Chapter 2
Deterministic Unit Commitment Models and Algorithms

This chapter introduces the basic formulations of unit commitment problems which are generally proposed to optimize the system operations by mixed integer linear programming. Meanwhile, the formulations target a series of external factors that affect electrical power generation schedules, such as ramping capacity, reserve requirement, transmission capacity, fuel constraint and emission. This chapter also introduces the solution approaches to solve the deterministic unit commitment problems, especially using Lagrangian Relaxation and Benders' Decomposition. The SCUC cases are provided to illustrate the UC modeling and decomposition processes. All formulation notations are listed in Appendix B for reference.

2.1 Introduction

Generally, unit commitment is defined to optimize the ON/OFF status of generating units to meet the forecasted loads and reserve requirements, so as to provide a least-cost power generation schedule. The unit commitment problems namely consider how to optimally operate generators under physical conditions, such as generation capacity, minimum ON time, minimum OFF time, ramp up/down rate, reserve requirements, as well as generation costs, such as startup/shutdown cost and fuel costs.

Since the electric power generation is not an isolated component in the power system, the real-time dispatch levels are also subject to demand changes, transmission capacity and corresponding transmission conditions. Assuming that the real-time loads follow the expectations of forecasted loads, when the transmission outage possibly occurs at a time, it would cause to transmission congestions in some lines and change the original transmission flows on current networks, and meanwhile, will likely affect the original power generation schedule (real-time unit commitment). This correlation reveals the importance of the co-optimization of generation and transmission in practice. Although the unit commitment problems combined with a transmission constrained network become more complicated, these studies are very helpful to guide unit commitment scheduling from the perspective of a whole power system's operations.

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2.2 Objective Function

The objective function of unit commitment usually achieve the minimum total operational cost over a planned time horizon, the maximum social welfare or the maximum total profit for a GENCO.

A generic UC objective function is composed of two component costs, related to two-stage decisions. The first component cost is determined by day-ahead decisions, i.e. the startup decision and shutdown decision on each generator (in first stage). We here assume there will be no reschedule of units occurring during next-day operating hours. The first-stage decision includes the start-up decision v_{gt} and the shutdown decision w_{gt} that indicate when generation units will be turned on or shut down, and other operational determinations for operation services. The second component cost comes from the total operational costs in the second stage, which is primarily made up of fuel cost and possible unserved energy penalty. And, this unserved energy penalty is usually produced by load-shedding losses when scheduled generators are not able to satisfy real-time demands. There is a list of parameter definitions in Table 2.2 for reference.

$$\min \sum_{g \in G} \sum_{t \in T} (SU_g v_{gt} + SD_g w_{gt}) + \sum_{g \in G} \sum_{t \in T} F_g(p_{gt}) + VOLL \sum_{i \in N} \sum_{t \in T} \delta_{it} \tag{2.1}$$

where

SU_g	start up cost of unit g
SD_g	shut down cost of unit g
$F_g(\cdot)$	fuel cost function for unit g
p_{gt}	the thermal power generation/dispatch amount of unit g at time t
$VOLL$	value of loss load [$/MWh]
δ_{it}	load loss at but i at time t

It should be noted that the fuel cost in the second-stage objective function is a quadratic function highly associated with power dispatch on a generator and fuel price. In general, the fuel cost function can be presented as a quadratic function of the dispatch/production level, p, i.e., for a generator g, $F_g(p) = a + bp + cp^2$, where a, b and c are usually positive cost coefficients. We know that the quadratic mixed 0–1 integer programming problem is not easy to solve in practice, especially when a lot of generators are involved. Further, due to the presence of binary decisions, this could bring extra computational burden on solving a nonlinear fuel cost function.

Instead of solving the mixed integer quadratic problem, an alternative method is to apply the piecewise linear approximation method to gain very close solutions for computational convenience. In other words, the original objective function is reformulated to generate a piecewise linear approximation and become a mixed integer linear programming problem. For gaining the piecewise linear approximation, the sum of squares (SOS) techniques are often used to substitute the fuel cost function $F_g(p)$ by the summation $\sum_{k=1}^{K} C_k\lambda_k$ with additional constraints,

$$\{p_g = \sum_{k=1}^{K} \Delta_k\lambda_k, \sum_{k=1}^{K} \lambda_k = u_g, \lambda_k \geq 0, k = 1, \ldots, K\},$$

where u is the commitment status of generator g, and $C_k$and Δ_k are coefficients used to approximate the quadratic curve.

Based on two status of a generator, we can know that when a generator is online and commits to supply capacity, i.e. $u_g = 1$, the UC model will be introduced with the following constraints,

$$\begin{aligned}
&p_g = \sum_{k=1}^{K} \Delta_k\lambda_k, \\
&\sum_{k=1}^{K} \lambda_k = 1, \\
&\lambda_k \geq 0, \quad k = 1, \ldots, K.
\end{aligned}$$

When the generator commitment status is in an "off" state, i.e., $u_g = 0$, the power dispatch level p_g become zero and has no any operational cost $F_g(p)$.

Because the cost function itself is convex (see Fig. 2.1), the piecewise linear approximation function is still convex. The solution obtained from the MILP is very close to the real optimal solution [HROC01, ZWPG13].

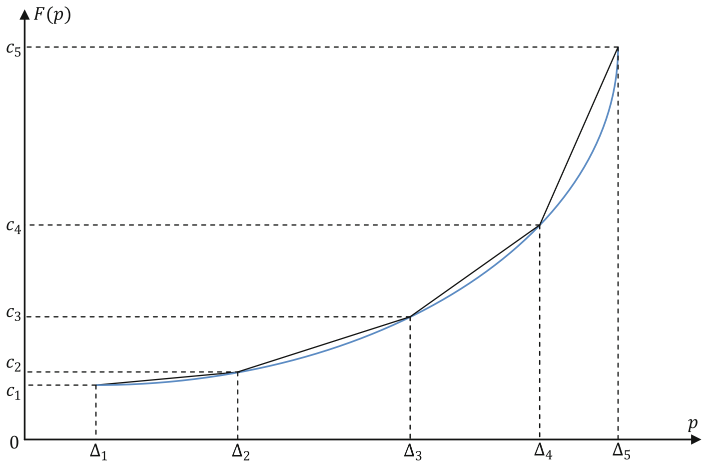

Fig. 2.1 Piecewise linear approximation of the fuel cost function [ZWPG13]

2.3 Constraints

In this sections, we introduce several common sets of UC constraints and variables from two-stage mixed integer linear programming models in details. From the most recent studies, we separate those typical constraints to address based on operation characteristics and service requirements.

2.3.1 Unit Commitment Constraints

In the day-ahead markets, an ISO determines an unit commitment schedule based on forecast demands and bids before the operating day, and designates power plants to prepare to generate electricity for next-day demands. As the first stage of operation scheduling, the UC constraints state generator status restricted by specific operation requirements, such as minimum ON time and minimum OFF time, and also specify startup action and shutdown action on each unit at a time period t, respectively.

Because a generator can't be started up or shut down arbitrarily in consecutive hours, Constraints (2.2) and (2.3) respectively indicate two generator's requirements: the shortest ON duration has to be met before a generator being shut down and the shortest OFF duration is also required before a generator being restarted up.

minimum ON time constraint:

$$u_{gt} - u_{g(t-1)} \leq u_{g\tau} \qquad \forall\, g \in G,\ t \in T, \tau = t, \ldots, min\{t + L_g - 1, |T|\} \tag{2.2}$$

minimum OFF time constraint:

$$u_{g(t-1)} - u_{gt} \leq 1 - u_{g\tau} \qquad \forall\, g \in G,\ t \in T, \tau = t, \ldots, min\{t + l_g - 1, |T|\} \tag{2.3}$$

where

u_{gt}: commitment decision, a generator commits online, if $u_{gt} = 1$; otherwise, $u_{gt} = 0$.
L_g: the minimum-ON duration
l_g: the minimum-OFF duration
τ: time alias, a possible operating time period starting from time t
$|T|$: the duration of a planning horizon

The startup action v_{gt} and the shutdown action w_{gt} are determined by the generator commitment statuses in the previous time period $t-1$ and the current time period t. Any operational actions can incur startup or shutdown costs, which are considered in the objective function.

Startup action constraint:

$$v_{gt} \geq u_{gt} - u_{g(t-1)} \qquad \forall\, g \in G,\ t \in T \tag{2.4}$$

Shutdown action constraint:

$$w_{gt} \geq -u_{gt} + u_{g(t-1)} \qquad \forall\, g \in G,\ t \in T \tag{2.5}$$

$$u_{gt},\ v_{gt},\ w_{gt} \in \{0, 1\} \qquad \forall\, g \in G,\ t \in T \tag{2.6}$$

where

v_{gt}: binary variable, startup action of unit g at time t
w_{gt}: binary variable, shutdown action of unit g at time t

2.3.2 Thermal Generation Constraints

According to given unit commitment schedules, power generation is to fulfill system operations through available generation resources and then to provide the least-cost generation outputs to serve demand. A generator output in a hour is subject to the maximum generation limit P_g^{max} and the minimum generation limit P_g^{min}. When a generator is scheduled online ($u_{gt} = 1$), the generation capacity is active giving bounds on dispatch level, shown in (2.7); otherwise, a generator output is forced to zero.

$$P_g^{min} u_{gt} \leq p_{gt} \leq P_g^{max} u_{gt} \qquad \forall\, g \in G,\ t \in T \tag{2.7}$$

$$p_{gt} \geq 0 \qquad \forall\, g \in G,\ t \in T \tag{2.8}$$

In addition, a generator output can be adjusted, increasing or decreasing between two successive time periods. The generation difference between two adjacent time periods is called ramping. A basic constraint to address generation ramping is presented in (2.9).

$$-RD_g \leq p_{gt} - p_{gt-1} \leq RU_g \qquad \forall\, g \in G, t \in T \tag{2.9}$$

where

RD_g: ramp-down rate of unit g
RU_g: ramp-up rate of unit g

We also take into account some specific ramping situations, in which ramping rate is a changeable value and affected by the previous time period of commitment status. T Some recent models have addressed this situation [WWG13c, WWG13b]. If a generator has a startup ramping, i.e. the dispatch level ramping up from 0 MW to P_g^{min}, the regular ramp up rate is not suitable under this condition, but can be replaced with P_g^{min}. In addition to startup ramping or shutdown ramping, the regular ramp up rate and the ramp down rate are applied to consecutive online status. Therefore, constraint (2.9) can be modified as follow:

$$p_{gt} - p_{gt-1} \le P_g^{min}(2 - u_{gt} - u_{g(t-1)}) + RU_g(1 + u_{g(t-1)} - u_{gt}) \quad \forall\, g \in G, t \in T \tag{2.10}$$

$$p_{gt-1} - p_{gt} \le P_g^{min}(2 - u_{gt} - u_{g(t-1)}) + RD_g(1 - u_{g(t-1)} + u_{gt}) \quad \forall\, g \in G, t \in T \tag{2.11}$$

where constraint (2.10) describes two following situations:

- If a unit is ON at time $t-1$ and ON at time t, the ramp up rate is RU_g;
- If a unit is OFF at time $t-1$ and ON at time t, the ramp up rate is P_g^{min}.

Similarly, constraint (2.11) describes other two situations:

- If a unit is ON at time $t-1$ and ON at time t, the ramp down rate is RD_g;
- If a unit is ON at time $t-1$ and OF at time t, the ramp down rate is P_g^{min}.

2.3.3 Operating Reserve Constraints

Operating reserve is one type of ancillary operations to support the power balance on the demand sides. The ISO promote ancillary services not only to enlarge the pool of energy resources and introduce advanced techniques that effectively and actively participate in the ISO market, but also to support the renewable energy integration as a complementary tool.

The current operating reserve services being offered in electric energy markets include synchronous or non-synchronous, regulation reserves, spinning reserves, and non-spinning reserves. The sources of energy provided from different reserve services are different: regulation service mainly supplied from online generators, partial spinning reserve provided from generators already connected to the grid or system resources, and non-spinning reserve provided from quick-start generators, system resources or interruptible loads. The response times of reserve services cn vary from a few seconds to 30 min, up to 60 min, depending on the control reserve deployment time.

To achieve the optimization of energy and reserve in practice, one can obtain an efficient energy and reserve offering strategy by Heuristic method [NLR04] or consider the reserve determination on pre-contingency and post-contingency conditions [BGC05]. In the fact that the durations of reserve services are often less than 30 min, if the reserve duration is considered as a significant factor, a sub-hourly unit commitment model become necessary to handle this time transition issue [YWGZ12]. Here, we primarily focus on hourly unit commitment formulations based on an optimization method.

The spinning reserve is generally accounted for partial online generating capacity or off-line generation resources. Their outputs are constrained by predetermined maximum spin reserve, shown as

$$0 \le s_{gt} \le S_g^{max} \qquad \forall\, g \in G, t \in T. \tag{2.12}$$

where

s_{gt}: spinning reserve of unit g at time t
S_g^{max}: maximum spinning reserve limit of unit g

Meanwhile, the generators that participate in biding spinning reserve must meet the spin reserve requirements given by ISOs. Constraint (2.13) describes an operating condition that the total spinning reserve at bus i should not less than the fixed reserve requirement.

$$\sum_{g \in G_i} s_{gt} \ge RS_{it} \qquad \forall\, i \in N,\ t \in T \tag{2.13}$$

where

RS_{it}: spinning reserve requirement for bus i at time t.

More typical constraints regarding spinning and non-spinning reserve requirements are shown in constraints (2.14)–(2.18). The provisions of spinning reserve are expended, not only from internal spinning reserves (e.g. from synchronized generators) but also from external spinning reserve (purchased from spinning reserve not served). The maximum spinning reserve can be estimated through the response time of spinning reserve at ON status, which is shown in (2.15).

$$\sum_{g \in G_i} s_{gt} + (sn)_t \ge RS_{it} \qquad \forall\, i \in N,\ t \in T \tag{2.14}$$

$$0 \le s_{gt} \le SRT \times MSR_g \times u_{gt} \qquad \forall\, g \in G,\ t \in T \tag{2.15}$$

The non-spinning reserve has a more complicated situation, in fact, divided into two types of reserves: nonspinning reserve if a unit is ON and nonspinning reserve if a unit is OFF. The former nonspinning reserve is similar to regular spin reserve from online generators, and the latter nonspinning reserve is provided from off-line quick start generators with a higher level of nonspinning capacities. Either of nonspinning reserve is also necessary to satisfy the non-spin reserve requirements by the total provisions of non-spin reserve resources. The corresponding formulations are given in (2.16)–(2.18).

$$\sum_{g \in G_i} (ns_{gt}^{ON} + (ns)_{gt}^{OFF}) + (nsn)_t \ge NRS_t \qquad \forall\, i \in N,\ t \in T \tag{2.16}$$

$$0 \le (ns)_{gt}^{ON} \le NSRT \times MSR_g \times u_{gt} \qquad \forall\, g \in G,\ t \in T \tag{2.17}$$

$$0 \le (ns)_{gt}^{OFF} \le QSC_g(1 - u_{gt}) \qquad \forall\, g \in G,\ t \in T \tag{2.18}$$

All reserves mentioned above are dominant in ancillary service markets. Meanwhile, more and more new products like flexible ramping products will be added to ancillary services and enrich the ancillary services market. ISOs also expect to benefit from the co-optimization by the effective determination of market clearing prices, the enhancement of reserve shortage pricing, the identification of units for system re-dispatch and proper compensation, etc.

2.3.4 Transmission Constraints

Power flows in a transmission network are usually considered in UC optimization problems, because they can be used to address power losses occurring in a network and eventually affect real-time power dispatch at a bus. Generally, Kirchhoff's current and voltage laws in a nodal way are applicable to find out electricity characteristics of transmission and distribution systems. Through simplifying calculation processes, one can present the power transmission using a DC linear approximation of power flows. In addition to voltage magnitudes, MVA or MVAR flows, the DC power flow method actually is often used to determine the MW flows on transmission lines in optimization models.

Measuring load-shedding losses is to help decision makers identify possible load losses at a specific bus. We can introduce a loss variable δ_{it} into the DC approximation of KCL constraints, in which the loss appeared at a bus for each time period will cause unserved energy penalty. The modified DC approximation of KCL involves in-bound and out-bound flow, thermal generation, forecasted demands, renewable energy generation as well as load-shedding loss, shown in constraint (2.19). The power transmission line from bus i to j also has a flow limit given in (2.20). In some cases, the load-shedding loss is not allowed in a specific location and thus δ_{it} needs to be restricted to zero.

$$\sum_{(i,j)\in A_i^+} f_{ijt} - \sum_{(j,i)\in A_i^-} f_{jit} = \sum_{g\in G_i} p_{gt} + R_{it} - D_{it}^0 + \delta_{it} \quad \forall\, i \in N,\ t \in T \tag{2.19}$$

$$-F_{ij}^{max} \le f_{ijt} \le F_{ij}^{max}, \quad \forall\,(i,j) \in A,\ t \in T \tag{2.20}$$

$$l_{it} \ge 0, \quad \forall\, i \in N,\ t \in T \tag{2.21}$$

where

f_{ijt}: unrestricted variable, a bi-direction flow between bus i and bus j
δ_{it}: load-shedding loss at bus i at time t
A_i^+: the set of flow starting at bus i
A_i^-: the set of flow ending at bus i
R_{it}: renewable energy output at bus i at time t
F_{ij}^{max}: transmission flow limit between bus i and bus j

Additionally, a DC approximation of Kirchhoff's voltage law is presented in constraint (2.22). The renewable energy output R_{it}, demand D_{it}, and phase angle β_{it} are usually given as parameters in the transmission constraints.

$$(f_{ijt} - f_{jit}) - B_{ijt}(\beta_{it} - \beta_{jt}) = 0 \qquad \forall\, (i, j) \in A,\ t \in T \tag{2.22}$$

$$\beta_{it} \text{ unrestricted,} \qquad \forall\, i \in N,\ t \in T \tag{2.23}$$

where

β_{it}: a phase angle at interconnected bus i
B_{ijt}: susceptance of an transmission line (i, j)

The system voltage and transformer tap limits are shown in constraint (2.24) and (2.25), respectively.

$$\mathbf{V}^{min} \leq \mathbf{V} \leq \mathbf{V}^{max}, \tag{2.24}$$

$$\mathbf{B}^{min} \leq \mathbf{B} \leq \mathbf{B}^{max}, \tag{2.25}$$

where

$\mathbf{V}$: system voltage vector
$\mathbf{B}$: transformer tap vector
$\mathbf{V}^{min}, \mathbf{V}^{max}$: system voltage lower and upper limit vector
$\mathbf{B}^{min}, \mathbf{B}^{max}$: transformer tap lower and upper limit vector

2.3.5 Emission Constraints

Environmental factor is one of operation considerations and usually addressed as a system level or regional emission limit in general. The emission control is mainly executed on these emission gases, i.e. CO_2, SO_2, NO_x. Also, the allowable emission amount highly depends on the fuel type of generating unit, for example, a coal-burning electric generating unit has a higher emission level than a gas-turbine unit. A system level emission limit over a planning horizon [FSL05b] is formulated as

$$\sum_{g \in G} \sum_{t \in T} \left(F_g^e(p_{gt}) u_{gt} + SU_g^e v_{gt} + SD_g^e w_{gt} \right) \leq E^{max}, \tag{2.26}$$

where

$F_g^e(\cdot)$: emission function of unit g
SU_g^e: startup emission of unit i at time t
SD_g^e: shutdown emission of unit i at time t
E^{max}: system emission limit

This constraint is applied to one emission gas and the emission function may vary according to the fuel type of generating units. In addition, this constraint can be tailored for regional emission limit based on the location area of generating units.

2.3.6 Unserved Energy Constraint

In some circumstances, load loss is allowed to occur and may come with unserved energy penalty reflected in the objective function. While the unserved energy constraint imposes a performance bounding to control the expected total load losses within an expected loss allowance.

$$E(\sum_{i \in N} \delta_{it}) \leq \varepsilon_t, \ \forall\, t \in T \tag{2.27}$$

where

$E(\cdot)$: the expectation of load loss in a power system
ε_t: loss allowance for time t

2.3.7 Reactive Power Constraints

Relative to real power generation, this subsection briefly introduce reactive power generation in current system operating, including generation limit, load bus balance and operating reserve requirement [].

$$Q_g^{min} u_{gt} \leq q_{gt} \leq Q_g^{max} u_{gt} \qquad \forall\, g \in G,\ t \in T \tag{2.28}$$

$$\sum_{g \in G} Q_g^{max} u_{gt} \geq D_t^Q, \qquad \forall\, t \in T \tag{2.29}$$

$$\textbf{load bus balance} \tag{2.30}$$

where

q_{gt}: reactive power generation of unit g at time t
Q_g^{min}: lower limit of reactive power generation of unit g
Q_g^{max}: upper limit of reactive power generation of unit g
D_t^Q: reactive power flow demand at time t

2.4 Case Studies

This section provides two selected cases to illustrate basic unit commitment problems and their solution analyses. Both cases are based on a modified 7-bus system, which are taken from Reference [HZW14]. The test system includes 4 generators, 1 wind farm, and 10 transmission lines with given capacities, shown on Fig. 2.2. The bus parameters corresponding to generating units are listed on Table 2.1. The generating unit parameters and their bid prices are given on Table 2.2. The transmission line parameters are given in Table 2.3. Here, line congestion is not considered in both case studies. The daily forecasted Loads are shown in Fig. 2.3 and the wind energy output is in Fig. 2.4. All models can be coded in C++ and solved by commercial solvers like CPLEX.

Here are two UC cases discussed as follow:

- Case 1: Joint energy and ancillary service optimization
- Case 2: Security-Constrained unit commitment with transmission contingency

Based on the given system, the case studies do not consider the impacts of transformers, phase shifter for MW control as well as contingency, i.e. generator outages, line outage.

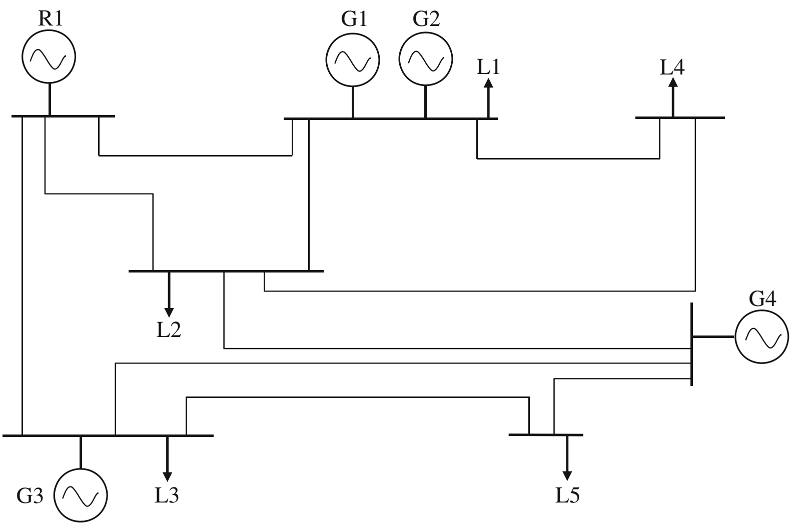

Fig. 2.2 The 7-bus system

Table 2.1 Bus parameters

Bus ID	Type	Unit ID	Gen Capacity (MW)	Spin Reserve limits (MW)	ES Cap. (MW)
B1	Wind	R1	100	–	20
B2	Coal	G1	90	10	20
B2	Coal	G2	60	–	–
B3	–	–	–	–	–
B4	Gas	G3	100	–	10
B5	–	–	–	–	–
B6	Coal	G4	90	–	–
B7	–	–	–	–	–

[a] The symbol, '–', represents no generation unit available at a corresponding bus

Table 2.2 Generator parameters and costs

	G1	G2	G3	G4
Min-ON (h)	2	1	2	2
Min-OFF (h)	2	2	2	1
Ramp-Up Rate(MW/h)	30	15	60	15
Ramp-Down Rate (MW/h)	15	15	60	15
P^{min} (MW)	20	10	20	15
P^{max} (MW)	90	50	90	60
S^{max} (MW)	15	10	15	10
Startup ($)	500	500	800	300
Shutdown ($)	500	500	800	300
Fuel Cost a ($)	6.78	6.78	31.67	10.15
Fuel Cost b ($/MWh)	12.888	12.888	26.244	17.820
Fuel Cost c ($/MWh^2)	0	0	0	0

2.4.1 *Case 1: Joint Energy and Ancillary Service Optimization*

This case focuses on the co-optimization of energy and ancillary service at a same planning horizon. This energy-reserve co-optimization aims to clear both markets simultaneously in a least-cost way. Although energy and spinning reserve come from the same physical resources, the same amount of electricity provided have different prices between energy market and ancillary service market. The problem is formulated in a two-stage mixed integer linear program. The UC schedule is modeled

Table 2.3 Transmission line parameters

Line ID	From	To	Flow capacity (MW)	Voltage (V)	Susceptance
L1	B1	B2	50	500	1
L2	B1	B3	160	500	1
L3	B1	B4	80	500	1
L4	B2	B3	100	500	1
L5	B2	B5	50	500	1
L6	B3	B5	30	500	1
L7	B3	B6	100	500	1
L8	B4	B6	50	500	1
L9	B4	B7	60	500	1
L10	B6	B7	50	500	1

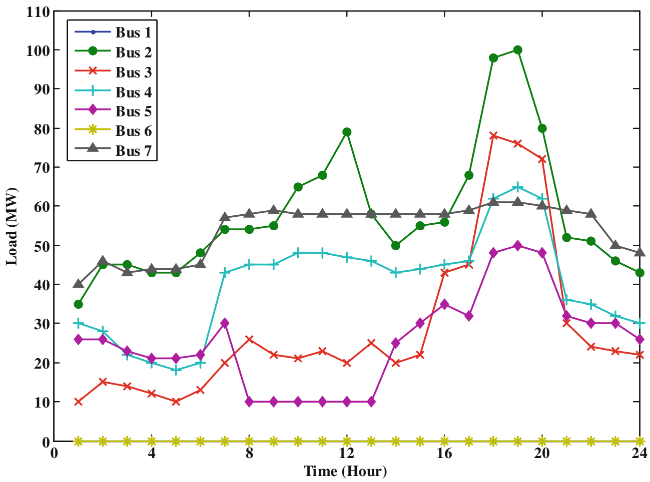

Fig. 2.3 Hourly loads on 7 buses

in the first stage, while both economic dispatch and spinning reserve are scheduled in the second stage.

The length of the planing horizon is 24 h and the forecasted wind energy output is given in one scenario. The wind farm is located at Bus 1 with a generating capacity of 100 MW. The hourly wind energy output was truncated in the range of [5, 80] MW with assumptions of a minimum production output and a maximum production output. Therefore, index sets for Case 1 are shown below and the hourly wind energy outputs are plotted in Fig. 2.4.

$G = 4$ Generators
$T = 24$ Hours

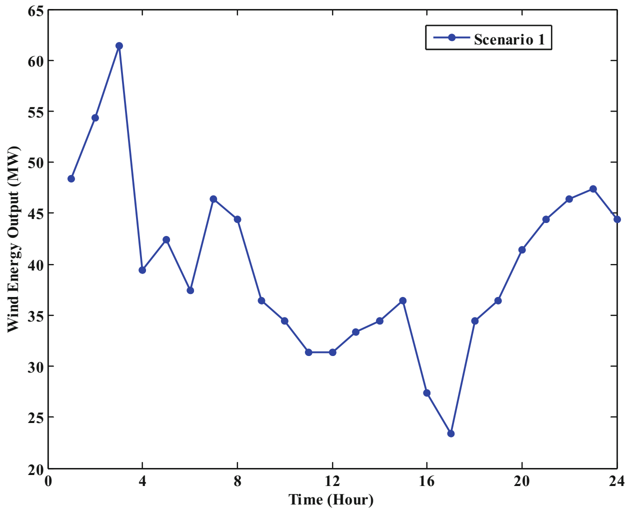

Fig. 2.4 Case 1: hourly wind energy output

$N = 7$	Buses
$S = 1$	Scenario
$\|\mathscr{A}\| = 10$	Transmission Lines

Then the determinist UC problem for energy and ancillary service is formulated.

$$\min \sum_{g\in G}\sum_{t\in T}(SU_{gt}v_{gt} + SD_{gt}w_{gt}) + \sum_{t\in T}\sum_{g\in G}[(b_{gt}p_{gt} + a_{gt}u_{gt}) + (b'_{gt}s_{gt} + a'_{gt}u_{gt})]$$
$$+ VOLL\sum_{t\in T}\sum_{i\in N}\Delta_{it}$$

s.t. The first-stage constraints

$$u_{gt} - u_{g(t-1)} \le u_{g\tau}, \quad \forall g \in G,\ t \in T, \tau = t, \ldots, min\{t + L_g - 1\}$$
$$u_{g(t-1)} - u_{gt} \le 1 - u_{g\tau}, \quad \forall g \in G,\ t \in T, \tau = t, \ldots, min\{t + l_g - 1\}$$
$$v_{gt} \ge u_{gt} - u_{g(t-1)}, \quad \forall g \in G,\ t \in T$$
$$w_{gt} \ge -u_{gt} + u_{g(t-1)}, \quad \forall g \in G,\ t \in T$$
$$u_{gt},\ v_{gt},\ w_{gt} \in \{0, 1\}, \quad \forall g \in G,\ t \in T$$

The second-stage constraints

$$P_g^{min}u_{gt} \le p_{gt} \le P_g^{max}u_{gt}, \quad \forall g \in G,\ t \in T$$
$$-RD_g \le p_{gt} - p_{gt-1} \le RU_g, \quad \forall g \in G,\ t \in T$$
$$0 \le s_{gt} \le S_g^{max}, \quad \forall g \in G, t \in T$$
$$p_{gt} + s_{gt} \le P_g^{cap}u_{gt}, \quad \forall g \in G, t \in T$$

Table 2.4 Objective value and unit commitment for 7-bus system

Objective value	Unit ID	Hour (1–24)
\$60615.6	G1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	G2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	G3	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0
	G4	0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

$$\sum_{g \in G_i} s_{gt} \geq RS_{it}, \quad \forall\, i \in N, t \in T$$

$$\sum_{(i,j)\in A_i^+} f_{ijt} - \sum_{(j,i)\in A_i^-} f_{jit} - \sum_{g\in G_i} (p_{gt}+s_{gt}) - \Delta_{it} = W_{it} - D_{it}, \quad \forall\, i \in N,\ t \in T$$

$$(f_{ijt} - f_{jit}) - B_{ijt}(\beta_{it} - \beta_{jt}) = 0, \quad \forall\, (i,j) \in A,\ t \in T$$

$$p_{gt}, s_{gt} \geq 0, \quad \forall\, g \in G,\ t \in T$$

$$\Delta_{it} \geq 0, \quad \forall\, i \in N, t \in T$$

$$f_{ijt}, \quad \forall\, (i,j) \in \mathscr{A}, t \in T$$

We can obtain the computational results using the solver CPLEX, in which the Brand-and-Cut-and-Price algorithm is used to solve mixed integer linear programs. The total generation cost for this recommended UC schedule is \$60615.6. Table 2.4 lists the objective value and the optimal UC schedule according to the forecasted (known) hourly wind energy outputs and loads. Figures 2.5 and 2.6 are the optimal generator dispatches and spinning reserve levels, respectively.

Without consideration of line congestion, load-shedding loss can be resulted from the physical generation conditions, such as generation limits or ramping constraints. The solution shows no loss occurs under this wind scenario. Therefore, the current generation capacities and ancillary service requirements are able to provide power balance in this system.

2.4.2 Case 2: SCUC with Transmission Contingency

This case focuses on the N-1 reliable DC optimal dispatch under transmission line outage. This problem bases on the Case 1's model and further considers the impacts of transmission contingency on operation scheduling. The model remains a mixed integer linear program and includes the transmission flow capacity constraint (2.31) subject to a line outage during a period $[t,\ t+a]$.

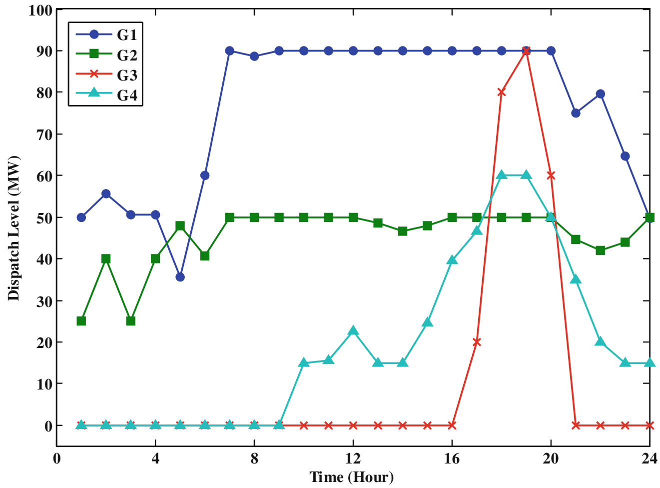

Fig. 2.5 7-bus system: dispatch level for each generator

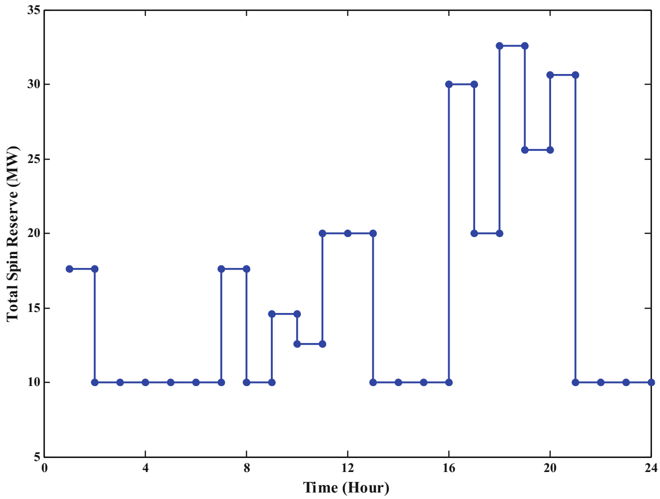

Fig. 2.6 7-bus system: total spin reserve

$$-F_{ij}^{max}\alpha_{ijt} \leq f_{ijt} \leq F_{ij}^{max}\alpha_{ijt}, \qquad \forall\,(i,j) \in A,\ t \in T \tag{2.31}$$

where

α_{ijt}: Binary parameter, if $\alpha_{ijt} = 1$, line outage occurs between bus i and bus j at time t; otherwise, $\alpha_{ijt} = 0$.

Note that the practical method to deal with transmission line outage is not limited to UC operation scheduling, including common transmission switching, whereas the state of transmission element (line or transformer), voltage and phase angle are fully taken into account in transmission switching. For the simplicity of case, here we do not consider such factors except transmission line.

Case 2 uses the same 7-bus system and shares the same parameters with Case 1. Assuming that the occurrence of line outage can be predicted in advance, only one line outage occurs in line (4, 7) at 11 am. During the line outage, the number of available transmission lines is reduced to 9 and the flow capacity $F^m_{47(11)}ax$ becomes zero. The deterministic UC model for Case 2 is shown as follow.

$$
\begin{aligned}
\min \sum_{g\in G}\sum_{t\in T}(SU_{gt}v_{gt} + SD_{gt}w_{gt}) + \sum_{t\in T}\sum_{g\in G}[(b_{gt}p_{gt} + a_{gt}u_{gt}) + (b'_{gt}s_{gt} + a'_{gt}u_{gt})] \\
+ VOLL\sum_{t\in T}\sum_{i\in N}\Delta_{it}
\end{aligned}
$$

s.t. The first-stage constraints

$$
\begin{aligned}
&u_{gt} - u_{g(t-1)} \le u_{g\tau}, \quad \forall\, g \in G,\ t \in T, \tau = t, \ldots, min\{t + L_g - 1, |T|\} \\
&u_{g(t-1)} - u_{gt} \le 1 - u_{g\tau}, \quad \forall\, g \in G,\ t \in T, \tau = t, \ldots, min\{t + l_g - 1, |T|\} \\
&v_{gt} \ge u_{gt} - u_{g(t-1)}, \quad \forall\, g \in G,\ t \in T \\
&w_{gt} \ge -u_{gt} + u_{g(t-1)}, \quad \forall\, g \in G,\ t \in T \\
&u_{gt},\ v_{gt},\ w_{gt} \in \{0, 1\}, \quad \forall\, g \in G,\ t \in T
\end{aligned}
$$

The second-stage constraints

$$
\begin{aligned}
&P^{min}_g u_{gt} \le p_{gt} \le P^{max}_g u_{gt}, \quad \forall\, g \in G,\ t \in T \\
&-RD_g \le p_{gt} - p_{gt-1} \le RU_g, \quad \forall\, g \in G,\ t \in T \\
&0 \le s_{gt} \le S^{max}_g, \quad \forall\, g \in G, t \in T \\
&p_{gt} + s_{gt} \le P^{cap}_g u_{gt}, \quad \forall\, g \in G, t \in T \\
&\sum_{g\in G_i} s_{gt} \ge RS_{it}, \quad \forall\, i \in N, t \in T \\
&\sum_{(i,j)\in A_i^+} f_{ijt} - \sum_{(j,i)\in A_i^-} f_{jit} - \sum_{g\in G_i}(p_{gt} + s_{gt}) - \Delta_{it} = -D_{it}, \quad \forall\, i \in N,\ t \in T \\
&-F^{max}_{ij}\alpha_{ijt} \le f_{ijt} \le F^{max}_{ij}\alpha_{ijt}, \quad \forall\, (i, j) \in A,\ t \in T \\
&p_{gt}, s_{gt} \ge 0, \quad \forall\, g \in G,\ t \in T \\
&\Delta_{it} \ge 0, \quad \forall\, i \in N, t \in T \\
&f_{ijt}, \quad \forall\, (i, j) \in \mathscr{A}, t \in T
\end{aligned}
$$

Because of the disruption of line (4, 7) at 11 am, the transmission flows in the network would be changed as well as the dispatch levels for some specific generating units. Table 2.5 shows that the objective value for UC with a line outage is increased to \$336, 438, in which 76.6% of costs come from the loss penalty. When the line outage happens, the new line capacities are not able to satisfy the surge in flow and line congestions also occur between some buses. Therefore, all units are required

Table 2.5 Objective value and unit commitment for 7-bus system with line outage

Objective value	Unit ID	Hour (1–24)
\$336, 438	G1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	G2	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	G3	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
	G4	1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1

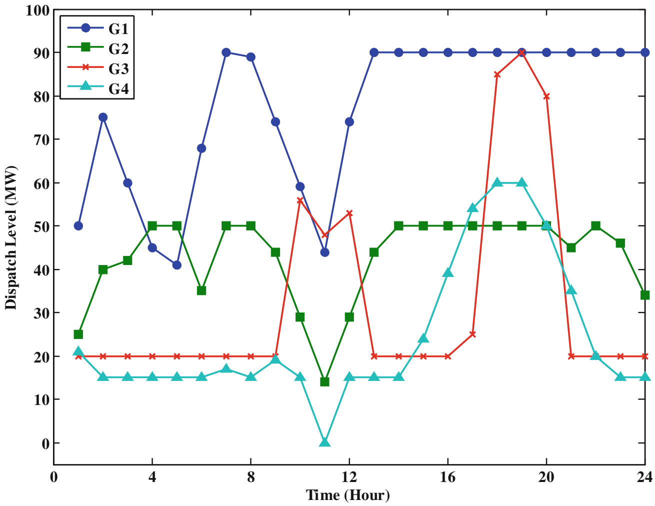

Fig. 2.7 7-bus system: dispatch level for each generator

online and try to meet the local demands first so as to mitigate line congestions. Meanwhile, the line outage forces G4 shut down at 11 am since the outflow of Bus 7 would be terminated.

Figure 2.7 describes the dispatch levels for each generator. Compared to the generation outputs in normal state (Fig. 2.5), these dispatch levels are more fluctuating to accommodate the flow changes. Also, in this case, ramp up/down capabilities regarding online units appear more important to adopt sudden changes in power systems.

Figure 2.8 shows the total spinning reserve in the whole system. Apparently, the overall reserve level is much higher than that of normal state and also the reserve level changes have higher frequency. In the normal state, there is no load-shedding loss within 24 h. However, the line outage results in load-shedding losses gathering at Bus 7 over on-peak hours (Fig. 2.9). Meanwhile, the line outage leading to sudden power supply changes can also trigger losses at other buses, e.g. Bus 3 and Bus 7.

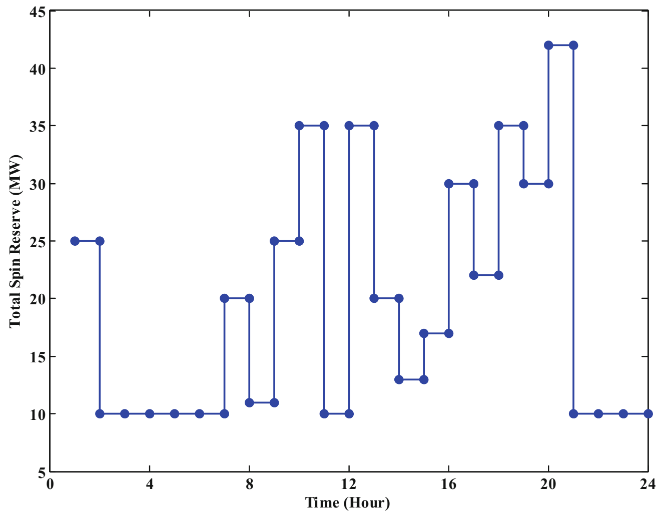

Fig. 2.8 7-bus system: total spin reserve

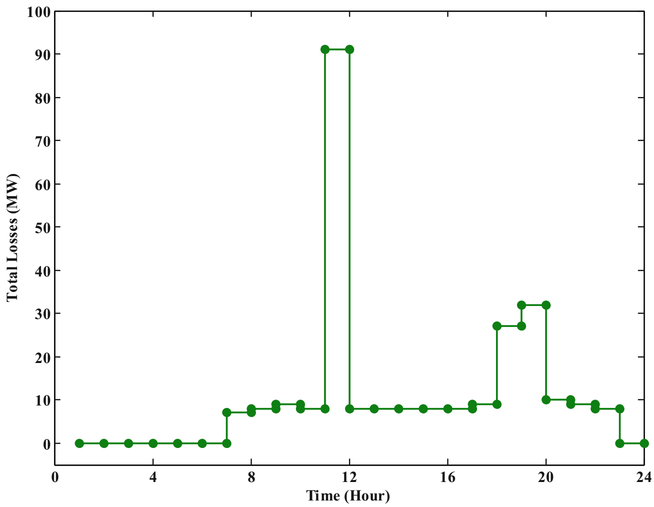

Fig. 2.9 7-bus system: total load-shedding losses

In fact, the unexpected line outage is a very serious contingency event, and thus, it's more applicable to arrange the forced line outage for transmission maintenance to mitigate an unexpected event. What's more, the ISOs/RTOs execute some operating reserves (non-spinning reserve or contingency reserve) to remove the transmission violation, not limited to adjusting tap transformers, phase shifters, predetermined dispatch levels and loads within given time limits.

2.5 Solution Approaches for Deterministic Unit Commitment

We mentioned the unit commitment problems which mainly consider physical generating requirements and power balance as a classic unit commitment problem. In the absence of uncertainty, the classic UC problem is modified to implement some hard operation requirements, i.e. must ON/OFF [OS92, FGL09a, FGL11], operating reserve [SNS01], maintenance [FSL07], emissions [Gje96, FSL05a]. These studies make the classical UC models become more realistic and applicable.

According to the nature of formulated UC problems, there are several common solution approaches for deterministic unit commitment summarized as follow.

- Priority list, including evolutionary programming,
- Dynamic programming,
- Mixed integer linear programming (e.g. Lagrange relaxation method, decomposition method), and
- Heuristics methods.

The above solution approaches have been applied to solve UC problems in the study and the reality. Priority list is one of initial solution methods and dynamic programming is also widely used to obtain UC schedule and optimal generation costs. Mixed integer linear programming has been employed in recent years as the most efficient solution optimization techniques to solve classical UC problems, which will be introduced in Sect. 2.5.3 with more details. Heuristics methods and evolutionary programming have been attempted to solve deterministic UC problems. However, their applications are limited to deterministic cases since they may have lower computational performance when the UC problems face a large-scale power system, and yet both methods can't guarantee for solution optimality. A overview regarding these two solution approaches refers to [Zhu09] for interests.

2.5.1 Priority List

For a generic priority list method, generators are committed in ascending order of the fuel cost so that the most economic base load units are committed first and the most costly units are scheduled last. The priority list method has a very fast computation process, but it is highly heuristic and only generate schedules with relatively high operation cost [SSUF03]. For solving simple UC examples using priority list, the interested reader is referred to [Zhu09, WW12] for more examples.

The usage of priority list is very simple, but is restricted to the basic economic dispatch constraints. This method thus has been extended to accommodate more complicating constraints. The main function of priority list becomes to generate initial solutions due to fast computation speed. Then the initial solutions will go through the improvement process with fast heuristic methods, and eventually one can obtain an economic dispatch schedule and the total generation cost.

In the recent studies, priority list is collaborated with evolutionary algorithms to solve UC problems. Some developed solution approaches provide attempts to solve basic UC problems, such as Evolution Programming, Hybrid Evolution Programming, Gbest based Artificial Bee Colony (ABC) optimization algorithms, Particle Swarm Optimization (PSO) and Differential Evolution technique [GR12a, GR12b].

One of proposed heuristics-based evolutionary algorithm is to evolve an initial population made of good solutions which is obtained by priority list method. Whereas the evolution is characterized by the elimination of the less fit, the survival of the fittest, a reproduction ability based on the fitness, and the genetic operators: crossover, mutation and time-window swap [SC05].

In addition, a hybrid ant system/priority list method is to cooperate the priority list method with the feature of ant system [WCN+09]. The priority list method gives a set of heuristics to be used for UC committing process under the operating constraints. Meanwhile, the ant system can gain the benefit of using a set of heuristic rules provided by the priority list method as directional bias information for improving its evolving process.

What's more, a study proposes an advanced quantum-inspired evolutionary unit commitment algorithm to develop a new searching initialization method based on unit priority list and a special Q-bit expression, which ensures the diversity in the initial search area for improving the efficiency of solution searching. Considering any prior knowledge of UC problem and the characteristics of the generator units, the evolutionary optimization process can be initialized better and carried on by a group-search for QEA-UC [CYW11].

2.5.2 Dynamic Programming

Dynamic programming (DP) is one of main solution techniques to optimize the thermal unit commitment schedule. Dynamic programming with an implicit enumeration approach is a common solution process to solve UC subproblems. Considering an example, there are n generators in a power system, so it has $2^n - 1$ ON-OFF statuses for determining an optimal UC solution. Using DP, it will go through all possible combinations and then pick the best solution(s). The computation times are also increased exponentially, thus the DP applications can't be easily applied in large-scale power systems due to its computational performance.

Some DP introductions with UC applications are clearly given in [Zhu09, WW12]. Generally, dynamic programming is not an unique method employed to produce unit commitment schedules on the whole system. In fact, DP remains its own computational advantages as many studies have proposed DP integrated with other strategies or methods, such as

- Priority list [HHWS88, SPR87, LST+97]
- Lagrangian relaxation method [WSK+95, LS05a, FSL05b]
- Artificial neural network algorithm [OS92]
- Artificial intelligence technique [WS93]
- Expert system [SNS01]
- Branch and bound algorithm [Che08]

The combination of DP with other techniques aims to improve the computation performance. Particularly, within [WSK+95, LS05a, FSL05b, GNLL97], DP is used to solve specific UC subproblems in which the objective is required to determine the optimal unit status cross hours. For the detailed solution process through the DP-Lagrangian relaxation method, one can refer Sect. 2.5.4.3.

2.5.3 Mixed Integer Linear Programming

Compared to other mentioned solution approaches, MILP is the most promising solution technique and has been successfully applied in UC problems. The classic unit commitment problem in abstract form is shown on the following mixed 0–1 linear program (2.32a):

$$[\mathbf{P}] : \min\ \mathbf{c_1^T x} + \mathbf{c_2^T y} \tag{2.32a}$$
$$\text{s.t.}\ \ \mathbf{A_1 x} = \mathbf{b}_1 \tag{2.32b}$$
$$\mathbf{A_2 x} + \mathbf{E y} = \mathbf{b}_2 \tag{2.32c}$$
$$\mathbf{x} \in \{0, 1\}^{n_1} \tag{2.32d}$$
$$\mathbf{y} \in \mathbb{R}_+^{n_2} \tag{2.32e}$$

where $\mathbf{c}_1 \in \mathbb{R}^{n_1}, \mathbf{c}_2 \in \mathbb{R}^{n_2}, \mathbf{b}_1 \in \mathbb{R}^{m_1}, \mathbf{b}_2 \in \mathbb{R}^{m_2}, \mathbf{A}_i \in \mathbb{R}^{n_1 \times m_i} (i = 1, 2)$, $\mathbf{E} \in \mathbb{R}^{n_2 \times m_2}$, and m_1, m_2 are scalars.

The mixed integer program contains an integer variable vector, $\mathbf{x}$, and a continuous variable vector, $\mathbf{y}$. The set of constraint (2.32b) represents unit commitment constraints only involving binary variables, while the set of sconstraint (2.32c) mainly covers the generation limits, operating reserve, ramping limits and emission constraints.

The main applications of MILP in UC can be extended with helps from two aspects: problem reformulation and algorithm modification, both of which aim to improving solution process as well as achieving optimal solution easier and faster.

As solving UC representations purely through dynamic programming would cause computational issues, the reformulations to UC problem can be completed using MILP. This aspiration of better formulations promotes seeking alternative representations to get rid of some computational obstacles, such as nonlinear structures. For instance, the original fuel cost function is a mixed integer quadratic function of dispatch/production level, but there exists some situations where directly solving this function may lead to solutions hardly reaching the global optima. To reduce the computational burden, the piecewise linear approximation technique is used to obtain an approximated value for generation variables [CA06, ZWPG13]. In addition, the traditional thermal generator constraints regarding to minimum ON/OFF time are reformulated through pure integer programming (i.e. unit commitment and startup/shutdown action constraint); ramping up/down constraints are simplified from the general ramping constraint describing the relationship between ramping and load level; similar to fuel cost function, the emission constraints can be linearized and show in mixed integer linear programs. Other recent studies on reformulations have reported alternative UC reformulations from mixed integer nonlinear programs and how to make MILP approximations more close to real solutions [FGL09b, Jab12, MELR13].

Regarding the second aspect (algorithm modifications), solution algorithms have been being developed for several years and also bring a lot of vitality to the application of MILP in UC problems. The traditional solution algorithms have been tailored and customized in the way of integrating basic solution algorithms with other solution strategies or decomposing original problems to master problem and multiple subproblems, so that they can be more suitable for applying in new developed UC models. As for deterministic UC problems, solving corresponding mixed integer linear programs in the last decade utilized one or two following solution technique(s) for better computational performance.

- Lagrangian relaxation technique [LB99, MMSN05, MMSN06, FLS+09, LS05b, WSF10],
- Benders' Decomposition [GGZ05b, GGZ05a, FSL05b, LS05a, LS05b],
- Branch-and-Cut method [WSF10],
- Augmented Lagrangian relaxation (LR) method and dynamic programming [FSL05b, LS05a],
- Tabu search [MMSN05, MMSN06],
- Hybrid subgradient and Dantzig–Wolfe decomposition approach [FSL05a]

Here, we mainly introduce two of effective solution techniques, i.e. Lagrangian relaxation (LR) and Benders' Decomposition (BD). Since recent UC problems involving uncertainties and their optimization models become more complicated, Lagrangian relaxation and Benders' Decomposition methods work as fundamental solution theories that provide help for developing other advanced solution algorithms.

Taking the benefits from decomposition methods, a large MILP model can be decomposed into smaller subproblem(s) which can be solved by existing solution algorithms easily, so that computation performance is improved.

Generally, a UC original problems can be solved directly via Brand-and-Cut-and-Price algorithms using solver CPLEX. After breaking down the original problem, if the subproblem is a MILP, the Branch-and-Cut method is suitable for solving this subproblem as well as Branch-and-Price method. If the subproblem is a linear program, many well-known linear algorithms can handle it easily. For solving the optimal commitments in master problems, LR and DP together can be applied to solve short-term UC problems; meanwhile, Tabu Search can be used in the attempt to solve a small size of UC. While solving a long-term UC might be still a challenge for current optimization methods, Fu et al. thus proposed a hybrid subgradient and Dantzig–Wolfe decomposition approach to tackle this issue.

2.5.4 Lagrangian Relaxation

Lagrangian Relaxation (LR) is a powerful relaxation technique, which is often used to solve UC problems. As many UC problems are complicated by a number of coupling constraints, their original problems can be modeled as (relatively) easy solving Lagrangian problems. More specifically, the problem reformulation is to replace the complicated constraints with penalty terms in the objective function, in which penalty terms are represented by the violation of constraints and their Lagrangian multipliers. In a Lagrangian problem, a lower bound can be obtained for the optimal value of the minimum non-convex UC problem [FLW13].

As an example, the general solution process for the SCUC model is shown on Fig. 2.10. The solution process starts from solving the master problem (MP), in which the constraints namely include unit commitment, economic dispatch, energy reserve, emission limit and unserved energy limit constraints. In the normal case without any outage, if the MP is found to be feasible, the incumbent solution (UC & ED) is passed to the subproblem for network security evaluation (NSE). If the incumbent solution satisfies the transmission requirements, the ED solution continues to be checked for contingency in NSE. If there is any incumbent solution that fails in NSE for both cases, the MP will be resolved for another solution.

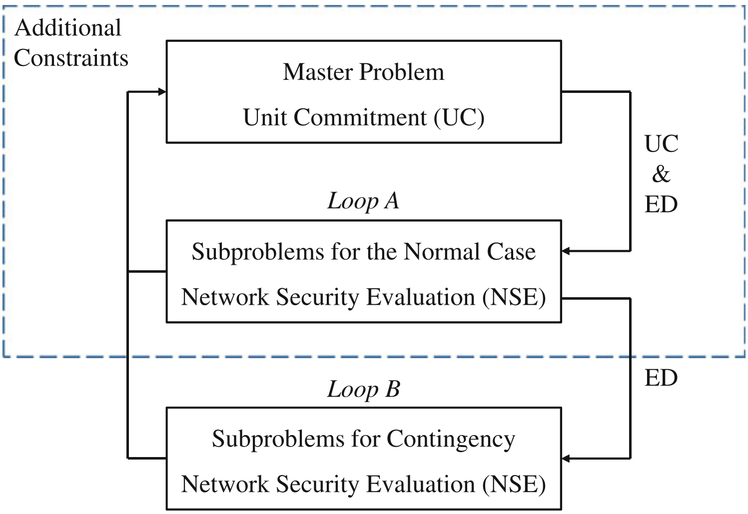

Fig. 2.10 The decomposition approach for SCUC [FLW13]

2.5.4.1 Application of Lagrangian Relaxation in UC Problem

Generally the abstract LR-based UC model can be written in (2.33).

$$[\textbf{LR-OP}] : \min \mathbb{F}(x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots, y_{n_2}) \tag{2.33a}$$
$$\text{s.t.} \ \mathbb{H}(x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots, y_{n_2}) \leq \mathbf{d} \qquad \breve{} \tag{2.33b}$$
$$\mathbb{G}_i(x_i, y_i) \leq b_i, \quad \forall\, i \tag{2.33c}$$
$$\mathbf{x} \in \{0, 1\}^{n_1} \tag{2.33d}$$
$$\mathbf{y} \in \mathbb{R}_+^{n_2} \tag{2.33e}$$

where constraints (2.33b) represent a set of coupling constraints, such as reserve requirements, emission constraints, fuel constraints and unserved energy limits, and constraints (2.33c) involve other non-coupling generation constraints, such as minimum ON/OFF constraints, startup/shutdown constraints, generation capacities, ramping limits, spinning/nonspinning constraints and so on.

Here we address the process how to create a Lagrangian problem to solve UC model. We first let non-negative ˘ denote the Lagrangian multipliers for the system coupling constraints (2.33b). The Lagrangian relaxations of the original problem 2.33 is to move the coupling constraint (2.33b) to the objective function, shown as

$$\begin{aligned} VD^* = \min & \ \mathbb{F}(x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots, y_{n_2}) \\ & + \breve{}(\mathbb{H}(x_1, x_2, \ldots, x_{n_1}, y_1, y_2, \ldots, y_{n_2}) - \mathbf{d}) \end{aligned} \tag{2.34}$$

subject to the unit constraints in (2.33c). When ˘ is a fixed value, the term $-\breve{}^T\mathbf{d}$ becomes constant and is discarded. Then the LR-based objective function can be decomposed into n_1 subproblems, where each subproblem 2.35 bases on a corresponding generator, shown as follow:

$$[\textbf{LR-SP}] : \min \mathbb{F}(x_i, y_i) + \breve{}^T \mathbb{H}_i(x_i, y_i) \tag{2.35a}$$
$$\text{s.t.} \ \mathbb{G}_i(x_i, y_i) \leq b_i, \tag{2.35b}$$
$$x_i \in \{0, 1\} \tag{2.35c}$$
$$y_i \in \mathbb{R}_+ \tag{2.35d}$$

As for solving the decoupled subproblems for each generator, dynamic programming (DP) has been verified as one of effective ways to generate every possible state at each DP stage. Many general discussions of DP can be found in the literature.

In the **LR-SP**, a state space is made up with all possible generator status and then DP will execute searching the best strategy from possible strategies of each stage. Once the generator state x_i and its power dispatch y_i over the planning horizon are determined, we can obtain the objective value for VD^*. This is the lower bound of the UC problem and will be used as the dual value. We then examine the relaxed coupling constraints to be satisfied. If these constraints can not be satisfied, the Lagrangian multipliers ˘ will be updated through another method (e.g., subgradient method).

If they are satisfied, based on the given UC solution, the economic dispatch problem will be solved to determine power dispatch amount on each generator.

$$
\begin{aligned}
VP^* = \min & \mathbb{F}(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_{n_1}, y_1, y_2, \ldots, y_{n_2}) \\
\text{s.t.} & \mathbb{H}(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_{n_1}, y_1, y_2, \ldots, y_{n_2}) \leq \mathbf{d} && (2.36) \\
& \mathbb{G}_i(\hat{x}_i, y_i) \leq b_i, \quad \forall\, i && (2.37) \\
& \mathbf{y} \in \mathbb{R}_+^{n_2} && (2.38)
\end{aligned}
$$

The objective value of VP^* is the primal value as the upper bound of the UC problem. We then compare the primal value with the dual value and examine their difference met within the range of duality gap. If the current difference exceeds the duality gap, Lagrange multiplier will be updated until another feasible solution is obtained and the duality gap stays in an acceptable range. So far the LR method has been applied to some specific coupling constraints relaxation, usually for ramping, hydropower generation, transmission network, and emission constraints.

What's more, due to the non-convexity of UC optimization problem, the performance of LR is highly affected by the multipliers and less sufficient to finding a global optimal solution with reasonable convergence speed. Then the augmented Lagrangian method can be applied to deal with the non-convexity in the means of adding quadratic penalty terms to the Lagrangian function. For general UC models, the main difference between Lagrangian Relaxation and Augmented Lagrangian Relaxation exists in the Lagrangian function. In order to improve the convexity of problem, in general we add a quadratic penalty term $-(c/2)\sum_{t\in T}(\sum_{g\in G} p_{gt}u_{gt} - D_t)^2$, which stands for the gap between supply and demand [FSL05b, SYL03].

2.5.4.2 LR Example

To illustrate the implementation of LR in UC problem, we construct a typical UC model to show the LR-based model and its solution process in details. We consider the following UC model with partial prevailing constraints and decompose the original model via the Lagrangian Relaxation method.

$$
\min \sum_{g\in G}\sum_{t\in T}(SU_{gt}v_{gt} + SD_{gt}w_{gt}) + \sum_{t\in T}\sum_{g\in G}(b_{gt}p_{gt} + a_{gt}u_{gt})
$$

$$
\text{s.t.}\; u_{gt} - u_{g(t-1)} \leq u_{g\tau}, \quad \forall\, g \in G,\; t \in T, \tau = t, \ldots, min\{t + L_g - 1, \|T|\} \tag{2.39a}
$$

$$
u_{g(t-1)} - u_{gt} \leq 1 - u_{g\tau}, \quad \forall\, g \in G,\; t \in T, \tau = t, \ldots, min\{t + l_g - 1, \|T|\} \tag{2.39b}
$$

$$
v_{gt} \geq u_{gt} - u_{g(t-1)}, \quad \forall\, g \in G,\; t \in T \tag{2.39c}
$$

$$w_{gt} \geq -u_{gt} + u_{g(t-1)}, \quad \forall\, g \in G,\ t \in T \tag{2.39d}$$

$$u_{gt},\ v_{gt},\ w_{gt} \in \{0, 1\}, \quad \forall\, g \in G,\ t \in T \tag{2.39e}$$

$$\sum_{g \in G} p_{gt} u_{gt} = D_t + \Delta_t, \quad \forall\, t \in T \tag{2.39f}$$

$$\sum_{g \in G} s_{gt} u_{gt} \geq RS_t, \quad \forall\, t \in T \tag{2.39g}$$

$$P_g^{min} u_{gt} \leq p_{gt} \leq P_g^{max} u_{gt}, \quad \forall\, g \in G,\ t \in T \tag{2.39h}$$

$$p_{gt} - p_{gt-1} \leq P_g^{min}(2 - u_{gt} - u_{g(t-1)}) + RU_g(1 + u_{g(t-1)} - u_{gt}), \quad \forall\, g \in G, t \in T \tag{2.39i}$$

$$p_{gt-1} - p_{gt} \leq P_g^{min}(2 - u_{gt} - u_{g(t-1)}) + RD_g(1 - u_{g(t-1)} + u_{gt}), \quad \forall\, g \in G, t \in T \tag{2.39j}$$

$$\sum_{g \in G} \sum_{t \in T} \left(F_g^e(p_{gt}) u_{gt} + SU_g^e v_{gt} + SD_g^e w_{gt} \right) \leq E^{max} \tag{2.39k}$$

$$p_{gt}, s_{gt} \geq 0, \quad \forall\, g \in G,\ t \in T \tag{2.39l}$$

From the given UC model, all constraints are categorized with the same features into separable constraints, i.e. (2.39a)–(2.39e), (2.39h)–(2.39j), and coupling constraints i.e. (2.39f), (2.39g) and (2.39k). Since these coupling constraints have the common feature that all units are aggregated in one constraint for operational requirement. In the consideration of system-level operation, if one generation variable get changed, other generation variables will be affected simultaneously. According to the LR framework, these coupling constraints are relaxed and placed in the objective function associated with Lagrangian multipliers. In doing so, we can construct a Lagrangian function for this UC problem as follows:

$$\begin{aligned} & L(v_{gt}, w_{gt}, u_{gt}, p_{gt}, \lambda_t^b, \lambda_t^r, \lambda^e) \\ = & \sum_{g \in G} \sum_{t \in T} (SU_{gt} v_{gt} + SD_{gt} w_{gt}) + \sum_{t \in T} \sum_{g \in G} (b_{gt} p_{gt} + a_{gt} u_{gt}) \\ & - \sum_{t \in T} \lambda_t^b \sum_{g \in G} p_{gt} u_{gt} - \sum_{t \in T} \lambda_t^r \sum_{g \in G} s_{gt} u_{gt} \\ & - \lambda^e \sum_{g \in G} \sum_{t \in T} \left(F_g^e(p_{gt}) u_{gt} + SU_g^e v_{gt} + SD_g^e w_{gt} \right) \end{aligned} \tag{2.40}$$

This Lagrangian function of UC problem is subject to separable constraints (2.39a)–(2.39e), (2.39h)–(2.39j), based on each individual generator.

During the LR decomposition process, when the commitment decision u_{gt} and generation decision p_{gt} are determined for all units over the planning horizon, the objective value of (2.40) in $k + 1^{th}$ iteration can be obtained as the lower bound of original UC problem. Next, we use the current solution $(\hat{\mathbf{u}}, \hat{\mathbf{p}})$ and check for the coupling constraints. When the current solution is not satisfied with that constraints, the

Lagrangian multiplier ˘ will be updated through the subgradient method. Otherwise, we solve the problem 2.41 with fixed $\hat{\mathbf{u}}$

$$\min \sum_{g\in G}\sum_{t\in T}(SU_{gt}\hat{v}_{gt} + SD_{gt}\hat{w}_{gt}) + \sum_{t\in T}\sum_{g\in G}(b_{gt}p_{gt} + a_{gt}\hat{u}_{gt})$$

$$\text{s.t.} \sum_{g\in G} p_{gt}\hat{u}_{gt} = D_t + \Delta_t, \quad \forall\, t \in T \tag{2.41a}$$

$$\sum_{g\in G} s_{gt}\hat{u}_{gt} \geq RS_t, \quad \forall\, t \in T \tag{2.41b}$$

$$P_g^{min}\hat{u}_{gt} \leq p_{gt} \leq P_g^{max}\hat{u}_{gt}, \quad \forall\, g \in G,\ t \in T \tag{2.41c}$$

$$p_{gt} - p_{gt-1} \leq P_g^{min}(2 - \hat{u}_{gt} - \hat{u}_{g(t-1)}) + RU_g(1 + \hat{u}_{g(t-1)} - \hat{u}_{gt}), \quad \forall\, g \in G, t \in T \tag{2.41d}$$

$$p_{gt-1} - p_{gt} \leq P_g^{min}(2 - \hat{u}_{gt} - \hat{u}_{g(t-1)}) + RD_g(1 - \hat{u}_{g(t-1)} + \hat{u}_{gt}), \quad \forall\, g \in G, t \in T \tag{2.41e}$$

$$\sum_{g\in G}\sum_{t\in T}\left(F_g^e(p_{gt})\hat{u}_{gt} + SU_g^e\hat{v}_{gt} + SD_g^e\hat{w}_{gt}\right) \leq E^{max} \tag{2.41f}$$

$$p_{gt}, s_{gt} \geq 0, \quad \forall\, g \in G,\ t \in T \tag{2.41g}$$

and obtain the corresponding solution **p** as well as the upper bound of original UC problem. Then check for the difference between the lower bound and upper bound. If the difference is within a specific gap, the UC final solution is obtained. Otherwise, update ˘ again until the optima is found.

2.5.4.3 LR-Based Solution Process

Here we briefly introduce the augmented Lagrangian relaxation integrated dynamic programming approach to solve UC problem within reasonable computation times [FSL05b]. The flow chat for this solution process is shown in Fig. 2.11 and the solution approach is explained as follow.

LR-Based Solution Approach:

- Step 1: Initiate Lagrangian multipliers, to support with power balance equalities, reserve requirements, system fuel limits, system emission limits, and system security constraints (Benders cuts).
- Step 2: Decouple the relaxed problem into several subproblems to represent individual generators (20). Taking the current values of multipliers, apply DP to solve the UC for each unit over a 24-h planning horizon.
- Step 3: Check all power balance, reserve, fuel, and emission constraints as well as Benders cuts produced from the network security check subproblem.

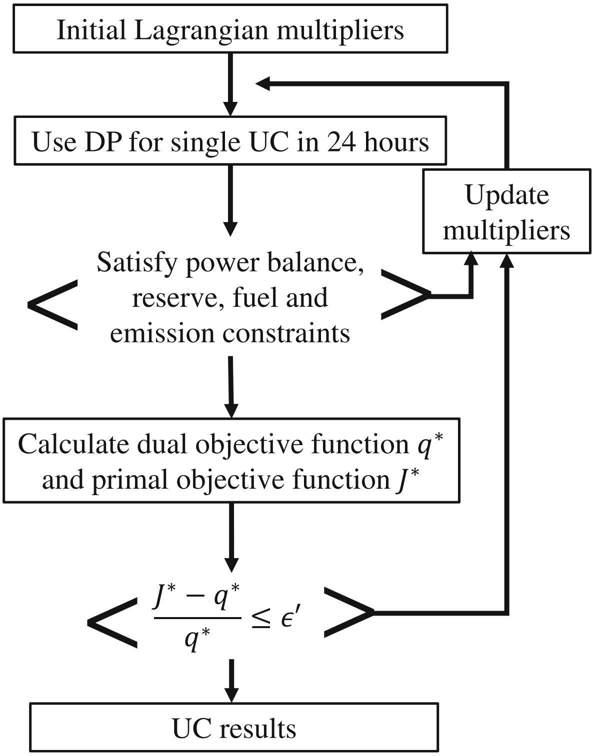

Fig. 2.11 The flow chart of augmented Lagrangian relaxation [FSL05b]

Update multipliers through the subgradient method. Go back to Step 2 if one of constraints cannot be met; otherwise, the solution process will move to Step 4.

- Step 4: Calculate the dual objective in the Lagrangian function and the primal objective (i.e., ED over a 24-h period). Terminate the solving process in the master problem when if the relative duality gap falls in the tolerance; otherwise, keep updating multipliers via the subgradient method, and return to Step 2.

2.5.5 Benders' Decomposition

The main use of Benders' decomposition is to decompose an original single large problem into a master problem (MP) and one/multiple smaller subproblems (SP) to alleviate the computational difficulty from directly solving an optimization problem. After decomposition, the algorithm process goes through serval steps: solving MP to get a lower bound, passing its current solutions to SP, solving SP to get a upper bound and then generating Bender's cuts for MP until LB and UB are converged.

As for decomposition, we target to build the subproblem as a linear program (LP) or a convex nonlinear program [CGB06] in that it applies the theory of duality to get a feasible solution, and allow the master problem include all discrete variables, such as binary variables or integer variables. In some cases, one can also keep some of the continuous variables in the master problem according to the needs of master problem and the program structure of subproblem.

2.5.5.1 Principles of Benders' Decomposition

In this section, we consider a MILP-based UC problem and use it as an example to illustrate the procedure of Benders' decomposition. The original UC has two types of decision variables, **x** and **y**, which are vectors of integer and continuous variables. For fixing values of **x** variables, the original problem is given by

$$\min \{\mathbf{f}(\hat{\mathbf{x}}) + \mathbf{c_2^T y} \mid \mathbf{Ey} \geq \mathbf{b_2} - \mathbf{A_2}\hat{\mathbf{x}},\ \mathbf{y} \in \mathbb{R}_+,\ \mathbf{y} \geq 0\}. \tag{2.42}$$

Since the value of function **x** is fixed in the objective function and moved out from the function **y**, the problem (2.42) can be written as follow:

$$\mathbf{f}(\hat{\mathbf{x}}) + \min \{\mathbf{c_2^T y} \mid \mathbf{Ey} \geq \mathbf{b_2} - \mathbf{A_2}\hat{\mathbf{x}},\ \mathbf{y} \in \mathbb{R}_+,\ \mathbf{y} \geq 0\}, \tag{2.43}$$

where the inner minimization problem is defined to be subproblem (SP).

Let ¯ denote dual variables (extreme points in a feasible region) associated with the specific constraint, $\mathbf{Ey} \geq \mathbf{b_2} - \mathbf{A_2}\hat{\mathbf{x}}$. If $\mathbf{y} \in \mathbb{Y}$ is a nonempty polytope, there exists an extreme point for optimal solution in SP. We can further formulate the dual SP as

$$\min \{z \mid z \geq (\mathbf{b_2} - \mathbf{A_2}\hat{\mathbf{x}})^{T\,¯},\ \mathbf{E}^{T\,¯} \leq \mathbf{c}_2,\ ¯ \geq 0\}. \tag{2.44}$$

Solving the inner minimization problem means enumerating all extreme points of Y in the subproblem. If there are partial k ($k < Q$) extreme points selected, the MP becomes a relaxed master problem (RMP) with less constraints given by

$$\min \{\mathbf{f}(\mathbf{x}) + z \mid \mathbf{x} \in \mathbb{X},\ z \geq (\mathbf{b_2} - \mathbf{A_2}\mathbf{x})^{T\,\hat{}}{}_j,\ \text{for } j = 1, 2, \ldots, k\}. \tag{2.45}$$

Define $(\bar{x}, \bar{z})$ as an optimal solution to RMP. In this situation with given partial extreme points, $(\bar{x}, \bar{z})$ can only be considered as a feasible solution to the master problem ($k = Q$). To check this optimality condition, we equivalently check if this solution can make the inequality (2.46) at all extreme points hold true.

$$\bar{z} \geq (\mathbf{b_2} - \mathbf{A_2}\bar{\mathrm{x}})^{T\,¯}{}_j,\ \text{for } j = 1, 2, \ldots, Q \tag{2.46}$$

If the current solution of RMP, $(\bar{x}, \bar{z})$, violates one or partial constraints in SP, an *optimality cut* (2.47) will be imposed to RMP.

$$z \geq (\mathbf{b} - \mathbf{Dy})^T \hat{\mathbf{u}}_{k+1}. \tag{2.47}$$

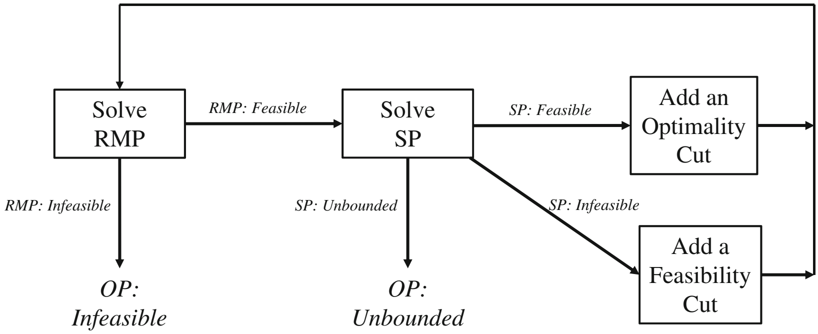

Fig. 2.12 Solution types for master problem and subproblems in Benders' Decomposition

If SP has infeasible solutions, a *feasibility cut* (2.48) will be added to RMP.

$$0 \geq (\mathbf{b} - \mathbf{D}\mathbf{y})^T \hat{\mathbf{u}}_{k+1}. \tag{2.48}$$

During the solving process, MP and SP may experience one or more solution types, shown in the Fig. 2.12. After solving RMP, it may have a feasible solution which will be passed to SP for the next-step solution, or may have an infeasible solution that indicates the original problem to be infeasible. Then the suproblem is solved with three possible cases: feasible, infeasible and unbounded. Based on the solution type of SP, an optimality cut or a feasibility cut will be generated and then added to RMP for next iterations. If the SP has the unbounded case, it also shows that the original problem is unbounded.

To solve a classical MILP problem with L-shaped structure, we outline a traditional Benders' Decomposition algorithm as follow:

- **Initialization**: Let $\hat{\mathbf{x}}$:= initial feasible solution, only solve for the function of x to get the initial LB and then fix $\mathbf{x}$ to solve for UB.
- **Step 1**: Solve the RMP, $\min_x\{f(\mathbf{x}) + z |\ \mathbf{x} \in X$, cuts, z unrestricted$\}$.
  If RMP is feasible, get solutions $(\bar{\ }, \bar{z})$ and $LB := f(\bar{\mathbf{x}}) + \bar{z}$; otherwise, the algorithm is terminated.
- **Step 2**: Solve the SP, $\max_\mu\{\mathbf{f}(\hat{\mathbf{x}}) + (\mathbf{b_2} - \mathbf{A_2}\hat{\mathbf{x}})^T \bar{\ } | \mathbf{A}^T \bar{\ } \leq \mathbf{c}, \bar{\ } \geq 0\}$.
  If SP is feasible, get dual solutions $\hat{\ }$ and $UB := \mathbf{f}(\hat{\mathbf{x}}) + (\mathbf{b_2} - \mathbf{A_2}\hat{\mathbf{x}})^T \hat{\ }$.
  Add optimality cut $z \geq (\mathbf{b_2} - \mathbf{A_2}\mathbf{x})^T \hat{\ }$ to RMP.
  If SP is infeasible, add feasibility cut $0 \geq (\mathbf{b_2} - \mathbf{A_2}\mathbf{x})^T \hat{\ }$ to RMP.
- If $(UB - LB)/UB \leq \epsilon$, the current solution is optimal and the algorithm is terminated.
  If $(UB - LB)/UB > \epsilon$, perform next iteration and go to Step 1.

2.5.5.2 Application of Benders' Decomposition in UC Problem

Based on the above decomposition approach, we can obtain the decomposed UC problems: an integer master problem (BD-MP) and a linear subproblem (BD-SP), which are given by

$$[\mathbf{BD\text{-}MP}] : LB = \min_{x,\pi} \ \mathbf{c_1^T x} + \mathbf{ß} \tag{2.49a}$$
$$\text{s.t.} \quad \mathbf{A_1 x} = \mathbf{b}_1 \tag{2.49b}$$
$$\mathbf{x} \in \{0, 1\}^{n_1} \tag{2.49c}$$
$$\pi \geq \mathscr{O}(\mathbf{x}) \tag{2.49d}$$
$$0 \geq \mathscr{F}(\mathbf{x}) \tag{2.49e}$$

$$[\mathbf{BD\text{-}SP}] : UB = \min_{y} \ \mathbf{c_2^T y} \tag{2.50a}$$
$$\text{s.t.} \quad \mathbf{Ey} = \mathbf{b}_2 - \mathbf{A_2 \hat{x}} \tag{2.50b}$$
$$\mathbf{y} \in \mathbb{R}_+^{n_2} \tag{2.50c}$$

where π is a free variable; constraints (2.49d) and (2.49e) represents a set of optimality cuts and feasibility cuts, respectively.

In the review of decomposition strategies of UC problems the decomposition strategy depending on the types of decision variables has been used a lot, as shown in 2.49 and 2.50.

- Solve the MP with unit commitment and generated cuts;
- Given the current solutions from MP, solve the SP including economic dispatch, operating reserve, emission, transmission, reactive power and unserved energy constraints. Generate Benders' cut(s) according to solution type of SP in current iteration.

Another common strategy of Benders' Decomposition is to solve general security-constrained unit commitment (SCUC) in two operation stages:

- Solve the MP with unit commitment, economic dispatch, operating reserve and emission constraints;
- Given the current solutions from MP, solve the SP only regarding to transmission, reactive power and unserved energy constraints. Check if any network violations occur and generate Benders' cuts.

For both decomposition schemes, the MP includes new generated cuts, and the SP are solved iteratively and checked for convergence. When using the second decomposition scheme, the MP becomes a mixed integer program while the SP is built as a simple linear program and used for meeting network constraints.

From the literature, the network security check is usually arranged in the SP. In particular, the DC network security check focuses on the power flow balance and flow

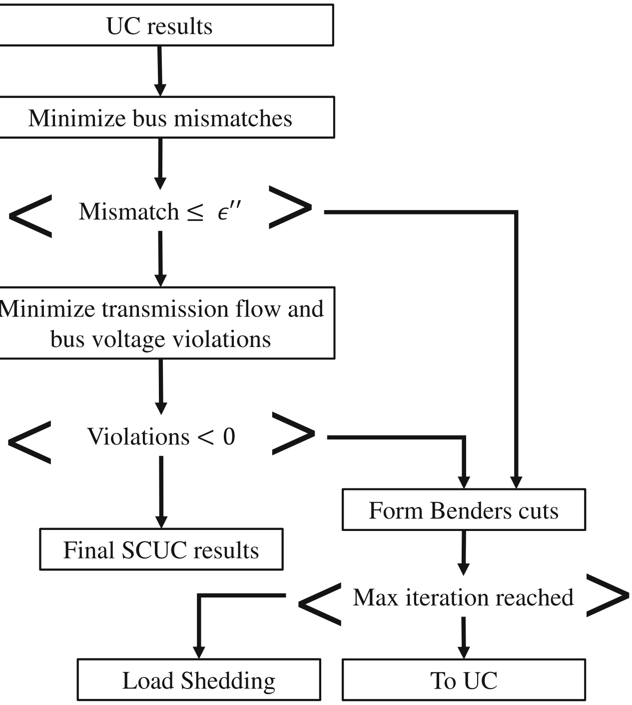

Fig. 2.13 BD-SP: the flow chart of AC network security check [FSL05b]

restrictions on transmission lines. If the DC network constraint is replaced by more complicated AC network constraint, the scheme remains suitable for AC network security check. Because the DC network constraints only consider the power flow balance at a bus and have several limitations, such as ignoring bus voltage violations, feasible distribution of reactive power and interactions between real and reactive power conditions. When the AC network considers such requirements left behind, it is more appropriate to handle them in SP through the security check. The flow chart for a comprehensive network security check in subproblem is shown on Fig. 2.13. This decomposition strategy has also been testified to solve a deterministic large-scale UC problem effectively, i.e. 118 bus system [FSL05b].

2.5.5.3 BD Example

We take the same UC problem shown in Sect. 2.5.4.2 and decompose it using the first strategy of Benders' decomposition. The UC problem is decomposed into a MP and a SP, shown in 2.51 and 2.52. For the second strategy of BD, interested readers can find some explicit examples in [SYL03].

[BD-MP] :

$$\min \quad \sum_{g\in G}\sum_{t\in T}(SU_{gt}v_{gt} + SD_{gt}w_{gt} + a_g u_{gt}) + \pi \tag{2.51a}$$

$$\text{s.t.} \quad u_{gt} - u_{g(t-1)} \le u_{g\tau}, \quad \forall\, g \in G,\ t \in T, \tau = t, \ldots, min\{t + L_g - 1, |T|\} \tag{2.51b}$$

$$u_{g(t-1)} - u_{gt} \le 1 - u_{g\tau}, \quad \forall\, g \in G,\ t \in T, \tau = t, \ldots, min\{t + l_g - 1, |T|\} \tag{2.51c}$$

$$v_{gt} \ge u_{gt} - u_{g(t-1)}, \quad \forall\, g \in G,\ t \in T \tag{2.51d}$$

$$w_{gt} \ge -u_{gt} + u_{g(t-1)}, \quad \forall\, g \in G,\ t \in T \tag{2.51e}$$

$$u_{gt},\ v_{gt},\ w_{gt} \in \{0, 1\}, \quad \forall\, g \in G,\ t \in T \tag{2.51f}$$

$$\pi \ge \mathscr{O}(\mathbf{u}) \tag{2.51g}$$

$$0 \ge \mathscr{F}(\mathbf{u}) \tag{2.51h}$$

[BD-SP] :

$$\min \quad \sum_{t\in T}\sum_{g\in G} b_{gt}p_{gt} \tag{2.52a}$$

$$\text{s.t.} \quad \sum_{g\in G} p_{gt} = D_t + \Delta_t, \quad \forall\, t \in T \quad \rightarrow \alpha_t \tag{2.52b}$$

$$\sum_{g\in G} s_{gt} \ge RS_t, \quad \forall\, t \in T \quad \rightarrow \beta_t \tag{2.52c}$$

$$p_{gt} \ge P_g^{min}\hat{u}_{gt}, \quad \forall\, g \in G,\ t \in T \quad \rightarrow \gamma_{gt} \tag{2.52d}$$

$$p_{gt} \le P_g^{max}\hat{u}_{gt}, \quad \forall\, g \in G,\ t \in T \quad \rightarrow \varepsilon_{gt} \tag{2.52e}$$

$$p_{gt} - p_{gt-1} \le P_g^{min}(2 - \hat{u}_{gt} - \hat{u}_{g(t-1)}) + RU_g(1 + \hat{u}_{g(t-1)} - \hat{u}_{gt}), \quad \forall\, g \in G, t \in T \quad \rightarrow \vartheta_{gt} \tag{2.52f}$$

$$p_{gt-1} - p_{gt} \le P_g^{min}(2 - \hat{u}_{gt} - \hat{u}_{g(t-1)}) + RD_g(1 - \hat{u}_{g(t-1)} + \hat{u}_{gt}), \quad \forall\, g \in G, t \in T \quad \rightarrow \kappa_{gt} \tag{2.52g}$$

$$\sum_{g\in G}\sum_{t\in T} F_g^e(p_{gt}) \le E^{max} - \sum_{g\in G}\sum_{t\in T}(SU_g^e\hat{v}_{gt} + SD_g^e\hat{w}_{gt}) \quad \rightarrow \nu \tag{2.52h}$$

$$p_{gt}, s_{gt} \ge 0, \quad \forall\, g \in G,\ t \in T \tag{2.52i}$$

where power balance constraint (2.39f) and spinning reserve requirement (2.39g) are replaced with (2.52b) and (2.52c). The major difference between these two expressions is that the bilinear terms ($p_{gt}u_{gt}$, $s_{gt}u_{gt}$) are eliminated and only linear terms (p_{gt}, s_{gt}) remain for simplifying the solving process in SP. In addition, the power balance has the same restriction as (2.39f), while the spinning reserve sources are expanded not only from online units but also offline units. In doing so, these modifications can simplify the computation process in SP.

We define several dual variables, such as $\alpha_t,\ \beta_t,\ \gamma_{gt},\ \varepsilon_{gt},\ \vartheta_{gt},\ \kappa_{gt},\ \nu$ corresponding to constraints (2.52b)–(2.52h), respectively. Then the optimality cut, $\pi \geq \mathscr{O}(\mathbf{u})$, is formed in (2.53) through the dual solution of **BD-SP**.

$$\begin{aligned}\pi \geq &\sum_{t\in T}\hat{\alpha}_t(D_t+\Delta_t)+\sum_{t\in T}\hat{\beta}_t RS_t+\sum_{g\in G}\sum_{t\in T}\hat{\gamma}_{gt}P_g^{min}u_{gt}+\sum_{g\in G}\sum_{t\in T}\hat{\varepsilon}_{gt}P_g^{max}u_{gt}\\ &+\sum_{g\in G}\sum_{t\in T}\hat{\vartheta}_{gt}\left[P_g^{min}(2-\hat{u}_{gt}-\hat{u}_{g(t-1)})+RU_g(1+\hat{u}_{g(t-1)}-\hat{u}_{gt})\right]\\ &+\sum_{g\in G}\sum_{t\in T}\hat{\kappa}_{gt}\left[P_g^{min}(2-\hat{u}_{gt}-\hat{u}_{g(t-1)})+RD_g(1-\hat{u}_{g(t-1)}+\hat{u}_{gt})\right]\\ &+\hat{\nu}[E^{max}-\sum_{g\in G}\sum_{t\in T}(SU_g^e v_{gt}+SD_g^e w_{gt})]\end{aligned} \tag{2.53}$$

This cut is associated with binary variables (u_{gt}, v_{gt}, w_{gt}) and given with incumbent dual values in k^{th} iteration.

2.6 Summary

This chapter introduces basic UC formulations in terms of optimization methods, including objective function and their essential constraints: unit commitment constraints, electricity dispatch, operating reserve constraints, transmission constraints, emission constraints, unserved energy constraints, and reactive power constraints. To address UC problems by optimization approaches, we chose two typical case studies to illustrate how to model UC problems and analyze optimal solutions for better decision making. For the improvement of solution process of UC models, we also provided a overview of solution approaches and summarized their recent development. Particularly, we provided a detailed introduction on the most widely used methods for solving moderate power systems, involving MILP, LR decomposition method and BD decomposition method.

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