Preface

Purpose

This book is the outgrowth of many years of trying to quickly teach engineers and mathematics majors Fourier Analysis at University of Florida. The major constraints were the following:

1. The students will not necessarily have a prior course in modern analysis; therefore, Lebesgue measure is suppressed. Linear Algebra and calculus is all that is assumed.
2. The students generally do not have two or three semesters to master the subject. They have one semester to get an overview of the subject. While those are the constraints in Gainesville, I believe that this book will support a two-semester course taught slower with complete detail. In ten years, I have never covered everything in this book.
3. Most of the students will not be interested in an analysis class which does not cover some applications. Understanding some applications needs to be a goal of the class.
4. The course should have mathematical rigor, but this rigor should not be emphasized at the cost of applications.
5. The students should be exposed to the concepts of isometry, sampling, interpolation, and aliasing.
6. When I wrote the first test for this class, many of the questions had been on previous Numerical Analysis Ph.D. qualifying exam. Thus, I had to find a way to let undergraduate students learn, without expecting heavy proof ability as would be expected at a much later date.
7. Some exercises which involve the calculation of Fourier Series or integrals are necessary and helpful. Calculation of complicated integrals should not be the only emphasis of the course.

8. Projects seem to be the best way of having them "get their feet wet". Most notably numerical projects are using MATLAB, although other packages could suffice. I let them program in any language they wish, but refuse to answer questions about anything other than MATLAB. I have included some of the projects I used for one semester. Please feel free to alter them.

While there were many mathematics students in these courses, many of them are EE, CS, ME, Physics, etc. Thus, the purpose is to give a mathematical introduction to Fourier Analysis, while emphasizing the applications. The goal is not to give a final course on Fourier Analysis with complete mathematical rigor. The goal is not to make the course a final course on any of the applied subject topics. The goal is to set the plate for further investigation by the students into the topics which they find most interesting or which their individual study plans demand.

Suggestions to Professors

Covering the Material

One obvious observation is that there is a lot of material in the book for one semester. I may have come close to covering all of it in one semester, but that is never my goal. I would love to be at a place where the students do not need to graduate, and we could take two semesters to cover the material. In a thirteen-week semester, I generally spend 7–8 weeks on Chapters 2-4. I try to emphasize to the students my belief that if they know this material, they can read the rest easily. I do emphasize Chapter 5, since sampling is at the forefront of most of modern signal processing, communications, etc.

After Chapters. 2-4, things become far easier for the students, and generally, it is fun time to play with what they learned. I usually cover the later chapters in a week or so. I pick and choose them according to what the students are interested in. Normally, I move to Image Processing after Chapter 5, because nearly all students want to be able to play with images. The Medical Imaging chapter is of personal interest to me, and I find it a lot of fun. I believe that students do also. I generally end up with some cleanup time, i.e., just covering the highlights of chapters which we did not cover, sometimes one lecture per chapter.

In summary, I do not feel it is necessary to cover more than one or two chapters after the base knowledge in Chapters 2-5. If I have time, I keep going.

Projects

You will notice that the chapter projects are merely collections of exercises. There is no magic in how I have arranged them. They seem to work, but they are continually evolving. You may want to change small things because code from prior semesters will always hang around and is hard for students to resist. For instance, merely changing expansions on $[-\pi, \pi]$ to $[-2, 2]$ will insure that the students have to put some of their own thought into the projects.

It is completely viable to build custom projects from individual exercises. I tend to stay with ones that have traditionally worked. Whether other projects are designed is certainly up to the individual instructor.

Testing

Part of the reason I wrote this book was that testing on this broad material was difficult, and probably above the level of an undergraduate class. On the other hand, I do give one test per semester, generally over the material in Chapter 4. The purpose of that test is to make sure that the students know some of Fourier Analysis off the top of their head. Required material includes the basics of dilation, translation, differentiation, convolution, etc. I refer to these as the "easy" problems. I challenge them to know some of the "hard" problems, such as the uncertainty principle, the proof of the Inverse Fourier Transform, and the Shannon Sampling Theorem. I do not require all of them.

There is no magic here, just what I found necessary to feel that I had transferred the knowledge. Please experiment with a mix of testing vs. projects. If you find something that works, please let me know.

Acknowledgements and History

Undergraduate and Graduate Years

This book is somewhat of an oral history of my training in mathematics. I was an undergraduate student at University of Montana in Missoula. It was only through the efforts of some dedicated Professors, most notably George McRae and William Derrick, that I survived my time as an undergraduate with some academic integrity in place. They convinced me that I could succeed at Mathematics if only I decided to study.

I attended graduate school at Auburn University and began to look at Mathematics as a career and something that I loved, rather than a distraction from my other pursuits. The entire faculty at Auburn was very engaged in educating the graduate students, and I have to commend all of them. My thesis advisor Dr. Richard Zalik treated me as one of his family and guided me through my experience there. My experience at Auburn included a wide number of the Professors, and I would like to thank all of them. I was also blessed with a great number of excellent colleagues at Auburn, and I have to mention Saady, Bobby, and Jeffy, who received their Ph.Ds with me.

While at Auburn, I studied two rather different disciplines. I wrote my Master's Thesis on Tomography [20] and learned a great deal about dealing with real data and applied numerical analysis. I then wrote a fairly pure mathematical thesis on wavelet-type basis for Fourier Analysis [21]. These two different approaches, one very applied and one fairly pure, are hopefully illustrated in this book.

The Dartmouth Years

I was privileged to leave Auburn to work with Dennis Healy et al at Dartmouth on a postdoctoral fellowship. Dartmouth was a great place to be at the time. Dennis and I worked on a number of applications, medical imaging, radar processing, statistical pattern recognition, as well as others.

I also had the blessing of working with a great group of others, including Dan Rockmore and John Weaver. We eventually named our group the "Wavelet Warriors." While we rarely did anything directly with wavelets, we believed that Fourier Analysis, appropriately used through wavelets or otherwise, could solve problems. We solved some, and we left many unsolved, but it was a great time and place to work. Most of the Wavelet Warriors are shown in Figure 1, including the wavelet dogs, Charlie Olson and Digger Rockmore. While at Dartmouth, we were a family that generally played and worked together. I will always remember it as a very special time.

University of Florida

I spent a year at Johns Hopkins, studying under Carey Priebe. We enjoyed the time and he enlightened me to the wonders of spatial statistics. I have been at University of Florida since 1997. My colleagues have all been great. I must acknowledge the great luck I had to first study at Dartmouth with the Wavelet Warriors, and then to end up at Florida. John Klauder has done his best to keep my knowledge of Fourier Analysis alive and keep me out of trouble. Both are full-time jobs. Sergei Shabanov has tolerated my ignorance of physics and answered any questions I had.

Figure 1: Wavelet Warriors are depicted in this photograph from June of 1996. In the top row are Peter Kostelec, Doug Warner, Sumit Chawla, Geoff Davis, and Dennis Healy. At the bottom are Dan Rockmore, Digger Rockmore, Charlie Olson, and Tim Olson. The Wavelet Warrior Viking regalia is circa 1995 and brought much strength to the group.

Dedication:

This book is dedicated to Professor Dennis Healy and the other Wavelet Warriors, at Dartmouth and beyond. Professor Healy unfortunately departed this world in September of 2009, but he will be with all of us forever.

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