

Preface

Analysis forms an essential basis of both mathematics and statistics, as well as most of the natural sciences. Moreover, and to an ever increasing extent, mathematics has been used to underpin our understanding of the social sciences. It was Galileo's insight that "Nature's great book is written in the language of mathematics." And it is the theory of analysis (specifically, differentiation and integration) that was created for the express purpose of describing the universe in the language of mathematics. Working out the precise mathematical theory took almost 300 years, with a large portion of this time devoted to creating definitions that encapsulate the essence of limit and continuity. This task was neither easy nor self-evident.

In postsecondary education, analysis is a foundational requirement whenever mathematics is an integral component of a degree program. Mastering the concepts of analysis can be a difficult process. This is one of the reasons why introductory analysis courses and textbooks introduce the material at many different levels and employ various methods of presenting the main ideas. This book is not meant to be a first course in analysis, for we assume that the reader already knows the fundamental definitions and basic results of one-variable analysis, as is discussed, for example, in [7]. In most of the cases we present the necessary definitions and theorems of one-variable analysis, and refer to the volume [7], where a detailed discussion of the relevant material can be found.

In this volume we discuss the differentiation and integration of functions of several variables, infinite numerical series, and sequences and series of functions. We place strong emphasis on presenting applications and interpretations of the results, both in mathematics itself, like the notion and computation of arc length, area, and volume, and in physics, like the flow of fluids. In several cases, the applications or interpretations serve as motivation for formulating relevant mathematical definitions and insights. In Chapter 8 we present applications of analysis in apparently distant fields of mathematics.

It is important to see that although the classical theory of analysis is now more than 100 years old, the results discussed here still inspire active research in a broad spectrum of scientific areas. Due to the nature of the book we cannot delve into such

matters with any depth; we shall mention only a small handful of unsolved problems.

Many of the definitions, statements, and arguments of single-variable analysis can be generalized to functions of several variables in a straightforward manner, and we occasionally omit the proof of a theorem that can be obtained by repeating the analogous one-variable proof. In general, however, the study of functions of several variables is considerably richer than simple generalizations of one-variable theorems. In the realm of functions of several variables, new phenomena and new problems arise, and the investigations often lead to other branches of mathematics, such as differential geometry, topology, and measure theory. Our intent is to present the relevant definitions, theorems, and their proofs in full detail. However, in some cases the seemingly intuitively obvious facts about higher-dimensional geometry and functions of several variables prove remarkably difficult to prove in full generality. When this occurs (for example, in Chapter 5, during the discussion of the so-called integral theorems) with results that are too important for either the theory or its applications, we present the facts, but not the full proofs.

Our explicit intent is to present the material gradually, and to develop precision based on intuition with the help of well-designed examples. Mastering this material demands full student involvement, and to this end we have included about 600 exercises. Some of these are routine, but several of them are problems that call for an increasingly deep understanding of the methods and results discussed in the text. The most difficult exercises require going beyond the text to develop new ideas; these are marked by (*). Hints and/or complete solutions are provided for many exercises, and these are indicated by (H) and (S), respectively.

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