

2

Stylized Facts of Financial Markets and Bubbles

2.1 Introduction to Stylized Facts

2.1.1 Definition

A casual examination of the financial press reveals the tendency to explain market movements on the bases of “events”. That is, price movements are often rationalized on some economic or political innovation.¹ Even though a priori it might seem like different assets in different markets are not necessarily influenced by the same information sets, the findings of seventy years of empirical studies indicates that this is the case if one focuses on their statistical properties. Seemingly random variations in asset prices share non-trivial statistical properties and financial markets are characterized by some generic features of the financial time series of the instruments traded within them. These properties, common across a wide range of instruments, markets, and time periods are called *stylized empirical facts*. Hence stylized facts are the general properties expected to be present in any set of returns that we need to understand in order to model financial market dynamics.

The term “stylized facts” was coined by Nicholas Kaldor who in 1961 suggested that economic theory construction should begin with a summary of the relevant facts. However, given that “*facts as recorded by statisticians, are always subject to numerous snags and qualifications, and for that reason are*

¹Cutler, D.M., Poterba, J.M. and Summers, L. (1989). What moves stock prices? *Journal of Portfolio Management*, pp. 4–12.

incapable of being summarized", he proposes theorists "*should be free to start off with a stylized view of the facts – i.e. concentrate on broad tendencies, ignoring individual detail*". It is with respect to the broad tendencies that result from such a process, that Kaldor coins the term "*stylized facts*".² Thus, in economics the acceptance of *stylized facts* is used to provide a simplified representation of empirical findings³ so consistent that are accepted as "true" due to their generality. For instance, most sets of daily returns present three major stylized facts pervasive across time and markets: their distribution is not normal, there is almost no correlation between returns for different days, and the correlations between the magnitudes of returns on nearby days are positive and statistically significant.

To summarize, *stylized facts* was coined for stable patterns that emerge from many different sources of empirical data.⁴ They are obtained by taking a common denominator among the properties observed in different markets and instruments, presenting broad generalizations that summarize statistical calculations which, although true in "essence", may contain inaccuracies in the detail.^{5,6}

A key purpose behind the concept of stylized facts is to help build adequate models for the questions under investigation. To achieve this objective, a basic concern is to ensure the model remains parsimonious enough so no irrelevant details distract our attention, and still sufficiently rich to capture all relevant aspects. This was the obstacle Kaldor was facing and it was in this context, that he introduced the concept.

The stylized facts approach helps clarify the object of research⁷ by focusing on the extraction of the key characteristics of the phenomenon of interest. This has two important methodological implications. The first relates to the "*as if*" concept (cf. Boland 1987/1994:535⁸) which in the stylized facts

²Kaldor, N. (1961/1968). *Capital Accumulation and Economic Growth*. in Lutz, FA and Hague, DC edn: *The Theory of Capital* (Reprint, London: Macmillan), pp. 177–222.

³Cooley, T. (1995). *Frontiers of Business Cycle Research*. Princeton University Press, p. 3.

⁴Kaldor, N. (1961/1968). *Capital Accumulation and Economic Growth*. in Lutz, FA and Hague, DC edn: *The Theory of Capital* (Reprint, London: Macmillan) pp. 177–222.

⁵Sewell, M. (2011). Characterization of Financial Time Series. *Research Note RN/11/01*, University College London, London.

⁶Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance*, 1, pp. 223–236.

⁷In contrast to normative research positive research attempts to explain empirical observations, see Blaug, M. (1998). *The Positive-Normative Distinction*. in Davis, J.B., Hands, D.W. and Mäki, U., edn, *The Handbook of Economic Methodology* (Cheltenham: Elgar), pp. 370–374.

⁸Boland, L.A. (1987/1994). Stylized Facts, in Eatwell, J., Milgate, M. and Newman, P., edn: *The New Palgrave Dictionary of Economics*, 4, repr. with corrections (London: Macmillan Press), pp. 535–536.

case is used to state facts as if they truly represent the phenomenon of interest and, in so doing, steering model construction. This is in contrast with the neoclassical use in which *as if* is understood to imply that the facts are true (cf. Friedman 1953/1989⁹). That is, in the latter case we accept the assumptions required in the model and particularly those related to the predictions it can generate. Hence, the use made under the stylized facts framework refers to a prior step in which a given level of abstraction is chosen.

The second implication refers to the fact that stylized facts are compatible with the idea that these can be known (cf. Mäki 1998:407¹⁰), while simultaneously implying that models are instruments of thought, thus promoting an analytical approach in model construction. The caveats deriving from these implications are that steps in the scientific process can now be affected by having the researcher determine the stylized facts he wishes to inspect, and building a model which might affect results. Solow pointed to these issues when, with respect to Kaldor's stylized facts of macroeconomic growth, he stated "[...] *there is no doubt that they are stylized, though it is possible to question whether they are facts.*" (Solow 1969/1988:2¹¹).

2.1.2 Asymptotic and Convergence

The availability of large data sets of high-frequency price series and the application of computer-intensive methods for analyzing the statistical properties of stocks, commodities, and market indexes have opened new horizons for financial research. One of the challenges generated by the study of these newly available vast data sets is the possibility of capturing the information contained in a meaningful way. The goal is to 'let the data speak for itself'. Hence, in this analysis, *non-parametric* methods can be used as these make no quantitative assumptions about the properties of the stochastic process generating the data. In addition, some semi-parametric methods may be used as well as these do not completely specify the form of the price process even if they imply the existence of a parameter which describes a property of the process.

⁹Friedman, M. (1953/1989). The Methodology of Positive Economics, in Friedman M., edn: Essays in Positive Economics, 15th edn (Chicago: Univ. of Chicago Press), pp. 3–43.

¹⁰Mäki, U. (1998). Realism, in Davis, J.B., Hands, D.W. and Mäki, U., edn: The Handbook of Economic Methodology (Cheltenham: Elgar), pp. 404–409.

¹¹Solow, R.M. (1969/1988). Growth Theory: an Exposition. Paperback edn (New York, NY: Oxford Univ. Press).

Statistical inference implies using samples from populations and drawing conclusions about those populations from the sample data. All brochures describing financial investment opportunities add the following warning to their potential readers: ‘Past returns do not imply future performance’. Nonetheless, it would be pointless statistically analyze market data if none of their statistical properties were to remain stable over time.¹² Strictly speaking, “invariant” means estimates remain unchanged when measurements and parameters are transformed in a compatible way. The invariance of statistical properties of the return process in time corresponds to the *stationarity* hypothesis.

The term *ergodic* describes a dynamical system which behaves in the same way regardless if averaged over time or over the space of all the system’s states. In econometrics a stochastic process is considered ergodic if its statistical properties such as the mean and variance can be inferred from a single adequately sized sample. The rationale is that any sample must represent the average statistical properties of the entire process, rather than those of a section. Thus, while stationarity is necessary, it is not sufficient as averages need to converge to the quantities they are supposed to estimate and the ergodic property is necessary to ensure that happens. Even though ergodicity is often satisfied by iid (Independent and identically distributed random variables) observations, this is not obvious for processes with complex dependences such as returns and systems with long-range dependence.¹³ The multifractal processes recently introduced to model high frequency returns might also fall into this category, as well as processes which suffer chaotic changes.^{14,15} In these scenarios, the relationship between sample averages and model expectations remains an unsettled matter.¹⁶

Parametric methods are statistical techniques that make certain assumptions about the parameters of the sampled populations, such as the population being normally distributed. Nonetheless, these assumptions cannot always be validated, and that is why nonparametrics methods developed techniques which do not share the restrictions of their parametric counterparts. Nonparametrics methods are sometimes called “distribution free”

¹²<http://www.cmap.polytechnique.fr/~rama>.

¹³Bouchaud, J.P. (2001). Power laws in economics and finance. *Quantitative Finance*, 1, pp. 105–112.

¹⁴Mandelbrot, B., Fisher, A. and Calvet, L. (1997). A multifractal model of asset returns. *Cowles Foundation for Economic Research Working Paper*, <http://ssrn.com/abstract=78588>.

¹⁵Muzy, J.F., Delour, J. and Bacry, E. (2000). Modelling fluctuations of financial time series: from cascade process to stochastic volatility model. *The European Physical Journal B-Condensed Matter and Complex Systems*, 17(3), pp. 537–548.

¹⁶https://en.wikipedia.org/wiki/Ergodic_theory.

tests, although they still have some underlying distributional requirements. In general, parametric procedures have nonparametric counterparts, even though the hypothesis tested may not be exactly the same. For example, a parametric two-sample test for differences in means has a nonparametric counterpart test for differences in medians.¹⁷

Asymptotic or *large sample theory* is a generic structure for the evaluation of the properties of estimators and statistical tests in which it is assumed that the sample size n grows indefinitely. Asymptotic theory produces results that cannot be obtained with “finite-sample theory” given these are treated as approximately valid for finite sample sizes. The properties of statistical procedures are evaluated in the limit as $n \rightarrow \infty$, the *limit* being the value a function “approaches” as the input gets close to some value. The underlying assumption of asymptotic theory is that a data set of size n can be increased so that the sample size grows infinitely: $n \rightarrow \infty$. Hence, results unavailable for samples of finite sizes can be obtained.

The *law of large numbers* (LLN) says that the average of the result of repeating the same experiment many times should be close to the expected value, and will become closer as the number of trials is increased. Specifically, the LLN states that for a sequence of iid random variables X_1, X_2, \dots , the sample averages \bar{X}_n converge in probability to the population mean $E[X_i]$, as $n \rightarrow \infty$. For a finite n on the other hand, we cannot claim the likely distribution of \bar{X}_n if the distributions of individual X_i 's are unknown.

2.1.3 Modes of Convergence of Random Variables

In probability theory, there are different notions of *convergence of random variables* to some limit random variable. The same concepts are known in more general mathematics as *stochastic convergence* formalizing the idea that a sequence of random events can be expected to settle down into an essentially stable behavior. The various notions of convergence relate to how such behavior can be characterized: two commonly known behaviors are that the sequence eventually takes a constant value and that values in the sequence continue to change but can be described by an unchanging probability distribution. The patterns into which a sequence of random events may settle are:

¹⁷<http://mvpprograms.com/help/mvpstats/NonparametricTests>.

- a. Convergence to a fixed value;
- b. Outcomes that increasingly resemble those a deterministic function would produce;
- c. An increasing preference towards a certain outcome;
- d. An increasing aversion against roaming far away from a given outcome.
- e. That the probability distribution describing the next outcome may grow increasingly similar to a given distribution;
- f. That the series resulting from estimating the expected value of the outcome's distance from a given value may converge to 0;
- g. That the variance of the random variable describing the next event grows increasingly smaller.

The convergence of a single series to a limiting value is important, as is the notion that two series may converge towards each other. This latter case can be approached by analyzing the sequence resulting from the difference or the ratio of the two series. For example, if the average of n independent random variables Y_i , $i = 1, \dots, n$, all having the same finite mean and variance, is given by

$$X_n = \frac{1}{n} \sum_{i=1}^n Y_i \quad (2.1)$$

then as n tends to infinity, X_n converges *in probability* to the common mean of the random variables Y_i . This result is the well-known *weak law* of large numbers.

Most modeling situations involve stochastic regressors, nonlinear models, or nonlinear estimation techniques. Instances when it is difficult to obtain exact statistical results, such as the expected value or true distribution, make our prior discussion on convergence relevant as we need to rely on approximations based on what we know about certain statistics in large samples. In this scenario the concept of convergence is most useful. For instance, as n grows large we can obtain convergence to a constant. Two examples are the sample mean converges to the population mean, or convergence of the variance to zero, meaning that the random variable is getting closer to something that is not random. The following are some types of convergence:

- a. *Convergence in distribution.* With this type of convergence, we increasingly expect to see the next outcome in a sequence of random experiments becoming better modeled by a given probability distribution. Convergence in distribution is the weakest form of convergence.

However it is frequently used as it arises from the application of the central limit theorem.

- b. *Convergence in probability.* The essence of this type of convergence is that the probability of an “unusual” outcome decreases as the sequence enlarges. An estimator is called consistent if it converges in probability to the quantity being estimated. Convergence in probability is also the type of convergence established by the weak law of large numbers.
- c. *Almost sure convergence.* Almost sure convergence is often denoted by adding the letters *a.s.* over an arrow indicating convergence:
 $X_n \xrightarrow{a.s.} X$. The sequence X_n converges *almost surely* or *almost everywhere* or *with probability 1* or *strongly* towards X means that $\Pr \left(\lim_{n \rightarrow \infty} X_n = X \right) = 1$. That is, the values of X_n approach the value of X in the sense that events for which X_n does not converge to X have probability 0.
- d. *Sure convergence.* The sequence of random variables (X_n) over the same probability space converges *surely* or *everywhere* or *pointwise* towards X :

$$\lim_{n \rightarrow \infty} X_n(\omega) = X(\omega), \quad \forall \omega \in \Omega \quad (2.2)$$

where Ω is the sample space of the underlying probability space over which the random variables are defined. *Sure convergence* is rarely used as it implies all the other kinds of convergence already stated and only exists on sets with probability zero. Given that random variables are functions, the notion of pointwise convergence of sequence functions is extended to sequence of random variables:

$$\{\omega \in \Omega \mid \lim_{n \rightarrow \infty} X_n(\omega) = X(\omega)\} = \Omega \quad (2.3)$$

- e. *Convergence in r -th mean.* Convergence in r -th mean says that the expected value (E) of the r -th power of the difference between X_n and X , converges to zero. Given a real number $r \geq 1$, the sequence X_n converges *in the r -th mean* towards the random variable X , if the r -th absolute moments $E(|X_n|^r)$ and $E(|X|^r)$ of X_n and X exist, and the

$$\lim_{n \rightarrow \infty} E(|X_n - X|^r) = 0 \quad (2.4)$$

This type of convergence is sometimes indicated as follows:

$$X_n \xrightarrow{L^r} X \quad (2.5)$$

Convergence in the r -th mean, for $r \geq 1$, implies convergence in probability. The most important cases of convergence in r -th mean are when X_n converges in r -th mean to X for $r = 1$, in this case we say that X_n converges in mean to X , and when X_n converges in r -th mean to X for $r = 2$, and we say that X_n converges in mean square to X .

In general, an estimator is a function of the data. Hence, the key property of estimators is that are *consistent*. This happens when the estimator converges in probability to the true value of the parameter it intends to estimate:

$$\hat{\theta}_n \xrightarrow{P} \theta_0 \quad (2.6)$$

Most often, the estimators we encounter in practice have the *asymptotically normal* distribution:

$$\sqrt{n} \left(\hat{\theta}_n - \theta_0 \right) \xrightarrow{d} N(0, V) \quad (2.7)$$

$\hat{\theta}_n$ being the sequence of estimators.

2.2 Stylized Facts of Financial Markets

2.2.1 Introduction

Markets are efficient in the weak-form sense if past returns cannot predict future returns (Fama 1970¹⁸). However, even though the efficient market hypothesis has been a cornerstone of financial theory, evidence points to market inefficiencies such as serial correlation, momentum and mean reversion.

A decade before the 2008 crises began Case and Shiller (1989, 1990)^{19,20} had already questioned the efficiency of the market for single-family homes by providing evidence of positive serial correlation in year-to-year changes in prices, and negative serial correlations at lags of two-to-four years.

¹⁸Fama, E. (1970). Efficient capital markets: a review of theory and empirical work. *The Journal of Finance*, 25, pp. 383–417.

¹⁹Case, K.E. and Shiller, R.J. (1989). The Efficiency of the Market for Single-Family Homes. *National Bureau of Economic Research*, Inc, NBER Working Papers 2506.

²⁰Case, K.E. and Shiller, R.J. (1990). Forecasting Prices and Excess Returns in the Housing Market. *Real Estate Economics, American Real Estate and Urban Economics Association*, 18(3), pp. 253–273.

In addition, these authors showed that future house price changes can be predicted with rents and other lagged fundamental variables. Other researchers investigated additional markets providing equivalent evidence for housing prices in other countries.²¹

Mean reversion patterns in returns for stocks, bonds, exchange rates, and precious metals had also been exposed by Cutler, Poterba, and Summers in their 1991 paper.²² Previously, in their 1990²³ work they had already explained this pattern of short-term return momentum and long-term mean reversion by demonstrating that interactions between rational investors and noise traders following positive feedback strategies that result in *stylized facts* later described in these pages. That same year, work by De Long, Shleifer, Summers, and Waldmann²⁴ confirmed that rational traders can destabilize the market by driving up prices beyond fundamentals to then sale out to feedback traders at even higher prices.

In 2003, Abreu and Brunnermeier²⁵ reckoned that jointly rational agents have the ability to correct the mispricing generated by noise traders. However, bubbles defined as in our first chapter persist as attempts to adjust prices by rational agents are discouraged given uncertainty about bubble duration and the expectation of profiting from following a positive feedback strategy. Among others, works by Hong and Stein (1999),²⁶ and Frankel and Froot (1991)²⁷ provide further evidence. To explain short-run momentum and long-run mean reversion in markets the former introduces a model with news watchers and momentum traders, while the latter replicates the observed prolonged periods of overvaluation by building an agent model with trend chasers and investors trading on mean reversion to fundamentals.

²¹Englund, P. and Ioannides, Y.M. (1997). House Price Dynamics: An International Empirical Perspective. *Journal of Housing Economics, Elsevier*, 6(2), pp. 119–136.

²²Cutler, D.M., Poterba, J.M. and Summers, L.H. (1991). Speculative Dynamics. *Review of Economic Studies*, 58(3), pp. 529–546.

²³Cutler D.M., Poterba J.M. and Summers L.H. (1990). Speculative Dynamics and the Role of Feedback Traders. *American Economic Review*, 80(2), pp. 63–68.

²⁴De Long, J.B., Shleifer, A., Summers, L.H. and Waldmann, R.J. (1991). The Survival of Noise Traders in Financial Markets. *The Journal of Business, University of Chicago Press*, 64(1), pp. 1–19.

²⁵Abreu, D. and Brunnermeier, M.K. (2003). Bubbles and Crashes. *Econometrica*, 71(1), pp. 173–204.

²⁶Hong, H. and Stein, J.C. (1999). A Unified Theory of Underreaction, Momentum Trading, and Overreaction in Asset Markets. *The Journal of Finance*, 54(6), pp. 2143–2184.

²⁷Frankel, J. and Froot, K. (1991). Exchange Rate Forecasting Techniques, Survey Data, and the Implications for the Foreign Exchange Market. *Working Paper Series, Department of Economics, Institute for Business and Economic Research, UC Berkeley*.

Consistent with positive feedback trading, Bange (2000)²⁸ shows that stock portfolio adjustments of individual investors reflect past market movements. In addition, Keim and Madhavan (1995)²⁹ document momentum trading by institutional investors, whereas, in the housing market, Case and Shiller (1988)³⁰ find individuals base their expectations largely on past price movements rather than fundamentals. Finally, Clayton (1997)³¹ shows that prices in the Canadian apartment market move opposite to predictions based on rational expectations, probably due to the influence of noise traders and trend chasing.

Given the widespread evidence of positive feedback trading among market participants, in 2011 Kouwenberg and Zwinkels³² tried to improve forecasts for housing market prices by estimating a behavioral heterogeneous agent model with two types of traders: chartists acting as positive feedback traders, and fundamentalists betting on mean-reversion to a rent-based fundamental value estimate. This model directly relates the behavioral characteristics of traders at the micro level to the resulting market price at the macro level. In this work investors can switch strategies depending upon the recent performance of their forecasting rules, positive feedback versus fundamental based. As such, results not only explore the underreaction-overreaction anomaly, but also show bubble periods when the housing price deviates from a rent-based fundamental value. Overall, their findings showed that expectations based on short-term momentum and mean reversion to fundamentals can both predict well future changes in the USA house price index. In the latter part of the sample period, 1992–2005, the proportion of investors following the positive feedback trading rule was consistently above average, while prices moved far above the rent-based fundamental value. From 2006 onwards, however, the mean reversion rule regained importance during the housing market downturn. Simulation results showed that the estimated model produces regular boom-bust cycles.

²⁸Bange, M.M. (2000). Do the Portfolios of Small Investors Reflect Positive Feedback Trading? *Journal of Financial and Quantitative Analysis*, 35(2), pp. 239–255.

²⁹Keim, D.B. and Madhavan, A. (1995). Execution Costs and Investment Performance: An Empirical Analysis of Institutional Equity Trades. *Rodney L. White Center for Financial Research Working Papers*, Wharton School Rodney L. White Center for Financial Research (Revised: 9–95), pp. 26–94.

³⁰Case, K.E. and Shiller, R.J. (1988). The behavior of home buyers in boom and post-boom markets. *New England Economic Review*, Federal Reserve Bank of Boston, pp. 29–46.

³¹Clayton, J. (1997). Are housing price cycles driven by irrational expectations? *Journal of Real Estate Finance and Economics*, 14(3), pp. 341–363.

³²Kouwenberg, R. and Zwinkels, R. (2011). Chasing Trends in the USA Housing Market. *Technical report. Working Paper*, Erasmus University Rotterdam, available at: <http://ssrn.com/abstract=1539475>.

An essential component of the above mentioned models is the presence of “chartists” who use historical prices or levels to forecast future trends. Contrary to fundamentalists, chartists do not believe in random price movements but rather think prices can be to a certain extent predicted through the study of past trends and patterns such as “head-and-shoulders” or “support and resistance” levels. Hence, they look at the history of a security’s trading pattern rather than at external drivers such as economic, fundamental and news events.

Independently of the merits of these beliefs and strategy, many investors acting collectively to replicate a patterned behavior affect a price action to repeat itself. That is, if a large number of market participants follow a trading rule, such as buy when the price goes up, then the effect of this trading rule is a type of “self-fulfilling hypothesis” mechanism. A number of the biases reviewed in Chap. 3 help explain why investors disregard probability rules and rather choose to trust that past trends characterize fair expectations for the future³³ distorting prices away from their fundamental baseline. However, other effects are also generating equity market returns with heavy tails, excess kurtosis and volatility clustering as we shall review in these pages.

A whole range of such stylized facts are now widely acknowledged, and many attempts are made to reproduce them with mathematical models in the hope of improving our understanding of financial markets. Here we take a closer look at the literature on stylized facts.

2.2.2 Statistics of Financial Time Series

The Theory of the Random Walk claims that financial asset’s price changes or returns cannot be determined using historical price information. Under this introduction of price returns, two main models are commonly used. The first is the Bachelier-Osborne 1959 model which declares that returns have a constant finite volatility over a given period of time. Under this theory, the results of a log-normal distribution for price returns have a volatility that is proportional to the square root of the time lag “ τ ”, say one minute, one day, one week, one month, and so on.³⁴ Nonetheless, current research shows returns do not follow a Gaussian distribution, as kurtosis and fat

³³De Bondt, W.F.M. (1993). Betting on Trends: Intuitive Forecasts of Financial Risk and Return. *International Journal of Forecasting*, 9(3), pp. 355–371.

³⁴Bachelier, L. (1900). *Theorie de la speculation*, reprinted in Cootner, P. edn, *The Random character of stock market prices* (Cambridge, Massachusetts: MIT Press), pp. 17–78.

tails are present, evidencing volatility in higher frequency than predicted by a normal distribution.^{35,36} A whole range of such stylized facts of financial markets are now acknowledged. Consequently attempts are geared to reproduce them with mathematical models so that a better understanding of financial markets can be achieved.³⁷ In the following pages, we will briefly review some of the most widely acknowledged stylized facts of financial return series.^{38,39,40}

2.2.3 Returns

The first stylized fact of financial time series is that returns are not significantly autocorrelated. The autocorrelation (also called serial correlation, serial dependence or mean reversion) of price changes is found to be mainly insignificant as even for a few lags of 1 min the autocorrelation function decays very rapidly to zero.^{41,42} Nonetheless, some anomalies exist, as findings show that weekly and monthly returns are weakly negatively correlated, whilst daily, weekly and monthly index returns are positively correlated. Furthermore, high frequency market returns exhibit negative autocorrelation.

³⁵Brock, W.A., Hsieh, D.A. and LeBaron, B. (1991). *Nonlinear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence* (The MIT Press).

³⁶Miljković, V. and Radović, O. (2006). Stylized Facts Of Asset Returns; Case Of Belex. *Facta Universitatis, Series: Economics and Organization*, 3(2), pp. 189–201.

³⁷Bollerslev, T., Engle, R.F. and Nelson, D.B. (1994). GARCH Models, in: Engle R.F. and McFadden; D.L. edn *Handbook of Econometrics*, 4 (Amsterdam: Elsevier), pp. 2961–3038; Pagan, A. (1996). The econometrics of financial markets. *Journal of Empirical Finance*, 3(1), pp. 15–102; Guillaume, D.M., Dacorogna, M.M., Davé, R.R., Müller, U.A., Olsen, R.B. and Pictet, O.V. (1997). From the bird's eye to the microscope: A survey of new stylized facts of the intra-daily foreign exchange markets. *Finance and Stochastics*, 129(1), pp. 95–129; Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance*, 1(2), pp. 223–236.

³⁸Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance*, 1(2), pp. 223–236.

³⁹Chakraborti, A., Toke, I.M., Patriarca, M. and Abergel, F. (2011). Econophysics review: I. Empirical facts. *Quantitative Finance*, 11(7), pp. 991–1012.

⁴⁰Guillaume, D.M., Dacorogna, M.M., Davé, R.R., Müller, U.A., Olsen, R.B. and Pictet, O.V. (1997). From the bird's eye to the microscope: A survey of new stylized facts of the intra-daily foreign exchange markets. *Finance and Stochastics*, 129(1), pp. 95–129.

⁴¹Pagan, A. (1996). The econometrics of financial markets. *Journal of Empirical Finance*, 3(1), pp. 15–102.

⁴²Cont, R., Potters, M. and Bouchaud, J.P. (1997). Scale invariance and beyond. In *Proceedings of the CNRS Workshop on Scale Invariance*. (edited by Dubrulle, F.G.B. and Sornette, D., Springer: Berlin). Available online at: <http://ssrn.com/abstract=39420> or doi:10.2139/ssrn.39420.

These results have been explained to mean that there are large positive cross-autocorrelations across individual securities across time.⁴³

With respect to the autocorrelation of squared returns, already in 1963 Mandelbrot⁴⁴ wrote that “*large price changes tend to be followed by large changes of either sign and small changes tend to be followed by small changes*”. In other words, the autocorrelation of absolute or squared returns is positive. This is sometimes referred to as a *long memory effect* or *clustered volatility*, a statistical fact that has been verified in different financial markets. Thus, in contrast to the lack of dependence in returns, the autocorrelation for the absolute and squared returns is always positive and significant, and decays slowly. In addition, the autocorrelation in the absolute returns is generally higher than the autocorrelation in the corresponding squared returns. The central stylized fact about autocorrelation in squared returns is that it decays slowly with increasing lags, perhaps like a Power Law.

The assumption of random returns being normally distributed is convenient because the Gaussian distribution is characterized by only two parameters: the mean and the standard deviation. This is an example of *stylized fact*. Nonetheless, price returns do not converge to the Gaussian, the “*stylized fact*.” Rather, they are non-stationary, and exhibit skewness, and fat tails resulting from a level of volatility higher than that predicted by a normal distribution.^{45,46,47} The distributions are increasingly fat-tailed as data frequency increases (smaller interval sizes) whereas annual returns are approximately normal.^{48,49}

Thus the distribution tends to be non-normal, sharp peaked and heavy tailed, with these being more pronounced for intraday values. With respect to the skewness, Cont (2001) reports normalized values ranging between -0.4 and -0.1 for five-minute returns of three different financial instruments

⁴³Campbell, J.Y., Lo A.W. and Mackinlay, A.C. (1996). *The Econometrics of Financial Markets*. Princeton, New Jersey: Princeton University Press, p. 74.

⁴⁴Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36(4), pp. 394–419.

⁴⁵Kendall, M.G. (1953). The analysis of economic time-series—Part I: Prices. *Journal of the Royal Statistical Society, Series A, General*, 116(1), pp. 11–25.

⁴⁶Houthakker, H.S. (1961). Systematic and random elements in short-term price movements. *The American Economic Review*, 51(2), pp. 164–172.

⁴⁷Osborne, M.F.M. (1962). Periodic structure in the Brownian motion of stock prices. *Operations Research*, 10(3), pp. 345–379.

⁴⁸Gopikrishnan, P., Plerou, V., Amaral, L.A., Meyer, M. and Stanley, H.E. (1999). Scaling of the distribution of fluctuations of financial market indices. *Physical Review E*, 60(5), pp. 5305–5316.

⁴⁹Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36(4), pp. 394–419.

which he explains: “one observes large drawdowns in stock prices and stock index values but not equally large upward movements” (Cont 2001, p. 224⁵⁰). However, Cont (2001) himself, and others later assert that standard estimators may overestimate the magnitude of skewness as the stylized facts may not accurately describe the “true” underlying values.^{51,52}

It has been observed that as one increases the time scale, τ , over which the returns are calculated, the fat-tail property becomes less pronounced, and the distribution approaches the Gaussian form. Aggregational normality is another fourth ‘stylized-fact’. Thus, as the time scale increases, the more Gaussian the distribution becomes. The fact that the shape of the distribution changes with τ makes it clear that the random process underlying prices has a non-trivial temporal structure. More generally, one can find quite different results by sampling return series using different “clocks”, revisiting the stylized facts with a new clock can give additional insight into return series. This could be an essential consideration in designing mathematical models for explaining the stylized facts thus we dedicate a small section to describe the most general findings of the “scaling” effect.

2.3 Stylized Facts of Bubbles

2.3.1 Introduction

The history of financial markets is full of periods in which asset prices differ greatly from fundamentals. Nonetheless, as related in our prior chapters, there is little consensus on the precise factors generating such occurrences. Part of the problem may derive from the fact that most analysis focus exclusively on asset price behavior, whereas efficient market advocates stress that, on retrospect, valuations are almost always inaccurate. In addition, bubble periods are often associated with times of technological or financial innovations and the temporal expectation of high growth helps clutter the correct assessment of the assets’ value.

⁵⁰Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance*, 1(2), pp. 223–236.

⁵¹Tae-Hwan, K. and White, H. (2004). On more robust estimation of skewness and kurtosis. *Finance Research Letters*, 1(1), pp. 56–73.

⁵²Bonato, M. (2011). Robust estimation of skewness and kurtosis in distributions with infinite higher moments. *Finance Research Letters*, 8(2), pp. 77–87.

Heterogeneous beliefs ensure the coexistence of optimists and pessimists. However, the cost asymmetry between long and short positions imply that optimists' views are expressed more completely. Thus, even when opinions are on average unbiased, prices are biased upwards. Furthermore, the self fulfilling prophecy and a myriad of physiological mechanisms result in optimistic expectations being constantly updated by further hopes that, in the future, yet more optimistic buyers may appear. This can translate into prices above and beyond the discounted value of estimated future payoffs to capture the hoped rewards.

The value of the resale option, the difference between what is actually paid for a stock and the estimated discounted value of the expected cash flows derived from holding the asset, is the bubble portion of the price.⁵³ Bubble episodes are associated with increases in trading volume because increases in the volatility of beliefs result in boosts of the value of the resale option. Hence, broadening the disparity between prices and fundamental's valuation results in a larger volume of trade. Thus, a well established stylized fact is that bubble episodes are associated with increases in trading volume.⁵⁴

However, not all bubble models may capture this effect. For instance, rational bubbles are characterized by continuous price increases.⁵⁵ In this scenario, agents hold the expected rate of price increase will suffice to compensate for any risk of the bubble bursting. So in contrast to models based on heterogeneous beliefs and costly short selling, rational bubble theories do not explain the relationship between bubbles and high trading volume, nor those with assets with a known value at a future date T , such as many credit instruments.

When beliefs are heterogeneous capital availability plays a key role, even through additional leverage. The reason is that market prices are settled by the amount the marginal buyer is willing to pay. Consequently, an increase in the individual's power to buy the asset allows more extreme optimists to acquire the full supply of the asset, furthering the value of the resale option. The opposite situation arises when agents have limited access to capital or capacity to bear risk. In these instances, the additional supply of the asset is lead by less optimistic marginal buyers who attribute a smaller fundamental value to the asset. In these circumstances, supply increases are accompanied

⁵³Brunnermeier, M.K. (2008). Bubbles, in *The New Palgrave Dictionary of Economics*, edited by Durlauf S. and Blume L. (Basingstoke: Palgrave Macmillan).

⁵⁴Scheinkman, J.A. (2013). Speculation, Trading and Bubbles. Economic Theory Center, Princeton University, Research Paper No. 050–2013. <http://ssrn.com/abstract=2227701> or <http://dx.doi.org/10.2139/ssrn.2227701>.

⁵⁵Santos, M.S. and Woodford, M. (1997). Rational asset pricing bubbles. *Econometrica*, pp. 19–57.

by declines in asset values. When unexpected increases in the supply happen, the difference between the price and the fundamental valuation of the marginal buyer diminishes. The reason is that a larger quantity shall have to be absorbed reducing the value of the resale option. Thus, another stylized fact is that increases in asset supply help implode bubbles.

Lastly, it has also been proposed that “technological innovations” is a bubble generating mechanism, our third stylized fact. This is backed by evidence and as an argument motivated by the idea that market wide innovations generate optimism. The excitement about a recent innovation is a “precipitating factor” amplified by feedback mechanisms.⁵⁶

In summary, the three key stylized facts resume in the fact that bubbles coincide with increases in trading volume and financial or technological innovations, and that implosions coincide with increases in assets supply.

2.3.2 Relationships Between Stylized Facts

2.3.2.1 Volatility and Volume

Already in the 70s, it was noted that trading volume and price change variance have a curvilinear relationship.⁵⁷ This property was further elucidated by the discovery that the variance of log-returns after N trades, i.e. over a time period of N in trade time, is proportional to N .^{58,59,60} Studying the variance of price changes in *trade time* suggests that the number of trades is a good proxy for the unobserved volatility.⁶¹

Trading volume is correlated with all measures of volatility. There is a positive correlation between conditional volatility and volume; conditioning on lagged volume strongly attenuates the “leverage” effect; and after conditioning

⁵⁶Shiller, R.J. (2006). Irrational exuberance (Crown Business).

⁵⁷Clark, P.K. (1973). A subordinated stochastic process model with finite variance for speculative prices. *Econometrica*, 41, pp. 135–155.

⁵⁸Chakraborti, A., Toke, I.M., Patriarca, M. and Abergel, F. (2011a). Econophysics review: I. Empirical facts. *Quantitative Finance*, 11(7), pp. 991–1012;

Chakraborti, A., Toke, I.M., Patriarca, M. and Abergel, F. (2011b). Econophysics review: II. Agent-based models. *Quantitative Finance*, 11(7), pp. 1013–1041.

⁵⁹Plerou, V., Gopikrishnan, P., Amaral, L.A.N., Gabaix, X. and Stanley, H.E. (2000). Economic fluctuations and anomalous diffusion. *Physical Review E*, 62(3), pp. R3023–R3026.

⁶⁰Silva, A.C. and Yakovenko, V.M. (2007). Stochastic volatility of financial markets as the fluctuating rate of trading: An empirical study. *Physica A*, 382, pp. 278–285.

⁶¹Gopikrishnan, P., Plerou, V., Gabaix, X. and Stanley, H.E. (2000b). Statistical properties of share volume traded in financial markets. *Physical Review E*, 62, pp. R4493–R4496.

on lagged volume, there is a positive risk-return relation.⁶² Nonetheless it is very difficult to disentangle the volume effect from the volatility effect given that these quantities are strongly positively correlated.⁶³

2.3.2.2 Returns and Volume

Trading volume plays a key role in rational models describing the information flow⁶⁴ and the speed of adjustment of information.⁶⁵ For instance, it has been argued that serial correlation is due to changing risk-aversion and that in days of high trading volume serial correlation is larger.⁶⁶ In some instances this finding has been explained as profit taking.⁶⁷ Other studies have shown that high volume is associated with negative autocorrelation, while low volume is associated with positive autocorrelation.⁶⁸

Some empirical observations on the relation between price changes and trading volume establish that volume is positively related to the magnitude of the price change and, in equity markets, to the price change per se⁶⁹ including absolute changes in prices and dividends.^{70,71} Overall it has been found that large price movements are followed by high volume and that even after conditioning on lagged volume, there is a positive risk-return relation.⁷²

⁶²Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance*, 1(2), pp. 223–236.

⁶³Bianco, S. and Renò, R. (2006). Dynamics of intraday serial correlation in the Italian futures market. *Journal of Futures Markets*, 26(1), pp. 61–84.

⁶⁴Admati, A. and Pfleiderer, P. (1988). A Theory of Intraday Patterns: Volume and Price Variability. *Review of Financial Studies*, 1, pp. 3–40.

⁶⁵Chordia, T. and Swaminathan, B. (2000). Trading Volume and Cross-Autocorrelations in Stock Returns. *The Journal of Finance*, 55, pp. 913–935. doi: [10.1111/0022-1082.00231](https://doi.org/10.1111/0022-1082.00231).

⁶⁶Campbell, J.Y., Grossman, S.J. and Wang, J. (1993). Trading Volume and Serial Correlation in Stock Returns. *The Quarterly Journal of Economics*, 108, pp. 905–939.

⁶⁷Säfvenblad, P. (2000). Trading volume and autocorrelation: Empirical evidence from the Stockholm Stock Exchange. *Journal of Banking & Finance*, 24, pp. 1275–1287.

⁶⁸Conrad, J., Hameed, A. and Niden, C. (1994). Volume and autocovariances in short-horizon individual security returns. *Journal of Finance*, 49, pp. 1305–1329.

⁶⁹Gallant, A.R., Rossi, P.E. and Tauchen, G. (1992). Stock prices and volume. *The Review of Financial Studies*, 5, pp. 199–242.

⁷⁰Wang, J. (1994). A Model of Competitive Stock Trading Volume. *The Journal of Political Economy*, 102, pp. 127–168.

⁷¹Karpoff, J.M. (1987). The Relation Between Price Changes and Trading Volume: A Survey. *The Journal of Financial and Quantitative Analysis*, 22(1), pp. 109–126.

⁷²Bianco, S. and Renò, R. (2006). Dynamics of intraday serial correlation in the Italian futures market. *Journal of Futures Markets*, 26(1), pp. 61–84.

2.3.2.3 Volatility and Returns

The “leverage effect” refers to the tendency for changes in stock prices to be negatively correlated with changes in stock volatility.⁷³ Fixed costs such as financial and operating leverage provide a partial explanation for this phenomenon. A firm with debt and equity outstanding typically becomes more highly leveraged when the value of the firm falls. This raises equity returns volatility if the returns on the firm as a whole are constant. However, it has been argued that the response of stock volatility to the direction of returns is too large to be explained by leverage alone.^{74,75,76}

⁷³Black, F. (1976). Studies of Stock Price Volatility Changes. Proceedings of the Business and Economics Statistics Section. *American Statistical Association*, pp. 177–181.

⁷⁴Christie, A. (1982). The Stochastic Behavior of Common Stock Variances: Value, leverage, and Interest Rate Effects. *Journal of Financial Economics*, 10, pp. 407–432.

⁷⁵Schwert, W. (1989). Why Does Stock Market Volatility Change Over Time? *Journal of Finance*, XLIV, pp. 1115–1153.

⁷⁶Bollerslev, T., Engle, R.F. and Nelson, D.B. (1994) *GARCH Models*, in: Engle, R.F. and McFadden, D.L., edn, *Handbook of Econometrics*, 4 (Amsterdam: Elsevier), pp. 2961–3038.

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