relative) (%)	NPV change (%)
Calibrated IR volatility	94.0075	0.0000
IR reversion $+0.0010$	94.0095	$+0.0020$
Recalibrated IR volatility	94.0043	-0.0032

2.5 Amortizing Structures

While in a bullet structure the full notional is paid back at maturity, there are also bonds that pay back part of the notional during the lifetime, most commonly together with the coupon payments. For example, the 10-year bond example from above may be modified such that on each coupon payment date 10% of the notional is paid back to the investor, leaving a final payment of only 100,000 USD on an initial investment of 1,000,000 USD. The coupon amounts are then computed with respect to the current notional of the period, that is these amounts shrink in the same proportion as the notional itself. Table 2.8 gives an example of such a bond's payment schedule.

Callability works in the same fashion for amortizing structures as it does for bullet structures: at an exercise date, the outstanding notional may be paid back to the investor.

Other than the amortizing notional payments which are scheduled to be paid back, the early redemption amounts are paid back if and only if the issuer chooses to do so. This distinction is important to remember. In this sense, amortization does not constitute an additional positive value to the issuer, while the one-sided early redemption rights do. It is important to distinguish these two kinds of repayments.

Table 2.8 Fix rate amortizing bond schedule

Accrual start	Accrual end	Payment date	Notional	Payment
2016-05-25	2017-05-25	2017-05-25	1,000,000.00	50,000.00
2017-05-25	2018-05-25	2018-05-25	900,000.00	45,000.00
2018-05-25	2019-05-25	2019-05-27	800,000.00	40,000.00
2019-05-25	2020-05-25	2020-05-25	700,000.00	35,000.00
2020-05-25	2021-05-25	2021-05-25	600,000.00	30,000.00
2021-05-25	2022-05-25	2022-05-25	500,000.00	25,000.00
2022-05-25	2023-05-25	2023-05-25	400,000.00	20,000.00
2023-05-25	2024-05-25	2024-05-27	300,000.00	15,000.00
2024-05-25	2025-05-25	2025-05-26	200,000.00	10,000.00
2025-05-25	2026-05-25	2026-05-25	100,000.00	5,000.00
		2026-05-25		100,000.0

Table 2.9 Asset swap of an amortizing fix rate bond, fixed leg schedule

Accrual start	Accrual end	Payment date	Notional	Payment
2016-05-25	2017-05-25	2017-05-25	1,000,000.00	−50,000.00
2017-05-25	2018-05-25	2018-05-25	900,000.00	−45,000.00
2018-05-25	2019-05-25	2019-05-27	800,000.00	−40,000.00
2019-05-25	2020-05-25	2020-05-25	700,000.00	−35,000.00
2020-05-25	2021-05-25	2021-05-25	600,000.00	−30,000.00
2021-05-25	2022-05-25	2022-05-25	500,000.00	−25,000.00
2022-05-25	2023-05-25	2023-05-25	400,000.00	−20,000.00
2023-05-25	2024-05-25	2024-05-27	300,000.00	−15,000.00
2024-05-25	2025-05-25	2025-05-26	200,000.00	−10,000.00
2025-05-25	2026-05-25	2026-05-25	100,000.00	−5,000.00

The natural hedge swap for an amortizing bond is again a fix versus float swap, but with the same amortizing notional structure as the bond (on both legs). Again, if the bond is callable, the hedge swap is made callable in the same way and the same remarks about the imperfectness of the hedge (and making it perfect by credit-linking it) apply. Table 2.9 displays the schedule of the fixed leg of the asset swap, and Table 2.10 the schedule of the floating leg, assuming an Euribor 6M index and a zero margin.

Concerning model calibration, similar remarks as in Sect. 1.3 apply; a suitable set of market instruments to calibrate the valuation model can be found by a Delta-Gamma best-matching approach (Andersen and Piterbarg 2010b, 19.4, Caspers 2013), or other matching methods as outlined in Andersen and Piterbarg (2010b), 19.4.4, 19.4.5.

We compute a best-matching calibration basket for the amortizing bond again following Andersen and Piterbarg (2010b), 19.4. Table 2.11 shows the

Table 2.10 Asset swap of an amortizing fix rate bond (zero margin), floating leg schedule, Euribor 6M rates are estimated on a flat forward curve @2%

Accrual start	Accrual end	Payment date	Notional	Payment
2016-05-25	2016-11-25	2016-11-25	1,000,000.00	10,133.19
2016-11-25	2017-05-25	2017-05-25	1,000,000.00	9,967.15
2017-05-25	2017-11-27	2017-11-27	900,000.00	9,219.50
2017-11-27	2018-05-25	2018-05-25	900,000.00	8,870.83
2018-05-25	2018-11-26	2018-11-26	800,000.00	8,150.83
2018-11-26	2019-05-27	2019-05-27	800,000.00	8,018.00
2019-05-27	2019-11-25	2019-11-25	700,000.00	7,015.75
2019-11-25	2020-05-25	2020-05-25	700,000.00	7,015.75
2020-05-25	2020-11-25	2020-11-25	600,000.00	6,079.91
2020-11-25	2021-05-25	2021-05-25	600,000.00	5,980.29
2021-05-25	2021-11-25	2021-11-25	500,000.00	5,066.59
2021-11-25	2022-05-25	2022-05-25	500,000.00	4,983.58
2022-05-25	2022-11-25	2022-11-25	400,000.00	4,053.28
2022-11-25	2023-05-25	2023-05-25	400,000.00	3,986.86
2023-05-25	2023-11-27	2023-11-27	300,000.00	3,073.17
2023-11-27	2024-05-27	2024-05-27	300,000.00	3,006.75
2024-05-27	2024-11-25	2024-11-25	200,000.00	2,004.50
2024-11-25	2025-05-26	2025-05-26	200,000.00	2,004.50
2025-05-26	2025-11-25	2025-11-25	100,000.00	1,007.78
2025-11-25	2026-05-25	2026-05-25	100,000.00	996.72

Table 2.11 Calibration basket for an amortizing bond, following the representative basket approach

Expiry	Maturity	Nominal	Strike
2017-05-18	2023-05-22	693,427.09	0.019946
2018-05-18	2023-10-23	620,913.74	0.019936
2019-05-20	2024-03-22	545,833.24	0.020013
2020-05-18	2024-08-20	472,931.84	0.019962
2021-05-18	2024-12-20	400,376.58	0.019953
2022-05-18	2025-05-20	327,004.67	0.019921
2023-05-18	2025-09-22	251,921.83	0.019884
2024-05-20	2026-01-22	173,156.94	0.020173
2025-05-19	2026-05-21	95,848.96	0.020133

result. The strike of the calibrating swaption is again close to 2% for all expiries. The maturity, however, for the first calibrating swaption is 3 years shorter than the bond's maturity, which is due to the decreasing notional over the lifetime of the bond. Likewise, the nominal is reduced. The same applies to the following calibrating swaptions, but to a lesser degree until the last swaption, which has its maturity date close to the bond maturity.

2.6 Callable Floater and Inverse Floater

A plain vanilla floating rate bond pays a Libor rate, plus possibly a spread, as the coupon. The accrual periods have the same tenor as the Libor rate itself, which in turn is fixed at the beginning of the period, usually a few business days before the accrual start date. We also call such a bond a floating rate note or floater for short.

The valuation of the floater is given by

$$V^{\text{Float Bond}} = N\left(\sum_{i=1}^{m} \tau_i (F_T(\tilde{T}_i, \tilde{T}_{i+1}) + s_i) DF_B(0, T_i) e^{-\lambda T_i} + DF_B(0, T_m) e^{-\lambda T_m}\right) \tag{2.1}$$

The natural hedge swap of a (callable) floater with price given by (2.1) would be a (callable) Libor against Libor swap, but with possibly different tenors and spreads on each of its legs. For example, an Euribor 1Y + 100bp floater may be hedged by a Euribor 1Y + 100bp against Euribor 6M + 212bp swap. Here, the margin on the Euribor 6M leg is determined by the tenor basis spread between Euribor 6M and Euribor 1Y and the credit spread of the bond, such that the package trades at par (assuming that a par asset swap is used for hedging).

The valuation of the hedge swap is given by

$$\begin{aligned} V^{\text{Float-Float Swap}} = &\sum_{i=1}^{n} \tilde{\tau}'_i (F_T(\tilde{T}'_i, \tilde{T}'_{i+1}) + s'_i) DF_{\text{OIS}}(0, \tilde{T}'_i) \\ &- \sum_{i=1}^{m} \tilde{\tau}_i (F_T(\tilde{T}_i, \tilde{T}_{i+1}) + s_i) DF_{\text{OIS}}(0, \tilde{T}_i) \end{aligned} \tag{2.2}$$

with m, n the number of coupon periods of the payer, resp. the receiver side of the swap, $\tilde{\tau}'_i$, $\tilde{\tau}_i$ the day count fractions of the coupons, T, T' the tenors of the two legs, $\tilde{T}'_i$, $\tilde{T}_i$ the index estimation start dates, $\tilde{T}'_{i+1}$, $\tilde{T}_{i+1}$ the index estimation end dates, s'_i, s_i the spreads and T'_i, T_i the payment times of the coupons.

At this point, it becomes particularly apparent that the plain hedge swap construction (without credit-linking) gives an imperfect hedge construction.

We illustrate the situation by looking at a 10-y floater paying half-yearly coupons at Euribor 6M + 300bp, which is callable yearly. Table 2.12 shows the estimated flows, and Table 2.13 the call schedule.

Table 2.14 shows the valuation of this floater. Looking at the effect of rate and volatility shifts in the model's interest rate and credit parameters, see Table 2.16,

Table 2.12 Euribor 6M floater schedule without the notional repayment at the end

Accrual start	Accrual end	Payment	Est. fixing (%)	Spread (%)	Est. amount
2016-05-25	2016-11-25	2016-11-25	1.9826	3.0000	25,466.52
2016-11-25	2017-05-25	2017-05-25	1.9824	3.0000	25,050.49
2017-05-25	2017-11-27	2017-11-27	1.9827	3.0000	25,743.89
2017-11-27	2018-05-25	2018-05-25	1.9825	3.0000	24,773.96
2018-05-25	2018-11-26	2018-11-26	1.9826	3.0000	25,605.21
2018-11-26	2019-05-27	2019-05-27	1.9825	3.0000	25,189.16
2019-05-27	2019-11-25	2019-11-25	1.9826	3.0000	25,189.71
2019-11-25	2020-05-25	2020-05-25	1.9825	3.0000	25,189.16
2020-05-25	2020-11-25	2020-11-25	1.9826	3.0000	25,466.52
2020-11-25	2021-05-25	2021-05-25	1.9824	3.0000	25,050.49
2021-05-25	2021-11-25	2021-11-25	1.9826	3.0000	25,466.52
2021-11-25	2022-05-25	2022-05-25	1.9824	3.0000	25,050.49
2022-05-25	2022-11-25	2022-11-25	1.9826	3.0000	25,466.52
2022-11-25	2023-05-25	2023-05-25	1.9824	3.0000	25,050.49
2023-05-25	2023-11-27	2023-11-27	1.9827	3.0000	25,743.89
2023-11-27	2024-05-27	2024-05-27	1.9825	3.0000	25,189.16
2024-05-27	2024-11-25	2024-11-25	1.9826	3.0000	25,189.71
2024-11-25	2025-05-26	2025-05-26	1.9825	3.0000	25,189.16
2025-05-26	2025-11-25	2025-11-25	1.9826	3.0000	25,328.12
2025-11-25	2026-05-25	2026-05-25	1.9824	3.0000	25,050.49

we see that both the interest rate level and the interest rate volatility have comparatively little impact on the valuation than the credit spread level and volatility. Table 2.15 shows the impact of the IR-CR correlation underlying's NPV: an increased correlation decreases the value of the float coupons.

It is interesting to look at the exercise region for a callable floater, see Fig. 2.2, and compare it to the corresponding picture for a callable fix bond, see Fig. 2.1. The interest rate level now has practically no impact on the exercise decision; it is solely driven by the credit spread level.

Finally, what would be a suitable calibration basket built from market swaptions? The answer is that there is not really a particularly appropriate one, since the call right in the floater does not have a significant sensitivity to the interest rate levels (it has sensitivity to the credit spread level, but regarding this the market instruments have no sensitivity on the other hand). This corresponds to the fact that there is no effective natural hedge for the callable floater.

It is market practice to just calibrate to a strip of ATM co-terminal swaptions to pick up the general market volatility in the model.

We should mention that the pay-off of a Libor floater is usually floored at an effective coupon rate of zero percent, so that negative coupon payments are avoided. This is not much of a difference if interest rates or the margin paid on top of the Libor fixing is sufficiently high, but for low interest rate markets and a low margin the embedded floor becomes a significant price driver. While

Table 2.13 Euribro 6M Floater call schedule, the settlement dates are chosen to be equal to the accrual start date of the first period that is part of the exercise right, the exercise date has a 5 business days (TARGET) notice period

Exercise date	Settlement date
2016-11-18	2016-11-25
2017-05-18	2017-05-25
2017-11-20	2017-11-27
2018-05-18	2018-05-25
2018-11-19	2018-11-26
2019-05-20	2019-05-27
2019-11-18	2019-11-25
2020-05-18	2020-05-25
2020-11-18	2020-11-25
2021-05-18	2021-05-25
2021-11-18	2021-11-25
2022-05-18	2022-05-25
2022-11-18	2022-11-25
2023-05-18	2023-05-25
2023-11-20	2023-11-27
2024-05-20	2024-05-27
2024-11-18	2024-11-25
2025-05-19	2025-05-26
2025-11-18	2025-11-25

Table 2.14 NPV of a callable floater

	NPV (EUR)	NPV (relative) (%)
Bond underlying	996,509.47	99.6509
Bond call option	−36,135.67	−3.6136
Callable bond total	960,373.81	96.0374

Table 2.15 NPV of a floating rate bond under different IR-CR correlations

	NPV (relative) (%)	NPV change (%)
IR-CR correlation 0	99.8909	0.0000
IR-CR correlation 0.5	99.6509	−0.2400
IR-CR correlation −0.5	100.1294	+0.2385

we ignored such a floor up to now, we go into more detail about this in the following, discussing inverse floaters.

An inverse floater has a similar pay-off as a vanilla floater, but it reverses the sign of the Libor part of the pay-off and typically introduces a multiplier. An example would be a pay-off of the form $9\% - 2 \cdot$ Euribor 6M, where the coupon is floored at 0% to avoid negative coupons in the floater.

Roughly assuming a fixing of 2% for the Euribor 6M one arrives at a coupon of 5%, just as in the case of the floater above.

Table 2.16 NPV of a callable floating rate bond under different model parameter scenarios and IR and CR rate shifts

	NPV (relative) (%)	NPV change (%)
Original parameters	96.0374	0.0000
IR rate level $+0.0010$	96.0529	$+0.0155$
CR spread level $+0.0010$	95.5608	-0.4766
IR volatility $+0.0010$	96.0100	-0.0274
CR volatility $+0.0010$	95.5631	-0.4743

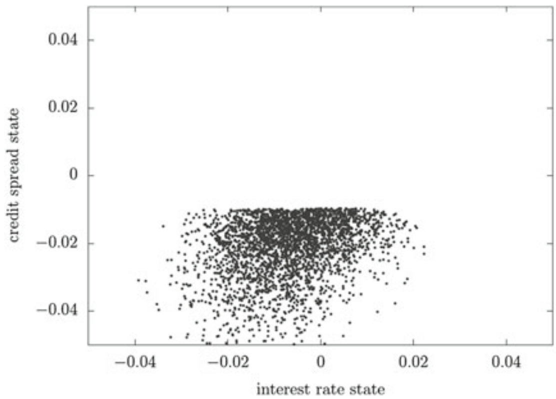

Fig. 2.2 Exercise region for 10th exercise date (2020-11-28) of a floating rate bond; exercise states are taken from the Monte Carlo simulation used for pricing

One interesting point in the pay-off is the embedded floor. Obviously, it is activated if Euribor 6M fixings exceeding 4.5%, that is the floored coupon rate can actually be understood as a long position in a 2% standard market cap from the bond's investors perspective. Table 2.17 shows an example of realized cashflows under simulated Euriobr 6M fixings. The embedded floors produce a pay-off for the accrual periods starting on 2022-05-25, 2022-11-25, 2023-11-27 and 2024-05-27.

From the valuation perspective, for a non-callable inverse floater, the embedded cap can be valued separately using the market Black76 formula. However, note that the discounting has to be done on the bond's benchmark curve with the credit spread added. The latter is also called the Asset Swap spread since it is tied to the bond.

If the inverse floater is callable, the situation gets more complicated. The model we used so far for interest rates, the LGM model, has not enough flexibility to be calibrated to both caplet smiles and a strip of co-terminal swaptions. Possible alternatives are models with local volatility like the Markov functional model, or a full Libor market model.

Table 2.17 Inverse floater schedule with a simulated series of fixings

Accrual start	Accrual end	Payment	Sim. fixing (%)	Spread (%)	Sim. amount
2016-05-25	2016-11-25	2016-11-25	1.9000	9.0000	26,577.78
2016-11-25	2017-05-25	2017-05-25	2.1000	9.0000	24,133.33
2017-05-25	2017-11-27	2017-11-27	2.5000	9.0000	20,666.67
2017-11-27	2018-05-25	2018-05-25	3.0000	9.0000	14,916.67
2018-05-25	2018-11-26	2018-11-26	3.1000	9.0000	14,388.89
2018-11-26	2019-05-27	2019-05-27	2.8000	9.0000	17,188.89
2019-05-27	2019-11-25	2019-11-25	3.5000	9.0000	10,111.11
2019-11-25	2020-05-25	2020-05-25	3.7000	9.0000	8,088.89
2020-05-25	2020-11-25	2020-11-25	3.9000	9.0000	6,133.33
2020-11-25	2021-05-25	2021-05-25	4.0000	9.0000	5,027.78
2021-05-25	2021-11-25	2021-11-25	3.8000	9.0000	7,155.56
2021-11-25	2022-05-25	2022-05-25	4.1000	9.0000	4,022.22
2022-05-25	2022-11-25	2022-11-25	4.6000	9.0000	0.00
2022-11-25	2023-05-25	2023-05-25	5.0000	9.0000	0.00
2023-05-25	2023-11-27	2023-11-27	4.3000	9.0000	2,066.67
2023-11-27	2024-05-27	2024-05-27	4.8000	9.0000	0.00
2024-05-27	2024-11-25	2024-11-25	4.5000	9.0000	0.00
2024-11-25	2025-05-26	2025-05-26	4.0000	9.0000	5,055.56
2025-05-26	2025-11-25	2025-11-25	3.9000	9.0000	6,100.00
2025-11-25	2026-05-25	2026-05-25	4.0000	9.0000	5,027.78

2.7 Zero Bonds, Callable Zeros and Accreting Swaps

A Zero Coupon Bond does not pay coupons during his lifetime but only the final capital. There are two types: the first paying a deterministic amount at maturity and the second a stochastic amount.

Although there are no coupon payments, it is useful to think of these bonds as coupon paying bonds with the coupons immediately reinvested in the bond again.

For example, consider a fixed rate bond with a notional of 1,000,000 USD paying a yearly coupon of 5% which is immediately reinvested in the notional of the bond. Table 2.18 shows the reinvested coupons in the nominal column as well as the actual, physical flow of the instrument at maturity. This is an example of a compounding zero bond.

Table 2.19 shows an equivalent bond, but with rescaled nominal, such that the final nominal corresponds to the notional of the bond. This is in contrast to the compounding zero bond where the initial notional equals the notional of the bond. This type of zero bond is called discounted zero bond.

In case the reinvested coupon is not deterministic that is not known in advance, as is the case for "Libor plus spread" coupons as the most basic

Table 2.18 Fixed rate compounded zero bond schedule, the only physical payment occurs at maturity

Accrual start	Accrual end	Payment	Nominal	Amount
2016-05-25	2017-05-25	2017-05-25	1,000,000.00	0.00
2017-05-25	2018-05-25	2018-05-25	1,050,000.00	0.00
2018-05-25	2019-05-25	2019-05-27	1,102,500.00	0.00
2019-05-25	2020-05-25	2020-05-25	1,157,625.00	0.00
2020-05-25	2021-05-25	2021-05-25	1,215,506.25	0.00
2021-05-25	2022-05-25	2022-05-25	1,276,281.56	0.00
2022-05-25	2023-05-25	2023-05-25	1,340,095.64	0.00
2023-05-25	2024-05-25	2024-05-27	1,407,100.42	0.00
2024-05-25	2025-05-25	2025-05-26	1,477,455.44	0.00
2025-05-25	2026-05-25	2026-05-25	1,551,328.22	1,628,894.63

Table 2.19 Fixed rate discounted zero bond schedule; the repayment at maturity is equal to the nominal of the bond

Accrual start	Accrual end	Payment	Nominal	Amount
2016-05-25	2017-05-25	2017-05-25	613,913.25	0.00
2017-05-25	2018-05-25	2018-05-25	644,608.91	0.00
2018-05-25	2019-05-25	2019-05-27	676,839.36	0.00
2019-05-25	2020-05-25	2020-05-25	710,681.33	0.00
2020-05-25	2021-05-25	2021-05-25	746,215.40	0.00
2021-05-25	2022-05-25	2022-05-25	783,526.16	0.00
2022-05-25	2023-05-25	2023-05-25	822,702.47	0.00
2023-05-25	2024-05-25	2024-05-27	863,837.60	0.00
2024-05-25	2025-05-25	2025-05-26	907,029.48	0.00
2025-05-25	2026-05-25	2026-05-25	952,380.95	1,000,000.00

example or for other structured coupons (for example CMS coupons, CMS spread coupons or PIK note coupons), only a compounding zero bond can be constructed. That is a zero bond with unknown final notional to be paid back. Table 2.20 shows an example with simulated fixings, based on a coupon of Euribor 6M plus 300 basis points spread.

Callability of zero bonds has the same mechanics as for fixed rate bonds, but the notional be paid back if the early redemption right is exercised follows a non-constant schedule. Usually, this is equal to the current accreted nominal. Table 2.23 shows an example corresponding to the zero bond shown in Table 2.18.

The natural hedge instrument of a (callable) zero bond is a (callable) accreting swap, which exchanges the virtual coupon of the zero bond, but this time as a physical payment in the swap, against an Libor plus spread coupon. In

Table 2.20 Floating zero bond schedule with simulated fixings

Accrual start	Accrual end	Payment	Sim. fixing (%)	Spread (%)	Sim. nominal	Sim. amount
2016-05-25	2016-11-25	2016-11-25	1.9000	3.0000	1,000,000.00	0.0
2016-11-25	2017-05-25	2017-05-25	2.1000	3.0000	1,025,044.44	0.0
2017-05-25	2017-11-27	2017-11-27	2.5000	3.0000	1,051,328.29	0.0
2017-11-27	2018-05-25	2018-05-25	3.0000	3.0000	1,081,203.54	0.0
2018-05-25	2018-11-26	2018-11-26	3.1000	3.0000	1,113,459.44	0.0
2018-11-26	2019-05-27	2019-05-27	2.8000	3.0000	1,148,363.30	0.0
2019-05-27	2019-11-25	2019-11-25	3.5000	3.0000	1,182,035.87	0.0
2019-11-25	2020-05-25	2020-05-25	3.7000	3.0000	1,220,878.88	0.0
2020-05-25	2020-11-25	2020-11-25	3.9000	3.0000	1,262,232.76	0.0
2020-11-25	2021-05-25	2021-05-25	4.0000	3.0000	1,306,747.50	0.0
2021-05-25	2021-11-25	2021-11-25	3.8000	3.0000	1,352,737.76	0.0
2021-11-25	2022-05-25	2022-05-25	4.1000	3.0000	1,399,752.91	0.0
2022-05-25	2022-11-25	2022-11-25	4.6000	3.0000	1,449,720.20	0.0
2022-11-25	2023-05-25	2023-05-25	5.0000	3.0000	1,506,033.77	0.0
2023-05-25	2023-11-27	2023-11-27	4.3000	3.0000	1,566,609.80	0.0
2023-11-27	2024-05-27	2024-05-27	4.8000	3.0000	1,625,697.10	0.0
2024-05-27	2024-11-25	2024-11-25	4.5000	3.0000	1,689,803.75	0.0
2024-11-25	2025-05-26	2025-05-26	4.0000	3.0000	1,753,875.48	0.0
2025-05-26	2025-11-25	2025-11-25	3.9000	3.0000	1,815,943.18	0.0
2025-11-25	2026-05-25	2026-05-25	4.0000	3.0000	1,879,637.39	0.0
		2026-05-25				1,945,790.19

Table 2.21 Asset swap for fixed rate compounded zero bond schedule, fixed leg

Accrual start	Accrual end	Payment	Nominal	Amount
2016-05-25	2017-05-25	2017-05-25	1,000,000.00	50,000.00
2017-05-25	2018-05-25	2018-05-25	1,050,000.00	52,500.00
2018-05-25	2019-05-25	2019-05-27	1,102,500.00	55,125.00
2019-05-25	2020-05-25	2020-05-25	1,157,625.00	57,881.25
2020-05-25	2021-05-25	2021-05-25	1,215,506.25	60,775.31
2021-05-25	2022-05-25	2022-05-25	1,276,281.56	63,814.08
2022-05-25	2023-05-25	2023-05-25	1,340,095.64	67,004.78
2023-05-25	2024-05-25	2024-05-27	1,407,100.42	70,355.02
2024-05-25	2025-05-25	2025-05-26	1,477,455.44	73,872.77
2025-05-25	2026-05-25	2026-05-25	1,551,328.22	77,566.41

addition, the notional of the swap is not constant, but increases by each virtual coupon. Tables 2.21 and 2.22 show the schedules of the legs of such an asset swap.

Finally, the representative basket method can be applied here as well to determine a calibration basket. Table 2.24 lists the calibration instruments for the callable zero bond given by Tables 2.18 and 2.23. We observe that the strike

Table 2.22 Asset swap for fixed rate compounded zero bond schedule, floating leg (amount is estimated on a flat 2%) forward curve

Accrual start	Accrual end	Payment	Nominal	Est. amount
2016-05-25	2016-11-25	2016-11-25	1,000,000.00	9,711.11
2016-11-25	2017-05-25	2017-05-25	1,000,000.00	10,558.33
2017-05-25	2017-11-27	2017-11-27	1,050,000.00	13,562.50
2017-11-27	2018-05-25	2018-05-25	1,050,000.00	15,662.50
2018-05-25	2018-11-26	2018-11-26	1,102,500.00	17,563.44
2018-11-26	2019-05-27	2019-05-27	1,102,500.00	15,606.50
2019-05-27	2019-11-25	2019-11-25	1,157,625.00	20,483.53
2019-11-25	2020-05-25	2020-05-25	1,157,625.00	21,654.02
2020-05-25	2020-11-25	2020-11-25	1,215,506.25	24,229.09
2020-11-25	2021-05-25	2021-05-25	1,215,506.25	24,445.18
2021-05-25	2021-11-25	2021-11-25	1,276,281.56	24,788.22
2021-11-25	2022-05-25	2022-05-25	1,276,281.56	26,309.13
2022-05-25	2022-11-25	2022-11-25	1,340,095.64	31,507.14
2022-11-25	2023-05-25	2023-05-25	1,340,095.64	33,688.52
2023-05-25	2023-11-27	2023-11-27	1,407,100.42	31,261.08
2023-11-27	2024-05-27	2024-05-27	1,407,100.42	34,145.64
2024-05-27	2024-11-25	2024-11-25	1,477,455.44	33,612.11
2024-11-25	2025-05-26	2025-05-26	1,477,455.44	29,877.43
2025-05-26	2025-11-25	2025-11-25	1,551,328.22	30,755.08
2025-11-25	2026-05-25	2026-05-25	1,551,328.22	31,198.93

Table 2.23 Zero fix compounded call schedule, the exercise amount is equal to the compounded nominal schedule, the exercise date is chosen to have a notice period of 5 business days (TARGET) before the settlement date

Exercise date	Settlement date	Exercise amount
2017-05-18	2017-05-25	1,050,000.00
2018-05-18	2018-05-25	1,102,500.00
2019-05-20	2019-05-27	1,157,625.00
2020-05-18	2020-05-25	1,215,506.25
2021-05-18	2021-05-25	1,276,281.56
2022-05-18	2022-05-25	1,340,095.64
2023-05-18	2023-05-25	1,407,100.42
2024-05-20	2024-05-27	1,477,455.44
2025-05-19	2025-05-26	1,551,328.22

is again, and expected close to 2% for all calibrating instruments, while the notional increases over the lifetime and the maturity decreases starting from a date beyond the maturity of the zero bond for the first calibration instruments, matching the maturity for the last (two) calibration instruments.

Table 2.24 Zero fix compounded calibration basket, following the representative basket approach

Expiry	Maturity	Nominal	Strike
2017-05-18	2026-08-24	1,096,497.57	0.019967
2018-05-18	2026-08-24	1,143,772.29	0.019964
2019-05-20	2026-07-22	1,191,195.80	0.019999
2020-05-18	2026-07-20	1,242,027.65	0.019969
2021-05-18	2026-06-22	1,294,664.97	0.019970
2022-05-18	2026-06-22	1,348,914.20	0.019969
2023-05-18	2026-06-22	1,409,093.35	0.019962
2024-05-20	2026-05-22	1,455,776.38	0.020127
2025-05-19	2026-05-21	1,487,009.51	0.020133

2.8 Conclusion and Summary

In this section, we introduced callability as an important exotic feature that is often added to bonds and swaps, typically in the form of multiple ("Bermudan") call rights. We gave valuation examples and explain how different model parameters influence the pricing of the call option of a bond. We explained the asset swap construction for callable bonds via callable swaps and go into some detail about how a model can be calibrated to liquid vanilla market options representing the bond's call right as suitable as possible. We covered callable fixed rate bonds as well as amortizing structures, floater and zero bonds as the corresponding asset swaps.

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