Chapter 2
Rectangular Beams and One-Way Slabs

2.1 Introduction

This chapter covers the analysis (checking the strength) and the design (sizing the concrete and steel) of reinforced concrete beams and slabs that span primarily one way.

The previous chapter emphasized that concrete is very weak in tension, but strong in compression. As a result, reinforcements are used to supply tensile strength in concrete members (most commonly in the form of round reinforcing bars or *rebars*). Like any other building system, reinforced concrete structures have advantages and disadvantages.

2.2 Advantages of Reinforced Concrete

1. *Can be cast into any shape* This is the main advantage of reinforced concrete compared to other building materials. Concrete members can be made into any desired shape by using forms. Figure B2.1 in Appendix B shows the pleasing exterior of a reinforced concrete building.
2. *Has great resistance to fire and water* Concrete loses its structural integrity much more slowly than wood or steel when subjected to high temperature. In fact, concrete is often used as fireproofing material. Concrete also better resists exposure to water, does not corrode like steel, and does not lose strength as wood does. Certain chemicals in water, however, can harm concrete.
3. *Is a low-maintenance material* Concrete does not corrode, so it does not need to be painted and regularly maintained when exposed in the environment.
4. *Has very long service life* Reinforced concrete structures that are well designed and built last a very long time.

M. Setareh, R. Darvas, *Concrete Structures*, DOI 10.1007/978-3-319-24115-9_2

2.3 Disadvantages of Reinforced Concrete

1. *Has very low tensile strength* Concrete has a very low tensile strength in comparison to its compressive strength. Consequently, reinforcing steel bars are needed to counteract the development of tensions in concrete structures.
2. *Requires shoring and forms* This is a major disadvantage of concrete because it raises the cost of concrete structures, especially in countries such as the United States where labor costs are high. Shoring and formwork often constitute more than half the total cost of the structure.
3. *Has variations in properties* The mechanical and physical properties of concrete are sensitive and require careful proportioning, mixing, curing, and so on. Eliminating large variation in these properties demands carefully monitored procedures.
4. *Results in heavy structural members* Reinforced concrete structures are heavier than similar steel or wood structures. This results in larger building dead loads, which in turn result in larger foundations. Concrete structures are also more sensitive to differential settlements. Thus, concrete structures require relatively good soil conditions.

2.4 On the Nature of the Design Process

Before attending to the main topic of this chapter, which is the analysis and design of bending members, a discussion on the concept of *design* is appropriate.

Ask ten people about the meaning of the word "design" and you probably will get ten different answers. Design also has very different meanings to architects and to engineers. And to top it all off, design is often viewed as synonymous with sizing of members. So we hope that readers will forgive the rather loose usage of the term *design*.

Structural design of reinforced concrete structures is an iterative process. It begins with the layout of the structure or, in other words, with the selection of the structural system. Any practitioner will admit that this initial step is by far the hardest part of the process. It requires the designer to come up with a synthesized whole for the building, laying out all the component elements (columns, girders, beams (or joists), and slabs). Furthermore, the designer must also estimate the sizes of the elements within the space in order to go to the next step, that is, to *analysis*.

The flowchart of Figure 2.1 presents a somewhat simplified picture of the process. Oddly enough, it begins with a step in synthesis, or the conception of the structure. This step is nonmathematical, for the aim of the study at this point is to look at what the building structure should do. What spaces are required? What is the minimum column spacing required to fit the architectural program?

But before we reach the part designated as "Analysis" or "Design," we must complete another exercise: identifying the loads that the structure may be subjected to in its life span.

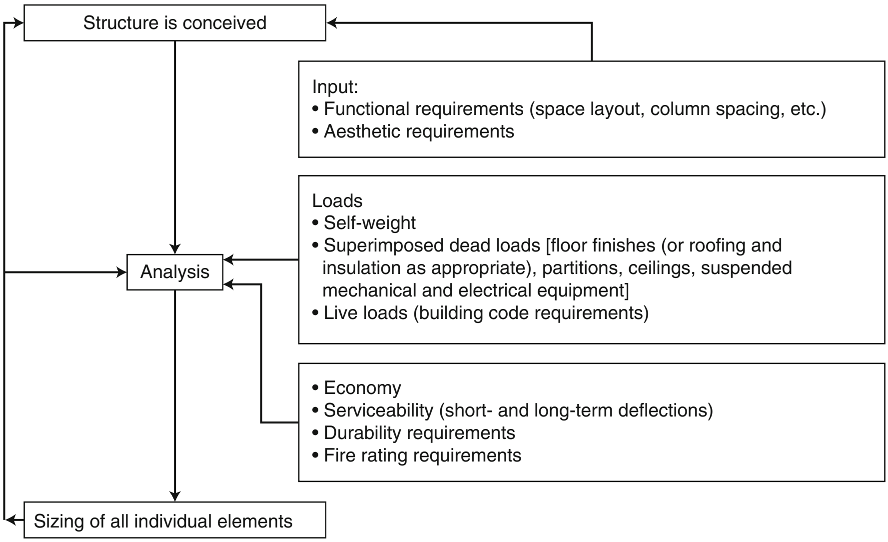


Figure 2.1 The iterative nature of structural design

Loads generally fall into two major categories: gravity loads and lateral loads. Gravity loads are further divided into two major groups: *dead loads* and *live loads*. One can only guess how this nomenclature came into usage. Perhaps people originally identified loads that were stationary as "dead," and loads that moved as "live." Today, we make a somewhat different distinction between these two loads. Dead loads are those that remain permanently attached to the structure, while other loads that are transitory in nature are referred to as live loads. Thus, furniture and stored items as well as loads from people's activities are in the latter category. For example, most of the weight in a library's stack area is from the stored books with only a very small part of the floor loads coming from the visitors; nevertheless, the stacks and the books are considered live loads. In addition, environmental effects such as moisture or temperature changes may create stresses in the structure, so they also may be loosely defined as loads that the structure must safely withstand.

Before any meaningful analysis can be performed to calculate and appropriately size any component element within a structure, designers must establish the loads that such an element can safely support, or at least must reasonably approximate them.

In a concrete structure, the *self-weight* is a very significant part of the dead loads. Because self-weight depends on the size of the particular member, a reasonable estimate must be made on the size. After the designer estimates the size, he or she can calculate the loads from the self-weight, assuming that reinforced concrete weighs about 150 lb/ft^3. At this point we do not want to tax the student's attention with detailed discussion on the selection of an appropriately sized beam or slab, and all of the reasons thereof. This subject will be discussed later in this chapter. In any case, if during the design process the designer determines that an initial estimate of the member's size, and thus the self-weight, was significantly in error, he or she has to re-analyze the member, taking into account the newly adjusted size; thus, the *iterative* nature of the design and sizing.

Superimposed dead loads (SDL) are somewhat ambiguous. Often these items and their precise location in space are not completely known at this stage of the design (see Figure 2.1). Partition layouts have not been decided yet, or may change in the future. Ductwork, piping, and light fixtures may go anywhere. So the designer is forced to make a blanket estimate on these. Most practitioners estimate that the combination of these items will exert about 15–20 lb/ft^2 of floor area. (The only areas that need more careful attention are those where some special flooring, such as stone or terrazzo, is planned. These items exert about 12–13 lb/ft^2/in. thickness. Thus, a 2 in. terrazzo flooring weighs about 25 psf.)

Live loads (LL) are prescribed by building codes for the particular usage of a space. These loads are listed as uniformly distributed minimum loads and represent the current professional wisdom. Because live loads are not uniformly distributed except in very isolated cases, they have very little, if anything, to do with the real loads that may occur on structures. Actual surveys show that total loads, uniformly averaged out over the whole floor area, amount to only about 15–20 % of the codes' mandated minimums in spaces like hotels, residential buildings, and offices. These minimums, however, represent a statistical probability of the loads that the structure may experience in a projected lifetime of 50 or 100 years. Furthermore, these code-prescribed live loads also try to account for the dynamic nature of many loads by treating them as equivalent static loads.

This discussion of loads should suffice to show that any calculation made during the load analysis phase will contain unavoidable inaccuracies and uncertainties. These errors are inevitable no matter how carefully the designer tries to evaluate the currently envisioned, but essentially future loads.

Example 2.1 In this simple floor plan, beams 12 in. wide and 20 in. deep are spanning 30 ft. The beams are located $9'-0''$ center to center. A 5-in. thick slab spans from beam to beam. (See Figure 2.2.) The floor structure will be used in a general office building, thus (per Code) the minimum uniformly distributed live load is 50 lb/ft^2. Calculate the dead and live loads that one interior beam has to carry. Assume 20 psf for the superimposed dead load for the partitions, mechanical and electrical systems, and so on.

Solution The beams are 9 ft apart, so each beam is assumed to be responsible for the loads that occur 4.5 ft from either side of the beam's centerline. Thus, each linear foot of beam will support loads from 9 ft^2 of floor in addition to the weight of the stem.

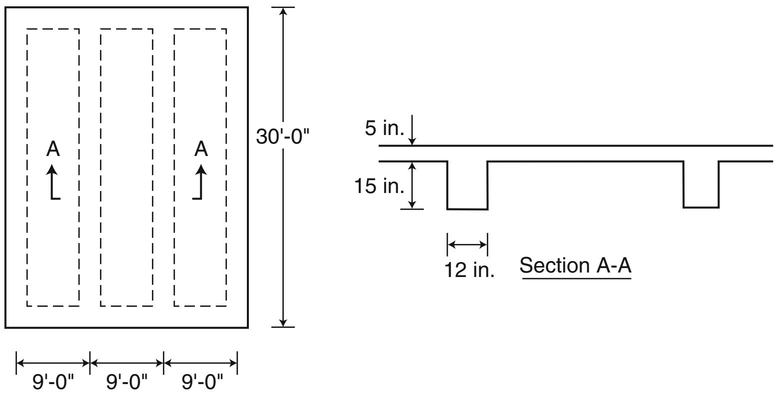


Figure 2.2 Floor plan and section

Loads from the slab:

5 in. slab self-weight (5/12) × 150	62.5 psf
Superimposed dead loads, estimated	20.0 psf
Total dead load on slab	82.5 psf
Dead loads on beam from slab: 9 ft × 82.5 =	742.5 lb/ft
Volume of stem per foot: (12 × 15)/144 × 1 ft = 1.25 ft^3/ft of beam	
Weight of stem: 1.25 × 150 =	187.5 lb/ft
TOTAL DEAD LOADS: w_D =	930 lb/ft

In addition, the beam will support live loads from 9 ft^2 of floor area on each linear foot of beam. Thus:

TOTAL LIVE LOADS: $w_L = 9 \times 50$ psf = 450 lb/ft

Summary: See Figure 2.3.

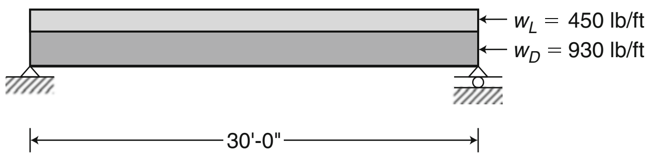


Figure 2.3 Floor beam

2.5 Live Load Reduction Factors

We complete this discussion of loads by dealing with the concept of *live load reduction factors*. These are derived from statistical analyses of the probability of having the maximum amount of live loads everywhere on a floor of a building.

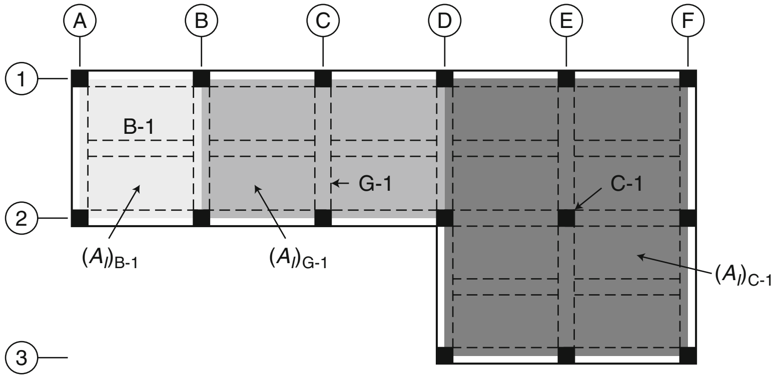


Figure 2.4 Influence areas for different structural members

Studies indicate that the larger the floor area that contributes loads to a particular member, the less likely it is that every square foot of that area will bear the maximum amount of live loads.

Different codes deal with this concept somewhat differently. Some codes relate the live load reduction to the *tributary area* (A_T), or the area directly loading the particular element under investigation. Other codes relate the live load reduction to the so-called *influence area* (A_I), the area in which a part, however small, of any load may contribute to the loading of a particular element under investigation. In other words, the influence area for a structural member is the part of the building structure that may fail if that member is removed.

As an example consider Figure 2.4, which shows the floor framing plan for a reinforced concrete building. To determine the influence area for beam B-1, assume that this beam is removed. This will cause the slabs supported by B-1 to fail. As a result, the influence area for B-1 is $(A_I)_{\text{B-1}}$, the area between column lines 1, 2, A, and B. Following this logic, if we remove girder G-1, the beams it supports will fail, and consequently the slabs supported by the beams. Thus, the area between column lines 1, 2, B, and D $(A_I)_{\text{G-1}}$ will collapse. A similar study will show that the influence area for column C-1 is the area between column lines 1, 3, D, and F.

One variation of the live load reduction formula is given in Equation (2.1):

$$L_{\text{red}} = L_0\left(0.25 + \frac{15}{\sqrt{A_I}}\right) \tag{2.1}$$

where

L_{red} = the reduced design live load per square foot of area supported by the member
L_0 = the unreduced design live load per square foot of area supported by the member
A_I = the influence area of the member in square feet

Equation (2.1) is applicable whenever $A_I > 400\ \text{ft}^2$. The usage of live load reduction is limited in that the reduction cannot exceed 50 % ($L_{\text{red}} \geq 0.5\, L_0$) for members supporting one floor and cannot exceed 60 % ($L_{\text{red}} \geq 0.4\, L_0$) for members supporting two or more floors. Live load reductions do not apply for live loads in excess of 100 psf, except for members supporting two or more floors, in which case the live load can only be reduced up to 20 %.

Example 2.2 For the interior beam of Example 2.1, determine the reduced live loads.

Solution The influence area, A_I, for the beam is:

$$A_I = 2 \times 9 \times 30 = 540\,\text{ft}^2$$

Because this area is larger than 400 ft^2, a reduced live load may be used in the design of the beam. The reduced design live load is:

$$L = 50\left[0.25 + \frac{15}{\sqrt{540}}\right] = 50 \times 0.895 = 44.8\,\text{psf}$$

Thus, the reduced design live load on this beam is:

$$w_L = 44.8 \times 9 = 403\,\text{lb/ft}$$

rather than the previously calculated load of 450 lb/ft.

2.6 Continuity in Reinforced Concrete Construction

Many readers may have encountered only statically determined structural elements. These are simply supported beams (with or without cantilevers at their ends), cantilevers fixed at one end and free to move at the other, simple posts, and so on. These elements are all characterized by needing only the equations representing static equilibrium ($\sum H = 0$, $\sum V = 0$, $\sum M = 0$) to solve for the reactions.

A review of what "reactions" means may be needed here. A building element does not exist in a stand-alone vacuum. It is connected to other elements. At a point of connection the free relative displacement between the element under study and the rest of the structure is denied. This denial of free movement results in the transmission of a force (or moment) at the connection between the supporting and the supported elements. Look at Figure 2.5a for example. Here a beam end is supported on a wall. Elsewhere within the span the beam is free to deflect, or move vertically. But this ability to displace vertically is denied at the place of the support.

Figures 2.5b, c show the symbols of a *hinge* type of support and a *roller*. In the hinge support, the two relative displacement components (vertical and horizontal) are denied between the beam (the member under investigation) and the support

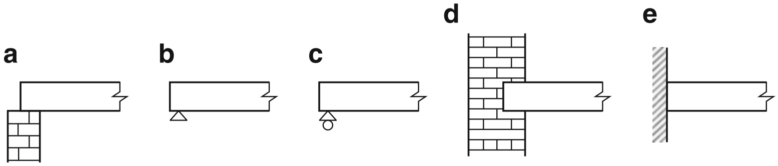
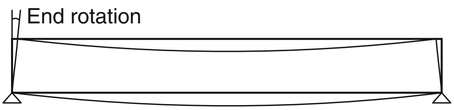


Figure 2.5 The meaning of the different support conditions: (**a**) wall supporting a beam (roller), (**b**) a hinge support, (**c**) a roller support, (**d**) wall supporting a beam (fixed), (**e**) a fixed support

Figure 2.6 Joist before and after deformation



below it. Thus, vertical and horizontal forces could be transmitted at the point between the beam and the support. (The forces coming from the support to the supported member are called *reaction forces*.) At a roller support (Figure 2.5c) only relative vertical displacement is denied; the beam could still freely roll horizontally without resistance. Correspondingly only a vertical force could be transmitted between the beam and the support. Figure 2.5d shows a beam end built into a large mass. The beam end cannot move horizontally or vertically, and it cannot rotate with respect to the mass. This condition is called *fixity*. The usual symbol of fixity is shown in Figure 2.5e. In this condition, horizontal force, vertical force, and a moment may be transmitted between the member and the support at that location.

All of these support conditions are quite familiar to students who have had a first course in structures. These support conditions represent what may be called *absolute conditions*: The displacement (vertical, horizontal, or rotational) is either freely available, or completely denied. As will be pointed out later, there is an infinite number of conditions in between, especially as related to rotations. Consider, for example, a flexible joist supported by a wall or beam at its ends (Figure 2.6). The mere supporting certainly precludes vertical displacement of the joist, thus a force transfer occurs. An action force is transmitted from the joist to the wall or beam, and an equal but opposite reaction force is transmitted from the supporting element to the joist. As the joist deflects under load, its supported ends can rotate freely; thus, the moments at the ends are zero.

Reinforced concrete construction is monolithic, which means that members are intimately built together with neighboring members. Slabs are continuous over supporting beams and girders; beams and girders are continuous over supporting interior columns, and so on.

Figure 2.7 illustrates the point. The slab in the beam and slab structure is continuous in both horizontal directions over the beams. The beams are continuous over other beams or columns.

A simple problem is presented here to clarify the concept. Admittedly, this problem does not occur in reinforced concrete structures, but it serves to illustrate the concept. A continuous structural member is represented by an imaginary center

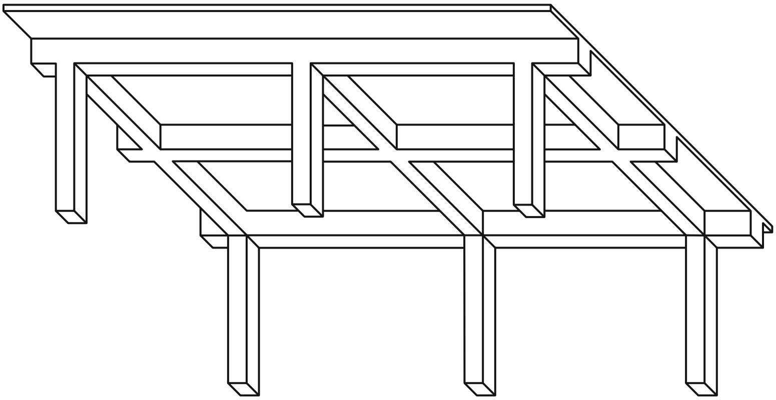


Figure 2.7 Beam and slab floor framing

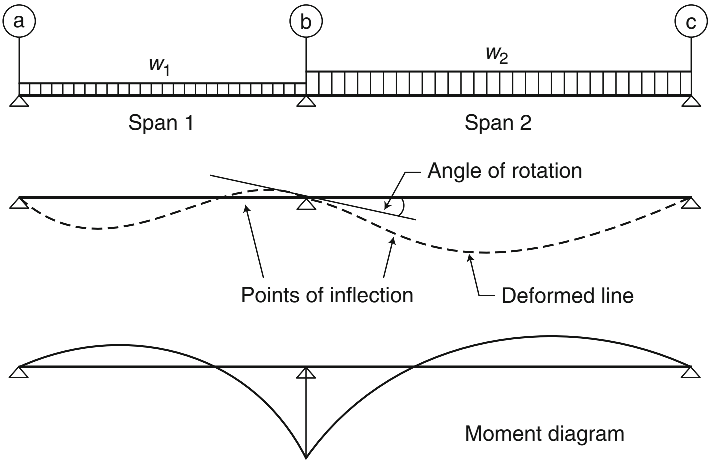


Figure 2.8 Deformations and moments in a two-span beam

line (see Figure 2.8). On this two-span beam, Span 2 is larger than Span 1. If the loads are about the same, Span 2 will deflect more. Consequently this deflection will try to force Span 1 to curve upward slightly near the center support to follow Span 2. (The tangent to the deformation curve will rotate toward Span 2.) Study of the deformation curve shows that the beam bends into an upward curvature, that is, tension develops at the top of the beam, between the two points of inflection (where the moment in the beam is zero), whereas elsewhere the beam bends downward, resulting in tensions at the bottom. The moment diagram is shown below the deformation line of the beam. The moments are referred to as positive when tension is on the bottom, and negative when tension is on the top.

The deformation line in Figure 2.8 shows that the longer span (Span 2) will force the beam to rotate toward itself at the center support. The resistance against this rotation comes from the bending stiffness of the member in Span 1. Stiffness is the ability of a member to resist deformation. There are several different types of stiffness, such as flexural, shear, axial, and torsional. Each type refers to a specific ability to resist a certain type of deformation. The greater the stiffness, the more is the effort required to bring about the specific deformation.

The flexural stiffness of a member is linearly related to the moment of inertia, I, which is a cross-sectional property, and to the modulus of elasticity, E, the ease of extendibility or compressibility of the material; and is inversely related to the length, ℓ, of the member. Thus, if K represents the flexural stiffness, $K = k\frac{EI}{\ell}$, where k is a numerical constant that depends on the support conditions of the other end of the member.

In the simple beam shown in Figure 2.8, if the flexural stiffness of Span 1 is infinitely large, it will resist any attempt by Span 2 to rotate the section over the center support toward itself. Hence the condition for Span 2 will approach that of full fixity at its left end. On the other hand, if the stiffness of Span 1 is very small, it will offer very little resistance against the efforts of Span 2 to rotate freely at the center support. Thus, as far as Span 2 is concerned, such a condition might be a "simple support," regardless of the continuity.

2.7 Propagation of Internal Forces

The free-body diagrams that resulted from the continuity are shown in Figure 2.9. Double subscripts identify the locations of shears and moments. Thus, if the first span is from a to b then V_{ab} represents the shear in that span at end a, and so on.

The two-span continuous beam is dissected to show the propagation of loads and moments. Each "cut" shows every force and every moment as they act on the part under consideration. For example, M_{ba} is shown as a clockwise arrow on Span 1, whereas it is shown as a counterclockwise arrow on the small part over the b support. These are two manifestations of the same moment, a concept well known from Newtonian physics (action and reaction). Similarly, V_{ba} is shown at the same cut as an upward force on Span 1 that comes from the support to the beam, as well as a downward force that comes from the beam to the support.

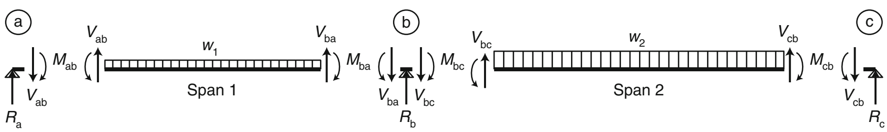


Figure 2.9 Propagation of internal forces on a two-span beam

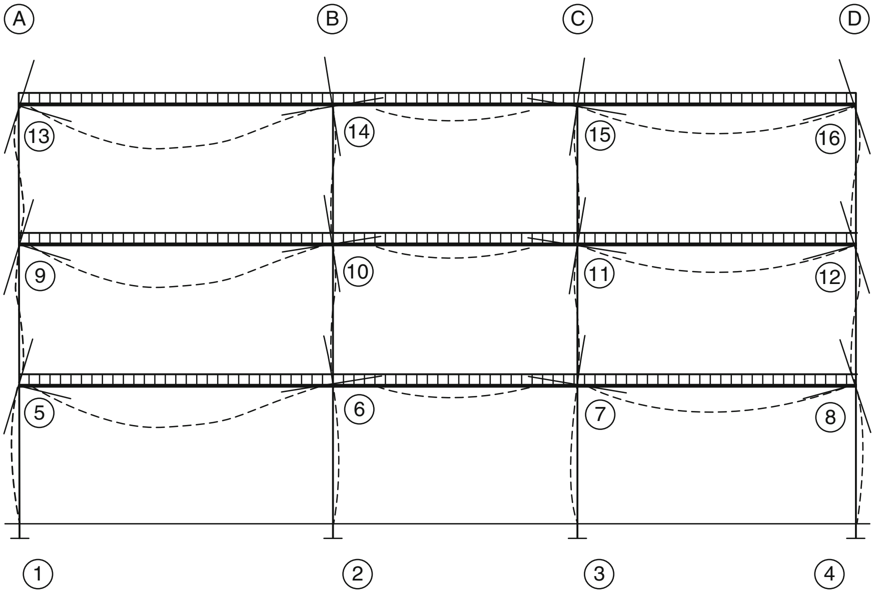


Figure 2.10 Deformations of a three-bay and three-story monolithic structure

Consider now the following self-evident statement: When a structure is in equilibrium, every part must be in equilibrium. Thus the well known equilibrium conditions of $\sum H = 0$, $\sum V = 0$, and $\sum M = 0$ apply for each individual part that is arbitrarily cut out of the structure. For example, the reaction force on the left-hand support, R_{a}, must equal the shear force, V_{ab}, transferred by the beam to that support. If we consider that $\sum M = 0$ on the same piece, we conclude that M_{ab} must equal zero, for there is no other moment on the piece to maintain equilibrium. On the small piece just above the *b* support, the reaction force from the support R_{b} must equal the sum of V_{ba} and V_{bc}. Note also that $M_{\mathrm{ba}} = M_{\mathrm{bc}}$ in order to satisfy equilibrium conditions.

Figure 2.10 shows a three-story-high, three-bay-wide reinforced concrete frame with all the joints numbered. The two outer bays are shown as somewhat wider than the inner bay. Thus, when they are all loaded in an approximately uniform way, the larger spans will try to rotate the ends of the inner bay (between column lines B and C) toward themselves. Thus, the joints on line B will rotate counterclockwise, and the joints on line C will rotate clockwise. At the exterior ends, the loads on the beams will rotate the joints on line A clockwise, and the joints on line D counterclockwise.

From the study of the deformation lines, we can draw some important general conclusions. The beams will have two curvature reversals (inflection points or points of counterflexure). They curve downward in their midspans, resulting in tensions at the bottom (positive moment region). They will curve upward near their ends, resulting in tensions at the top (negative moment region).

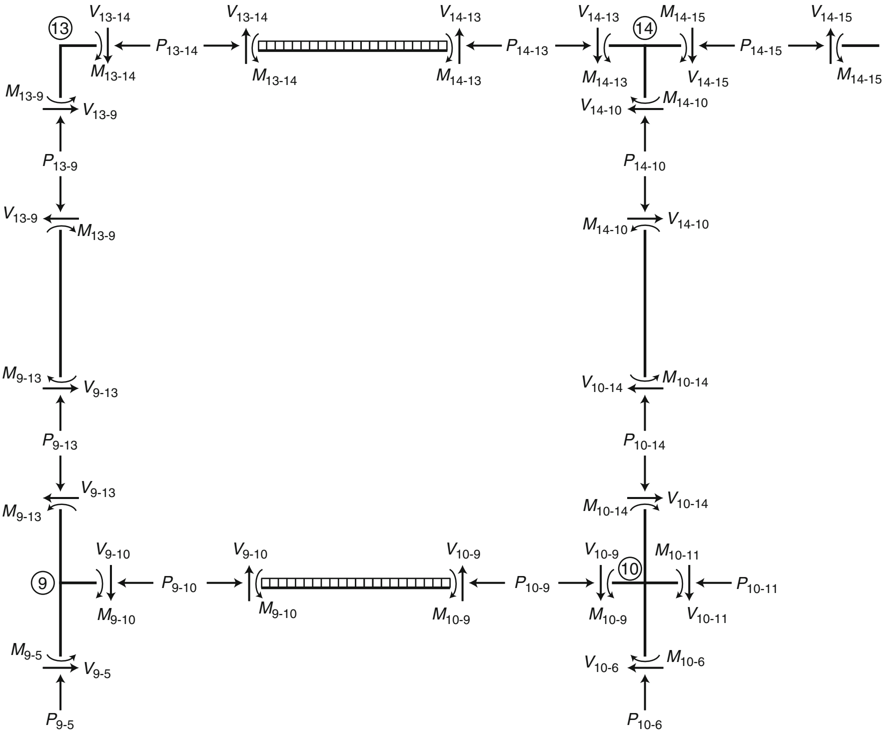


Figure 2.11 The propagation of forces and moments between beams and columns

The columns on the two upper floors, due to the forced rotations of their ends, will bend into a double curve (S curve). Depending on the amount of fixity available at the footing level, the lower columns will bend either into a double curve when the fixity at the base is significant, or into a single curve when the resistance against rotation at the base approaches that of a hinge.

Figure 2.11 shows free-body diagrams for part of the frame. Again $\sum H = 0$, $\sum V = 0$, and $\sum M = 0$ apply for each individual part. Thus, the axial force in beam 13–14 must equal the shear at the top of column 9–13 for Node 13 to be in equilibrium. The axial force in the column equals the shear at the left end of beam 13–14. And the moment at the end of column 9–13 must maintain equilibrium with the moment at the left end of beam 13–14. Mathematically:

$$\text{For} \sum H = 0 \quad V_{13\text{-}9} - P_{13\text{-}14} = 0$$
$$\text{For} \sum V = 0 \quad P_{13\text{-}9} - V_{13\text{-}14} = 0$$
$$\text{For} \sum M = 0 \quad M_{13\text{-}14} - M_{13\text{-}9} = 0$$

The reader may want to study and write out the equilibrium equations for other free-body parts.

2.8 On the “Fickleness” of Live Loads

As stated earlier, loads permanently attached to the structure are referred to as *dead loads,* and transitory loads are referred to as *live loads*. The nature of live loads is that sometimes they are there and sometimes they are not, so it is entirely possible that the live loads are fully present in one bay, while completely missing in other bays. Figures 2.12a–d show the effects of loading one span at a time on a four-bay continuous beam. In each case the deformation and the moment diagram are shown schematically under different *live loading* conditions. Deformations are shown as dashed lines.

A study of the deformation lines and the moment diagrams of these four different cases leads to the following conclusions:

1. The largest positive moments due to *live loads* in a given span occur when live loads are on that span and on every second span on either side. This is known as a *checker-board* pattern loading. See Figure 2.13a, b.
2. The largest negative moments due to *live loads* near a support occur when live loads are on neighboring spans and on every other span on either side. See Figure 2.13c–e.

Thus, on a continuous beam the number of live loading patterns that result in maximum moment effects equals the number of supports. For example, in a four-span beam with five supports, five different live loading patterns need to be considered to find the possible absolute maximums in each of the positive and negative moment zones.

These are only the moments that are due to the effects of the live loads. The cases, shown in Figure 2.13a–e must be combined with the moments resulting from the dead loads, that is, the loads that are permanently present on the structure, whose effects are not variable. The combinations of the dead load moments and the live load moments will result in a maximum possible moment at every location along the beam. The live and dead loads, when plotted into a graph such as the one shown in Figure 2.14, produce a diagram that represents all these combinations. This is called *the diagram of maximum moments* or *the moment envelope*.

Two important points must be noted here. Figure 2.14 shows that in some portions of each span, only positive moments occur, and in others, only negative moments, regardless of the distribution of the live loads. There are portions of each span, however, where either positive or negative moments may occur. This fact is significant in that it affects how a continuous beam must be reinforced.

The second point is that so far we have assumed that the continuous beam is similar to a mathematical line supported on knife-edge supports. The result of such a simplified assumption is that the reactions appear as concentrated forces and the moment diagram has a sharp peak (cusp) at those points. This result, however, is not in conformance with the physical reality. Supports (columns) have a width over which the reactions are distributed. This modifies the moment diagram within the width of the support to something similar to the sketch shown in Figure 2.15.

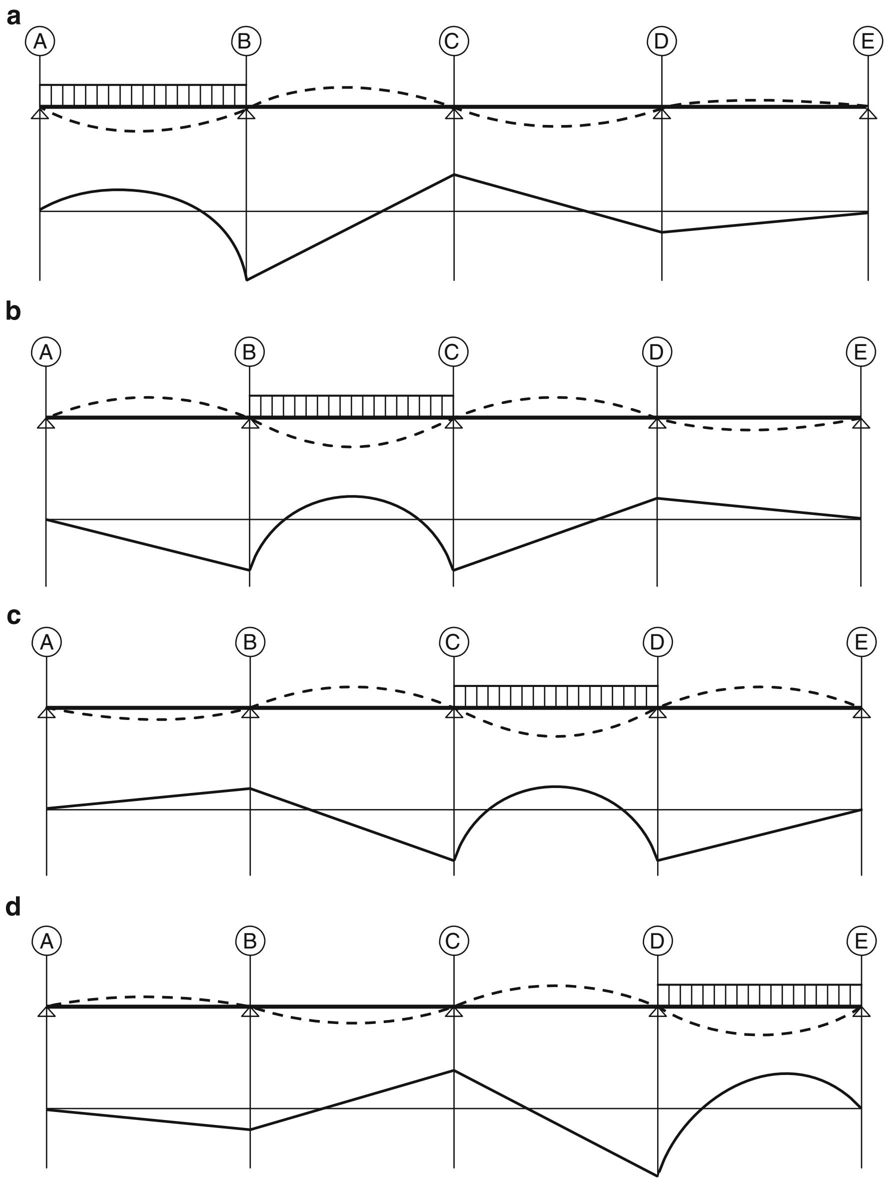


Figure 2.12 (**a**) The effects of live loads on span A-B. (**b**) The effects of live loads on span B-C. (**c**) The effects of live loads on span C-D. (**d**) The effects of live loads on span D-E

The exact shape of the moment diagram at this location is quite immaterial, for both theoretical studies and numerous test results clearly show that the critical negative moments in the beam occur at the faces of the supports. (Refer to ACI Code, Section 7.4.2.1 and Section 9.4.2.1)

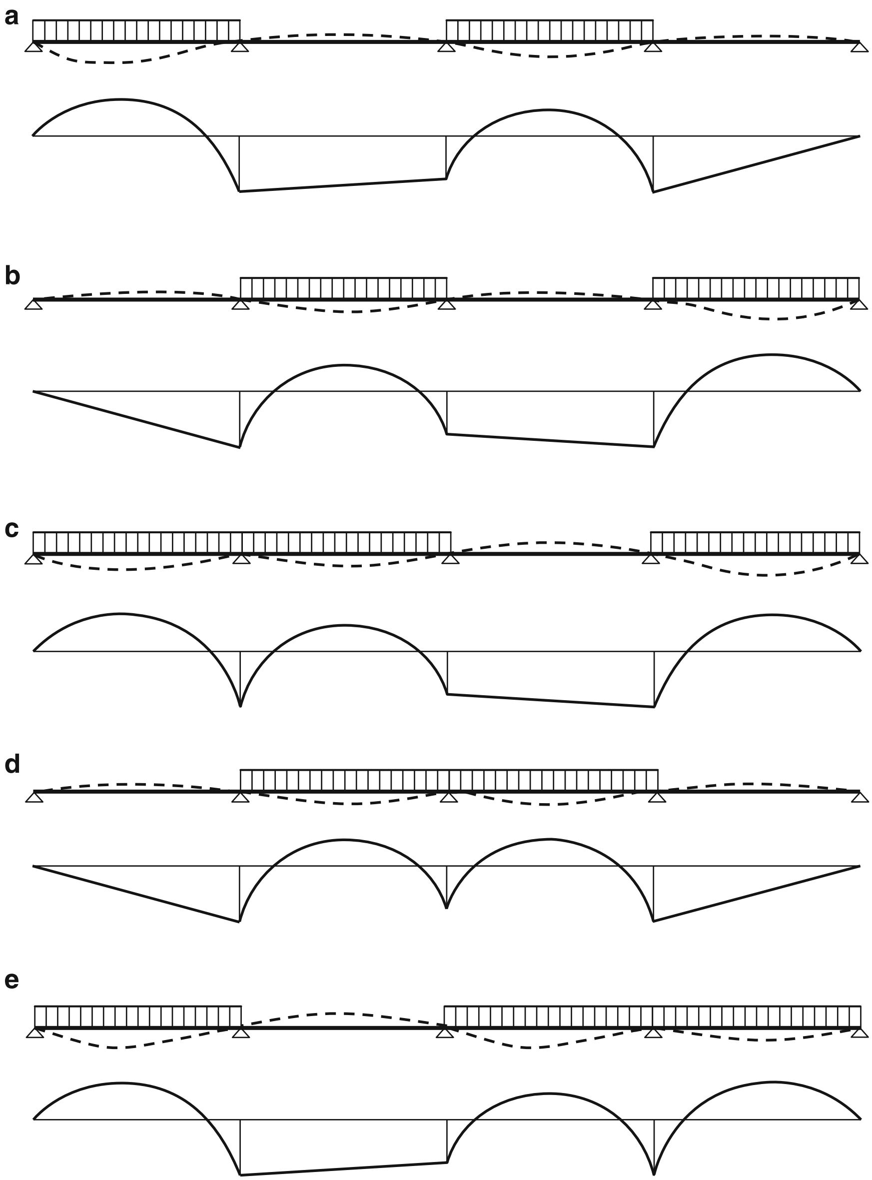


Figure 2.13 (**a**) Live loads in the first and third bays. Largest positive moments in first and third spans. (**b**) Live loads in the second and fourth bays. Largest positive moments in second and fourth spans. (**c**) Live loads in the first, second, and fourth bays. Largest negative moments at second support. (**d**) Live loads in the second and third bays. Largest negative moment at third support. (**e**) Live loads in the first, third, and fourth bays. Largest negative moment at fourth support

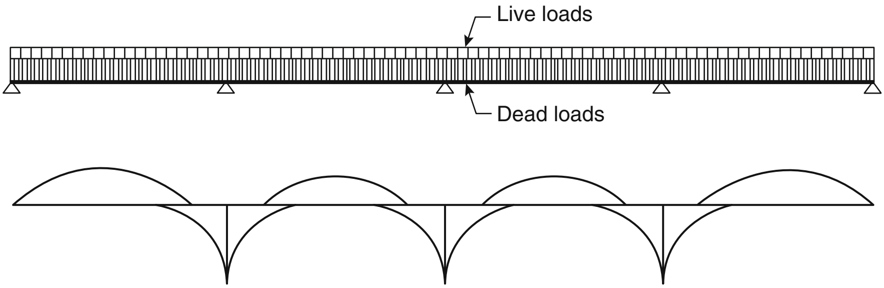


Figure 2.14 Maximum moments due to dead loads and different combinations of live loads

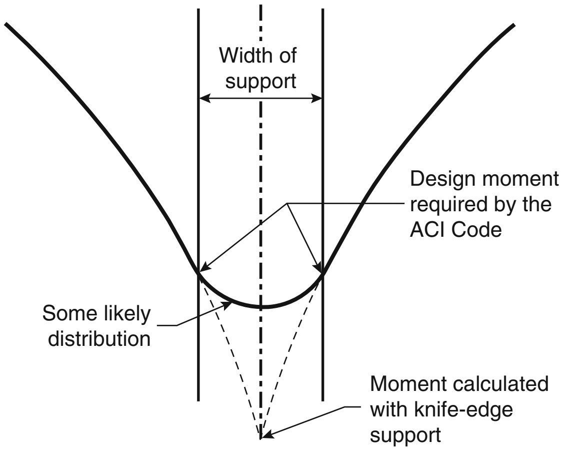


Figure 2.15 The true moments in beams at columns

2.9 The ACI Code Moment and Shear Coefficients

The complexities involved in the design of a very simple continuous beam may seem quite bewildering. In practice, however, a vastly simplified procedure is available in most cases.

Any moment along a span may be expressed as follows:

$$M_u = \text{coefficient} \cdot w_u \ell_n^2 \tag{2.2}$$

where

- w_u is the intensity of the total factored load (see Section 2.10), or the load per unit length. This variable should be evaluated and applied separately for each span if the live loads are different in each one
- ℓ_n is the net (clear) span for positive moment or shear, or the average of adjacent net (clear) spans for negative moment

When certain conditions are satisfied, the ACI Code permits the use of approximate moments and shears in the design of continuous beams and one-way slabs in lieu of the detailed analysis for maximum moments outlined in the previous section. Approximate moments and shears usually provide reasonable and sufficiently conservative values for the design of these horizontal flexural elements.

ACI Code Section 6.5.1 requires the following conditions for the use of these coefficients:

- *There are two or more spans* The beam or slab is continuous; that is, the approximation does not apply to a single span only.
- *Spans are approximately equal, with the longer of two adjacent spans not greater than the shorter by more than 20 %* The larger span tends to pull the shorter neighboring span upward if there are significant differences between adjacent spans.
- *Loads are uniformly distributed.*
- *Unit live load does not exceed three times the unit dead load* This is usually the case with reinforced concrete structures.
- *Members are prismatic* This means that the cross section is constant along the length of the span.

The ACI Code design moments and shears are applicable when these preconditions are satisfied. Table A2.1 and the accompanying figure list the coefficients for the moments and shears according to the end conditions and number of spans. In the authors' experience, the ACI coefficients are somewhat more conservative than values obtained from detailed computerized analysis; thus, their use will result in additional safety for the structure.

In actual practice the use of simplified methods to find the design moments and shears is in decline. Many proprietary computer programs are available that not only help evaluate all the most critical loading combinations, but also aid in the design of the required reinforcing. These programs require the sizes of the members as input, for the analysis of an indeterminate structure. (The result, or the output, depends on the relative stiffnesses of the members.) Thus, the application of these coefficients is still very useful for obtaining quick results that can be used in preliminary sizing of the members, which in turn enables the development of input data for a more detailed computerized analysis.

2.10 The Concept of Strength Design

The first design theory of reinforced concrete, developed near the end of the nineteenth century, simply borrowed its approach from the prevailing theory of elasticity. The method assumed that reinforced concrete elements at usual actual loads will have stress levels that might be considered to fall within the elastic zone. Figure 1.8 indicates that concrete in compression may follow an approximately linear stress/strain relationship as long as the stress level does not exceed 50 % of its ultimate strength level. Steel reinforcing behaves elastically below its yield point. So the concept of *working stress design* (WSD) was not an unreasonable methodology, and the underlying calculation technique is still used when estimating deformations (deflections) in structural elements. (See Section 3.3 for a more detailed discussion.)

The WSD method, however, has many conceptual drawbacks. First and foremost, it does not account for differences between dead and live loads. Rather, it simply lumps them together and assigns a "collective" margin of safety, regardless of the origin of the load. Dead loads can be estimated much more accurately than can live (transitory) loads; thus, logic dictates that the part of the load that comes from dead loads could use a much smaller safety factor against failure. On the other hand, the magnitude and the distribution of the live loads are much more uncertain.

Another, and equally important, drawback of the WSD method is that it inaccurately assumes that concrete behaves in a linear fashion with increasing stress levels. Merely knowing a stress level does not ensure a correct prediction of an undesirable level of stress (i.e., failure), because steel has a linear stress response to strain whereas concrete has a nonlinear one.

The third, and perhaps the most significant, drawback of the WSD method is that it is unimportant to know the stress level in a structure at a given loading. What is important is to know how much overload the structure can take before it fails.

Strength is needed to have a *safe design,* or *adequate strength,* so that the structure does not fail whether the actually occurring loads were underestimated or excess load is placed on the structure. Thus, *load factors* (i.e., values used to magnify the actual loads [called *working* or *service* loads]), or moments or shears therefrom, are used so as to create a *demand* on the strength. The concept of demand states, for example, that the structure (or, more precisely, a given element under investigation) must have an *ultimate strength* (i.e., before it fails) not less than those given by Equation (2.3a) (ACI Code, Section 5.3.1).

$$
\begin{aligned}
& U = 1.4D \\
\text{or}\quad & U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \\
\text{or}\quad & U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W) \\
\text{or}\quad & U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R) \\
\text{or}\quad & U = 1.2D + 1.0E + 1.0L + 0.2S \\
\text{or}\quad & U = 0.9D + 1.0W \\
\text{or}\quad & U = 0.9D + 1.0E
\end{aligned}
\tag{2.3a}
$$

where

U = required (ultimate) strength
D = effect from dead loads
L = effect from live loads
W = effect from wind loads
E = effect from seismic (earthquake) loads

L_r = effect from roof live loads
S = effect from snow loads
R = effect from rain loads

The multipliers applied to the effects in the various load combinations are the *load factors*. These guard against accidental overloading of the structure. They also take account of the imprecision in establishing the magnitude, or the distribution, of the loads. Thus, for example, greater load factors are assigned to live loads (or wind loads, or earthquake loads) than to dead loads to account for greater uncertainty.

Also, dead loads sometimes actually help to counteract the effect of wind or earthquake loads. For these conditions a more conservative approach is to presume that calculated dead loads are somewhat less than assumed. Such a concept is accounted for by the sixth and seventh load combinations in Equation (2.3a). These load combinations can be simplified by combining all live loads as L and using the larger load factor. In addition, for $U = 1.4D$ to govern the design, the condition of $D > 8L$ must exist, which is not very probable in most cases. Therefore, the load combination given below will be used for the member supporting floor loads (Equation (2.3b)), and for members supporting roof loads only (Equation (2.3c)) throughout this book (these typically include slabs, beams and girders):

$$U = 1.2D + 1.6L \tag{2.3b}$$

$$U = 1.2D + 1.6L_r \tag{2.3c}$$

where D includes the effects from all the dead loads and L is due to all the live loads. For members that support both floor and roof loads (neglecting the effects of wind or earthquake loads), the governing load combination from Equation (2.3a) are (these typically include columns and walls):

$$\begin{aligned} &U = 1.2D + 1.6L + 0.5L_r \quad (\text{if } \ L \geq 1.83L_r) \\ &\text{or} \\ &U = 1.2D + 1.6L_r + 1.0L \ (\text{if } \ L < 1.83L_r) \end{aligned} \tag{2.3d}$$

The effects of fluid, F, lateral earth pressure, H, and forces due to restraint of volume change and differential settlement, T, can also be incorporated in the load combination with their corresponding load factors. Refer to ACI Section 5.3 for details.

2.11 Design (Ultimate) Strength

The ultimate strength of a section within a structure (as discussed in detail later for separate and combined cases of bending moment, shear, torsion, and axial load) is calculated from the sizes (dimensions) of the section, the materials (steel and concrete) employed, and the amount of reinforcing used. This calculation gives us the *supply,* or the resisting strength furnished by the section. In flexural design, for example, this calculated quantity is designated as M_n, which is called *nominal moment strength* or *nominal resisting moment.* Nominal strength is the calculated strength, provided that everything goes according to plan; that is, the concrete is at least as strong as assumed in the design, the dimensions of the beam, slab, or any designed element is exactly as shown on the plans, the required reinforcing is placed exactly where it was assumed in the calculations, and so on. But experience shows that there is no such thing as perfectly executed plans, even in the best circumstances. ACI 117-90, "Standard Tolerances for Concrete Construction and Materials" lists tolerances that are reasonable to expect when good workmanship is provided. Furthermore, the calculation processes employ simplified mathematical models that should be considered as only reasonable approximations of reality. The design methodology also tries to reflect the relative importance of different structural components. The failure of columns, for example, may result in collapse of an entire building, but the failure of a beam typically causes only limited local damage.

In light of all these possible detrimental effects to the assumed strength, a *strength reduction factor* (ϕ-factor), sometimes referred to as an *under-strength* factor, is introduced to the above defined *nominal strength.* This factor accounts for the fact that the section's strength may be less than assumed in the analysis.

Thus, we arrive at the concept of *useable strength* (or supply), which is the product of the nominal strength and the strength reduction factor.

Different ϕ factors are used for different types of effects. Equation (2.4) gives some ϕ factors.

$$\begin{aligned} \text{Flexure} \quad & \phi = 0.90 \\ \text{Shear and torsion} \quad & \phi = 0.75 \\ \text{Axial compression (columns)} \quad & \phi = 0.65 \end{aligned} \tag{2.4}$$

Hence the *ultimate strength design* (USD) method can be stated as the following inequality:

$$\begin{aligned} \text{Demand} \; & \leq \text{Supply} \\ \text{or} \quad \text{required ultimate strength} \; & \leq \text{useable design strength} \\ \text{or} \quad \text{effects of loads} \; & \leq \text{resisting capacity of member} \end{aligned}$$

And so for a beam subjected to gravity (dead and live) loads, for example, Equations (2.5)–(2.8) represent this concept.

$$M_u = 1.2M_D + 1.6M_L \tag{2.5}$$

and

$$M_u \leq \phi M_n \tag{2.6}$$

Defining the design resisting moment, M_R, as

$$M_R = \phi M_n \tag{2.7}$$

the following must hold for the beam to be safe:

$$M_u \leq M_R \tag{2.8}$$

On the left side of Equation (2.8) is the demand. The demand depends only on the span, the type of support (e.g., simply supported, cantilevered, etc.), and the loads. All this information comes from the static analysis.

On the right side of Equation (2.8) stands the supplied strength of the section (design resisting moment, M_R), which depends on the size and shape of the cross section, the quality of the materials employed (f_c' and f_y), and the amount of reinforcing furnished. Thus, the left side of the inequality is unique, but the right side is undefined. An infinite number of different sizes, shapes, and reinforcing combinations could satisfy a given problem. *The only rule is that the supplied useable strength be larger than (or at least equal to) the required strength.*

Example 2.3 Assume that the beam in Example 2.1 is simply supported. Calculate the required ultimate flexural strength (factored moment from the loads). Use the permitted reduced live load.

Solution

$$M_D = 930 \times 30^2/8 = 104{,}625\,\text{lb-ft}$$
$$M_L = 403 \times 30^2/8 = 45{,}338\,\text{lb-ft}$$

Thus:

$$M_u = 1.2 \times 104{,}625 + 1.6 \times 45{,}338 = 198{,}091\ \text{lb-ft}\ \ (\text{or } 198.1\,\text{kip-ft})$$

The same result could be obtained by using factored loads (the loads multiplied by their respective load factors).

$$w_u = 1.2 \times 930 + 1.6 \times 403 = 1{,}761\,\text{lb/ft} = 1.761\,\text{kip/ft}$$

and

$$M_u = 1.761 \times 30^2/8 = 198.1\ \text{kip-ft}$$

Notice that when finding factored loads from service or working loads, the nature of the loads does not change; only their magnitudes are multiplied by the corresponding load factors. If a service load is distributed, its factored value is also distributed; if the service load is concentrated, its corresponding factored load is also concentrated. The following example clarifies this point.

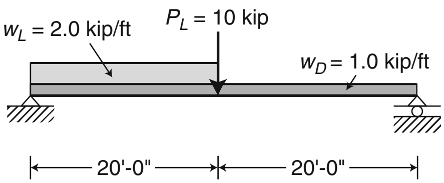


Figure 2.16 Example 2.4 (service loads)

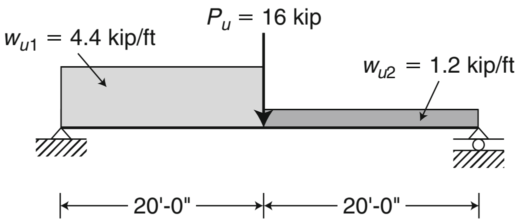


Figure 2.17 Example 2.4 (factored loads)

Example 2.4 Determine factored loads for the beam shown in Figure 2.16.

Solution For the left half of the beam:

$$w_{u1} = 1.2w_D + 1.6w_L$$
$$w_{u1} = 1.2 \times 1.0 + 1.6 \times 2.0 = 4.4\,\text{kip/ft}$$

For the right half of the beam:

$$w_{u2} = 1.2w_D + 1.6w_L$$
$$w_{u2} = 1.2 \times 1.0 + 1.6 \times 0 = 1.2\,\text{kip/ft}$$

The concentrated load is a live load only:

$$P_u = 1.2P_D + 1.6P_L$$
$$P_u = 1.2 \times 0 + 1.6 \times 10 = 16\,\text{kip}$$

The factored loads on the beam are shown in Figure 2.17.

2.12 Assumptions for the Flexural Design of Reinforced Concrete Beams

To this point we have discussed the calculations for the left side of the design Equation (2.8) (demand) in some detail. In this section we develop the right side of the design equation. To establish the *supply*, or the ultimate flexural strength, of a

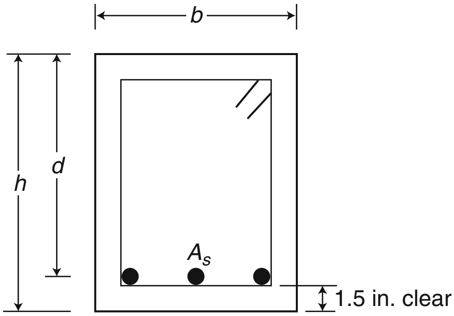


Figure 2.18 Definition of symbols used in a rectangular beam section

reinforced concrete section, we must discuss the stages of stress that a reinforced concrete section experiences before reaching failure. This discussion of these different stages of stress under increasing bending moments will also illuminate the assumptions made in developing expressions for calculating the ultimate strength of the section. To keep the discussion simple, we will examine a beam with a rectangular cross-section like the one shown in Figure 2.18.

The symbols in Figure 2.18 will be used throughout this book. They are the standard ACI symbols used with reinforced concrete. Thus:

b = width of the section
h = the overall depth of a section
d = the effective depth of a section, or the depth from the centroid of the tension reinforcement to the compression face
A_s = the sum of the cross-sectional areas of the reinforcing bars

Notice that the reinforcement is not placed at the very bottom of the beam. The first and foremost reason for this placement is to provide corrosion protection to the reinforcement. The inner environment of concrete is highly alkaline (high pH value) and helps to protect the reinforcement. The concrete cover also provides fire protection to the reinforcement. Furthermore, the concrete surrounds the reinforcing steel, which enables intimate bonding and allows the concrete and the steel, two individual materials, to work together. The required minimum concrete cover is given in Section 20.6.1.3.1 of the ACI Code. For unexposed beams it is 1.5 in. to the stirrups. (The stirrups, usually made out of #3 or #4 bars, will be discussed in Chapter 4.)

Figure 2.19 shows a simply supported beam that has a simple rectangular cross section made of plain concrete (homogeneous material). This type of beam is almost never used in an actual building, but it will give us insight into the behavior of concrete beams.

The uniformly distributed load (Figure 2.19a) represents the self weight plus some superimposed load. The slightly exaggerated deflected shape is shown in Figure 2.19b, and the moment diagram in Figure 2.19c. Attention will be directed to the section where the bending moment is the greatest. This location is where the stresses and the strains are also the largest.

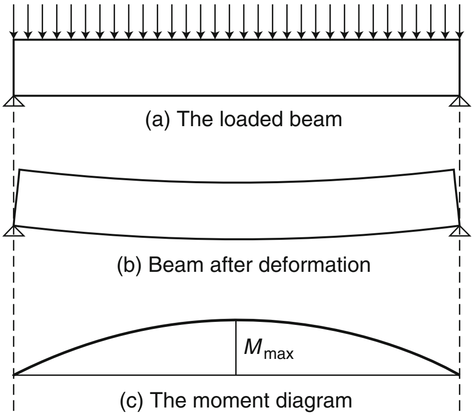


Figure 2.19 Elastic bending

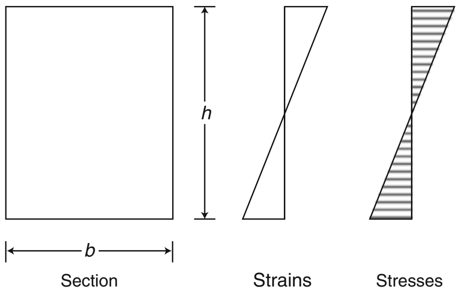


Figure 2.20 Linear distribution of strains and stresses

Figure 2.20 shows the cross section of the beam and the distribution of strains and stresses if the beam is unreinforced. Figure 2.21 illustrates the distribution of the strains and stresses in a 3-D form. As long as the bending moments are small, that is, the resulting tensile stresses at the bottom are less than the ultimate tensile strength of the concrete, the section will behave as if it were made of a homogeneous, quasi-elastic material. The bottom is in tension, and the top is in compression.

Direct your attention to the *strain* diagram first. Strain represents changes in length. The strain distribution is linear from bottom to top.

The farther up or down a point is from the imaginary center, the greater the strain in the beam. The largest tensile strains are at the bottom, whereas the largest compressive strains are at the top. There is a line across the section where the strain is zero. This is called the *neutral axis*. The straight-line distribution of strains is known as the *Bernoulli–Navier hypothesis*. This distribution is called a "hypothesis" because it results not from mathematical derivation, but from careful measurements made on countless tests of many different materials, including concrete. The distribution of *stresses* is also linear when the material follows Hooke's law, as

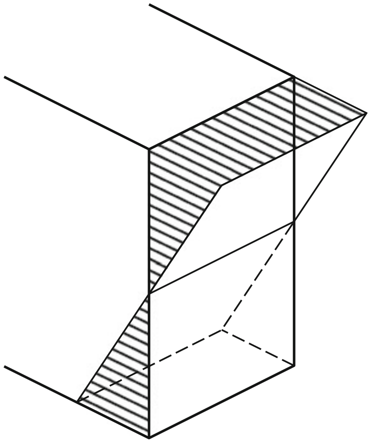


Figure 2.21 3D representation of linear strain or stress distribution

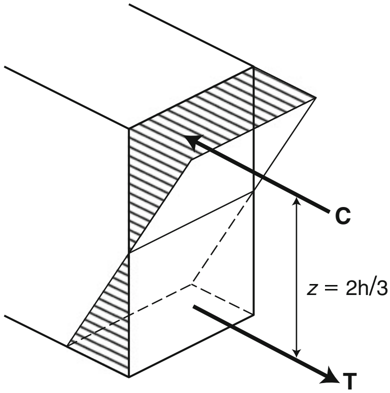


Figure 2.22 The internal couple in homogeneous beams

steel does below the so-called *proportional limit*. Stresses are forces acting on a unit area. Thus, it is possible to determine the resultant for these forces. *The resultant, which is a tensile (T) or compressive (C) force, is equal to the volume of the stress block,* For example, if the largest compressive stress is f_{cmax}, then the sum of all the compressive forces is given by Equation (2.9).

$$C = {}^{1}\!/_{2}\left[f_{\text{cmax}} \times \left({}^{h}\!/_{2}\right) \times b\right] \tag{2.9}$$

Similarly, the sum of all tensile forces is given by Equation (2.10).

$$T = {}^{1}\!/_{2}\left[f_{\text{tmax}} \times \left({}^{h}\!/_{2}\right) \times b\right] \tag{2.10}$$

These resultants will be located at the centroid of the wedge-shaped stress blocks, as shown in Figure 2.22. Equilibrium requires that these resultants be equal in magnitude, and together they form an *internal couple*. The internal couple is equivalent to the bending moment at the section.

Example 2.5 For the beam of Figure 2.22, assume $b = 12$ in., $h = 24$ in. and $M_{max} = 38.4$ kip-ft. Determine the bending stresses and the equivalent tensile and compression forces acting on the section.

Solution The section modulus is:

$$S = b \times h^2/6 = 1{,}152\,\text{in.}^3$$

Thus the maximum stresses are:

$$f_{max} = M_{max}/S = 38.4 \times 12/1{,}152 = 0.400\,\text{ksi}$$

Then

$$C = T = 1/2 \times [0.400 \times (24/2) \times 12] = 28.8\,\text{k}$$

The moment arm between the maximum stresses is $z = 2 \times 24/3 = 16$ in.

The moment equivalent of this couple is:

$$C \times z = T \times z = 28.8 \times 16 = \frac{460.8\,\text{kip-in.}}{12} = 38.4\,\text{kip-ft}$$

which agrees with the given moment, $M_{max} = 38.4$ kip-ft.

The concept of the internal couple will become a very important tool in considering a reinforced concrete beam. If the beam in Example 2.5 has enough tensile strength to withstand the applied 0.400 ksi (400 psi) tensile stress, the beam will not fail. As discussed earlier, concrete has a rather limited tensile strength. The modulus of rupture, which was said to represent the ultimate tensile strength of concrete in flexure, is given in Equation (1.3).

As mentioned previously, the modulus of rupture is a statistical average (with a considerable coefficient of variation) that is empirically derived from many laboratory tests. At increasing loads, a magnitude very soon is applied at which the beam's tensile strength is exhausted. At that point, somewhere near the maximum moment, the beam will crack. Without reinforcement, the crack will instantly travel upward and the beam will collapse, as shown in Figure 2.23.

In the following discussion the beam is assumed to have flexural reinforcement. Such a beam is shown in Figure 2.24. As long as the tensile stresses in the concrete at the bottom of the section are less than the modulus of rupture, there will be no

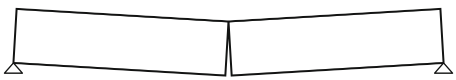


Figure 2.23 Bending failure of an unreinforced concrete beam

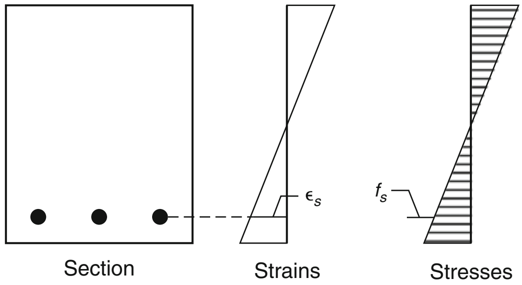


Figure 2.24 Strain and stress distribution of a reinforced concrete beam prior to cracking

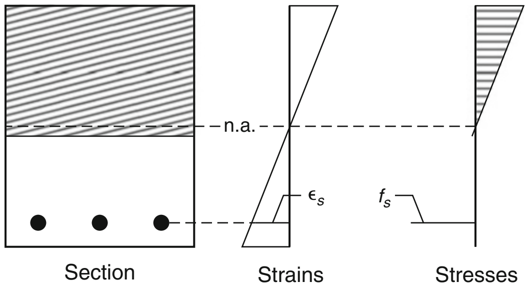


Figure 2.25 Strains and stresses after cracking

cracks. At the location of the reinforcing steel, the concrete and the steel have identical strains. The steel is bonded to the concrete, thus they must deform together. But the two different materials respond differently to deformation because they have a different modulus of elasticity, so the stresses will be different. In this particular case the stress in the steel will be much larger than that in the concrete.

For example, assume a concrete with $f_c' = 3{,}000\,\text{psi}$. Then $E_c = 57{,}000\sqrt{3{,}000} = 3{,}122{,}000\,\text{psi} = 3{,}122\,\text{ksi}$. The modulus of elasticity of the reinforcing steel is $E_s = 29{,}000$ ksi. According to Hooke's law the stress equals the product of the modulus of elasticity and the strain. So it follows that the stress in the steel will be about nine times higher (the ratio of the two moduli of elasticity values) than the stress in the concrete in the immediate vicinity. This ratio is usually designated as $n = E_s/E_c$ and is called the *modular ratio.*

The concrete cracks under increasing applied forces, and it is the reinforcement that carries the tension across the crack. The crack travels up to a height, then stops somewhere below the neutral axis as seen in Figure 2.25. The shaded area represents the uncracked part of the section. Where the strains are still small near the neutral axis, the concrete is still able to transfer some tensile stresses (albeit very small), even in the cracked section; *however, the amount of tensile force represented by the still un-cracked tensile stress volume is so small that it is simply neglected.*

Assuming, therefore, that the concrete does not carry any tension after cracking, the bending moment in the section is transferred across from one side of the crack to

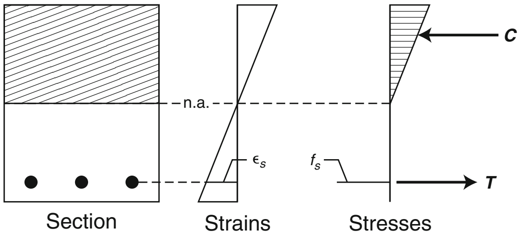


Figure 2.26 The internal couple after cracking

the other via the tension in the steel and the compression in the concrete, as seen in Figure 2.26. This assumption simplifies the development of an appropriate formula for the internal couple. The tensile component of this couple is at the centroid of the reinforcing steel, while the compressive component is at the centroid of the wedge-shaped compression block. Comparing Figures 2.22 and 2.26 indicates that the T force now is concentrated at the centroid of the reinforcing.

In Figure 2.26 the compression stress block is represented as a triangular wedge shape. This representation is more or less accurate as long as the compressive stresses in the concrete remain quite low. Figure 1.8 shows the generic shapes of the stress-strain curve of concrete in compression, and the assumption of linear distribution of stresses may be justified up to approximately $0.5f_c'$.

As the applied loads increase, there is a corresponding increase in bending moments throughout the beam. Thus, many more sections away from the location of the maximum moment will develop tensile stresses that exceed the concrete's ultimate tensile strength, resulting in the development of more cracks. While theoretically the spacing between cracks is very small, it does not happen that way, because the formation of a crack relieves tensile strains in the concrete in its immediate neighborhood. Initially the cracks are very fine hairline cracks, and a magnifying glass may be needed to locate them. These hairline cracks do not indicate that there is anything wrong with the beam: They occur naturally in reinforced concrete beams subjected to flexure under normal working load conditions. In fact, the reinforcement does not even do much work until after the concrete has cracked.

As the bending moment at the section increases, the magnitude of T and C, the tension and compression components of the internal couple, must also increase. In the reinforcement this is simply reflected as an increase in stresses. Correspondingly, the steel also will experience greater strains and elongation. As long as the strains in the reinforcing are less than the yield strain, the relationship between stresses and strains remains linear.

In the concrete, however, the increased compression strains result in a nonlinear response of the stresses while maintaining the required increase in the volume of the stress block. The concrete stress block becomes more and more bounded by a curvilinear surface. Ultimately, the contour will resemble the one shown in Figure 2.27. This diagram is the same as the ones shown in Figure 1.8, except the

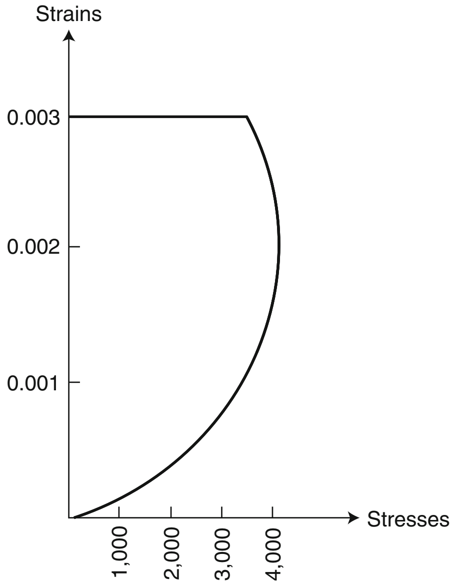


Figure 2.27 Typical curvilinear stress distribution in the concrete at ultimate strength

axes are reversed. At the origin, the strains and stresses are zero, just like on the beam section at its neutral axis. At the top there is a strain value of 0.003, which is a value selected by the ACI Code (somewhat arbitrarily) as the *ultimate useful strain*. Somewhere between these two limits (in the neighborhood of 0.002) the peak stress (the *maximum compressive strength* or simply *compressive strength*) occurs. In calculations this value is designated as f_c'; it is the specified compression strength of the concrete, as already mentioned in Section 1.6.1.

On the tension side (i.e., at the reinforcement), Figure 1.17 shows the stress-strain curve of the reinforcing steel, or the response of the steel to increasing strain values. This curve clearly shows that the steel has significant residual strength even after it has yielded, but this residual strength (the strength gained in the strain hardening zone) is neglected. Thus, we assume that the stresses will linearly increase with increasing strains up to yield, after which ever-increasing strains produce no corresponding increase in stresses. Scientifically, this curve is known as a *bilinear stress-strain diagram,* and the response of the steel as *elasto-plastic behavior*. Figure 2.28 shows the assumed stress-strain diagram for 40 and 60 ksi steel, respectively.

2.13 Different Failure Modes

As a first case assume that a beam has a relatively *small* amount of reinforcing steel. Such a beam is shown in Figure 2.29. With increasing demand on the internal couple the stresses in the steel will reach yield *before* the demand on the concrete

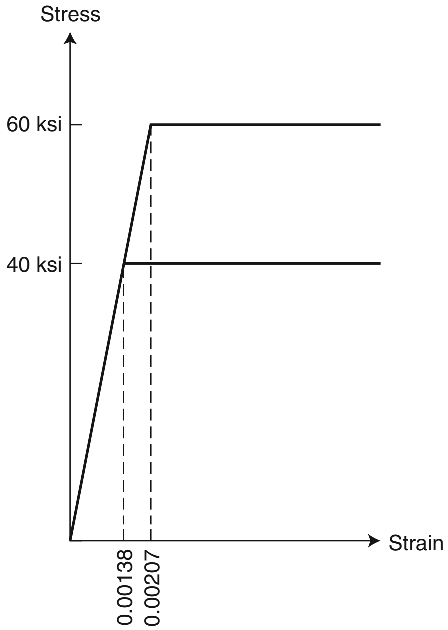


Figure 2.28 Assumed bilinear stress-strain diagram of reinforcing steel

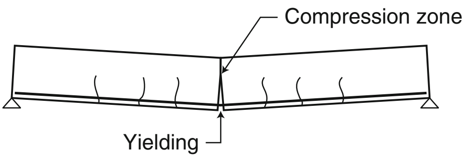


Figure 2.29 Tension-controlled failure of a reinforced concrete beam

compression block reaches the ultimate concrete compressive strength. With increasing elongation in the steel, still prior to yield, the cracks will become wider and more visible. When the steel starts to yield (i.e., elongate rapidly), the relatively narrow crack at the bottom opens up. This forms a wedge that shifts the neutral axis upward, thus decreasing the area available for the compressive stress block, until the concrete crushes on the compressive side as a *secondary* failure. The *primary* cause of failure was due to the yielding of the reinforcement. In a somewhat misleading way such sections are sometimes referred to as *underreinforced* sections. This unfortunate expression implies that the section is underreinforced as compared to the capacity of the compression part of the section. (In Section 2.17 we will discover that the behavior of an under-reinforced section is classified as *tension-controlled* or *transition-controlled* depending on the level of tensile strain in the steel at the time of failure.)

As a second case consider a beam that has a relatively large amount of reinforcing. For such a beam the steel will be able to develop the *T* part of the internal couple without yielding. As demand on the compression stress block

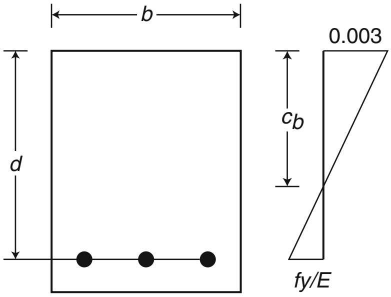


Figure 2.30 Strain distribution at "balanced" failure

increases, however, the capacity to provide a sufficiently large volume of concrete stresses will be exhausted, reaching the state shown in Figure 2.27. In such a case the *primary* failure occurs in the concrete. These types of sections are referred to as *overreinforced,* that is, the beam has more reinforcing in the section than what could be used with the largest possible compressive stress block.

A casual observer may care little about what initiated the failure of the beam. But the two modes of failure vastly differ. The first mode, in which the primary failure happens due to the yielding of the reinforcing, is a *ductile* process and is preceded by significant cracking, fairly large deflections, and similar warning signs. The beam, in a way, tells you that something bad is about to happen.

In the second mode there are no such obvious signs of impending failure. The reinforcement, in providing the tensile part of the internal couple, experiences relatively low strains, so the few hairline cracks do not serve as warning signs. Consequently when the failure occurs, it happens in a sudden, explosive way—the concrete failure in compression is very abrupt.

Between these two different failure modes is a special case, known in the literature as the *balanced-failure* condition. Balanced failure is a theoretical limit dividing the underreinforced and overreinforced failure modes. We feel that this is an unfortunate terminology, because the word *balance* (i.e., equilibrium) should not be used to describe a failure mode that is anything but the maintenance of balance. We would prefer to use the expression *simultaneous failure*. But whatever terminology is used, it refers to the amount of reinforcement in a section that causes the concrete at the compression side to fail at exactly the same time the steel begins to yield. So the strain in the steel will be the yield strain, and the strain at the extreme edge of the concrete will be 0.003. This balanced condition is depicted in Figure 2.30.

2.14 The Equivalent Stress Block

A quick look at Figure 2.27, or at its 3-D representation in Figure 2.31, should convince anyone that it would be impractical to calculate the value of *C* by figuring out the volume of the stress block. The calculation would require integral calculus,

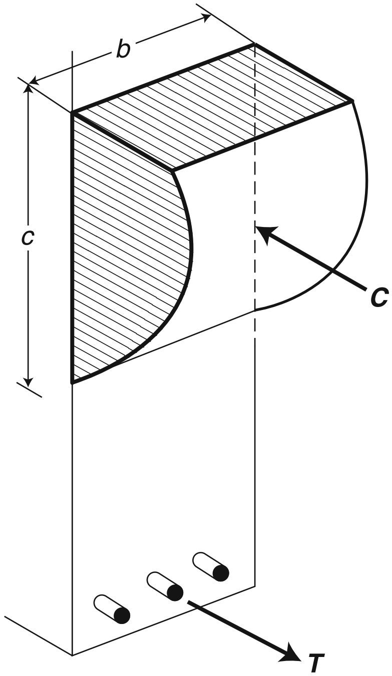


Figure 2.31 True stress distribution in the concrete at ultimate strength

even if there was an easy way to express the shape of the curve mathematically. A reasonable approximation can be obtained by substituting a stress block whose volume is about the same as the true stress volume enclosed in Figure 2.31, and whose centroid is fairly close to that of the true stress volume. This is known as the *equivalent stress block,* and is shown in Figure 2.32.

The relationship between the true stress block and the equivalent stress block has been established by studying many concrete stress-strain curves. The simple rectangular block has been adopted for its simplicity and ease of calculation. If a uniform stress value of $0.85f_c'$ is adopted, then only the relationship between the depth of the equivalent stress block a and the distance of the neutral axis from the top c is needed. This relationship is given in Equation (2.11).

$$a = \beta_1 c \tag{2.11}$$

To account for the somewhat different shapes of the stress-strain curves of different strengths of (refer to Figure 1.8) concrete, β_1 is given by the ACI Code (Section 22.2.2.4.3) as follows:

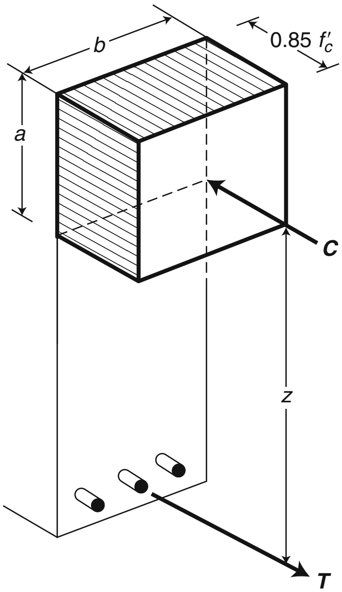


Figure 2.32 The equivalent stress block

$\beta_1 = 0.85$ for concrete strength f_c' up to and including 4,000 psi. For strengths above 4,000 psi, β_1 shall be reduced at a rate of 0.05 for each 1,000 psi of strength in excess of 4,000 psi, but β_1 shall not be taken less than 0.65

Equation (2.12) gives the expression to calculate β_1 for $f_c' > 4{,}000$ psi.

$$\beta_1 = 0.85 - 0.05\left(\frac{f_c' - 4{,}000}{1{,}000}\right) \geq 0.65 \tag{2.12}$$

The equivalent stress block makes it extremely easy to manipulate the expression to calculate the ultimate (design) resisting moment of a given section. The moment arm of the internal couple, z, can be calculated using Equation (2.13).

$$z = d - \frac{a}{2} \tag{2.13}$$

The numerical value of the internal couple can be expressed in two different ways, using the designation of M_n for the *nominal resisting moment* and M_R for the *design resisting moment*. These moments can be calculated using Equations (2.14) and (2.15), respectively.

$$M_n = Tz \quad \text{or} \quad M_n = Cz \tag{2.14}$$

$$M_R = \phi M_n = \phi Tz = \phi Cz \tag{2.15}$$

where

$T = A_s f_y$ (the area of the reinforcing multiplied by the yield stress of the steel)
$C = 0.85f_c'ab$ (the volume of the equivalent stress block)

Equilibrium requires that T be equal to C, thus

$$A_s f_y = 0.85f_c' ab \tag{2.16}$$

Solving this equation for a gives Equation (2.17) for calculating the depth of the equivalent stress block.

$$a = \frac{A_s f_y}{0.85f_c' b} \tag{2.17}$$

Note that a will increase as larger amounts of reinforcement, or reinforcing steel with greater strength is used. On the other hand a will be smaller if a wider section, or stronger concrete is used. Note, however, that a is independent of the depth of the section.

2.15 The Steel Ratio (ρ)

Sometimes it is useful to express A_s as a fraction of the working cross section, which is the product of the width b and the effective depth (or *working depth*) d. The term *steel percentage* or, more accurately, *steel ratio* refers to the ratio between the area of the reinforcing steel and the area of the working concrete section.

The steel ratio is calculated using Equation (2.18).

$$\rho = \frac{A_s}{bd} \tag{2.18}$$

Note that ρ is a nondimensional number, area divided by area, so it is not a percentage per se. But it can be made into a percentage by multiplying it by 100. For example, assume the following beam data: $b = 12$ in., $h = 24$ in., $A_s = 3$ #6 bars $= 3 \times 0.44 = 1.32$ in.^{2}, and #3 stirrups in the beam.

Then $d = 24 - 1.5$ in. (concrete cover) $- 0.375$ in. (diameter of the stirrup) $-$ 0.75 in. (diameter of #6 bar)/2 $= 21.75$ in. Thus, the steel ratio is $\rho = \dfrac{A_s}{bd} = \dfrac{1.32}{12(21.75)} = 0.00506$ (or 0.506 %).

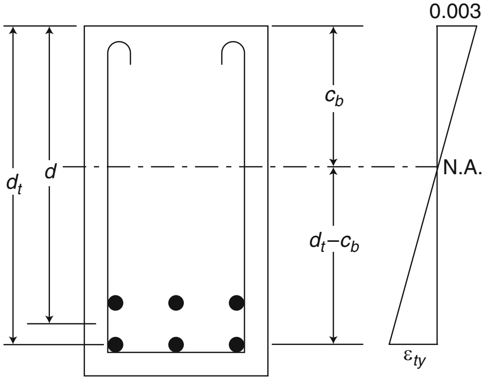


Figure 2.33 Strain distribution at balanced failure

2.16 The Balanced Steel Ratio

Section 2.13 discussed the two possible different failure modes of reinforced concrete beams in bending. The theoretical dividing point between them, the "balanced failure," was also discussed. In this case the steel in the outermost layer (if there is more than one layer) reaches its yield strain exactly when the maximum compressive strain in the concrete reaches the 0.003 value. The strain distribution at balanced failure resembles the one shown in Figure 2.33. In order to cover the more general (although not so frequent) case of multilayer reinforcing in the beam, a distinction is made between d, the working depth, and d_t, the depth to the outermost layer of reinforcing on the tension side. When there is only one layer of reinforcement, $d = d_t$.

From the similarity of the two triangles above and below the neutral axis, c_b, the depth of the neutral axis at balanced failure can be expressed as a function of d_t and f_y.

$$\frac{c_b}{d_t - c_b} = \frac{0.003}{\varepsilon_{ty}} \tag{2.19}$$

Solving for c_b

$$c_b = \frac{0.003 d_t}{0.003 + \varepsilon_{ty}} \tag{2.20}$$

because

$$\varepsilon_{ty} = \frac{f_y}{E_s} = \frac{f_y}{29{,}000{,}000} \tag{2.21}$$

We can substitute and rearrange to obtain

$$c_b = \frac{87{,}000}{87{,}000 + f_y} d_t \tag{2.22}$$

In these equations f_y is substituted in psi.

With this information the depth of the equivalent stress block at *balanced failure* can be calculated using Equation (2.23).

$$a_b = \beta_1 c_b = \frac{A_{sb} f_y}{0.85 f_c' b} \tag{2.23}$$

where A_{sb} is the theoretical amount of reinforcing needed to cause a balanced failure mode.

When c_b from Equation (2.22) and $A_{sb} = \rho_b bd$ from Equation (2.18) are substituted into Equation (2.23).

$$\beta_1 \frac{87{,}000 d_t}{87{,}000 + f_y} = \frac{\rho_b b d f_y}{0.85 f_c' b}$$

the steel ratio for balanced failure, ρ_b, can be calculated using Equation (2.24).

$$\rho_b = \frac{0.85 f_c'}{f_y} \beta_1 \frac{87{,}000}{87{,}000 + f_y} \frac{d_t}{d} \tag{2.24}$$

If $d_t = d$, which means there is only one layer of reinforcing steel (by far the most frequent case), then Equation (2.24) becomes Equation (2.25).

$$\rho_b = \frac{0.85 f_c'}{f_y} \beta_1 \frac{87{,}000}{87{,}000 + f_y} \tag{2.25}$$

Note that the value of ρ_b depends only on the selected materials (f_c' and f_y) and is independent of the size of the section. (The ratio $\frac{d_t}{d}$ becomes necessary only when there is more than one layer of reinforcement.)

2.17 Elaboration on the Net Tensile Strain in Steel (ε_t)

In an effort to generalize the approach for members subject to both bending and axial compressive forces, the ACI Code strives to treat these combination cases together. The different failure modes were discussed in Section 2.13. These modes are distinguished by whether the primary failure is due to yielding of the steel or to

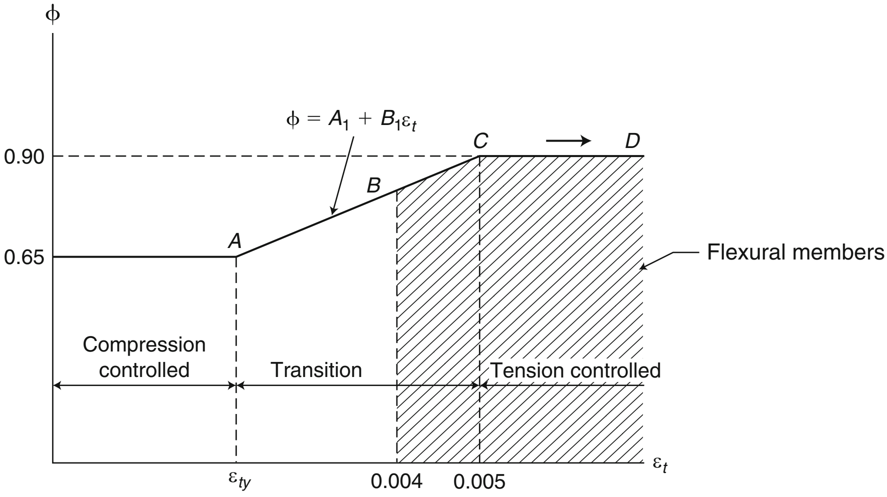


Figure 2.34 Variation of ϕ versus ε_t

crushing of the concrete. The former is called *tension-controlled* failure, and the latter is *compression-controlled* failure. It was also previously noted that tension-controlled failure results in highly desirable ductility, whereas compression-controlled failure is abrupt and nonductile in nature. Unfortunately, as will be discussed later in Chapter 5, the desire to have only ductile tension-controlled failure modes cannot always be satisfied. But in flexural members, at least we can control the failure behavior by using no more steel than an amount that ensures the desirable ductility. In the past this was accomplished by limiting the reinforcement ratio, ρ, to $75\rho_b$ in flexural members. Since 2002 the ACI Code has adopted a new approach that is a better integration of dealing with members subject to axial stresses whether from flexure, or axial compression, or both. If ductile failure mode cannot always be assured, then the use of a larger safety factor against a nonductile type of failure is warranted. This larger safety factor is obtained by regulating the ratio between the useful ultimate moment or design resisting moment ($M_R = \phi M_n$) and the nominal ultimate moment (M_n). This requires only an adjustment in the ϕ (strength reduction) factor.

The ACI Code (Section 21.2.2) defines three different types of section behavior: *tension-controlled, compression-controlled,* and a *transition zone,* which is the zone between the tension- and the compression-controlled failure zones. Figure 2.34 shows a graphical representation of these zones, and defines and separates the three regions. Theoretically the division between compression-controlled failure and tension-controlled failure is where $\varepsilon_t = \varepsilon_{ty}$. In other words, the section is compression-controlled if the strain in the steel is less than the yield strain; and is tension-controlled if the strain in the steel is greater than the yield strain when the compression strain in the concrete reaches the limit of 0.003. For design purposes, however, the ACI Code requires a safely assured tension-controlled section; thus, it

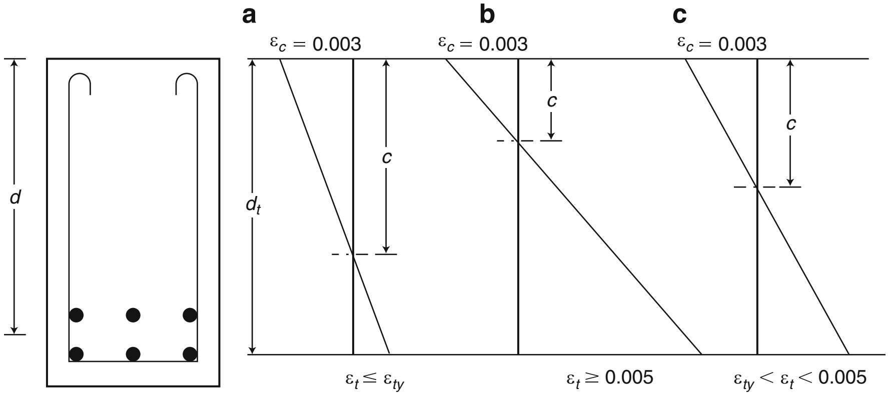


Figure 2.35 Strain distribution and net tensile strain (ε_t) at behavior limits: (**a**) compression-controlled sections; (**b**) tension-controlled sections; (**c**) transition-controlled sections

defines a section as tension-controlled only when the steel strain at ultimate strength is greater than 0.005. Between the two limits, yield strain (ε_{ty}) and 0.005, the Code defines a *transition zone* with lowered ϕ values.

Note that the ACI Code allows ε_t for flexural members to be as small as 0.004 at ultimate strength. A somewhat diminished ϕ factor, however, is required in conjunction.

It may be helpful here to repeat what was discussed in Section 2.13 in a somewhat different format. Figure 2.35 defines graphically the behavior of reinforced concrete sections.

1. *A compression-controlled section* is a reinforced concrete section in which the strain in the concrete reaches 0.003 at ultimate strength, but the strain in the steel (ε_t) is less than the yield strain (ε_{ty}). (See Figure 2.35a.) In other words, at the ultimate strength of the member, the concrete compressive strain reaches 0.003 before the steel in tension yields. This condition results in a brittle or sudden failure of beams and should be avoided. In reinforced concrete columns, however, a design based on compression-controlled failure behavior cannot be avoided. As shown in Figure 2.34, $\phi = 0.65$ is mandated for this case, which is considerably less than the $\phi = 0.90$ that is used for tension-controlled sections. The reasons for this additional factor of safety are: (1) compression-controlled sections have less ductility; (2) these sections are more sensitive to variations in concrete strength; and (3) the compression-controlled sections generally occur in members that support larger loaded areas than do members with tension-controlled sections.
2. *A tension-controlled section* is a reinforced concrete section in which the tensile strain in steel (ε_t) is more than 0.005 when the compression strain in concrete reaches 0.003 (see Figure 2.35b). In other words, when a section is tension-controlled at ultimate strength, steel yields in tension well before the strain in the concrete reaches 0.003. Flexural members with tension-controlled sections have

ductile behavior. As a result, these sections may give warning prior to failure by *excessive deflection* or *excessive cracking,* or both. Not all tension-controlled sections will give both types of warning, but most tension-controlled sections should give at least one type of warning. Both types of warnings, excessive deflection and cracking, are functions of the strain, particularly the strain on the tension side. Because tensile strains are larger than compressive strains in tension-controlled sections at failure, the ACI Code allows a larger ϕ factor (0.90) for these types of members.

3. *A transition-controlled section* is a reinforced concrete section in which the net tensile strain in the steel (ε_t) is between yield strain (ε_{ty}) and 0.005 when the compression strain in the concrete reaches 0.003. (See Figure 2.35c.) Some sections, such as those with a limited axial load and large bending moment, may have net tensile strain in the extreme steel (ε_t) between these limits. These sections are in a transition region between compression- and tension-controlled sections. In Figure 2.34, the line *AC* represents the Code-defined relationship between ϕ and ε_t in the transition-controlled zone. The value of ϕ in the transition zone can be calculated using Equation (2.26).

$$\phi = A_1 + B_1\,\varepsilon_t \tag{2.26}$$

where the coefficients A_1 and B_1 may be expressed as

$$A_1 = \frac{0.00325 - 0.9\,\varepsilon_{ty}}{0.005 - \varepsilon_{ty}}$$

$$B_1 = \frac{0.25}{0.005 - \varepsilon_{ty}}$$

Table A2.2a in Appendix A lists the values for the coefficients A_1 and B_1 for commonly used reinforcing steels.

2.18 The Location of the Neutral Axis and Limit Positions

Consider the strain diagram shown in Figure 2.36. The location of the neutral axis at ultimate strength (c) depends upon the net tensile strain of the steel. Observe the solid and the dotted lines. Because the strain at the compression face is constant (0.003), c becomes smaller as the steel strain increases. Using similar triangles of the strains above and below the neutral axis, an expression can be derived to calculate the depth of the neutral axis, c.

$$\begin{aligned} \frac{c}{d_t - c} &= \frac{0.003}{\varepsilon_t} \\ c\varepsilon_t &= 0.003(d_t - c) \end{aligned} \tag{2.27}$$

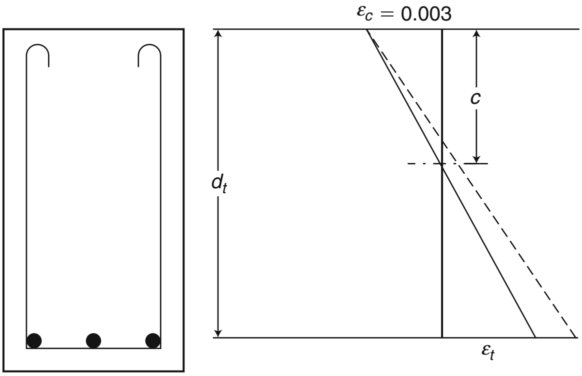


Figure 2.36 Variation of the location of the neutral axis (c) with the tensile strain in steel (ε_t)

Solving Equation (2.27) for c:

$$c = \frac{0.003}{0.003 + \varepsilon_t} d_t \tag{2.28}$$

The ratio of c/d_t, given in Equation (2.29), is often used to check if a section is tension-controlled.

$$\frac{c}{d_t} = \frac{0.003}{0.003 + \varepsilon_t} \tag{2.29}$$

Two values of ε_t are of special interest. The first one is $\varepsilon_t = 0.004$. This is the absolute minimum steel strain permitted by the ACI Code for members in flexure. (Refer to Figure 2.34 and ACI Code, Section 7.3.3.1 for one way slabs, and Section 9.3.3.1 for beams). Substituting this ε_t value into Equation (2.29) gives us Equation (2.30).

$$\frac{c}{d_t} = \frac{3}{7} = 0.429 \quad \text{or} \quad c = 0.429 d_t \tag{2.30}$$

Equation (2.30) gives the lowest permissible value of the neutral axis depth. In other words, this defines the largest permissible concrete area in compression ($c \leq 0.429 d_t$).

The second value of interest is $\varepsilon_t = 0.005$. Solving Equation (2.29) for this case, we obtain Equation (2.31) for the lowest location of the neutral axis depth for tension-controlled sections.

$$\frac{c}{d_t} = \frac{0.003}{0.003 + 0.005} = \frac{3}{8} = 0.375 \quad \text{or} \quad c = 0.375 d_t \tag{2.31}$$

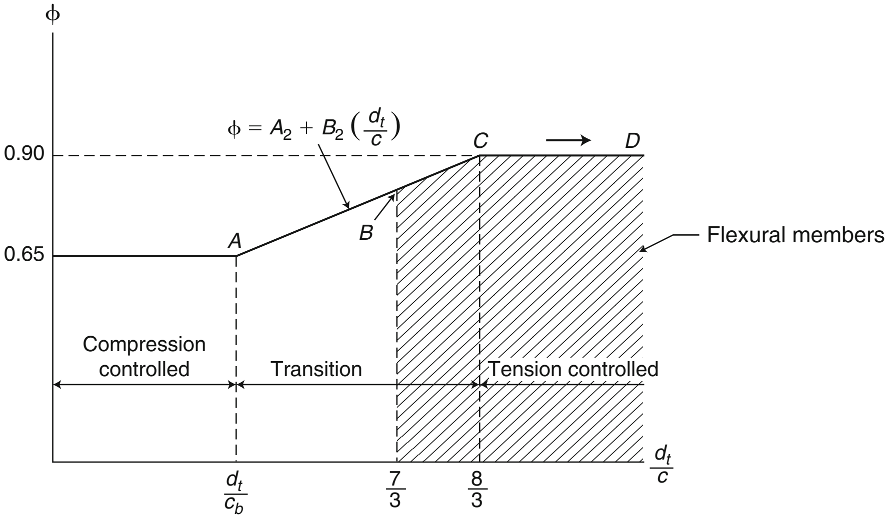


Figure 2.37 Variation of ϕ versus $\frac{d_t}{c}$

2.19 Relationship Between ϕ and d_t/c

Equation (2.29) shows that the ratio of either c/d_t, or its inverse, d_t/c, are in direct relationship with the steel tensile strain ε_t. Then it is possible to modify Figure 2.34 to show the ACI Code–prescribed strength reduction factor's (the ϕ factor's) variation in terms of the d_t/c ratio. (For convenience of graphing, the relationship is shown in terms of d_t/c.) Figure 2.37 expresses the changing ϕ values with respect to the ratio d_t/c. Note that the ratio d_t/c_b is the ratio of d/c at the balanced failure point.

Table A2.2b in Appendix A of this text lists the values for the coefficients A_2 and B_2 that describe the variations in ϕ values through the transition zone. The limiting ratios between the depth of the member and the location of the neutral axis (d_t/c) and its inverse at the balanced failure point (i.e., d_t/c_b or c_b/d_t) are also included.

2.20 Limitations on the Steel Percentage (ρ) for Flexural Members

With the help of Equations (2.30) and (2.31), the corresponding largest ρ values (i.e., the steel percentages that satisfy those limiting conditions) can be determined. For $\varepsilon_t = 0.004$ (lowest permitted steel strain value at ultimate strength of flexural members), the maximum depth of the neutral axis is calculated using Equation (2.32).

$$c_{\max} = \frac{3}{7} d_t \tag{2.32}$$

The corresponding depth of the equivalent stress block (refer to Equations (2.11) and (2.17)) is given by Equation (2.33).

$$a_{\max} = \frac{A_{s,\max} f_y}{0.85 f_c' b} = \frac{3}{7} \beta_1 d_t \tag{2.33}$$

where $A_{s,\max}$ is the amount of reinforcing steel necessary to have $\varepsilon_t = 0.004$.

Substituting for $A_{s,\max} = \rho_{\max}\, bd$ in Equation (2.33), then rearranging, the largest ρ value can be determined.

$$\frac{\rho_{\max} bdf_y}{0.85 f_c' b} = \frac{3}{7} \beta_1 d_t \tag{2.34}$$

$$\rho_{\max} = \frac{3}{7}(0.85)\beta_1 \frac{f_c'}{f_y} \cdot \frac{d_t}{d}$$

or

$$\rho_{\max} = 0.364 \beta_1 \frac{f_c'}{f_y} \cdot \frac{d_t}{d} \tag{2.35}$$

Equation (2.35) gives the maximum percentage of reinforcing steel permitted by the ACI Code in flexural members, unless the capacity is augmented by the use of compression reinforcing. (See more on that in Chapter 3.)

For sections with a single layer of reinforcing, $d_t/d = 1.0$, Equation (2.35) is simplified as indicated in Equation (2.36).

$$\rho_{\max} = 0.364 \beta_1 \frac{f_c'}{f_y} \tag{2.36}$$

In a similar way, we can determine the value of ρ that will ensure an $\varepsilon_t = 0.005$, the upper limit of ρ needed to ensure a tension-controlled (ductile) failure in beams at their ultimate strength. Designate this value of ρ as ρ_{tc}. After changing the right side of Equation (2.34) accordingly (see Equations (2.30) and (2.31)), then the value of ρ_{tc} can be calculated using Equation (2.38) (or Equation (2.39) for the special case of a section with only one layer of reinforcement).

$$\frac{\rho_{tc} bdf_y}{0.85 f_c' b} = \frac{3}{8} \beta_1 d_t \tag{2.37}$$

$$\rho_{tc} = 0.319\beta_1 \frac{f_c'}{f_y} \cdot \frac{d_t}{d} \tag{2.38}$$

$$\rho_{tc} = 0.319\beta_1 \frac{f_c'}{f_y} \tag{2.39}$$

Table A2.3 in Appendix A lists the values of $\rho_{\max}$ and ρ_{tc} for various grades of steel (f_y) and concrete strength (f_c') combinations. The value of the strength reduction factor (ϕ) is shown in the right column of the table. This value varies when $\rho_{tc} < \rho < \rho_{\max}$, or the beam's failure mode is in the transition zone (see Section 2.17). Table A2.3 indicates that not much is gained in terms of useable moment capacity with the required reductions in the ϕ values and when the reinforcing percentage is increased from ρ_{tc} to $\rho_{\max}$, especially when higher strength steels are used.

2.21 Minimum Steel Ratio ($\rho_{\min}$) for Reinforced Concrete Beams

When a reinforced concrete beam, for architectural or other reasons, is relatively large in cross section, or carries little load, the calculations may require only a very small amount of reinforcing steel. Such a section, if accidentally overloaded, will fail in a sudden, brittle manner. The reason is that the ultimate moment strength provided by the reinforced section is actually less than the strength of the same section without any reinforcing. Thus, the stress in the reinforcement will immediately reach yield at the first crack, causing the section to fail suddenly.

To ensure that reinforced beam's ultimate strength is larger than that of the unreinforced beam, Section 9.6.1.2 of the ACI Code requires a minimum amount of flexural steel in reinforced concrete beams. This requirement is given in Equation (2.40).

$$A_{s,\min} = \frac{3\sqrt{f_c'}}{f_y} bd \geq \frac{200}{f_y} bd \tag{2.40}$$

This minimum amount of steel ($A_{s,\min}$) provides enough reinforcement to ensure that the moment strength of the reinforced concrete section is more than that of an unreinforced concrete section, which can be calculated from its modulus of rupture.

In the past, the ACI Code required only an $A_{s,\min} = \frac{200}{f_y} bd$. For concrete strength greater than about 4,440 psi, however, this is not sufficient to ensure the desired aim;

$\frac{3\sqrt{f'_c}}{f_y} bd$ rectifies this condition. Because $A_{s,\min} = \rho_{\min}\, bd$, Equation (2.40) may be expressed mathematically in terms of $\rho_{\min}$ (minimum steel ratio) as shown in Equation (2.41).

$$\rho_{\min} = \max\left\{\frac{3\sqrt{f'_c}}{f_y}, \frac{200}{f_y}\right\} \tag{2.41}$$

Table A2.4 in Appendix A provides values of $\rho_{\min}$ for different grades of steel and compressive strengths of concrete.

2.22 Analysis of Rectangular Reinforced Concrete Sections

Analysis of a section means finding the $M_R = \phi M_n$ value. This may be necessary when checking an existing structure or element to determine if the strength provided by the section (*supply*) is sufficient to satisfy M_u that is calculated from the loads (*demand*). Finding M_R also makes it possible to calculate the maximum live load that may be permitted on the element.

An analysis can be performed only when all parameters that influence the ultimate strength of a section are known. There are five of these parameters, namely:

The dimensions of the section	b and d
The materials used in the beam	f'_c and f_y
The tensile reinforcement in the beam	A_s

Next we show two methods for calculating the value of M_R.

2.22.1 M_R *Calculation: Method I*

This method closely follows the already discussed and established formulae. Figure 2.38 shows the stress and strain distributions for a reinforced concrete rectangular beam at ultimate strength. For the most general case, a beam section with multilayer reinforcing is shown.

The resisting moment can be calculated from the internal couple and using Equations (2.42)–(2.44).

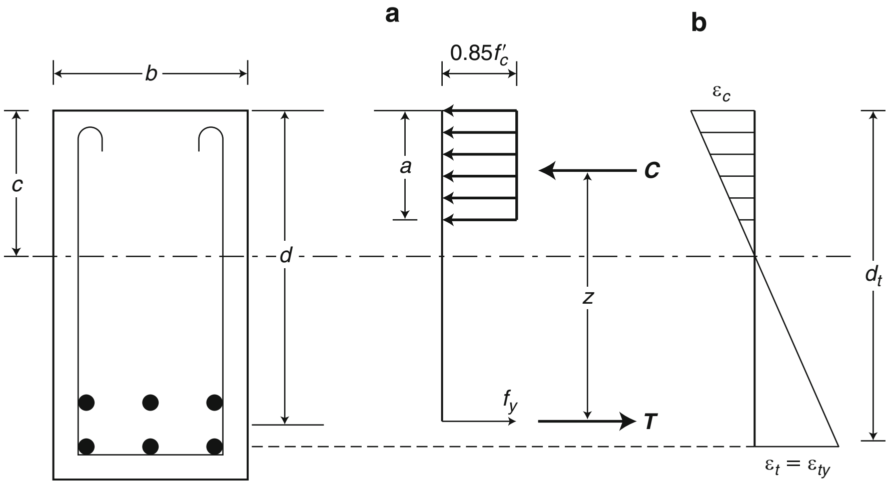


Figure 2.38 Stress and strain distributions on a reinforced concrete section

$$\begin{aligned} &T = A_s f_y \quad C = 0.85 f_c' ba \quad z = d - a/2 \\ &M_n = Tz = A_s f_y (d - a/2) \end{aligned} \tag{2.42}$$

$$M_n = Cz = 0.85 f_c' ba(d - a/2) \tag{2.43}$$

$$M_R = \phi M_n \tag{2.44}$$

The calculation proceeds as follows:

Step 1. Calculate $\rho = \dfrac{A_s}{bd}$ and check if $\rho \geq \rho_{\min}$ (from Table A2.4); if not, the beam does not satisfy the minimum requirements of the ACI Code, and its use for load carrying is not permitted. Determine whether $\rho \leq \rho_{\max}$ (from Table A2.3); if not, the beam has too much reinforcing and does not satisfy the latest ACI Code's limitations. A practical solution for this is to disregard the excessive amount of reinforcement, assume that the section is in the transition zone, and continue the calculations with the maximum permissible amount of reinforcing.

Step 2. Calculate the depth of the equivalent stress block from Equation (2.17):

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Step 3. Calculate the location of the neutral axis from Equation (2.11):

$$c = \frac{a}{\beta_1}$$

Step 4. Determine whether

$$\frac{c}{d_t} \leq \frac{3}{8}$$

If yes, the beam is in the tension-controlled failure zone; set $\phi = 0.90$ and go directly to step 5. If not $\left(\text{i.e., } \frac{3}{8} \leq \frac{c}{d_t} \leq \frac{3}{7}\right)$, the ϕ factor must be adjusted accordingly. Therefore, calculate the reduced ϕ:

$$\phi = A_2 + \frac{B_2}{\frac{c}{d_t}} (\text{refer to Table A2.2b for } A_2 \text{ and } B_2)$$

Step 5. Calculate $M_R = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$(refer to Equations (2.42) and (2.44))

Figure 2.39 summarizes the analysis steps.

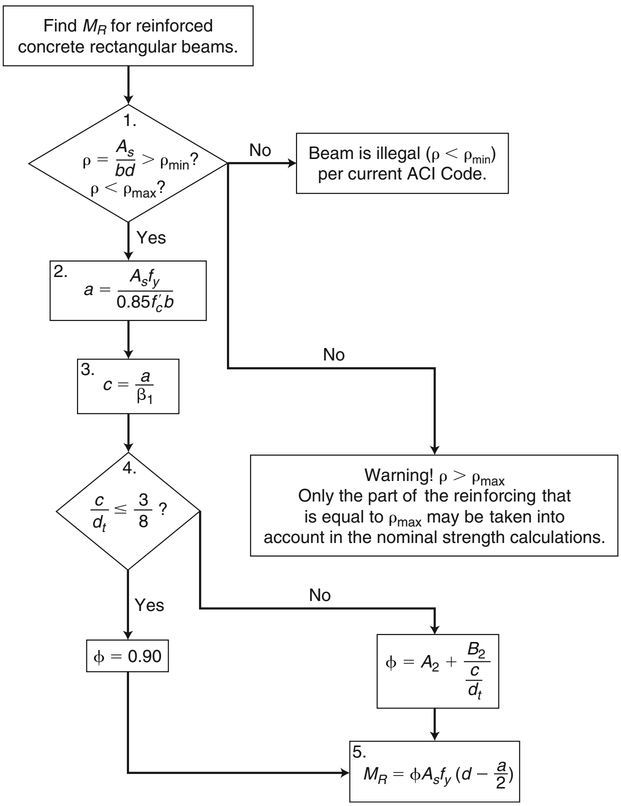
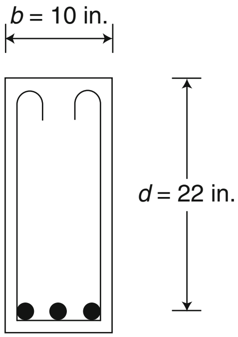


Figure 2.39 Flowchart to calculate M_R (Method I)

Example 2.6 Use Method I to determine the design resisting moment, M_R, of the reinforced concrete beam section shown below. $f_c' = 4\,\text{ksi}$, and $f_y = 40\text{ksi}$. The reinforcement is 3 #9 bars, $A_s = 3.00\ \text{in}^2$.



Solution Using the steps of Figure 2.39:

Step 1. Find the steel ratio, ρ:

$$\rho = \frac{A_s}{bd} = \frac{3}{10 \times 22} = 0.0136$$

$$\text{From Table A2.4} \rightarrow \rho_{\min} = 0.0050 < 0.0136 \quad \therefore \text{ ok}$$

$$\text{From Table A2.3} \rightarrow \rho_{\max} = 0.0310 > 0.0136 \quad \therefore \text{ ok}$$

Step 2. Calculate the depth of the compression zone, *a:*

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \times 40}{0.85 \times 4 \times 10} = 3.53\,\text{in.}$$

Step 3. From the depth of the equivalent stress block, determine the location of the neutral axis, c:

$$c = \frac{a}{\beta_1} = \frac{3.53}{0.85} = 4.15\,\text{in.}$$

Step 4. Determine whether the section is tension-controlled or is in the transition zone:

$$\frac{c}{d_t} = \frac{4.15}{22} = 0.189 < 0.375 \quad \therefore \text{ ok}$$

Therefore, the section is tension-controlled and the strength reduction factor $\phi = 0.90$.

Step 5. Calculate the resisting moment, M_R:

$$M_R = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_R = \frac{0.90 \times 3 \times 40\left(22 - \frac{3.53}{2}\right)}{12} = 182\,\text{ft-kip}$$

Example 2.7 Repeat Example 2.6 for $f_y = 60$ ksi, and $f'_c = 3$ ksi.

Solution

Step 1.

$$\rho = \frac{A_s}{bd} = 0.0136$$

$$\text{Table A2.4} \rightarrow \rho_{\min} = 0.0033 < 0.0136 \quad \therefore \text{ ok}$$

$$\text{Table A2.3} \rightarrow \rho_{\max} = 0.0155 > 0.0136 \quad \therefore \text{ ok}$$

Step 2.

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{3 \times 60}{0.85 \times 3 \times 10} = 7.06\,\text{in.}$$

Step 3.

$$c = \frac{a}{\beta_1} = \frac{7.06}{0.85} = 8.30\,\text{in.}$$

Step 4.

$$\frac{c}{d_t} = \frac{8.30}{22} = 0.377 > 0.375$$

$\therefore$ Section is in the transition zone (although just barely).

$$\phi = A_2 + \frac{B_2}{\frac{c}{d_t}}$$

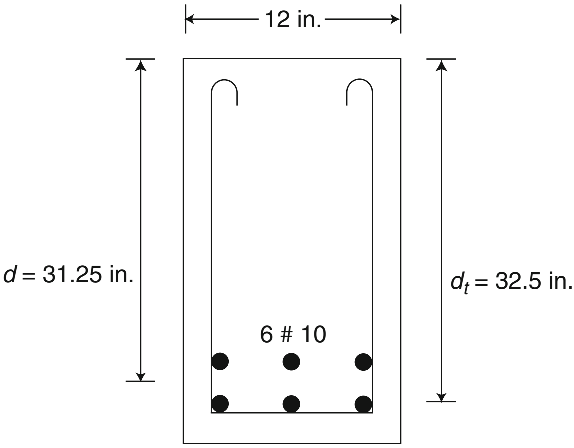
Use Table A2.2b to determine A_2 and B_2; then

$$\phi = 0.233 + \frac{0.25}{0.377} = 0.90$$

Step 5.

$$M_R = \frac{0.90 \times 3 \times 60\left(22 - \frac{7.06}{2}\right)}{12} = 249\,\text{ft-kip}$$

Example 2.8 Calculate M_R for the reinforced concrete beam section shown below. $f_y = 60$ ksi, $f_c' = 4$ ksi, $A_s = 7.62$ in^2.



Solution

Step 1.

$$\rho = \frac{A_s}{bd} = \frac{7.62}{12 \times 31.25} = 0.0203$$

$$\text{From Table A2.4} \rightarrow \rho_{min} = 0.0033 < 0.0203 \quad \therefore \text{ ok}$$

$$\text{From Table A2.3} \rightarrow \rho_{max} = 0.0207 > 0.0203 \quad \therefore \text{ ok}$$

Step 2.

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{7.62 \times 60}{0.85 \times 4 \times 12} = 11.21 \text{ in.}$$

Step 3.

$$c = \frac{a}{\beta_1} = \frac{11.21}{0.85} = 13.19 \text{ in.}$$

Step 4.

$$\frac{c}{d_t} = \frac{13.19}{32.5} = 0.406 > 0.375$$

$\therefore$ Section is in the transition zone. With the help of Table A2.2b:

$$\phi = A_2 + \frac{B_2}{\frac{c}{d_t}} = 0.233 + \frac{0.25}{0.406} = 0.85$$

Step 5.

$$M_R = \frac{0.85 \times 7.62 \times 60\left(31.25 - \frac{11.21}{2}\right)}{12} = 831\,\text{kip-ft}$$

2.22.2 M_R *Calculation: Method II*

This method results in the development of design aid tables, which are more user-friendly. The tables will also be useful when the aim is to design beam sections to satisfy a given M_u demand instead of analyzing.

The expressions for the components of the internal couple are

$$T = A_s f_y \quad C = 0.85 f_c' ba \quad z = d - a/2$$

Because $T = C$, the depth of the equivalent stress block is

$$a = \frac{A_s f_y}{0.85 f_c' b}$$

Substituting from Equation (2.18), $A_s = \rho bd$. Equation (2.45) can be used to calculate a.

$$a = \frac{\rho b d f_y}{0.85 f_c' b} = \frac{\rho d f_y}{0.85 f_c'} \tag{2.45}$$

Substituting from Equation (2.11), $a = \beta_1 c$, c can be determined using Equation (2.46).

$$\begin{aligned} \beta_1 c &= \frac{\rho d f_y}{0.85 f_c'} \\ c &= \frac{\rho f_y d}{0.85 f_c' \beta_1} \end{aligned} \tag{2.46}$$

or

$$\frac{c}{d_t} = \frac{\rho f_y}{0.85 f_c' \beta_1} \cdot \frac{d}{d_t} \tag{2.47}$$

Equation (2.47) is usually the preferred equation to check if a section is tension-controlled.

If $3/8 \leq c/d_t \leq 3/7$ the strength reduction factor, ϕ, must be adjusted accordingly.

$$\phi = A_2 + \frac{B_2}{\frac{c}{d_t}}$$

Substituting for c/d_t from Equation (2.47), Equation (2.48) can be used to calculate the adjusted strength reduction factor.

$$\phi = A_2 + B_2 \frac{0.85 f'_c \beta_1}{\rho f_y} \cdot \frac{d_t}{d} \tag{2.48}$$

Equation (2.48) provides the values of ϕ in the transition zone. In order to simplify the equation, introduce a new steel ratio, ρ_t:

$$\rho_t = \frac{A_s}{b d_t}$$

(Note that $d_t = d$ and $\rho_t = \rho$ when the beam has only a single layer of steel.)

Substituting $\rho = \rho_t \frac{d_t}{d}$ Equation (2.48) can be rewritten as Equation (2.48a).

$$\phi = A_2 + B_2 \frac{0.85 f'_c \beta_1}{\rho_t f_y} \tag{2.48a}$$

From Equation (2.43) (see also Figure 2.38):

$$M_R = \phi M_n = \phi C z$$
$$M_R = \phi (0.85 f'_c b a)\left(d - \frac{a}{2}\right)$$

Substituting from Equation (2.45) for a:

$$M_R = \phi \left(0.85 f'_c b \frac{\rho d f_y}{0.85 f'_c}\right)\left(d - \frac{\rho d f_y}{1.7 f'_c}\right)$$

Rearranging and simplifying:

$$M_R = b d^2 \left[\phi \rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c}\right)\right] \tag{2.49}$$

If the product in the bracket is designated as R (called the *resistance coefficient,* which has units of stress, psi or ksi) as shown in Equation (2.50),

$$R = \phi \rho f_y \left(1 - \frac{\rho f_y}{1.7 f_c'}\right) \tag{2.50}$$

the expression for M_R is simplified to Equation (2.51).

$$M_R = bd^2 R \tag{2.51}$$

It is clear from Equation (2.50) that R depends on the materials used (i.e., f_c', f_y and the steel ratio (ρ) in the beam), but it is independent of the dimensions of the section. Thus tables for R can be developed in terms of ρ for the various combinations of materials. Values of R can be found from Tables A2.5 through A2.7. $\rho_{\min}$ for beams are indicated on each table. Reinforcement ratio (ρ) values less than $\rho_{\min}$ may not be used in beams, but may be used in slabs and footings.

These tables were developed with R in psi. Using R in psi and beam dimensions b and d in inches results in lb-in. units for M_R. Because kip-ft are usually used in moment calculations, appropriate conversions must be made between lb-in. and kip-ft for the correct use of the tables.

$$M_R(\text{ft-kip}) = b\ \text{in.}(d\ \text{in.})^2 \frac{R(\text{psi})}{12{,}000}$$

The tables must be used with care, especially when large ρ values result in the section being in the transition zone. The value of ϕ depends on f_c', f_y, ρ, and $\frac{d_t}{d}$, thus if

$$\rho \leq \rho_{tc} \rightarrow \phi = 0.90$$

and if

$$\rho_{\max} \geq \rho > \rho_{tc} \rightarrow \phi = A_2 + B_2 \frac{0.85 f_c' \beta_1}{\rho f_y} \cdot \frac{d_t}{d}$$

The values of ρ_{tc} and $\rho_{\max}$ for common grades of steel and concrete strength are listed in Table A2.3.

An important note here is that Tables A2.5 to A2.7 have been developed based on ρ (i.e., beams with a single layer of reinforcement). If the beam has multiple layers of reinforcement ($\rho_t \neq \rho$), the R value must be modified by adjusting it to an R' value based on ρ_t. This can be easily done by using Equation (2.51a).

$$R' = \frac{\phi'}{\phi} R \tag{2.51a}$$

The values of ϕ', which are listed in Tables A2.5 to A2.7, correspond to the values of ρ_t.

The use of Method II for analysis of reinforced concrete beam sections involves the following steps:

Step 1. Determine whether $\rho \geq \rho_{\text{min}}$; if not, then the beam does not satisfy the minimum requirements of the ACI Code and its use for load carrying is not permitted.

Determine whether $\rho \leq \rho_{\text{max}}$; if not, the beam has too much reinforcing and does not satisfy the latest ACI Code's limitations. A practical solution for this is to disregard the excessive amount of reinforcement, assume that the section is in the transition zone, and continue the calculations with the maximum permissible amount of reinforcing.

Step 2. Use ρ, f_c' and f_y to obtain R and ϕ values from the appropriate Tables A2.5 to A2.7. If the beam has a single layer of steel or $\phi = 0.90$, find M_R from Step 3. Otherwise move to Step 4.

Step 3. Calculate $M_R = \dfrac{bd^2R}{12{,}000} \quad (b, d = \text{in.}; R = \text{psi}; M_R = \text{ft-kip})$

Step 4. For beams with multiple layers of reinforcement, calculate $\rho_t = \dfrac{A_s}{bd_t}$ and obtain the corresponding strength reduction factor (ϕ') from Tables A2.5 to A2.7.

Step 5. Calculate the modified value of the coefficient of resistance $\left(R' = R\dfrac{\phi'}{\phi}\right)$.

Step 6. Calculate $M_R = \dfrac{bd^2R'}{12{,}000}$.

The flowchart for Method II is shown in Figure 2.40.

Method II has fewer steps to follow, so it is easier to use. Method I, however, is more general as it does not require the use of design tables (which may not be readily available) and it is adaptable to any grade of steel or compressive strength of concrete, not just the ones listed in the tables.

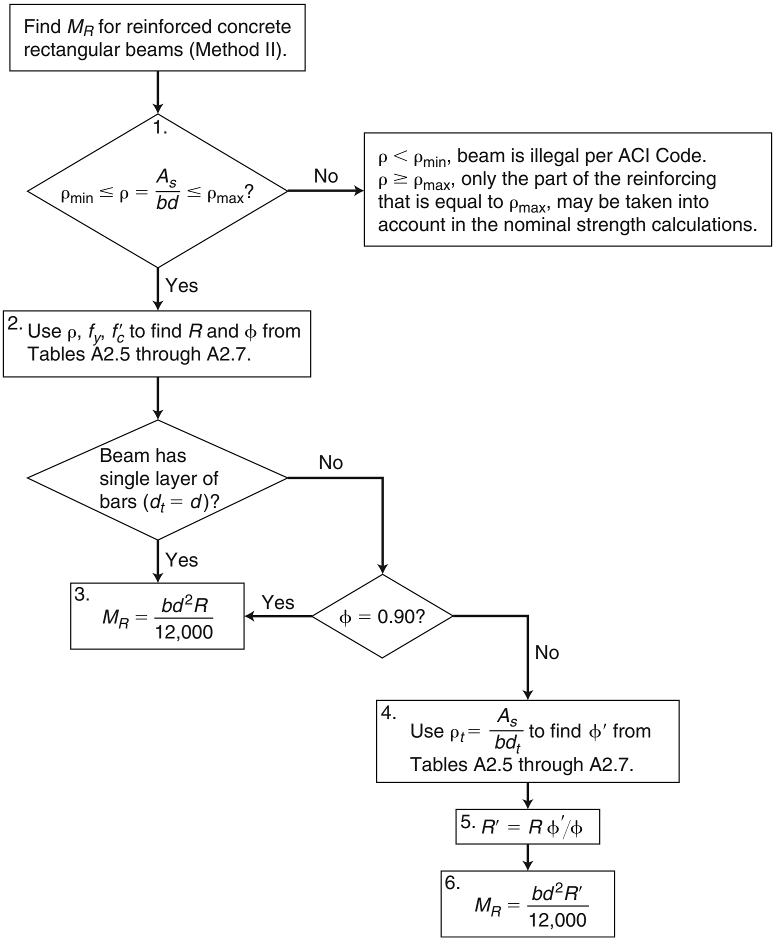


Figure 2.40 Flowchart to calculate M_R using Method II

Example 2.9 Solve Example 2.6 using Method II.

Solution

Step 1. From Example 2.6:

$$\rho = 0.0136 > \rho_{min} = 0.0050 \quad \therefore \text{ ok}$$
$$\text{From Table A2.3} \rightarrow \rho_{max} = 0.0310 > 0.0136 \quad \therefore \text{ ok}$$

Step 2. Using $\rho = 0.0136$, $f_y = 40$ ksi, and $f_c' = 4$ ksi, obtain the resistance coefficient, R, from Table A2.5b:

$$R = 450\,\text{psi}, \quad \phi = 0.90$$

Step 3. Because the beam has a single layer of reinforcement:

$$M_R = \frac{bd^2R}{12{,}000} = \frac{10 \times 22^2 \times 450}{12{,}000}$$
$$M_R = 182\,\text{ft-k}$$

which is the same as determined in Example 2.6.

Example 2.10 Solve Example 2.7 using Method II.

Solution

Step 1. From Example 2.7:

$$\rho_{\max} = 0.0155 > \rho = 0.0136 > \rho_{\min} = 0.0033 \quad \therefore \text{ ok}$$

Step 2. $\rho = 0.0136, f'_c = 3\,\text{ksi}$, and $f_y = 60$ ksi. From Table A2.6a:

$$R = 615\,\text{psi}, \quad \phi = 0.90$$

Step 3.

$$M_R = \frac{bd^2R}{12{,}000}$$
$$M_R = \frac{10 \times 22^2 \times 615}{12{,}000} = 248\,\text{ft-kip}$$

which is about the same as the result determined in Example 2.7.

Example 2.11 Solve Example 2.8 using Method II.

Solution

Step 1. From Example 2.8:

$$\rho = 0.0203 > 0.0033 \quad \therefore \text{ ok}$$

Because there are two layers of reinforcement, adjust $\rho_{\max}$ using Table A2.3:

$$\rho_{\max} = 0.0207\,\frac{d_t}{d} = 0.0207\frac{32.5}{31.25} = 0.0215 > 0.0203 \quad \therefore \text{ ok}$$

Step 2. Use ρ, f_y, and f'_c to obtain R from Table A2.6b.

$$\rho = 0.0203$$
$$f'_c = 4\,\text{ksi} \rightarrow \text{Table A2.6b} \rightarrow R = 825\,\text{psi}$$
$$f_y = 60\,\text{ksi} \qquad \phi = 0.82$$

Step 3. Because the beam has two layers of reinforcement and ϕ is not equal to 0.90, determine ρ_t and ϕ' and adjust the resistance coefficient, R:

Step 4.

$$\rho_t = \frac{A_s}{bd_t} = \frac{7.62}{12 \times 32.5} = 0.0195$$

$$\text{From Table A2.6b} \rightarrow \phi' = 0.85$$

Step 5. Adjusted value of the resistance coefficient (R') is:

$$R' = R\frac{\phi'}{\phi} = 825 \times \frac{0.85}{0.82} = 855\,\text{psi}$$

Step 6. $M_R = \dfrac{bd^2R'}{12{,}000} = \dfrac{12 \times 31.25^2 \times 855}{12{,}000} = 835\,\text{ft-kip}$

This result is about the same as that from using Method I. The difference is insignificant and is due to rounding errors in the calculations.

2.23 Selection of Appropriate Dimensions for Reinforced Concrete Beams and One-Way Slabs

2.23.1 Selection of Depth

The selection of a beam's depth is almost always a controversial issue. On the one hand, the building designer wants to minimize the depth of the structure in order to maximize the headroom without unduly increasing the height of the building. On the other hand, structural elements that are too shallow lead to increased short- and long-term deflections. These, in turn, may be detrimental to attached nonstructural building elements. Excessive deflections of concrete structures may result in cracked walls and partitions, non-functioning doors, and so on.

To guide in the design of well-functioning structures, the ACI Code (Sections 7.3.1.1 and 9.3.1.1) recommend a set of span/depth ratios, with the comment that the designer does not have to calculate deflections (an involved and somewhat uncertain process) if the utilized depth is at least equal to the values provided in ACI Table 7.3.1.1 for one-way slabs, and Table 9.3.1.1 for beams. These values are summarized graphically in Figures 2.41 and 2.42.

Note from Figures 2.41 and 2.42 that the recommended minimum depth for simply supported beams is span/16, whereas for one-way slabs this value is span/20. These types of support conditions are quite rare in monolithic reinforced concrete construction, because in most cases either continuity or some other type of restraint is available at the supports. If the member is continuous at both ends, $h_{\min} = \text{span}/21$ for beams and $h_{\min} = \text{span}/28$ for one-way slabs. Finally, if the beam is continuous at only one end, the minimum depth is span/18.5, and for one-way slabs is span/24.

A cautionary note is in order here. Span 2 (ℓ_2) in Figures 2.41 and 2.42 is shown as "Both ends continuous." This assumption is valid only if the cantilever at the left end of ℓ_2 is long enough to develop a significant end moment. Experience shows

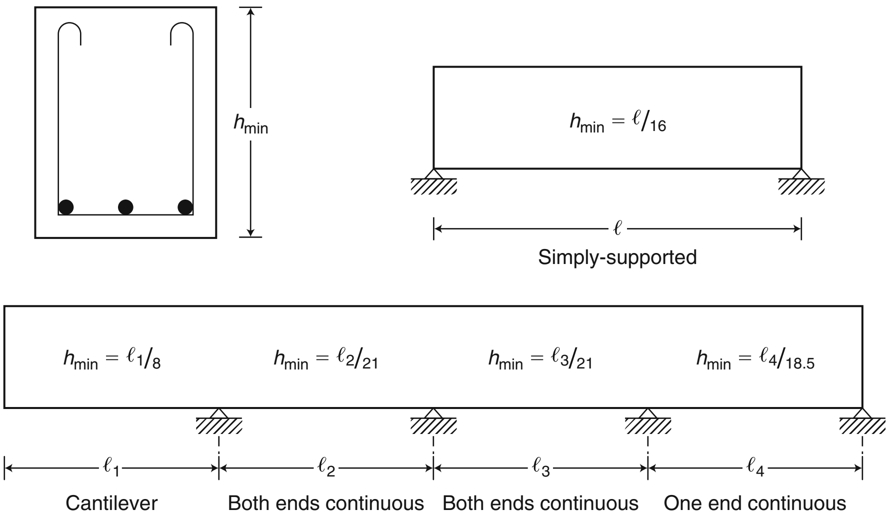


Figure 2.41 Minimum depth requirements for reinforced concrete beams

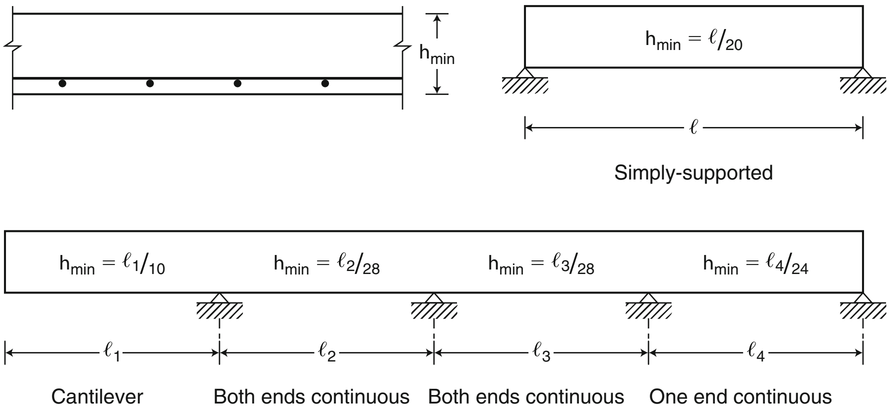


Figure 2.42 Minimum depth requirements for reinforced concrete one-way slabs

that when the cantilever length is at least $\ell_2/3$, the span ℓ_2 may safely be assumed as "both ends continuous" from the point of view of satisfactory deflection control.

The values shown in Figures 2.41 and 2.42 are applicable only to normal-weight concrete ($w_c = 145$ lb/ft^3) and Grade 60 reinforcement. For other conditions, the ACI Code Section 7.3.1 recommends the following modifications:

(a) For lightweight concrete in the range of 90–115 pcf, the values in Figures 2.41 and 2.42 need to be multiplied by $(1.65 - 0.005w_c)$ where w_c = unit weight of concrete in lb/ft^3. This factor should not be less than 1.09. For a typical lightweight structural concrete, $w_c = 115$ pcf. Then the multiplier is $1.65 - 0.005 \times 115 = 1.075 < 1.09$. Use a multiplier equal to 1.09.

(b) For f_y other than 60,000 psi, the values obtained from Figures 2.41 and 2.42 shall be multiplied by:

$$\left(0.4 + \frac{f_y}{100{,}000}\right) \tag{2.52}$$

If the selected beam depth is less than the recommended $h_{\min}$, the beam deflection has to be calculated and checked against the ACI Code requirements. Therefore, if a beam does not satisfy the minimum depth requirements, it may still be acceptable if computation of deflection proves it to be satisfactory.

2.23.2 Selection of Width

Minimum Bar Spacing in Reinforced Concrete Beams In Section 2.12 we discussed the role of concrete cover over the reinforcement. Reinforcing bars also need space between them to ensure adequate bond surface at their interface with the concrete. The space should also be larger than the size of the largest aggregate particle in the concrete.

Sections 25.2.1, 25.2.2 of the ACI Code require a minimum clear space for single and multiple layers of bars as follows:

Minimum Space ($s_{\min}$) for Single Layer of Bars The *minimum space* ($s_{\min}$) for a single layer of bars in beams (see Figure 2.43a) is the largest of the following: the diameter of bar (d_b), 1 in., and 4/3 of maximum size aggregate used in the concrete mixture.

Mathematically:

$$s_{\min} = \max\{d_b, 1\,\text{in.}, 4/3\,\text{max. aggregate size}\}$$

Note that in most building structure applications (save for footings and foundations) the usual concrete mix limits the size of the aggregate to ¾ in. Thus, a 1 in. minimum spacing satisfies the third of the spacing requirements.

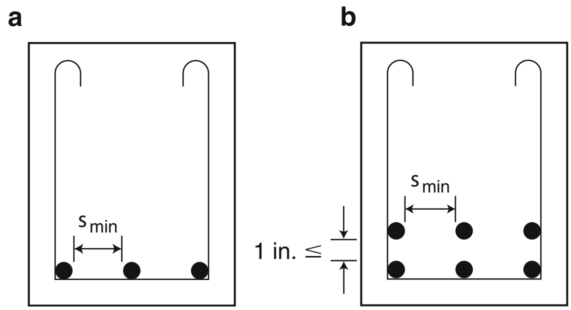


Figure 2.43 Minimum spacing between reinforcing bars: (**a**) single layer; (**b**) multiple layer

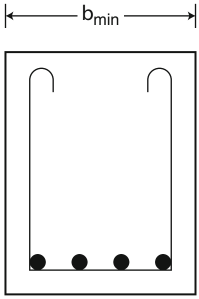


Figure 2.44 Minimum beam width ($b_{\min}$)

Minimum Space for Multiple Layers of Bars Where reinforcement is placed in two or more layers (see Figure 2.43b), bars in the upper layers shall be placed directly above bars in the lower layer with clear distance between layers not less than 1 in. In addition, the requirements of single-layer bars must also be satisfied.

Minimum Width ($b_{\min}$) of Reinforced Concrete Beams We use the minimum required space between bars in a single layer to calculate the *minimum beam width* needed to provide enough room for a specific number and size of bars. To compute $b_{\min}$, consider Figure 2.44. Usually #3 or #4 bars are used for stirrups. Also, the minimum cover for bars in beams is 1.5 in. Therefore, we can calculate $b_{\min}$ by adding the minimum required spaces and the bar diameters.

As an example, suppose that the beam in Figure 2.44 is reinforced with 4 #8 bars. Assuming #4 stirrups, the minimum width for this beam is:

$$b_{\min} = \underset{\substack{\uparrow \\ \text{Cover}}}{2 \times 1.5 \text{ in.}} + \underset{\substack{\uparrow \\ \text{Stirrups}}}{2 \times \frac{1}{2} \text{ in.}} + \underset{\substack{\uparrow \\ \text{Main bars}}}{4 \times 1 \text{ in.}} + \underset{\substack{\uparrow \\ s_{\min}}}{3 \times 1 \text{ in.}} = 11 \text{ in.}$$

Note that $s_{\min} = 1$ in. was used; this assumes that 4/3 of the maximum aggregate size is less than or equal to 1 in. Table A2.8, based on the above example, shows $b_{\min}$ for different numbers and sizes of bars in a single layer.

2.24 Crack Control in Reinforced Concrete Beams and One-Way Slabs

It was previously mentioned that a reinforced concrete member will always crack when subjected to bending. In fact, the reinforcing really starts working only after the development of cracks. Nevertheless, designers try to minimize the size of the cracks. Limitation of crack width is desirable for three main reasons: (1) appearance; (2) limitation of corrosion of the reinforcement; and (3) water-tightness.

Laboratory experiments have shown that several parameters influence the width and spacing of flexural cracks. The first is the *concrete cover* over the reinforcing. The smaller the cover, the smaller the crack width will be. The cover cannot be reduced beyond a certain limit, however, because a minimum cover is needed for fire and corrosion protection. Thus, the Code requires a minimum cover of 1.5 in. over the stirrups for interior beams, 2 in. for exposed exterior beams (see Figure B2.2 in Appendix B) and ¾ in. for joists and slabs. The 1.5 in. cover over the stirrups results in a cover of 1 ⅞ in. to 2 in. over the main reinforcement.

The second important parameter is the *maximum stress in the reinforcement* (directly related to the strain, or the elongation of the steel) at *service load* levels. This value may be assumed to be roughly $0.66f_y$. The higher the stress level is in the steel, the wider the cracks are expected to be. Thus, using more reinforcing than required to satisfy the ultimate strength capacity can reduce the width of cracks by reducing the stresses (and strains) at working load levels. This is not an economical choice, however. The same is true if steel with $f_y = 40{,}000$ psi is used instead of steel with $f_y = 60{,}000$ psi. The section would need 50 % more steel, but the much lower levels of stress at service load levels would help limit the crack width.

Another important parameter is the *maximum spacing* of the reinforcing bars. For minimizing the width of cracks, placing more and smaller bars closer together is preferable to placing a few large bars farther apart. The ACI Code (Sections 24.3.2, and 24.3.3) limits the maximum spacing of the tensile reinforcement in beams and one-way slabs. The empirical formula for maximum spacing, given in Equation (2.53), is based on the tensile stress in the steel and the concrete cover.

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right) \tag{2.53}$$

where s is the center-to-center spacing (in inches) of flexural tension reinforcement nearest to the extreme tension face; f_s is the calculated tensile stress (in psi) at service load in steel or $2/3 f_y$; and c_c is the least distance (in inches) from the surface of the reinforcement to the tension face. Equation (2.53) cannot address the control of cracking for all the different causes discussed.

If $f_y = 60{,}000$ ksi, the right side of Equation (2.53) is limited to 12 in. (since $f_s = 2/3\, f_y$). The left side of the inequality relates the maximum spacing (s) to the concrete cover (c_c). To better comprehend Equation (2.53), consider Figure 2.45, which shows the reinforcing bar with two different covers, c_{c1} and c_{c2}. If the concrete cover is *increased* from c_{c1} to c_{c2} and the crack width at the level of the reinforcement (w_s) is constant, the *surface crack width increases* from w_1 to w_2. Figure 2.45 clearly shows the relationship between surface crack width and amount of concrete cover.

We can use the maximum spacing limitation (s) given by Equation (2.53) to determine the maximum beam width ($b_{\max}$) as a function of the number of bars placed in the section. For example, for 4 #4 main bars, #4 stirrups, and $f_y = 60{,}000$ psi, the maximum permissible spacing of bars (s) is:

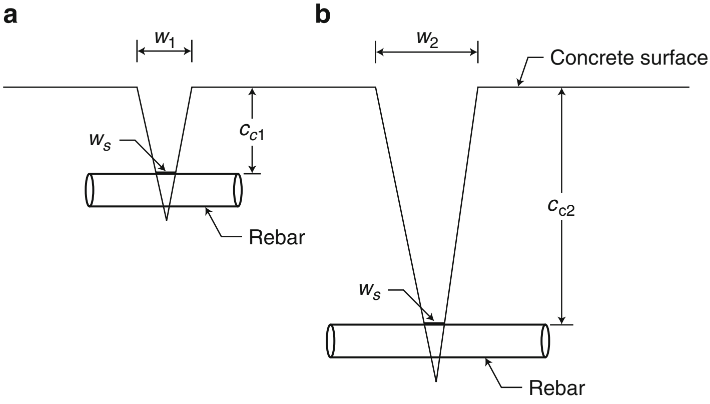


Figure 2.45 Relationship between crack width and concrete cover

$$s = 15\left(\frac{40{,}000}{f_s}\right) - 2.5c_c \leq 12\left(\frac{40{,}000}{f_s}\right)$$
$$s = 15\left(\frac{40{,}000}{\frac{2}{3} \times 60{,}000}\right) - 2.5(1.5 + 0.5) \leq 12\left(\frac{40{,}000}{\frac{2}{3} \times 60{,}000}\right)$$
$$s = 10\,\text{in.} \leq 12\,\text{in.} \rightarrow s = 10\,\text{in.}$$

and the maximum beam width ($b_{\max}$) is:

$$b_{\max} = \underset{\text{Cover}}{\underset{\uparrow}{2 \times 1.5\text{ in.}}} + \underset{\#4\text{stirrups}}{\underset{\uparrow}{2 \times \tfrac{1}{2}\text{in.}}} + \underset{\#4\text{bar}}{\underset{\uparrow}{\tfrac{1}{2}\text{ in.}}} + \underset{s}{\underset{\uparrow}{3 \times 10\text{ in.}}} = 34.5\text{ in.} \approx 34\text{ in.}$$

Note that in the above calculation, s is the *center-to-center* distance of the reinforcing bars. Therefore, only the diameter of one bar was used to determine $b_{\max}$. The last column of Table A2.8 lists $b_{\max}$ for different sizes and numbers of bars in a single layer. In practice, $b_{\max}$ is rarely a problem for beams; however, the maximum spacing limitation is an important issue when designing reinforcing layouts in slabs.

Table A2.9 shows the areas of reinforcing steel (A_s) for different sizes and numbers of bars.

2.25 Design of Beams

The ultimate strength of a beam depends on five parameters. These are the materials (f_c' and f_y), the dimensions of the section (b and d), and the amount of reinforcement (A_s). The last three parameters may be expressed in the form of the steel ratio $\rho = A_s/bd$.

Whichever way these parameters are expressed, they are always five in number. There is only one equation (or, more precisely, one inequality), however, that expresses the problem:

$$M_u \leq M_R$$

The left side of this inequality depends only on the applied loads. The right side of the inequality, on the other hand, depends on all five of the variables listed above. Thus, this problem has an infinite number of solutions. But if four out of the five parameters are preselected or assumed, the inequality can be readily solved.

As an example, contemplate the following considerations. In a floor of a given structure, it would be quite impractical to vary the quality of the concrete. Consequently, every beam and slab of the floors of the structure is usually cast with the same quality concrete (same f_c') throughout. (In columns, the use of a different quality concrete may be warranted; but even then all columns in a given floor level would have the same concrete mix.) So preselecting the concrete quality for the slabs and beams throughout a building is standard practice.

The same is true with the reinforcement. Labor is the dominant factor in the price of the "in-place" reinforcing steel. And the basic cost per ton of reinforcing steel with $f_y = 40$ ksi and $f_y = 60$ ksi is very near the same, so there is no economic incentive to use the former. In fact, 60 ksi steel provides 50 % more strength than 40 ksi steel, thus making it cheaper to use.

Of the three remaining variables, b (the width of the section), d (the working depth of the section), and A_s (the amount of reinforcement), two must still be preselected in order to solve for the remaining unknown quantity. Generally speaking, practitioners select a concrete section (b and h) and then solve for a minimum required amount of reinforcement to satisfy the demanded factored moment requirements. Often all beams have the same depth and width to enable the contractor to reuse the forms. In other cases keeping the depth of all beams uniform satisfies the minimum headroom requirement throughout the structure.

In general, two types of problems arise: (1) The beam's sizes (b and h) are set using the considerations stated above and the designer needs only to determine the required area of steel (A_s); this is by far the most common problem. (2) The beam's sizes (b and h) and area of steel (A_s) are all unknown and determined by the designer during the process; this problem is more academic than practical.

b, h = known, A_s = unknown

The flowchart in Figure 2.46 shows the steps for the design process.

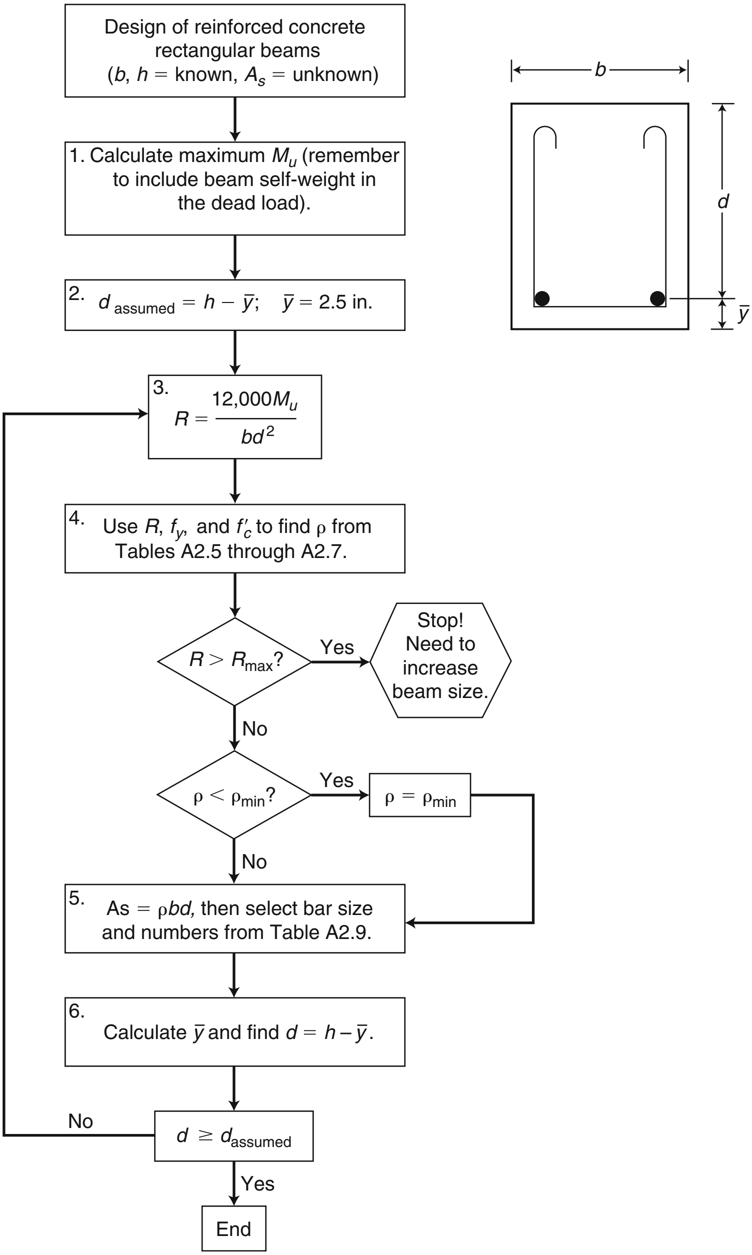


Figure 2.46 Flowchart for the design of reinforced concrete rectangular beams (b, h = known, A_s = unknown)

Step 1. Find the maximum factored bending moment, M_u.

Step 2. Because the bar sizes are not yet known, assume the distance from the edge of concrete in tension to the center of steel ($\bar{y}$) is 2.5 in. This is a reasonable assumption if the cover is 1.5 in., the stirrup diameter is 3/8 in. (#3) or ½ in. (#4), the main reinforcement is #8 to #10 bars or smaller, and there is only one layer of reinforcement.

Step 3. Use the assumed value of d to calculate the required resistance coefficient (R).

$$M_R = bd^2R\ (\text{refer to Equation}\,(2.51))$$

If b and d are in inches, and R in psi, M_u will need to be converted to in.-lb from its usual ft-kip units.

$$M_R = \frac{bd^2R}{12{,}000}$$

Set $M_u = M_R$:

$$M_u = M_R = \frac{bd^2R}{12{,}000}$$

$$R = \frac{12{,}000M_u}{bd^2}$$

Step 4. Use R, f_y, and f_c' to determine ρ from Tables A2.5 to A2.7. If R is greater than the maximum R value ($R_{\max}$) to be found in the tables, it means that the selected sizes are too small and must be increased.

If the value obtained is less than $\rho_{\min}$, it means that the beam sizes b and h are larger than needed to carry the loads with minimum reinforcement. This may happen when other considerations dictate the beam sizes. In this case use $\rho = \rho_{\min}$ from Table A2.4, because the beam *must* always have the required minimum reinforcement.

Step 5. Determine how much steel is needed and select bars using Table A2.9. It is also helpful to use Table A2.8 here, because it lists how many of a certain size of bar may be fitted into the selected b in a single layer.

Step 6. Once the bar sizes are known, the exact effective depth (d) can be calculated. If this depth is greater than what was assumed at the beginning of process, the design will be conservative as it will have more moment capacity than what was demanded. If the effective depth is less than the assumed value (e.g., the section needs multiple layers of reinforcements), then the process needs to be repeated with a new value of d. Insignificant differences in the assumed and recalculated values in d (less than 3/8 in. in slabs and 1/2 in. in beams) may be neglected and the reinforcing need not be redesigned.

Note that having multiple layers of reinforcing bars may influence the value of the strength reduction factor, ϕ.

Example 2.12 Figure 2.47a shows the partial framing plan of a beam-girder reinforced concrete floor system. The slab is 6 in. thick, and is subjected to a superimposed dead load of 30 psf. The floor live load is 100 psf. Beam B-2 has a width of 12 in. ($b = 12$ in.), and a total depth of 30 in. (including the slab thickness). Determine the steel required at Section 1.1. Use the ACI Code coefficients to calculate moments. Assume that the beam end is integral with the column. Use $f_c' = 4\,\text{ksi}, f_y = 60$ ksi, and assume that the unit weight of concrete is 150 pcf. Stirrups are #3 bars.

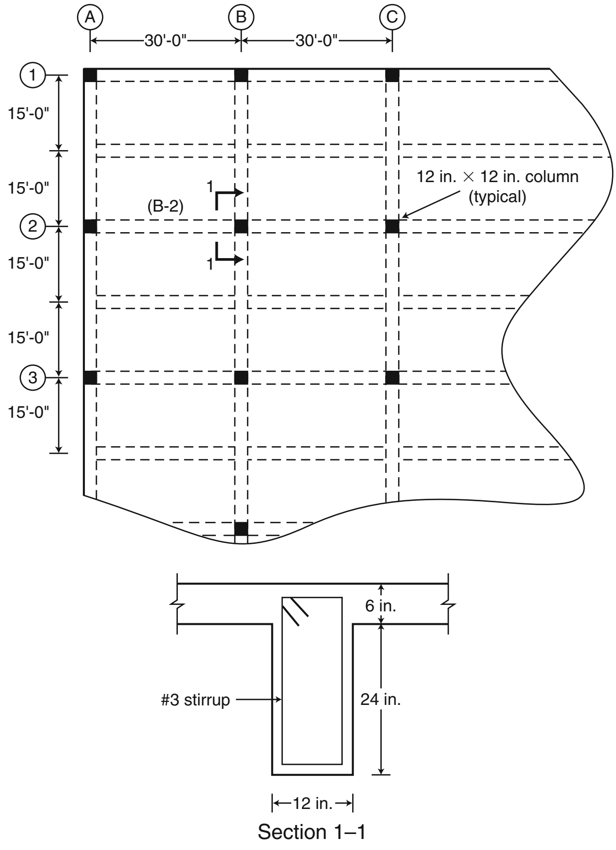


Figure 2.47a Framing plan and section for Example 2.12

Solution

Step 1. Before calculating the moments at the selected location, we must determine the floor loads:

$$\text{Weight of slab} = 150 \times \left(\frac{6}{12}\right) = 75\,\text{psf}$$
$$\text{Superimposed dead load} = 30\,\text{psf}$$
$$\text{Total dead load} = 105\,\text{psf}$$
$$\text{Live load} = 100\,\text{psf}$$

The tributary width for beam B-2 is 15′–0″; therefore, the uniform dead and live loads are:

$$w_D = \frac{105 \times 15}{1{,}000} + \frac{\overbrace{150\left(\frac{12}{12} \times \frac{24}{12}\right)}^{\text{Beam weight}}}{1{,}000} = 1.88\ \text{kip/ft}$$

$$w_L = \frac{100 \times 15}{1{,}000} = 1.5\,\text{kip/ft} \qquad \text{Note : Reduction of live load is neglected here.}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2 \times 1.88 + 1.6 \times 1.5 = 4.65\,\text{kip/ft}$$

The beam's clear span $\ell_n = 30\ \text{ft} - (0.5\ \text{ft} + 0.5\ \text{ft}) = 29\ \text{ft}$

Figure 2.47b shows the moments using the ACI coefficients from Table A2.1 for an exterior beam. Because the problem requires designing the reinforcement at Section 1.1:

$$(M_u)^- = \frac{w_u \ell_n^2}{10} = \frac{4.65(29)^2}{10} = 391\,\text{ft-kip}$$

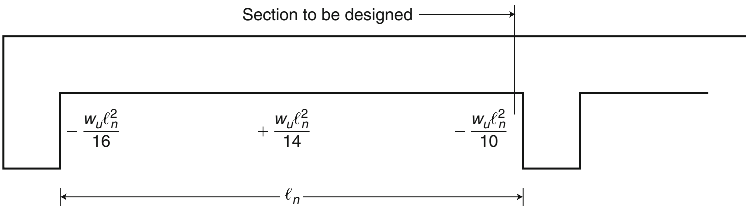


Figure 2.47b Moments using the ACI coefficients (Example 2.12)

Step 2. Assuming the distance ($\bar{y}$) from the edge of the beam in tension to the center of tensile steel is 2.5 in.:

$$d = h - \bar{y} = 30\,\text{in.} - 2.5\,\text{in.} = 27.5\,\text{in.}$$

Step 3. The required resistance coefficient, R, is:

$$R = \frac{12{,}000 M_u}{bd^2} = \frac{12{,}000 \times 391}{12(27.5)^2}$$

$$R = 517\,\text{psi}$$

Step 4.

$$R = 517\,\text{psi}$$
$$f'_c = 4\,\text{ksi} \rightarrow \text{Table A2.6b} \rightarrow \rho = 0.0106$$
$$f_y = 60\,\text{ksi}$$

Note that $\rho = 0.0106$ corresponding to $R = 519$ psi was conservatively selected.

$$\text{Table A2.4} \rightarrow \rho_{\min} = 0.0033 < \rho = 0.0106 \quad \therefore \text{ ok}$$

Step 5. Find the required amount of steel:

$$A_s = \rho b d = 0.0106(12)(27.5) = 3.50\,\text{in.}^2$$

From Table A2.9 → Try 3 #10 ($A_s = 3.81$ in.^{2})

The reinforcement is placed at the top of the beam, because the moment is negative at the section under investigation, which causes tension at the top. Figure 2.47c shows a sketch of the beam.

$$\text{Table A2.8} \rightarrow b_{\min} = 10.5\,\text{in.} < 12\,\text{in.} < b_{\max} = 24\,\text{in.} \quad \therefore \text{ ok}$$

Step 6. Check for the actual effective depth, *d:*

$$\bar{y} = \underbrace{1.5\text{ in.}}_{\text{Cover}} + \underbrace{3/8\text{ in.}}_{\text{Stirrup}} + \underbrace{1.27/2}_{\text{Bar diameter}} = 2.51\text{ in.}$$

$$d = h - y = 30\text{ in.} - 2.51\text{ in.} = 27.49\text{ in.} \approx d_{\text{assumed}} = 27.5\text{ in.} \quad \therefore \text{ok}$$

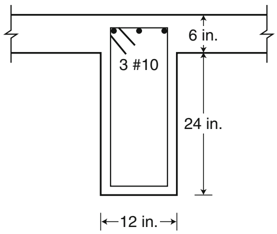


Figure 2.47c Sketch of beam for Example 2.12

b, h, A_s = unknown

There is still only one design equation, but the problem now is formulated differently. It is somewhat more "contorted" than the previous one, for if the designer does not like the results obtained with the assumed cross section and the corresponding reinforcement, he or she can just change the width or the depth (or both) and recalculate the reinforcement until satisfied with the design.

A first assumption may be an arbitrary selection of the steel ratio ρ. When ratios close to the ρ_{max} value are chosen, the amount of steel required creates a rather congested layout, especially in the positive moment regions (steel is placed in the bottom of the beam). On the other hand, an unnecessarily large concrete section may result if the section's moment requirement can be satisfied with ρ_{min}. Most practical designs have steel ratios somewhere between ρ_{max} and ρ_{min}.

Generally speaking, if ρ is assumed to be about $0.6\rho_{max}$ or less the beam proportions will likely be such that excessive deflection will not be a problem. Therefore, Table 2.1 is provided as an aid for the designer. In this table, ρ_{des} was calculated as $0.6\rho_{max}$ as a starting point.

Table 2.1 Design steel ratio (ρ_{des})

	ρ_{des}		
f_y **(psi)**	f_c' **=3,000 psi**	f_c' **=4,000 psi**	f_c' **=5,000 psi**
40,000	0.0139	0.0186	0.0218
60,000	0.0093	0.0124	0.0146
75,000	0.0074	0.0099	0.0116

Then the corresponding R value may be obtained from Tables A2.5 to A2.7. The value bd^2 can be determined using M_u:

$$M_u = Rbd^2 \rightarrow bd^2 = \frac{M_u}{R}$$

Two unknowns remain, however: b and d. There are no ACI Code requirements on the *geometrical proportioning* of beams. But it is more economical to design beams as deep and narrow rather than wide and shallow sections. This means that the effective depth, d, should be larger than the width, b. Generally speaking, the most economical beam sections for spans up to 25 ft usually have a d/b ratio between 1.5 and 2.5. For longer spans, a d/b ratio of 3–4 may be more suitable. Economy for a specific beam (or set of beams) is not the same as economy for the overall building. In fact, sometimes it is more economical to design wide and shallow beam sections due to the savings in the floor-to-floor height, even though this design will require more reinforcing steel.

Figure 2.48 summarizes the steps of the design process:

Step 1. Find the factored loads and moments.

Step 2. Use f_y and f_c' to select a ρ_{des} value from Table 2.1. Then find the corresponding R value from the appropriate design table (Tables A2.5 to A2.7).

Step 3. The formula for M_R is:

$$M_R = \frac{bd^2 R}{12{,}000}$$

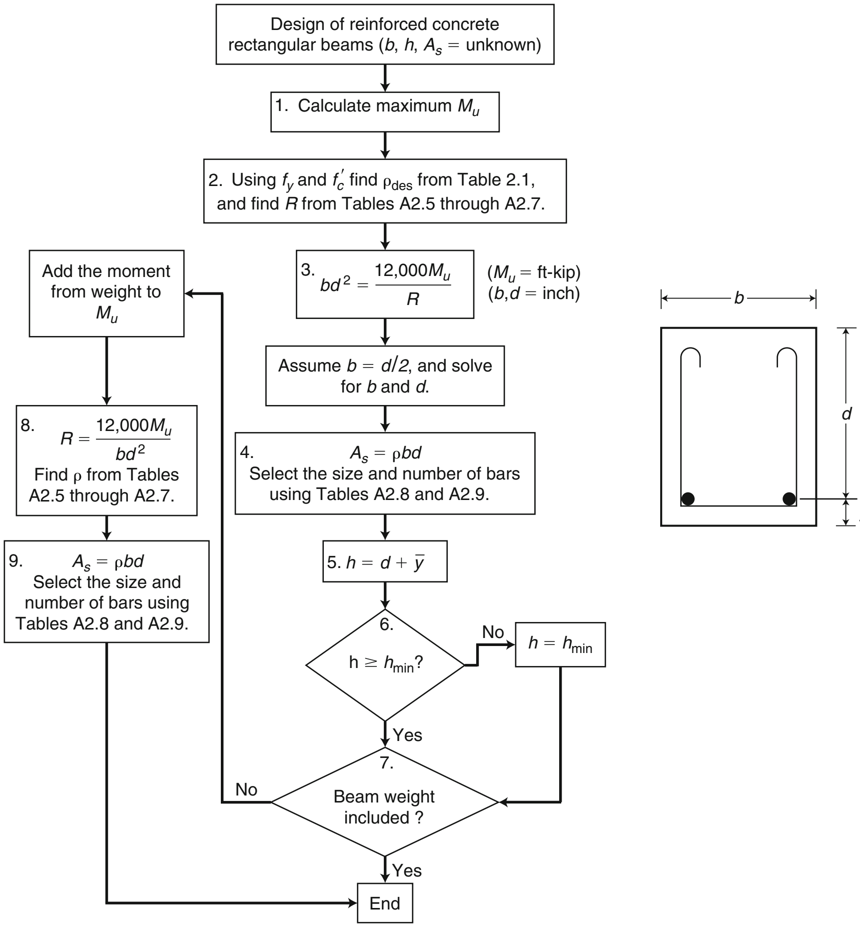


Figure 2.48 Flowchart for the design of reinforced concrete rectangular beams ($b, h, A_s =$ unknown)

and the design of the beam requires that $M_R \geq M_u$. For the most economical case, $M_u = M_R$; therefore

$$\frac{bd^2 R}{12{,}000} = M_u$$

Solving for bd^2:

$$bd^2 = \frac{12{,}000 M_u}{R}$$

Now we must preselect one dimension or the other: We either assume b and solve for d, or the other way around. A third possibility is to assume a certain proportion between d and b, for example, $d/b = 2$; then the problem again becomes straightforward.

Step 4. Use the values of b and d from above to find the required area of reinforcement (A_s):

$$A_s = \rho bd$$

and select the size and number of bars using Tables A2.8 and A2.9.

Step 5. Now find the beam's total depth (h) using the effective depth (d) from Step 3 and size of bars:

$$h = d + \bar{y}$$

Then round h up to the nearest 1 in.

Step 6. Check the beam depth for expected deformation performance by comparing it with h_{min} as recommended by the ACI Code (see Figure 2.41). If $h < h_{\text{min}}$, use h_{min}. In this case you may want to go back and recalculate A_s.

Step 7. Because the beam sizes were not known when the loads were calculated, the beam's self-weight could only be estimated. Experienced designers usually use their own rule of thumb for this purpose. For example, some engineers assume the beam's self-weight to be about 10–20 % of the loads it carries. Others estimate the total depth (h) to be roughly 6–8 % of the span, and $b \cong 0.5h$, and find a preliminary estimate for the beam's weight. But if we desire a more accurate value of the beam's weight, we can estimate it now and make corrections to the dead load and the total M_u.

Step 8. Find a new R value:

$$R = \frac{12{,}000 M_u}{bd^2}$$

and find the corresponding steel ratio (ρ) using Tables A2.5 to A2.7.

Step 9. Find the required area of steel:

$$A_s = \rho bd$$

and select the numbers and sizes of bars from Tables A2.8 and A2.9.

Example 2.13 Determine the required area of steel for a reinforced concrete rectangular beam subject to a total factored moment, $M_u = 400$ ft-kip, that already includes the estimated weight of the beam. $f'_c = 4{,}000\,\text{psi}$ and $f_y = 60{,}000$ psi and use $\rho_{\text{des}} = 0.0124$ from Table 2.1.

Solution

Step 1

$$M_u = 400\,\text{ft-kip}$$

Step 2

$$\text{For } f'_c = 4\,\text{ksi},\ f_y = 60\,\text{ksi} \text{ and } \rho = 0.0124$$
$$\text{using Table A2.6b} \rightarrow R = 596\,\text{psi}$$

Steps 3 and 4 Search now for the beam's sizes:

$$bd^2 = \frac{12{,}000 M_u}{R} = \frac{12{,}000 \times 400}{596}$$
$$bd^2 = 8{,}054\,\text{in.}^3$$

There are an infinite number of solutions, that is, an infinite number of concrete cross sections that will satisfy the design problem, even with the provision that $\rho = 0.0124$ (1.24 %). The table below lists a few solutions. Take your pick!

b	10 in.	12 in.	14 in.	16 in.	18 in.	20 in.
d	28.4 in.	26.0 in.	24.0 in.	22.5 in.	21.2 in.	20.1 in.
$A_{s,\text{ required}}$	3.52 in.^{2}	3.87 in.^{2}	4.17 in.^{2}	4.46 in.^{2}	4.73 in.^{2}	4.98 in^2
$h_{\text{practical}}$	32 in.	30 in.	28 in.	26 in.	24 in.	24 in.

A couple of important observations must be made here. All of these sections have approximately 1.24 % reinforcement, but the quantity of reinforcing grows as the beam becomes wider and shallower. Furthermore, the concrete cross-sectional area (and, consequently, the self-weight of the beam) also increase.

Another way to solve this same problem is to select a d/b ratio. For example, suppose that after determining that

$$bd^2 = 8{,}054\,\text{in.}^3$$

the designer selects a $d/b = 2.0$ ratio. Then:

$$b = \frac{d}{2}$$
$$\frac{d}{2}\left(d^2\right) = \frac{d^3}{2} = 8{,}054$$
$$d = \sqrt[3]{2 \times 8{,}054} = 25.3\,\text{in.}$$
$$b = \frac{d}{2} = \frac{25.3}{2} = 12.65\,\text{in.} \rightarrow \text{Select}\, b = 13\,\text{in.}$$
$$h = 25.3 + 2.5 = 27.8\,\text{in.} \rightarrow \text{Select}\, h = 28\,\text{in.}$$
$$A_s = \rho b d = 0.0124 \times 12.65 \times 25.3 = 3.97\,\text{in.}^2$$

Example 2.14 Use the floor framing plan and loadings of Example 2.12 (Figure 2.47a) to design the reinforced concrete rectangular beam along grid line 2. Assuming that the beam width $b = 12$ in., determine the beam depth, h, and required steel for the location of the maximum bending moment. Use ACI Code coefficients for calculation of moments. Assume that the beam end is integral with the column, $f_y = 60$ ksi, $f_c' = 4$ ksi, and the unit weight of the concrete is 150 pcf. The stirrups are #3 bars.

Solution

Step 1. Find the maximum ultimate moment, M_u.

From Example 2.12:

$$w_D = \frac{105 \times 15}{1{,}000} = 1.58 \text{ kip/ft} \ \ (\text{without the weight of the beam's stem})$$

$$w_L = \frac{100 \times 15}{1{,}000} = 1.5 \text{ kip/ft} \ \ (\text{without the use of live load reduction})$$

$$w_u = 1.2w_D + 1.6w_L = 1.2 \times 1.58 + 1.6 \times 1.5 = 4.3 \text{ kip/ft}$$

$$\ell_n = 30\,\text{ft} - \left(\frac{1}{2} + \frac{1}{2}\right) = 29\,\text{ft}$$

Using the ACI coefficients (Table A2.1) to calculate moments (Figure 2.49a), we determine that the maximum bending moment for the beam along line 2 is at the first interior column (negative moment):

$$M_u = \frac{w_u \ell_n^2}{10} = \frac{4.3(29)^2}{10} = 362\,\text{ft-kip}$$

Step 2. From Table 2.1 $\rightarrow f_c' = 4$ ksi, $f_y = 60$ ksi $\rightarrow \rho_{des} = 0.0124$ From Table A2.6b $\rightarrow R = 596$ psi

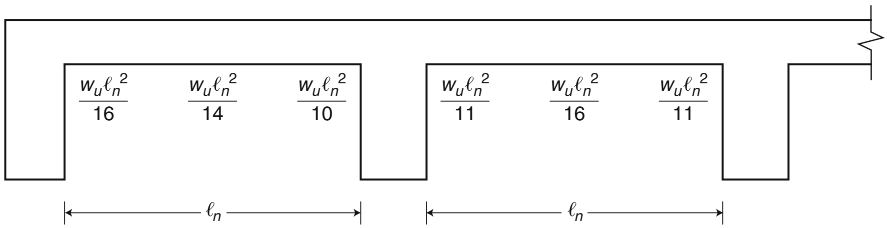


Figure 2.49 (**a**) Moments using the ACI coefficients (Example 2.14)

Step 3. Determine the beam's sizes:

$$bd^2 = \frac{12{,}000M_u}{R} = \frac{12{,}000 \times 362}{596}$$
$$bd^2 = 7{,}289\,\text{in.}^3$$
$$b = 12\,\text{in.} \rightarrow 12d^2 = 7{,}289$$
$$d^2 = 607 \rightarrow d = 24.7\,\text{in.}$$

Step 4. Calculate the required area of steel, and select the number and size of the reinforcing bars:

$$A_s = \rho bd = (0.0124)(12)(24.7) = 3.68\,\text{in.}^2$$
$$\text{From Table A2.9} \rightarrow \text{Try } 4\#9\ (A_s = 4\,\text{in.}^2)$$
$$\text{Table A2.9} \rightarrow b_{\min} = 12\,\text{in.} = 12\,\text{in.} \quad \therefore\ \text{ok}$$
$$\text{Table A2.9} \rightarrow b_{\max} = 34\,\text{in.} > 12\,\text{in.} \quad \therefore\ \text{ok}$$

Step 5. Use the selected bar sizes and the effective depth (d) to calculate the total beam depth (h):

$$\bar{y} = 1\frac{1}{2} + \frac{3}{8} + \frac{1.128}{2} = 2.44\,\text{in.}$$
$$h = d + \bar{y} = 24.7 + 2.44 = 27.14\,\text{in.}$$

This value is usually rounded up to the nearest 1 in. Thus:

$$h = 28\,\text{in.}$$

Step 6. Check to see if the beam depth is more than the recommended minimum for deflection control. The case for the beam with one end continuous results in the largest required depth (see Figure 2.41):

$$h_{\min} = \frac{\ell}{18.5} = \frac{30 \times 12}{18.5} = 19.5\,\text{in.} < 28\,\text{in.} \quad \therefore\ \text{ok}$$

Step 7. Calculate the correct beam weight. The total beam depth is 28 in. The concrete slab, however, is 6 in. thick; therefore, the beam depth (the stem) below the slab is 28 in. – 6 in. = 22 in.

$$\text{Stem weight} = \frac{150\left(\frac{12}{12} \times \frac{22}{12}\right)}{1{,}000} = 0.28\,\text{kip/ft}$$

The total uniform dead load acting on the beam (w_D):

$$w_D = 1.58 + 0.28 = 1.86\,\text{kip/ft}$$

$$w_u = 1.2 \times 1.86 + 1.6 \times 1.5 = 4.63\,\text{kip/ft}$$

$$(Mu)^- = \frac{w_u \ell_n^2}{10} = \frac{4.63(29)^2}{10} = 390\,\text{ft-kip}$$

Step 8.

$$R = \frac{12{,}000 M_u}{bd^2}$$

$$R = \frac{12{,}000 \times 390}{12(24.7)^2}$$

$$R = 639\,\text{psi}$$

From Table A2.6b → ρ = 0.0134 (this corresponds to $R = 638$ psi, which is very close).

Step 9.

$$A_s = \rho bd = (0.0134)(12)(24.7) = 3.97\,\text{in.}^2$$

From Table A2.9 → Use 4 #9 bars.

The selected reinforcement is the same as it was for the previous design cycle. Figure 2.49b shows the sketch of the beam.

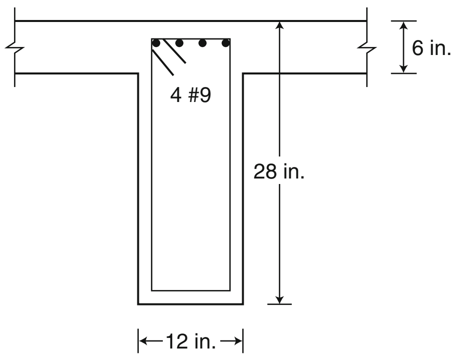


Figure 2.49 (**b**) Final design of Example 2.14

2.26 Slabs

Slabs or plates are very important components of reinforced concrete structures. The elements we have studied until now, could be described abstractly by a line: Bending of that line in a vertical plane by the loads described their behavior. These elements are called linear elements, because one of their three dimensions, the length, is much greater than the other two, i.e. the dimensions of the cross section.

Slabs (plates), on the other hand, cannot be described by a line. They have two dimensions, length and width, that are significantly larger than the third one, the thickness. Mathematically plates are described as planes. A mathematically exact analysis of slabs is not provided here but a discussion of their behavior is in order.

A slab can bend in two directions, so its bent shape is described not by the shape of a single line, but rather by the bent shape of a surface. A slab must carry the loads to the supports, hence it will bend accordingly. The behavior of a slab depends on the support conditions, that is, on how the designer chose to support it. The types of supports are:

(a) *Line supports* (beams, girders, walls) Slabs that are supported by these types of building elements are referred to as *one-* or *two-way slabs*. In this chapter we discuss only one-way slabs, although an attempt is made to explain the difference between one-way and two-way slabs. Chapter 6 discusses the different types of two-way slabs used as floor systems.
(b) *Point supports* (columns, posts, suspension points, etc.) Slabs supported by these types of supports are referred to as *flat slabs* or *flat plates*. We will discuss these in more detail in Chapter 6.
(c) *Continuous media* (slabs on grade)

The simple sketch in Figure 2.50 illustrates the behavior of a *one-way* slab. The beams that support the slab are poured together with the slab. Slabs are often not just single span, as shown here, but continuous over several spans defined by the beams' spacing. In the case of uniformly distributed loads, the most common for slabs (for it is quite rare to place large concentrated loads on slabs), every one-foot-wide strip of the slab is loaded identically; hence, the design is limited to only a one-foot-wide strip and the selection of the reinforcing for that strip. Then it is assumed that all the

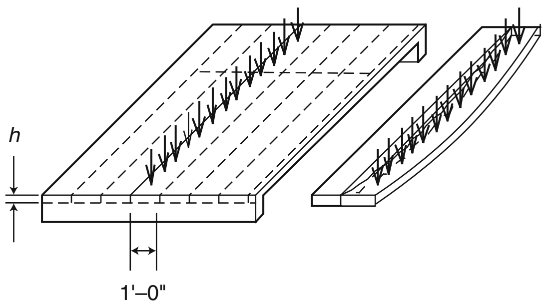


Figure 2.50 One-way slab behavior

other strips behave the same way, that is, they need the same amount of reinforcing. Figure 2.50 also illustrates that if only one imaginary strip is loaded, the adjacent slab strips will have to help. This is because it is impossible for a monolithic structure to get the deformation diagram shown on the right of the figure.

Figures 2.51 and 2.52 show the framing plan of different reinforced concrete floor/roof systems. In Figure 2.51a, slab S-1 is supported by the surrounding beams

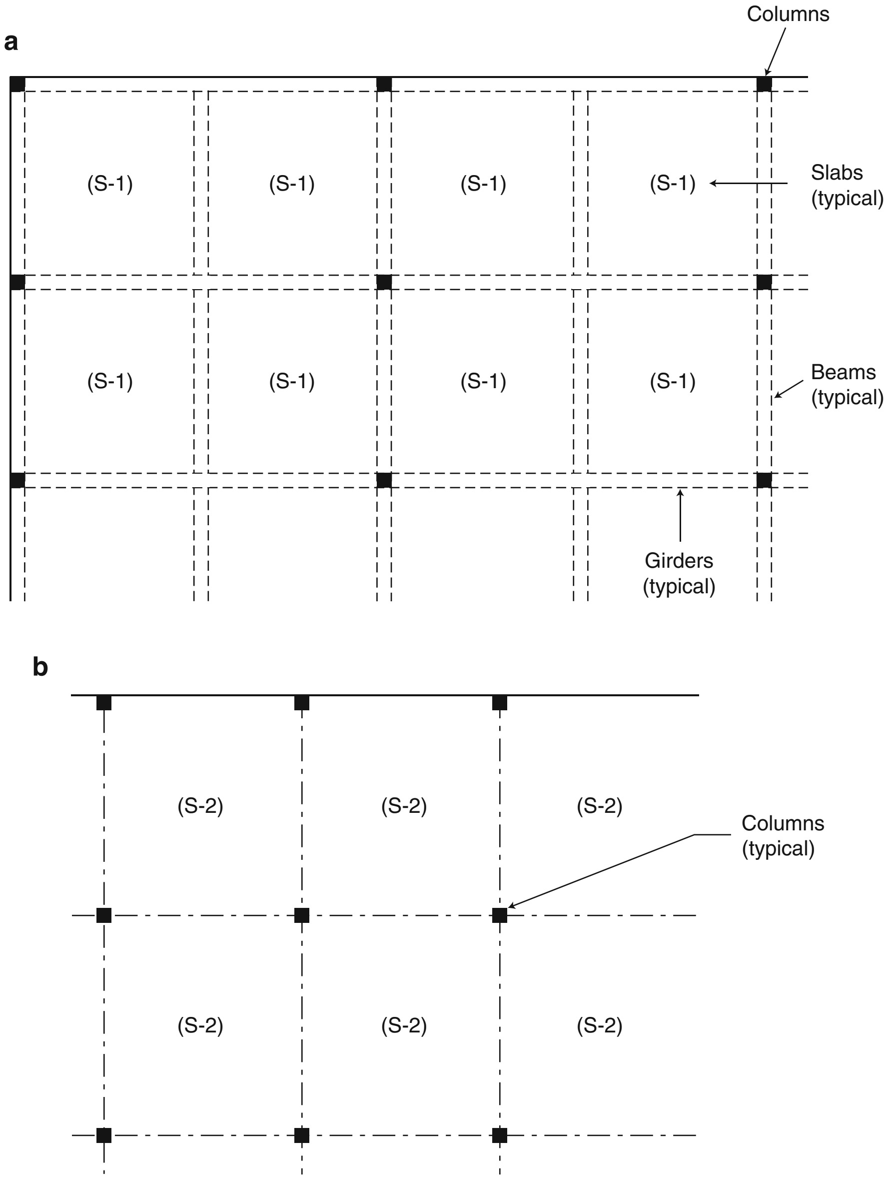


Figure 2.51 (**a**) Slabs in beam girder floor system; (**b**) flat plate slab

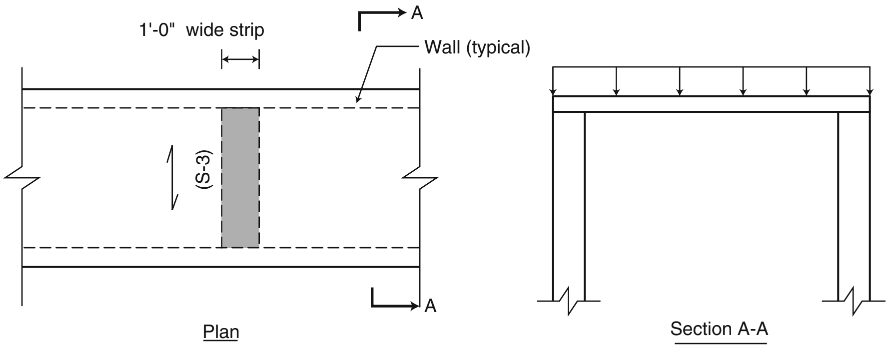


Figure 2.52 Slab supported by walls

and girders. In Figure 2.51b slab S-2 is part of a flat plate floor system, in which slabs are directly supported by columns. In Figure 2.52 slab S-3 is supported by two parallel walls, which can be made of concrete or masonry.

2.27 Behavior of Reinforced Concrete Slabs Under Loads

Depending on the geometry and location of the supports, most slabs are divided into two groups: one-way slabs, and two-way slabs.

One-way slabs bend mainly in one direction. If the supporting elements of the slab are only two parallel members such as beams or walls, the slab is forced to bend in a perpendicular direction. Figure 2.52 shows the plan view of a slab supported by two parallel walls. Because every 1 ft wide strip can be considered to be the same as all the others, only a single 1 ft wide strip of slab needs to be considered in analysis and design.

The slab's geometry is an important factor that affects its behavior under loads. Figure 2.53a shows a slab supported by edge beams B-1 and B-2. Determining the distribution of loads from the slab to the supporting beams can be simplified by assuming that the load is transferred to the nearest beam. Such an assumption is represented by drawing 45-degree lines from each slab corner. The enclosed areas show the tributary loads to be carried by each beam. Beam B-1 will carry large trapezoidal loads compared to the triangular loads that will be carried by beam B-2. As the ratio of longer span (ℓ_ℓ) to shorter span (ℓ_s) increases, B-1 carries more loads than does B-2, that is, more loads are transferred in the shorter span of the slab.

In fact, if the ratio $\frac{\ell_\ell}{\ell_s}$ is greater than or equal to 2.0 $\left(\frac{\ell_\ell}{\ell_s} \geq 2.0\right)$, the load carried by B-2 is quite small, and it can be neglected altogether. Therefore, if $\frac{\ell_\ell}{\ell_s} \geq 2.0$, the slab behaves as a one-way slab for all practical purposes, even though the slab is supported on all four edges.

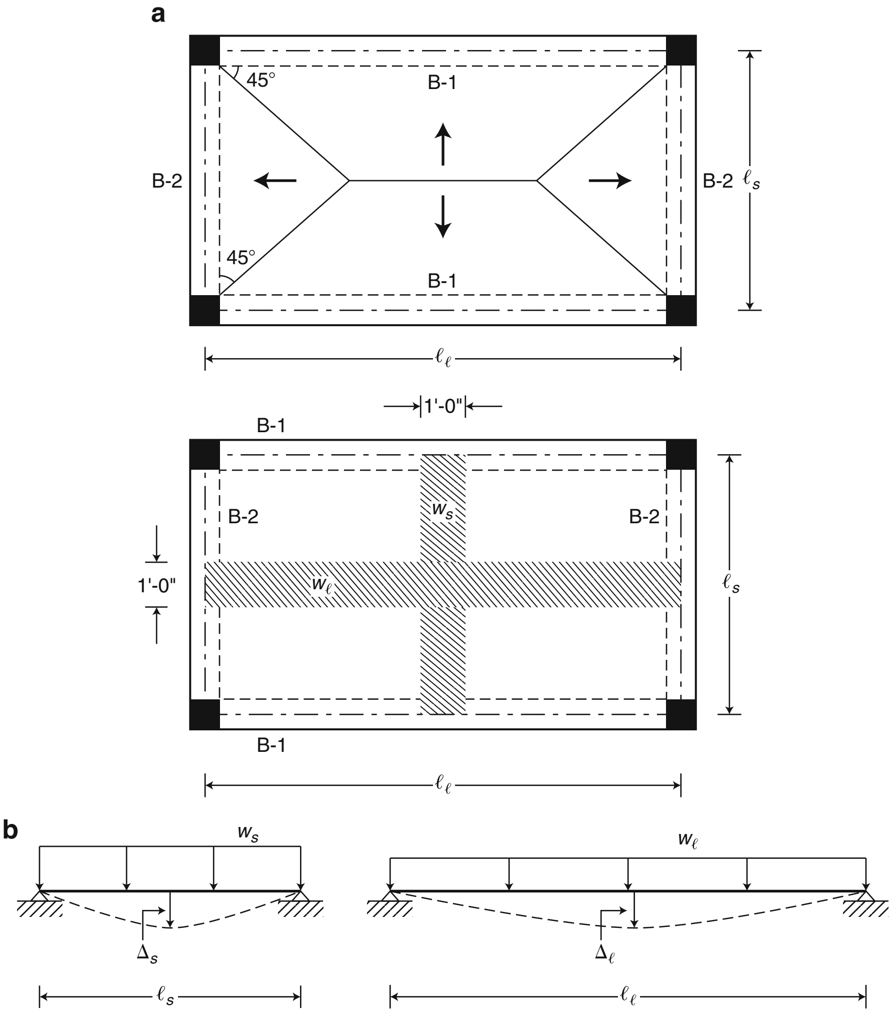


Figure 2.53 (**a**) Slab (edge supported); (**b**) slab load distribution

To better understand this assumption, consider Figure 2.53b, in which two 1 ft wide strips of slab in the long (ℓ) and short (s) directions are shown at midspan for both. The load carried by the short 1 ft wide strip is w_s, and the load carried by the long 1 ft wide strip is w_ℓ. If we assume that the slab is simply-supported along all edges, we can calculate the maximum mid-span deflections for the short (Δ_s) and long (Δ_ℓ) 1 ft wide strips from Equations (2.54) and (2.55).

$$\Delta_s = \frac{5 w_s \ell_s^4}{384 EI} \tag{2.54}$$

$$\Delta_\ell = \frac{5w_\ell \ell_\ell^4}{384EI} \tag{2.55}$$

The two deflections must be equal. Thus, an expression may be developed that relates the loads and spans, as shown in Equation (2.56).

$$\begin{aligned} \Delta_s &= \Delta_\ell \\ \frac{5w_s \ell_s^4}{384EI} &= \frac{5w_\ell \ell_\ell^4}{384EI} \\ w_s \ell_s^4 &= w_\ell \ell_\ell^4 \\ \frac{w_s}{w_\ell} = \frac{\ell_\ell^4}{\ell_s^4} &= \left(\frac{\ell_\ell}{\ell_s}\right)^4 \end{aligned} \tag{2.56}$$

The assumption for one-way behavior is $\ell_\ell/\ell_s \geq 2.0$. If $\ell_\ell/\ell_s = 2.0$ is substituted into Equation (2.56), w_s is equal to $16w_\ell$. Thus, the load transferred in the shorter direction (w_s) is 16 times larger than that transferred in the long direction (w_ℓ), when $\ell_\ell/\ell_s \geq 2.0$. Therefore, it is reasonable to assume that the loads are transferred mainly in the shorter direction.

Despite all the foregoing reasoning, structural engineers often design slabs as one-way slabs, even when the slabs' proportions do not satisfy the $\ell_\ell/\ell_s \geq 2.0$ requirement. The reason is that the shrinkage and temperature reinforcing needed in the long direction is usually quite enough to satisfy the small moment's requirements. Figure B2.3 in Appendix B shows a one-way slab supported by reinforced concrete beams. Design and analysis of floor systems with two-way slabs are discussed in Chapter 6.

2.28 Reinforcement in One-Way Slabs

In general, two types of reinforcement are used in one-way slabs: main reinforcement, and shrinkage and temperature reinforcement.

2.28.1 Main Reinforcement

The *main reinforcement* resists the bending moments. It is designed to act in the direction of the one-way slab's bending, which is along the shorter span length. Figure 2.54 shows the main reinforcement in a one-way slab supported by two parallel walls. The slab is assumed to be simply supported by the walls. In other words, no moment is transferred from the slab to the walls. Because the bottom portion of slab is in tension, the main reinforcement is placed in the bottom.

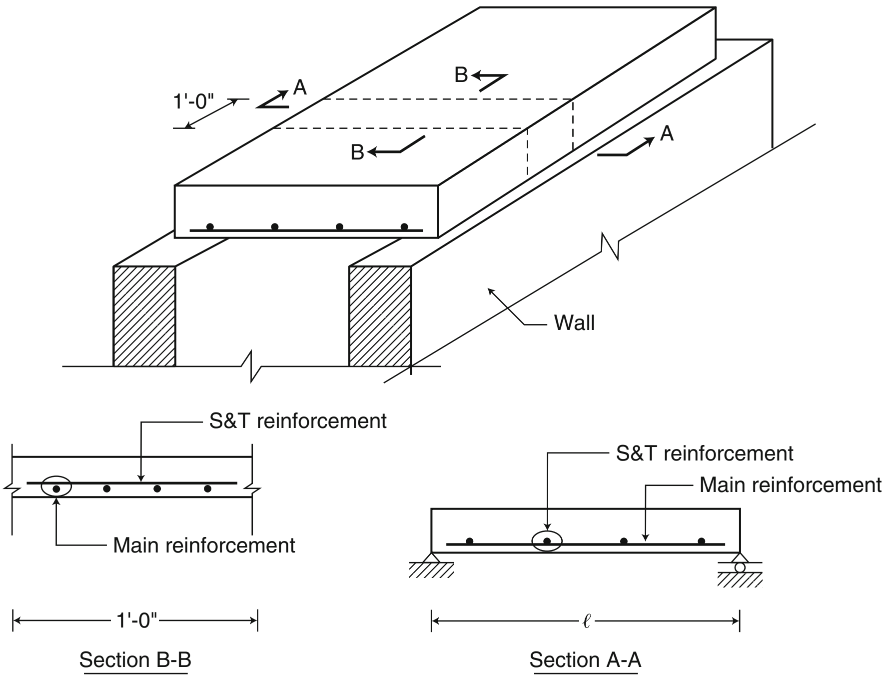


Figure 2.54 One-way slab reinforcement (simple span)

Similarly, the main reinforcement is placed in a continuous construction where tension develops. For this case, as shown in Figure 2.55, the main reinforcement is at the bottom of the slab in the midspan region (positive moment) and at the top of the slab over the supports (negative moment). Typically, #4 bars or larger are used as main reinforcement, #3 bars are susceptible to permanent distortion caused by the construction crew walking over them. This is more critical for the top (negative moment) bars as the slab effective depth (d) may be reduced.

2.28.2 Shrinkage and Temperature (S & T) Reinforcement

As discussed in Chapter 1, fresh concrete loses water and shrinks soon after placement. In addition, variations in temperature cause the concrete to expand and contract. These volume changes, when restrained, may result in cracking of concrete, especially in the early stages of strength development. Reinforcing bars are used to resist developing tensions in order to minimize cracks in concrete caused by shrinkage and temperature changes. The main longitudinal reinforcement in beams plays that role as well. Because the cross-sectional dimensions of beams are relatively small and beams may freely change their cross-sectional dimensions

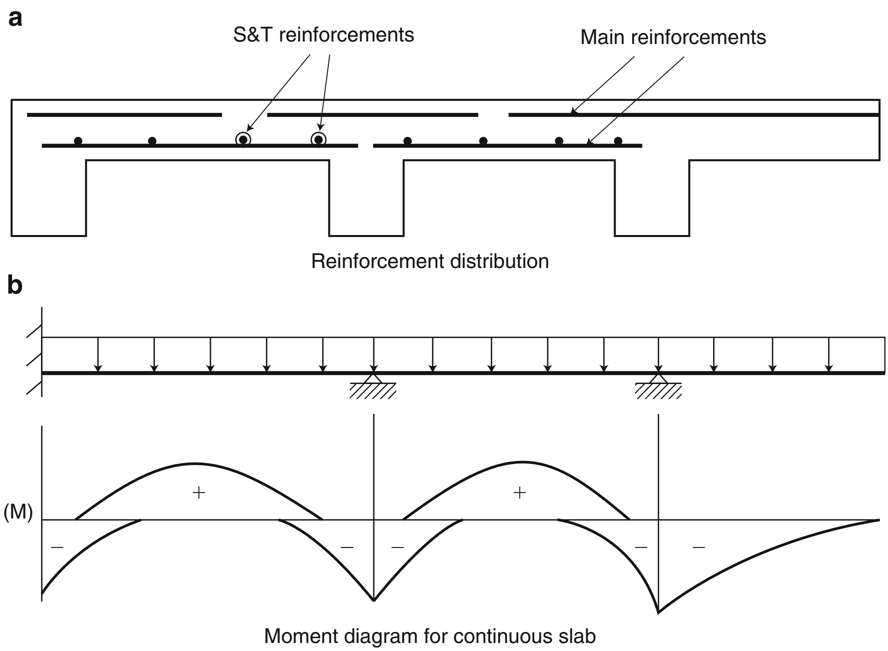


Figure 2.55 One-way slab reinforcement (continuous construction). (**a**) Reinforcement distribution. (**b**) Moment diagram for continuous slab (refer also to Figure 2.14)

without restraint, shrinkage and temperature reinforcement are not needed perpendicular to the main bars.

This is not the case in reinforced concrete slabs. Slabs typically have large dimensions in two directions, thus they need shrinkage and temperature reinforcement, which is placed in the direction perpendicular to the main reinforcement. Figures 2.54 and 2.55 show such reinforcement for simple-span one-way slabs and continuous one-way slabs, respectively. In addition, temperature and shrinkage reinforcement helps distribute concentrated loads to a wide zone transversely to the one-way direction. (This is necessary in bridges, for example, to distribute large wheel loads onto a much wider strip than the one directly affected by the concentrated load.)

2.28.3 Minimum Reinforcements for One-Way Slabs

As discussed above, two types of reinforcement are used in one-way slabs. The ACI Code sets the following minimum reinforcement criteria for both the main and the shrinkage and temperature reinforcements.

Minimum Main Reinforcement The minimum main reinforcement for slabs is equal to that required for shrinkage and temperature reinforcements (ACI Code, Section 7.6.1):

$$A_{s,\min} = A_{s(\mathrm{S\&T})} \tag{2.57}$$

In other words, if the calculated main reinforcement is less than that required for shrinkage and temperature reinforcement, the designer must use at least the latter amount.

Minimum Shrinkage and Temperature Reinforcement The ACI Code (Section 7.6.1.1) requires shrinkage and temperature reinforcement based on the grade of steel, as given in Equations (2.58)–(2.60).

$$\text{For } f_y = 40 \text{ or } 50\,\text{ksi} \rightarrow A_{s(\mathrm{S\&T})} = 0.002bh \tag{2.58}$$

$$\text{For } f_y = 60\,\text{ksi} \rightarrow A_{s(\mathrm{S\&T})} = 0.0018bh \tag{2.59}$$

$$\text{For } f_y > 60\,\text{ksi} \rightarrow A_{s(\mathrm{S\&T})} = \frac{0.0018 \times 60}{f_y} bh \geq 0.0014bh \tag{2.60}$$

In Equations (2.58)–(2.60), $b = 12$ in. (slab width), which corresponds to the width of the 1 ft wide strip, h is the overall thickness of the slab in inches, and $A_{s(\mathrm{S\,\&T})}$ is the area of steel in square inches per foot of width.

Minimum Concrete Cover for the Reinforcement in Slabs A minimum concrete cover is needed for the reinforcement to prevent various detrimental effects of the environment on reinforcing bars. Concrete cover is always measured from the closest concrete surface to the first layer of reinforcing. This is shown in Figure 2.56. Section 20.6.1.3 of the ACI Code requires a minimum concrete cover of ¾ in. for #11 and smaller bars, and 1.5 in. for #14 and #18 bars, provided that the concrete slab is not exposed to weather or not in contact with the ground.

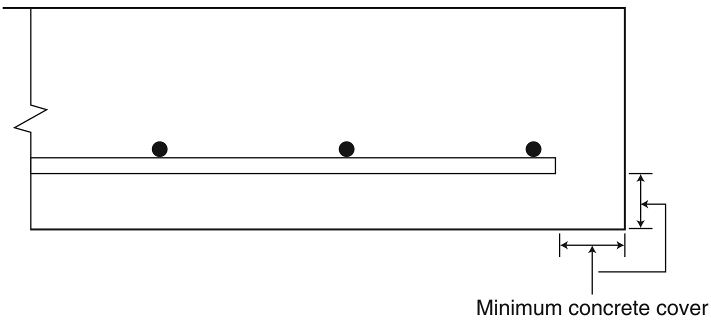


Figure 2.56 Minimum cover for slabs

Bar Spacing in Reinforced Concrete Slabs No specific minimum spacing of bars is required in slabs other than what was already discussed for beams. For practical reasons, however, bars are not placed closer than 3–4 in.

The ACI Code has different maximum spacing requirements for the main and the shrinkage and temperature reinforcements. These are as follows:

Maximum Spacing of Main Reinforcement Bars ACI 318-14 has two sets of requirements regarding maximum bar spacing for the main reinforcement in one-way slabs: (1) Section 7.7.2.3 requires that the maximum spacing of bars be limited to three times the slab thickness or 18 in., whichever is smaller; and (2) Section 24.3.2 limits the maximum main reinforcement spacing (s) of one-way slabs, as calculated by Equation (2.53), in order to control the width and spacing of flexural cracks.

We can use the required minimum cover of 3/4 in. for one-way slabs ($c_c = 0.75$ in.) and $f_s = 2/3\ f_y = 2/3\ (60{,}000) = 40{,}000$ psi to determine the maximum spacing for $f_y = 60$ ksi reinforcement. Substituting into Equation (2.53):

$$s = 15\left(\frac{40{,}000}{40{,}000}\right) - 2.5(0.75) \leq 12\left(\frac{40{,}000}{40{,}000}\right)$$

$$s = 13.1\,\text{in.} \leq 12\,\text{in.}$$

$$s = 12\,\text{in.}$$

Therefore, the maximum main reinforcement spacing with $f_y = 60$ ksi steel for one-way slabs is given by Equation (2.61a).

$$s_{\text{max, main}} = \min\{3h,\ 12\,\text{in.}\} \tag{2.61a}$$

Similarly, when using $f_y = 40$ ksi steel as main reinforcement, Equation (2.53) will simplify to Equation (2.61b).

$$s_{\text{max, main}} = \min\{3h,\ 18\,\text{in.}\} \tag{2.61b}$$

Maximum Bar Spacing of Shrinkage and Temperature Reinforcement ACI Code, Section 7.7.6.2.1 limits the spacing of the shrinkage and temperature reinforcements to five times the slab thickness or, 18 in., whichever is smaller:

$$s_{\text{max, (S\&T)}} = \min\{5h,\ 18\,\text{in.}\} \tag{2.62}$$

Minimum Thickness of Slab for Deflection Control The minimum recommended thickness for one-way slabs required to adequately control excessive deflections is based on Table 7.3.1.1 of the ACI Code, which is summarized graphically in Figure 2.42. Lesser thicknesses are permitted if the designer can show through a detailed deflection analysis that the Code's serviceability requirements are met.

2.29 Areas of Reinforcing Bars in Slabs

A 1 ft (12 in.) wide strip of slab is typically used for the analysis and design of one-way slabs. Thus, it is advantageous to define the amount of steel in a 1 ft wide strip as a function of the bar size and the spacing.

Table A2.10 lists spacing and bar sizes for slabs. The table provides the areas of reinforcement averaged out to 1 ft width for different sizes and spacing of bars. (One can interpolate for ½ in. spacing increments, if so desired.)

For example, with #5@8 in. o.c. (#5 bar at 8 in. on-center spacing), the table, under #5 bars spaced at 8 in., provides the area of steel per foot of section 0.47 in.^{2}. In other words, 0.47 in.^{2}/ft is equivalent to one #5 bar every 8 in.

Another example: If 0.50 in.^{2} of reinforcement is required for a 1 ft wide strip of a slab, the table offers several options, including #4@4 in. ($A_s = 0.60$ in.^{2}), #5@7 in. ($A_s = 0.53$ in.^{2}), #6@ 10 in. ($A_s = 0.53$ in.^{2}), and so on.

2.30 Analysis of Reinforced Concrete One-Way Slabs

In general, one-way slabs and reinforced concrete beams are analyzed very similarly. There are a few differences, however. These are listed below:

1. For the analysis of one-way slabs, b is always 12 in.
2. Slabs require a different amount of concrete cover over the reinforcement.
3. Slabs require shrinkage and temperature reinforcement.
4. The Code-specified minimum amounts of reinforcing steel for slabs and beams are different.
5. Minimum required depth/span ratios for adequate control of deflection are different.
6. Bar spacing requirements are different.

Figure 2.57 summarizes the steps for the analysis of reinforced concrete one-way slabs. They are as follows:

Step 1. Calculate the steel ratio, $\left(\rho = \dfrac{A_s}{bd}\right)$. A_s is the area of steel in a 1 ft wide strip of slab from Table A2.10. Compare ρ with $\rho_{\max}$ from Table A2.3. The maximum permitted steel ratio is the same for beams and slabs.

Step 2. Compare A_s with $A_{s,\min}$, which is the minimum required area of steel for the control of shrinkage and temperature-induced volumetric changes. If $A_s \leq A_{s,\min}$, the proportioning of steel and concrete is not acceptable according to the current ACI Code and the slab's use is illegal. If $A_s \geq A_{s,\min}$, however, then one of the following methods can be used to check the adequacy of the slab:

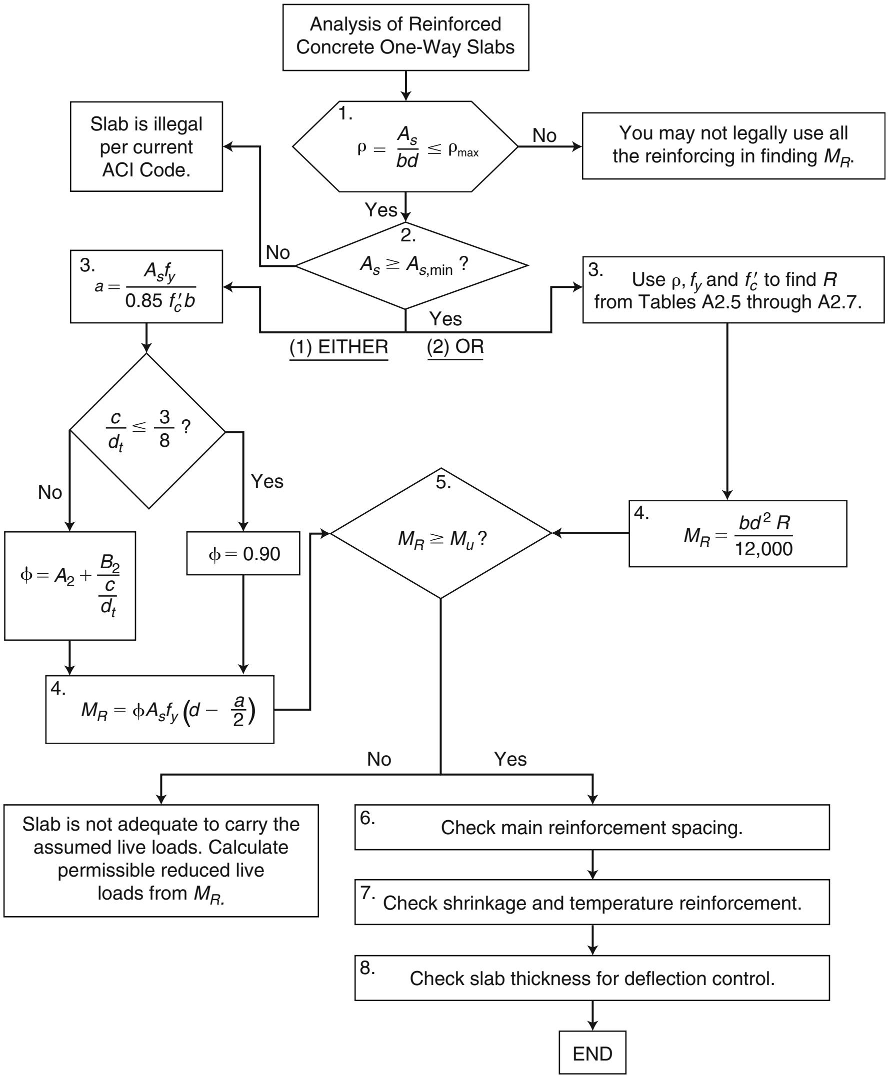


Figure 2.57 Flowchart for the analysis of reinforced concrete one-way slabs

Method I

Step 3. Calculate the depth of the compression zone:

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

Determine the location of the neutral axis (c):

$$c = \frac{a}{\beta_1}$$

If $\frac{c}{d_t} \leq \frac{3}{8}$, the section is tension-controlled and $\phi = 0.90$. Otherwise, the section will be in the transition zone. Calculate the strength reduction factor, ϕ:

$$\phi = A_2 + \frac{B_2}{\frac{c}{d_t}}$$

A_2 and B_2 are listed in Table A2.2b.

Step 4. Calculate the section's resisting moment (M_R):

$$M_R = \phi A_s f_y \left(d - \frac{a}{2}\right)$$

(If A_s is in in.^{2}, f_y ksi, d and a in., then M_R will have kip-in. unit. Divide the result by 12 to obtain M_R in the customary units of kip-ft).

Step 5. Compare M_R with the maximum factored moment from the applied loads. If $M_R < M_u$ the slab is not adequate to carry the *assumed* loads. Proceed to calculate a new permissible live load that the slab may legally support. If $M_R \geq M_u$, the section can take the assumed loads, but the reinforcing still needs to be checked for conformance with other Code requirements.

Step 6. Check spacing requirements. The maximum allowable spacing of main reinforcement is min{$3h$, 12 in.}, or min{$3h$, 18 in.} for $f_y = 60$ ksi and $f_y = 40$ ksi steel, respectively.

$$3\,\text{in.} \leq s \leq \min\{3h, 12\,\text{in.}\} \quad \text{for } f_y = 60\,\text{ksi}$$
$$3\,\text{in.} \leq s \leq \min\{3h, 18\,\text{in.}\} \quad \text{for } f_y = 40\,\text{ksi}$$

Step 7. Check the amount and spacing of shrinkage and temperature reinforcement, $(A_s)_{S\,\&T}$ (Refer to Equations (2.58)–(2.60).)

$$3\,\text{in.} \leq s_{S\&T} \leq \min\{5h, 18\,\text{in.}\}$$

Step 8. Check the thickness of the slab against the minimum thickness of one-way slabs for desirable deformation control (see Figure 2.42).

$$h_{\min} = \ell/20 \ \text{for simply-supported slabs}$$
$$h_{\min} = \ell/10 \ \text{for cantilevered slabs}$$
$$h_{\min} = \ell/28 \ \text{for both ends continuous slabs}$$
$$h_{\min} = \ell/24 \ \text{for one end continuous slabs}$$

If the slab thickness is less than the above limits, calculate the deflection and check it against the Code's serviceability requirements.

Method II

Steps 1 and 2 are the same as in Method I.

Step 3 Use f_y, f_c', and the calculated steel ratio (ρ) to obtain the resistance coefficient, *R*, from Tables A2.5 to A2.7.

Step 4 Use the *R* value to calculate the section's resisting moment.

$$M_R = \frac{bd^2R}{12{,}000}$$

R is in psi, $b = 12$ in., and *d* in inches. M_R will be in units of ft-kip. Steps 5, 6, 7, and 8 are the same as in Method I.

Example 2.15 Figure 2.58 shows a section through a reinforced concrete simply-supported one-way slab of an existing building. The maximum moment from dead loads, including the slab weight, is 3.0 (ft-kip)/ft, and that from live loads is 2.0 (ft-kip)/ft. Check the adequacy of the slab, including the shrinkage and temperature reinforcements, using (a) Method I, and (b) Method II.

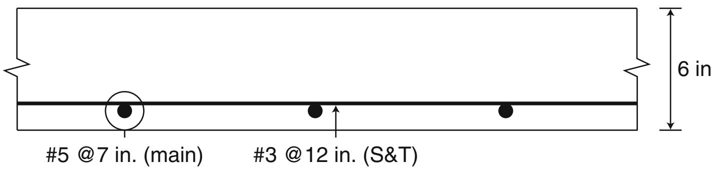


Figure 2.58 Sketch of one-way slab for Example 2.15

Use a concrete cover of ¾ in., $f_c' = 3.0$ ksi, and $f_y = 40.0$ ksi.

Solution

Step 1. Check the reinforcement ratio in the slab:

Diameter of #5 bars

$$\bar{y} = \frac{3}{4} + \frac{\frac{5}{8}}{2} = 1.06 \text{ in.}$$

Cover

$$d = h - \bar{y} = 6 \text{ in.} - 1.06 \text{ in.} = 4.94 \text{ in.}$$

#5@ 7 in. (main reinforcement) → Table A2.10 → $A_s = 0.53$ in.^{2}/ft

$$\rho = \frac{A_s}{bd} = \frac{0.53}{12 \times 4.94} = 0.00894$$

$f'_c = 3\,\text{ksi} \rightarrow$ Table A2.3 $\rightarrow \rho_{\max} = 0.0232 > 0.00894 \therefore$ok
$f_y = 40$ ksi

Step 2. Check the minimum area of main reinforcement. For slabs, this area is the same as the requirement for shrinkage and temperature reinforcement:

$$A_{s,\min} = A_{s(\text{S\&T})} = 0.002bh \quad \left(f_y = 40\,\text{ksi}\right)$$
$$A_{s,\min} = (0.002)(12)(6) = 0.14\,\text{in.}^2/\text{ft}$$
$$A_s = 0.53\,\text{in.}^2/\text{ft} > 0.14\,\text{in.}^2/\text{ft} \quad \therefore \text{ ok}$$

(a) *Method I*

Step 3. Calculate the depth of the compression zone:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.53 \times 40}{0.85 \times 3 \times 12}$$
$$a = 0.69\,\text{in.}$$

The neutral axis is located at c:

$$c = \frac{a}{\beta_1} = \frac{0.69}{0.85} = 0.81\,\text{in.}$$
$$d_t = d = 4.94\,\text{in.}$$
$$\frac{c}{d_t} = \frac{0.81}{4.94} = 0.164 < 0.375 \quad \therefore \ \phi = 0.90$$

Step 4.

$$M_R = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right)$$
$$M_R = (0.9)(0.53)(40)\left(4.94 - \frac{0.69}{2}\right)$$
$$M_R = \frac{87.7\,\text{in.-kip}}{12\,\text{in./ft}} = 7.3\,\text{ft-kip}$$

Step 5. Calculate the factored applied moment on the slab:

$$M_u = 1.2M_D + 1.6M_L$$
$$M_u = 1.2 \times 3.0 + 1.6 \times 2.0 = 6.8\,\text{ft-kip} < 7.3\,\text{ft-kip} \quad \therefore \text{ ok}$$

Step 6. Check the main reinforcement spacing:

$$3\,\text{in.} \le s \le \min\{3h, 18\,\text{in.}\}$$

The main reinforcement is #5@ 7 in.:

$$3\,\text{in.} < 7\,\text{in.} < \min\{3 \times 6\,\text{in.}, 18\,\text{in.}\}$$
$$3\,\text{in.} < 7\,\text{in.} < 18\,\text{in.} \quad \therefore \text{ ok}$$
$$\textit{Slab is ok.}$$

Step 7. Check the shrinkage and temperature reinforcements:

$$A_{s(\text{S\&T})} = 0.002bh = (0.002)(12)(6) = 0.14\,\text{in.}^2/\text{ft}$$

$$\text{From Table A2.10} \rightarrow \#3@12\,\text{in.} \rightarrow A_s = 0.11\,\text{in.}^2/\text{ft} < 0.14\,\text{in.}^2/\text{ft} \;\therefore\; \text{N.G.}$$

Therefore, the shrinkage and temperature reinforcement in the slab does not satisfy the current ACI Code's minimum requirement.

(b) *Method II*

$\rho = 0.00894$

Step 3. $f_c' = 3\,\text{ksi} \rightarrow \text{Table A2.5a} \rightarrow R = 299\,\text{psi}$ (by interpolation)
$f_y = 40$ ksi

Step 4.

$$M_R = \frac{bd^2R}{12{,}000}$$

$$M_R = \frac{(12)(4.94)^2(299)}{12{,}000}$$

$$M_R = 7.3\,\text{ft-kip}$$

This value is the same as the resisting moment we calculated in Step 4 using Method I. Steps 5, 6, and 7 are the same as those of Method I.

Example 2.16 Figure 2.59 shows the partial floor framing plan and section of a reinforced concrete floor system. The weight of the ceiling and floor finishing is 5 psf, the mechanical and electrical systems are 5 psf, and the partitions are 15 psf. The floor live load is 150 psf. The concrete is normal weight, $f_c' = 4\,\text{ksi}$, and $f_y = 60$ ksi. Check the adequacy of slab S-1 in the exterior bay at (a) midspan, and (b) over the interior supporting beam. Assume the slab is cast integrally with the supporting beams and use ACI code coefficients to calculate moments. Use ¾ in. cover for the slab.

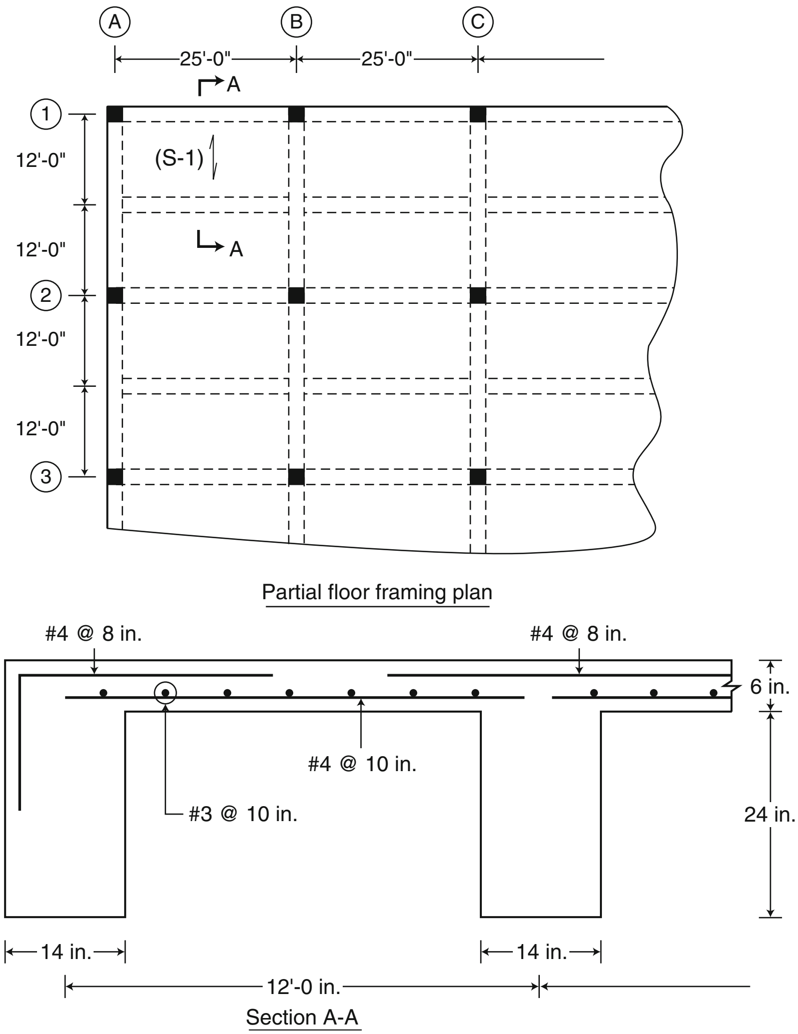


Figure 2.59 Framing plan and section for Example 2.16

Solution

(a) *Check the Slab at the Midspan*

Step 1. The main reinforcement at the midspan (positive moment) is #4@ 10 in.

$$\#4@10\,\text{in.} \rightarrow \text{Table A2.10} \rightarrow A_s = 0.24\,\text{in.}^2/\text{ft}$$

$$\bar{y} = \frac{3}{4} + \frac{\frac{4}{8}}{2} = 1.0\,\text{in.}$$
$$d = h - \bar{y} = 6\,\text{in.} - 1\,\text{in.} = 5\,\text{in.}$$

$$\rho = \frac{A_s}{bd} = \frac{0.24}{(12)(5)} = 0.0040$$
$$f'_c = 4\,\text{ksi} \rightarrow \text{Table A2.3} \rightarrow \rho_{\max} = 0.0207 > 0.004 \quad \therefore \text{ ok}$$
$$f_y = 60\,\text{ksi}$$

Step 2.

$$A_{s,\min} = A_{s(\text{S\&T})} = 0.0018bh \quad \left(f_y = 60\,\text{ksi}\right)$$
$$A_{s,\min} = (0.0018)(12)(6) = 0.13\,\text{in.}^2/\text{ft}$$
$$A_s = 0.24\,\text{in.}^2/\text{ft} > 0.13\,\text{in.}^2/\text{ft} \quad \therefore \text{ ok}$$

Method II is followed for the rest of the solution, as it requires fewer steps.

Step 3.

$$\rho = 0.0040$$
$$f'_c = 4\,\text{ksi} \rightarrow \text{Table A2.6b} \rightarrow R = 208\,\text{psi}$$
$$f_y = 60\,\text{ksi}$$

Step 4.

$$M_R = \frac{bd^2R}{12{,}000}$$
$$M_R = \frac{(12)(5)^2(208)}{12{,}000}$$
$$M_R = 5.2\,\text{ft-kip}$$

Step 5. The slab's dead and live loads are:

$$\text{Weight of slab} = 150\left(\frac{6}{12}\right) = 75\,\text{psf}$$
$$\text{Ceiling and the floor finishing} = 5\,\text{psf}$$
$$\text{Mechanical and electrical} = 5\,\text{psf}$$
$$\text{Partitions} = 15\,\text{psf}$$

$$\text{Total dead load} = 100\,\text{psf}$$
$$\text{Total live load} = 150\,\text{psf}$$

The slab's tributary width is 1′–0″:

$$w_D = \frac{100 \times 1}{1{,}000} = 0.10\,\text{kip/ft}$$

$$w_L = \frac{150 \times 1}{1{,}000} = 0.15\,\text{kip/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2 \times 0.10 + 1.6 \times 0.15$$

$$w_u = 0.36\,\text{kip/ft}$$

$$\ell_n = 12\,\text{ft} - \frac{14\,\text{in.}}{12} = 10.83\,\text{ft}$$

The maximum factored moment at the midspan of the exterior bay of the slab is:

$$M_u = \frac{w_u \ell_n^2}{14}$$

$$M_u = \frac{(0.36)(10.83)^2}{14}$$

$$M_u = 3.0\,\text{ft-kip} < M_R = 5.2\,\text{ft-kip} \quad \therefore \text{ ok}$$

Because M_R is much larger than M_u, the slab is overdesigned for positive moment.

Step 6. Check the spacing requirements for the main reinforcement:

$$3\,\text{in.} \le s \le \min\{3h, 12\,\text{in.}\}$$

$$3\,\text{in.} < 10\,\text{in.} < \min\{3 \times 6\,\text{in.}, 12\,\text{in.}\}$$

$$3\,\text{in.} < 10\,\text{in.} < 12\,\text{in.} \quad \therefore \text{ ok}$$

Step 7. Check shrinkage and temperature reinforcement:

$$A_{s(\text{S\&T})} = 0.0018bh\left(f_y = 60\,\text{ksi}\right)$$

$$A_{s(\text{S\&T})} = 0.0018(12)(6) = 0.13\,\text{in.}^2/\text{ft}$$

$$\#3@10\,\text{in.} \rightarrow \text{Table A2.10} \rightarrow A_s = 0.13\,\text{in.}^2/\text{ft} \quad \therefore \text{ ok}$$

Check the spacing of the shrinkage and temperature reinforcement:

$$3\,\text{in.} \le s \le \min\{5h, 18\,\text{in.}\}$$

$$3\,\text{in.} < 10\,\text{in.} < \min\{5 \times 6\,\text{in.}, 18\,\text{in.}\}$$

$$3\,\text{in.} < 10\,\text{in.} < 18\,\text{in.} \quad \therefore \text{ ok}$$

Step 8. For deflection control, the minimum recommended thickness (without calculating deflections) for the one-end-continuous slab is:

$$h_{\min} = \frac{\ell}{24} = \frac{12 \times 12}{24} = 6\,\text{in.} = 6\,\text{in.} \quad \therefore \text{ ok}$$

Slab is ok at mid-span.

(b) *Check the Slab at Supports*

Step 1. The main reinforcement at the supports (negative moment) is #4@8 in.

$$\#4@8\,\text{in.} \rightarrow \text{Table A2.10} \rightarrow A_s = 0.30\,\text{in.}^2/\text{ft}$$

$$\bar{y} = \frac{3}{4} + \frac{\frac{4}{8}}{2} = 1.0\,\text{in.}$$

$$d = h - \bar{y} = 6\,\text{in.} - 1\,\text{in.} = 5\,\text{in.}$$

$$\rho = \frac{A_s}{bd} = \frac{0.30}{(12)(5)} = 0.005$$

$$f'_c = 4\,\text{ksi} \rightarrow \text{Table A2.3} \rightarrow \rho_{\max} = 0.0207 > 0.005 \quad \therefore \text{ ok}$$

$$f_y = 60\,\text{ksi}$$

Step 2.

$$A_{s,\min} = A_{s(\text{S\,\&T})} = 0.0018bh\left(f_y = 60\,\text{ksi}\right)$$

$$A_{s,\min} = (0.0018)(12)(6) = 0.13\,\text{in.}^2/\text{ft}$$

$$A_s = 0.30\,\text{in.}^2/\text{ft} > 0.13\,\text{in.}^2/\text{ft} \quad \therefore \text{ ok}$$

Step 3.

$$\rho = 0.005$$

$$f'_c = 4\,\text{ksi} \rightarrow \text{Table A2.6b} \rightarrow R = 258\,\text{psi}$$

$$f_y = 60\,\text{ksi}$$

Step 4.

$$M_R = \frac{bd^2R}{12{,}000}$$

$$M_R = \frac{(12)(5)^2(258)}{12{,}000}$$

$$M_R = 6.5\,\text{ft-kip}$$

Step 5. The dead and live loads from part a are:

$$w_u = 0.36\,\text{kip/ft}(\text{from part a})$$
$$\ell_n = 10.83\,\text{ft}(\text{from part a})$$

The maximum factored moment at the first interior support for an exterior bay of the slab is (Table A2.1):

$$M_u = \frac{w_u \ell_n^2}{10}$$
$$M_u = \frac{(0.36)(10.83)^2}{10}$$
$$M_u = 4.2\,\text{ft-kip} < M_R = 6.5\,\text{ft-kip} \quad \therefore \text{ ok}$$

Step 6. Check the spacing of the main reinforcement:

$$3\,\text{in.} \leq s \leq \min\{3h, 12\,\text{in.}\}$$
$$3\,\text{in.} \leq s \leq \min\{3 \times 6\,\text{in.}, 12\,\text{in.}\}$$
$$3\,\text{in.} < 8\,\text{in.} < 12\,\text{in.} \quad \therefore \text{ ok}$$

The shrinkage and temperature reinforcement and the minimum depth for deflection were checked in part a.

Slab is ok at the support.

2.31 Design of Reinforced Concrete One-Way Slabs

The design process of one-way slabs is similar to that of reinforced concrete rectangular beams. Figure 2.60 summarizes the steps for the design of reinforced concrete one-way slabs. They are as follows:

Step 1. Select the slab thickness. The slab thickness is generally based on the minimum ACI requirements for deflection control (see Figure 2.42). This is usually rounded up to the nearest ½ in. for slabs with $h \leq 6$ in. and to the nearest 1 in. for those with $h > 6$ in.

Step 2. Calculate the factored loads (w_u), and then determine the maximum factored moment, M_u.

Step 3. Determine the slab's effective depth, d. Because the bar sizes are not yet known, assume #6 bars with 3/4 in. cover.

$$\bar{y} = 1.12\,\text{in.}$$

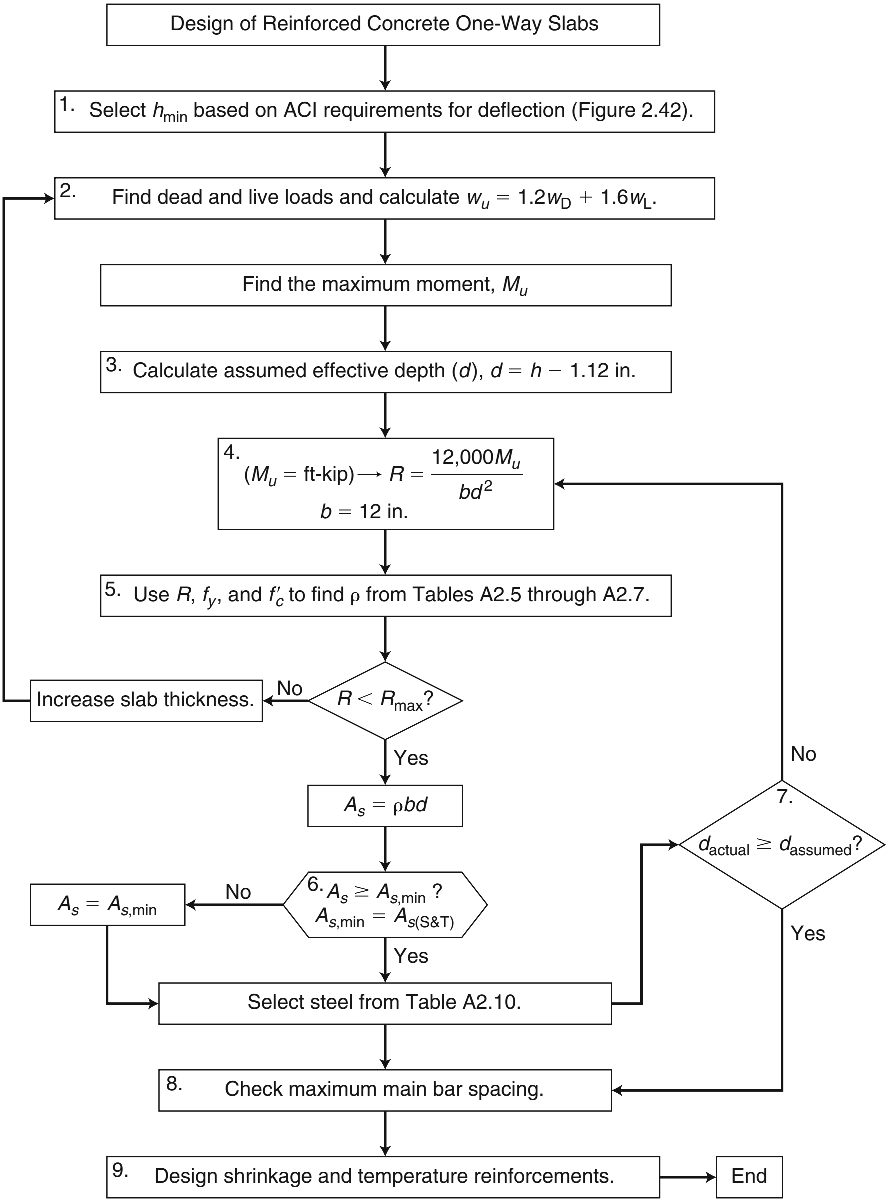


Figure 2.60 Flowchart for the design of reinforced concrete one-way slabs

Therefore, the assumed effective depth:

$$d = h - 1.12\,\text{in.}$$

Step 4. Determine the required resistance coefficient (R):

$$R(\text{psi}) = \frac{12{,}000 M_u}{bd^2}$$

$b = 12$ in., and d is in inches. M_u is in ft-kip and R in psi.

Step 5. Using R, f_y, and f'_c select ρ (steel ratio) from Tables A2.5 to A2.7. If the value of R is more than the maximum value shown in these tables ($R > R_{\max}$), the selected slab thickness is not adequate for the loads and needs to be increased. (Note that in most cases this does not happen. The required thickness for deflection control is usually more than what is required to carry the loads.)

$$A_s = \rho b d$$

Step 6. Check the minimum reinforcement requirement. The minimum area of steel for the main reinforcement must not be less than that required for shrinkage and temperature reinforcement:

$$A_{s,\min} = A_{s(\text{S\&T})}$$

If $A_s < A_{s,\min}$, the slab requires only a small amount of reinforcing steel, A_s. Use at least $A_{s,\min}$, however. Select the bar size and spacing from Table A2.10.

Step 7. Check for actual depth (d_{actual}) based on the bar selected. If $d_{\text{actual}} < d_{\text{assumed}}$, go back to Step 4 and revise. Repeat if the difference is too large (larger than 1/8 in. for slabs $h \leq 6$ in. and 1/4 in. for $h > 6$ in.).

Step 8. Check bar spacing. The spacing of bars selected in Step 6 has to be checked against the ACI Code requirements for maximum allowable spacing.

Step 9. Design the shrinkage and temperature reinforcements according to the ACI Code requirements.

Example 2.17 Design the one-way slab (S-1) of Example 2.16. Determine the reinforcement at (a) the midspan and (b) the supports.

Solution

(a) *Slab Design at the Midspan*

Step 1. Because S-1 is one end continuous, the minimum slab thickness ($h_{\min}$) is:

$$h_{\min} = \frac{\ell}{24} = \frac{12 \times 12}{24} = 6\,\text{in.}$$

Step 2. Determine the loads on the slab:

$$\begin{aligned}
\text{Weight of slab} &= 150(6/12) = 75\,\text{psf}\\
\text{Ceiling and floor finishing} &= 5\,\text{psf}\\
\text{Mechanical and electrical} &= 5\,\text{psf}\\
\text{Partitions} &= 15\,\text{psf}\\
\hline
\text{Total dead load} &= 100\,\text{psf}\\
\text{Total live load} &= 150\,\text{psf}
\end{aligned}$$

On a 1 ft wide strip

$$w_D = \frac{100 \times 1}{1{,}000} = 0.10\,\text{kip/ft}$$

$$w_L = \frac{150 \times 1}{1{,}000} = 0.15\,\text{kip/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2 \times 0.10 + 1.6 \times 0.15$$

$$w_u = 0.36\,\text{kip/ft}$$

$$\ell_n = 12\,\text{ft} - \frac{14\,\text{in.}}{12} = 10.83\,\text{ft}$$

The maximum factored moment at the midspan of S-1 (see Figure 2.61) is:

$$M_u = \frac{w_u \ell_n^2}{14}$$

$$M_u = \frac{(0.36)(10.83)^2}{14}$$

$$M_u = 3.0\,\text{ft-kip}$$

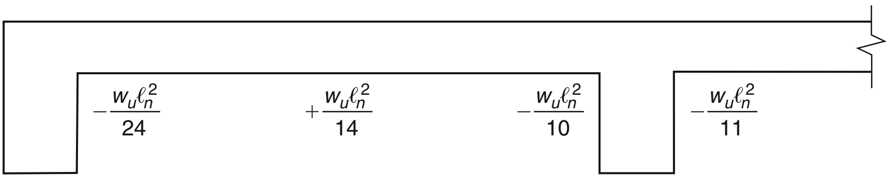


Figure 2.61 Design factored moments for slab S-1 of Example 2.17 using ACI Code coefficients from Table A2.1

Step 3. Assuming ¾ in. cover, calculate the slab's effective depth:

$$d = h - 1.12\,\text{in.} = 6\,\text{in.} - 1.12\,\text{in.} = 4.88\,\text{in.}$$

Step 4. Calculate the required resistance coefficient, R:

$$R = \frac{12{,}000 M_u}{bd^2}$$
$$R = \frac{12{,}000 \times 3.0}{(12)(4.88)^2} = 126\,\text{psi}$$

Step 5. Find ρ from Tables A2.5 through A2.7:

$$\begin{aligned} R &= 126\,\text{psi} \\ f_c' &= 4\,\text{ksi} \rightarrow \text{Table A2.6b} \rightarrow \rho = 0.0024 \\ f_y &= 60\,\text{ksi} \end{aligned}$$

Therefore, the required area of main reinforcement (A_s) is:

$$\begin{aligned} A_s &= \rho bd = (0.0024)(12)(4.88) \\ A_s &= 0.14\,\text{in.}^2/\text{ft} \end{aligned}$$

Step 6. The minimum amount of reinforcement for slabs cannot be less than the required shrinkage and temperature reinforcement steel:

$$A_{s,\min} = A_{s(\text{S\&T})} = 0.0018bh \quad \text{for } f_y = 60\,\text{ksi}$$
$$A_{s,\min} = (0.0018)(12)(6) = 0.13\,\text{in.}^2/\text{ft} < 0.14\,\text{in.}^2/\text{ft} \quad \therefore \text{ ok}$$
$$A_s = 0.14\,\text{in.}^2/\text{ft}$$
$$\text{From Table A2.10} \rightarrow \text{select \#4@17 in.} \quad (A_s = 0.14\,\text{in.}^2/\text{ft})$$

Note that according to Section 2.28, the smallest size bar for main reinforcement is #4.

Step 7. Check for the actual effective depth.

$$d_{\text{actual}} = 6 - \frac{3}{4} - \frac{\frac{4}{8}}{2} = 5.0\,\text{in.} > d_{\text{assumed}} = 4.88\,\text{in.} \quad \therefore \text{ ok}$$

Step 8. Check the main reinforcement spacing, s, ($f_y = 60$ ksi).

$$3\,\text{in.} \le s \le \min\{3h, 12\,\text{in.}\}$$
$$3\,\text{in.} < 17\,\text{in.} < \min\{3 \times 6\,\text{in.}, 12\,\text{in.}\}$$
$$3\,\text{in.} < 17\,\text{in.} < 12\,\text{in.} \quad \therefore \text{ N.G.}$$

Therefore:
Use #4@ 12 in. for the main reinforcement at the midspan.

Step 9. Calculate the required shrinkage and temperature reinforcement.

$$A_{s(\mathrm{S\&T})} = 0.0018bh = 0.13\,\mathrm{in.}^2/\mathrm{ft}$$
$$\text{From Table A2.10} \rightarrow \text{use \#3@10 in.}$$

The shrinkage and temperature reinforcement spacing (s) has to be within the following range:

$$3\,\mathrm{in.} \leq s \leq \min\{5h, 18\,\mathrm{in.}\}$$
$$3\,\mathrm{in.} < 10\,\mathrm{in.} < \min\{5 \times 6\,\mathrm{in.}, 18\,\mathrm{in.}\}$$
$$3\,\mathrm{in.} < 10\,\mathrm{in.} < 18\,\mathrm{in.} \quad \therefore \text{ ok}$$

Therefore,
Use #3@10 in. for the shrinkage and temperature reinforcement.

(b) *Slab Design at the Supports*

Step 1. From Step 1 of part a:

$$h_{\min} = 6\,\mathrm{in.}$$

Step 2. The factored uniformly distributed load on the slab (w_u) from Step 2 of part a is:

$$w_u = 0.36\,\mathrm{kip/ft}$$

and the clear span (ℓ_n) is:

$$\ell_n = 10.83\,\mathrm{ft}$$

From Figure 2.61, the moments at the exterior and interior supports are:

$$M_u^- = \frac{w_u \ell_n^2}{24} = \frac{(0.36)(10.83)^2}{24} = 1.76\,\text{ft-kip (exterior support)}$$
$$M_u^- = \frac{w_u \ell_n^2}{10} = \frac{(0.36)(10.83)^2}{10} = 4.22\,\text{ft-kip (interior support)}$$

Step 3.

$$\text{Assume } d = h - 1.12\,\mathrm{in.} = 6 - 1.12 = 4.88\,\mathrm{in.}$$

Step 4.

$$R = \frac{12{,}000 M_u}{bd^2} = \frac{12{,}000 \times 1.76}{(12)(4.88)^2} = 74\,\text{psi (exterior support)}$$

$$R = \frac{12{,}000 M_u}{bd^2} = \frac{12{,}000 \times 4.22}{(12)(4.88)^2} = 177\,\text{psi (interior support)}$$

Step 5.

$$\text{For exterior support} \begin{cases} R = 74\,\text{psi} \\ f_c' = 4\,\text{ksi} \rightarrow \text{Table A2.6b} \rightarrow \rho_{\text{ext.}} = 0.0014 \\ f_y = 60\,\text{ksi} \end{cases}$$

$$\text{For interior support} \begin{cases} R = 177\,\text{psi} \\ f_c' = 4\,\text{ksi} \rightarrow \text{Table A2.6b} \rightarrow \rho_{\text{int.}} = 0.0034 \\ f_y = 60\,\text{ksi} \end{cases}$$

Therefore:

$$(A_s)_{\text{ext.}} = \rho bd = (0.0014)(12)(4.88) = 0.082\,\text{in.}^2/\text{ft}$$
$$(A_s)_{\text{int.}} = \rho bd = (0.0034)(12)(4.88) = 0.20\,\text{in.}^2/\text{ft}$$

Step 6. From Step 6 of part a:

$$A_{s,\,\text{min}} = A_{s(\text{S\&T})} = 0.13\,\text{in.}^2/\text{ft}$$
$$(A_s)_{\text{ext.}} = 0.082\,\text{in.}^2/\text{ft} < 0.13\,\text{in.}^2/\text{ft} \quad \therefore\ \text{N.G.}$$

Therefore, use

$$(A_s)_{\text{ext.}} = 0.13\,\text{in.}^2/\text{ft}$$
$$(A_s)_{\text{int.}} = 0.20\,\text{in.}^2/\text{ft} > 0.13\,\text{in.}^2/\text{ft} \quad \therefore\ \text{ok}$$
$$\text{From Table A2.10} \rightarrow \text{Try \#4@12 in. (exterior supports)}$$
$$(A_s)_{\text{int.}} = 0.20\,\text{in.}^2/\text{ft}$$
$$\text{From Table A2.10} \rightarrow \text{Try \#4@12 in. (interior supports)}$$

Step 7. This is the same as in part a.

Step 8. Check the main reinforcement spacing:

$$3\,\text{in.} \leq s \leq \min\{3h, 12\,\text{in.}\}$$
$$3\,\text{in.} \leq s \leq \min\{3 \times 6\,\text{in.}, 12\,\text{in.}\}$$
$$3\,\text{in.} \leq s \leq 12\,\text{in.}$$
$$s_{\text{int.}} = s_{\text{ext.}} = 12\,\text{in.} = 12\,\text{in.} \quad \therefore \text{ ok}$$
$$\therefore \text{Use \#4@12 in. for the exterior and interior supports.}$$

Step 9. The shrinkage and temperature reinforcement was designed in part a. Figure 2.62 shows the slab as designed.

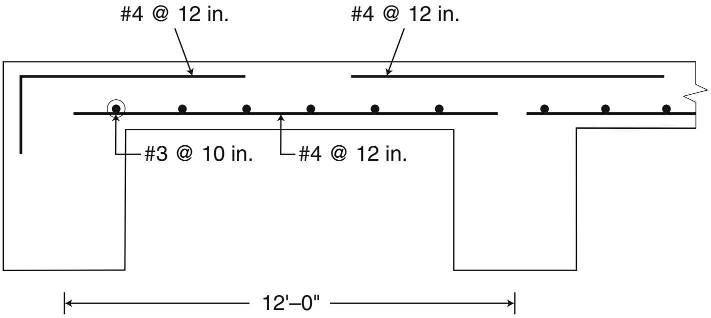


Figure 2.62 Slab S-1 designed in Example 2.17

Problems

In the following problems, unless noted otherwise, use normal weight concrete with a unit weight of 150 pcf, 1.5 in. for beam clear concrete cover, and 0.75 in. for slab clear concrete cover.

2.1 Consider a section with a width (b) of 14 in. and reinforced with 4 #9 bars in a single layer. $f_c' = 4{,}000$ psi, and $f_y = 60{,}000$ psi. Determine the moment capacity of the section, M_R, using Method I or II, for the following cases:

(a) $d = 28$ in.
(b) $d = 32$ in.
(c) $d = 36$ in.
(d) $d = 40$ in.

Show the changes in M_R with respect to the section's effective depth. Calculate the percentages of increase in M_R versus d.

2.2 Consider a rectangular reinforced concrete beam with an effective depth of 36 in. reinforced with 4 #9 bars. $f_c' = 4{,}000$ psi, and $f_y = 60{,}000$ psi. Determine M_R using Method I or II for the following cases:

(a) $b = 14$ in.
(b) $b = 16$ in.
(c) $b = 18$ in.
(d) $b = 20$ in.

Show the variation in M_R with b. For each case calculate the percentage of increase in M_R versus b.

2.3 Consider a reinforced concrete beam with a width (b) of 14 in. and an effective depth (d) equal to 36 in. $f_c' = 4{,}000$ psi, and $f_y = 60{,}000$ psi. Determine the moment capacity of this beam, M_R, for the following reinforcements:

(a) 4 #6 bars
(b) 4 #7 bars
(c) 4 #8 bars
(d) 4 #9 bars

Show the variation of M_R with respect to the area of reinforcements (A_s). For each case calculate the percentage of increase in M_R versus A_s.

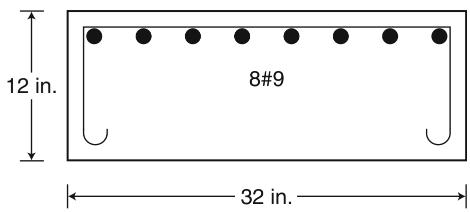
2.4 Consider a reinforced concrete beam with a width (b) of 14 in., and an effective depth (d) of 36 in. reinforced with 4 #8 bars. Use $f_y = 60{,}000$ psi. Determine the moment capacity, M_R, of this beam for the following cases:

(a) $f_c' = 3{,}000$ psi
(b) $f_c' = 4{,}000$ psi
(c) $f_c' = 5{,}000$ psi

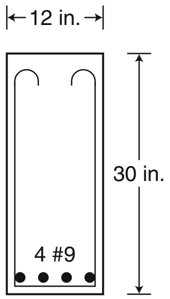
2.5 Rework Problem 2.4 for $f_c' = 4{,}000$ psi and for the following steel yield strengths:

(a) $f_y = 40{,}000$ psi
(b) $f_y = 60{,}000$ psi
(c) $f_y = 75{,}000$ psi

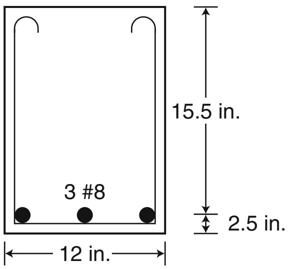
2.6 Determine the useful moment strength of the section shown below in accordance with the ACI Code. Use $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi, and #3 stirrups and follow Method II in the calculations.



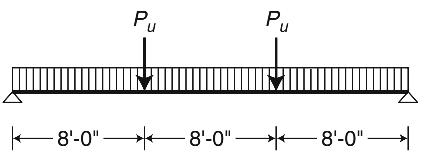
2.7 The rectangular reinforced concrete beam shown below is subjected to a dead load moment of 180 ft-kip and live load moment of 90 ft-kip. Determine whether the beam is adequate for moment capacity. $f_c' = 4{,}000$ psi, and $f_y = 60{,}000$ psi. The stirrups are #3 bars.



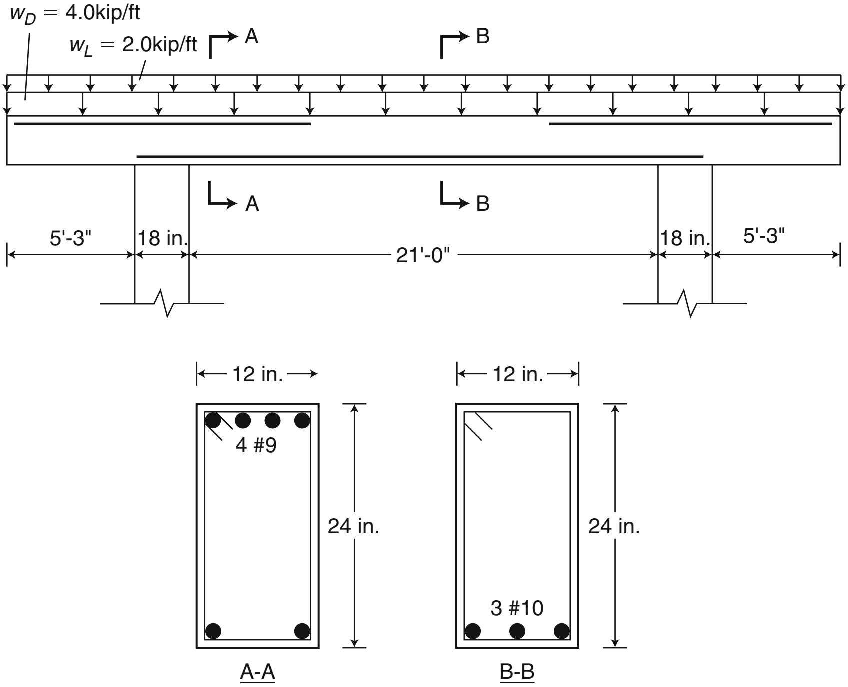
2.8 The beam below supports 500 lb/ft service dead loads and 600 lb/ft service live loads in addition to its self-weight. Calculate the maximum simply-supported span ($\ell = ?$) for the beam. Use Method II in the calculations. Use $f_c' = 5{,}000$ psi and $f_y = 60{,}000$ psi.



2.9 A rectangular beam carries uniformly distributed service (unfactored) dead loads of 3.0 kip/ft, including its own self-weight and 1.5 kip/ft service live loads. Based on the beam's moment capacity, calculate the largest factored concentrated loads, P_u, that may be placed as shown on the span in addition to the given distributed loads. The beam width is 18 in., and has a total depth of 30 in. with 5 #11 bars. Use $f_c' = 5{,}000$ psi, $f_y = 60{,}000$ psi, and #3 stirrups.

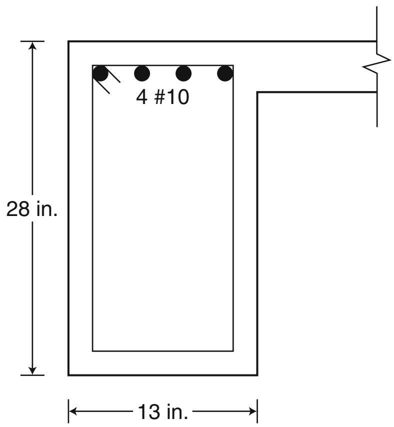


2.10 The beam shown below is part of a beam-girder floor system. It is subjected to a superimposed dead load of 4.0 kip/ft (excluding the beam weight) and a live load of 2.0 kip/ft. Check the adequacy of this beam. Use $f_c' = 4{,}000\,\text{psi}$, $f_y = 60{,}000$ psi, and #3 stirrups. Assume knife edge type supports at the centers of the walls.

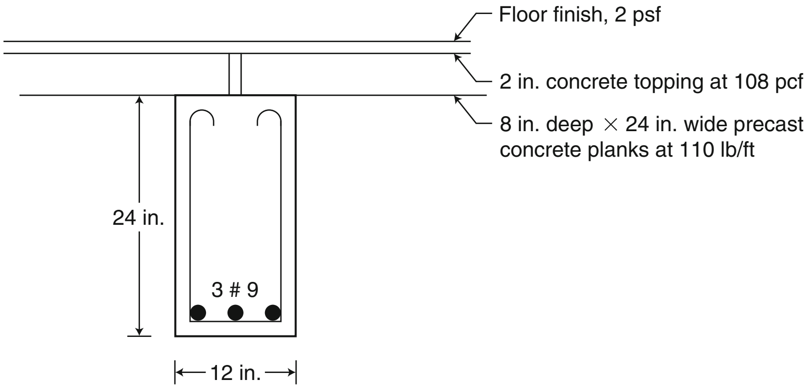


Note: Check both sections A-A and B-B. Neglect the reinforcement in the bottom of the beam at section A-A.

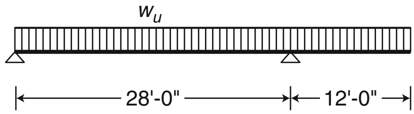
2.11 Determine the moment capacity, M_R, of the reinforced concrete section shown below if subjected to a negative moment. The stirrups are #3 bars. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000$ psi.



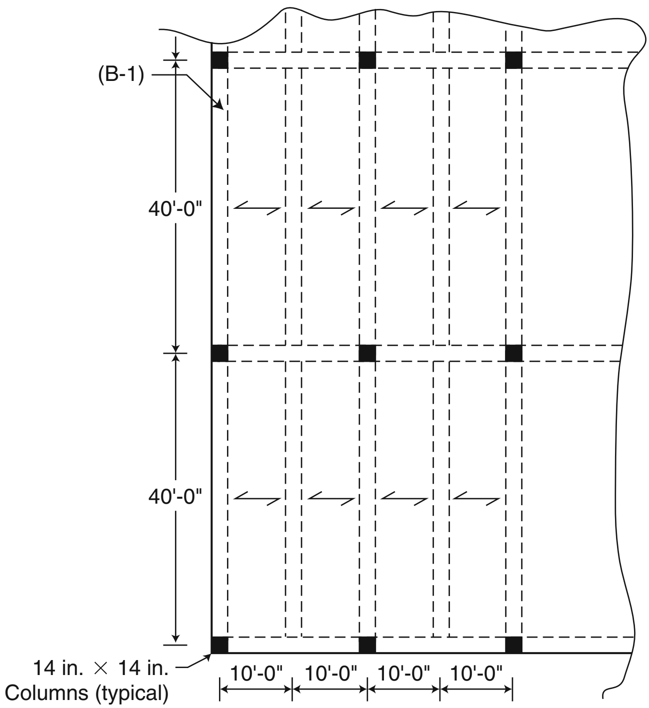
2.12 The figure below shows the cross section of a floor system consisting of a reinforced concrete beam supporting precast concrete planks. The beam span is 20′–0″ with 16′–0″ spacing. Calculate the maximum service live load per square foot of floor area. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000$ psi. The unit weight of lightweight (LW) concrete used is 108 pcf. Assume the beam is simply-supported.



2.13 The 16 in. × 27 in. rectangular reinforced concrete beam shown below is reinforced with 4 #10 bars in the positive moment region and 3 #11 bars in the negative moment region. Determine the maximum factored uniformly distributed load, w_u, for this beam. Stirrups are #4, $f_c' = 5{,}000\,\text{psi}$, and $f_y = 60{,}000$ psi.



2.14 The beam of Problem 2.11 is part of a beam-girder floor system shown below (beam B-1). The floor slab is 6 in. thick concrete, and the weight of the mechanical/electrical systems is 5 psf. Assume 15 psf for partition loads, and miscellaneous dead loads of 5 psf. What is the maximum allowable live load for this floor? Consider only the *negative* moment capacity of the section. (Note: Use the ACI moment coefficients. Live load is not to be reduced.)



2.15 Calculate the required areas of reinforcement for the following beams. Use $f'_c = 4{,}000$ psi and $f_y = 60{,}000$ psi.

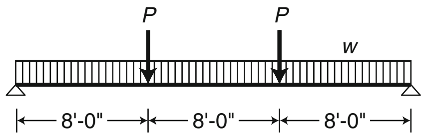
(a) $b = 10$ in., $d = 20$ in., $M_u = 200$ ft-kip
(b) $b = 12$ in., $d = 24$ in., $M_u = 300$ ft-kip
(c) $b = 18$ in., $d = 36$ in., $M_u = 500$ ft-kip

2.16 Design a rectangular reinforced concrete beam subjected to a factored load moment, $M_u = 250$ ft-kip. The architect has specified width $b = 10$ in. and total depth $h = 24$ in. Use $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi, and #3 stirrups.

2.17 Redesign the beam in Problem 2.16, assuming that the clear height for the building requires the total beam depth to be limited to 20 in. Determine the beam width (b) and the area of steel (A_s) in such a way that the section will be in the tension-controlled failure zone.

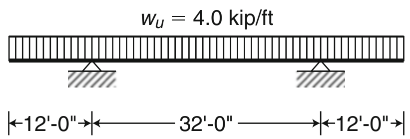
2.18 Design a rectangular beam for $M_u = 300$ kip-ft. Use $f_c' = 3{,}000$ psi, $f_y = 60{,}000$ psi, and #3 stirrups. Size the beam for $\rho = 0.01$ and $b/d = 0.5$ (approximate). Do not consider the beam's self-weight.

2.19 The 16 in. × 27 in. rectangular reinforced concrete beam shown below is subjected to concentrated loads of $P_D = 12.0$ kip and $P_L = 8.0$ kip. The uniformly distributed dead load, w_D, is 1.6 kip/ft (including the beam's self-weight), and the live load, w_L, is 1.0 kip/ft. Determine the required reinforcements. Sketch the section and show the selected bars. Use $f_c' = 5{,}000$ psi and $f_y = 60{,}000$ psi.



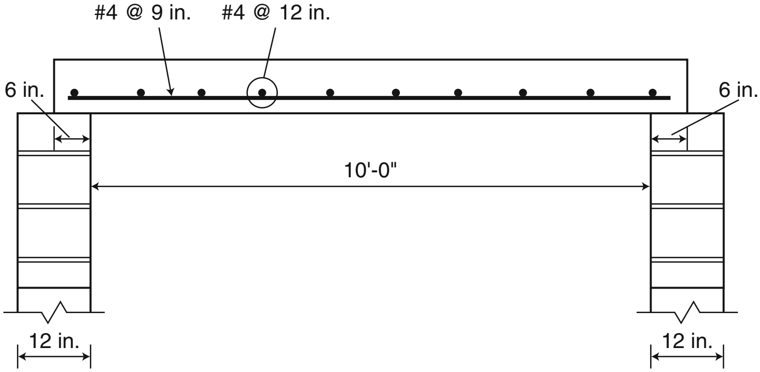
2.20 An artist is designing a sculpture that is to be supported by a rectangular reinforced concrete beam. The sculpture's weight is estimated to be 400 lb/ft (assumed as a live load). The beam section must be limited to $b = 8$ in. and $h = 12$ in. The artist wants to make his sculpture as long as possible. What is the maximum possible length of this cantilever beam without the use of compression reinforcement? Use $f_c' = 4{,}000$ psi, $f_y = 60{,}000$ psi, and #3 stirrups.

2.21 A 14 in. × 24 in. rectangular precast reinforced concrete beam supports a factored uniform load, $w_u = 4.0$ kip/ft, including the beam's self-weight. Determine the reinforcements required at the supports and the midspan. Use $f_c' = 4{,}000$ psi and $f_y = 60{,}000$ psi.

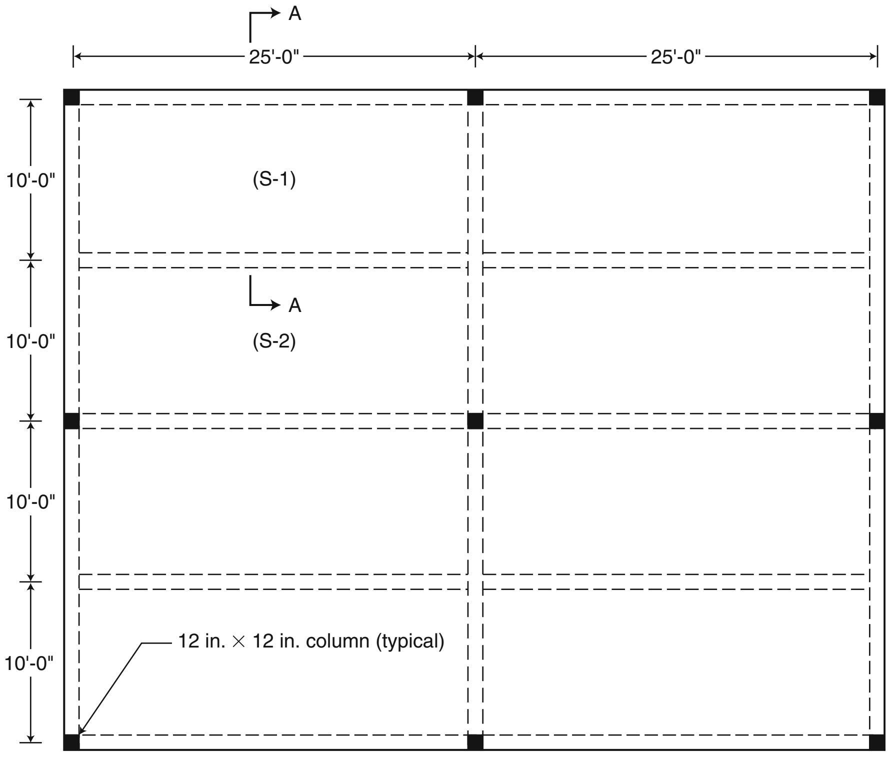


2.22 An 8 in. thick simply-supported reinforced concrete one-way slab is subjected to a live load of 150 psf. It has a 12 ft span and is reinforced with #4@8 in. as the main reinforcement and #4@12 in. as shrinkage and temperature reinforcement. Determine whether the slab is adequate. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000$ psi.

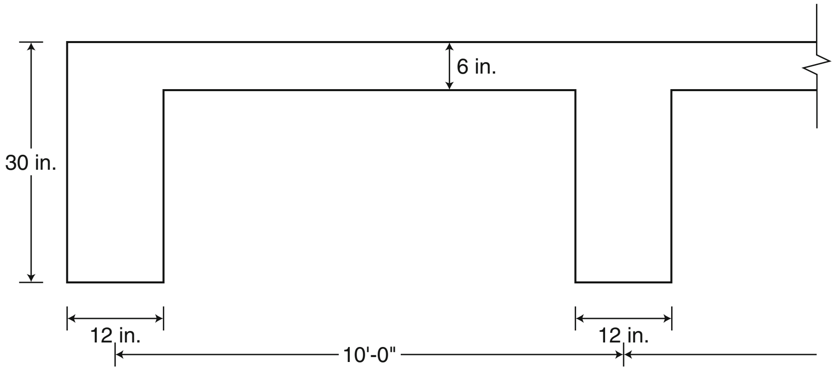
2.23 A 5 in.-thick simply-supported reinforced concrete one-way slab is part of a roof system. It is supported by two masonry block walls, as shown below. Assume a superimposed dead load (roofing, insulation, ceiling, etc.) of 15 psf and a roof snow load of 30 psf. Check the adequacy of the slab, including the required shrinkage and temperature reinforcement. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000$ psi. The bearing length of the slab on the wall is 6 in.



2.24 The figures below show the framing plan and section of a reinforced concrete floor system. The weight of the ceiling and floor finishing is 5 psf, that of the mechanical and electrical systems is 5 psf, and the weight of the partitions is 20 psf. The floor live load is 80 psf. The 6 in.-thick slab exterior bay (S-1) is reinforced with #6@9 in. as the main reinforcement at the midspan and #4@ 12 in. for the shrinkage and temperature reinforcement. Check the adequacy of the slab. Use the ACI moment coefficients. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000$ psi.



Framing Plan



Section A-A

2.25 Design a 6 in.-thick one-way slab for a factored moment, $M_u = 10$ ft-kip. Use $f_c' = 4{,}000\,\text{psi}$ and $f_y = 60{,}000$ psi.

2.26 Find the reinforcements for the midspan and supports for an interior 6 in.-thick slab (S-2) of the floor of Problem 2.24. Sketch the slab and show the reinforcements including the shrinkage and temperature reinforcement steel.

Self-Experiments

The main objective of these self-experiments is to understand the behavior of beams in bending (tension and compression) and changes in concrete strength with time, finding the modulus of rupture, and understanding the behavior of reinforced concrete beams under loading. The other objective is to understand the different aspects of concrete slabs. Remember to include all the details of the tests (sizes, time of day concrete was poured, amounts of water/cement/aggregate, problems encountered, etc.) with images showing the steps (making concrete, placing, forming, performing tests, etc.).

Experiment 1
In this experiment you learn about the behavior of beams in bending. Obtain a rectangular-shaped piece of Styrofoam with the proportions of a beam. Make slots on the top and bottom of the beam, as shown in Figure SE 2.1.

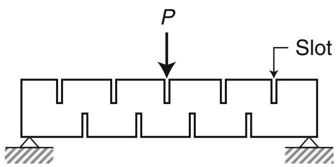


Figure SE 2.1 Styrofoam beam with slots

Place the beam on two supports and add a load at the center as shown in Figure SE 2.1. Answer the following questions:

1. What happened to the slots at the top and bottom of the beam?
2. Did the slots stay straight after adding the load?
3. Any other observations?

Experiment 2
You must start and perform Experiments 2 and 3 at the same time. In this experiment, you find the modulus of rupture for a plain concrete beam and learn about concrete curing and gaining strength with time.

For this experiment you will build four beams using concrete with w/cm ratio = 0.5. Size the beams as you wish, but do not make them excessively small or large (for practical reasons). After forming the beams (you can use cardboard or wood for your forms, depending on the beam size), spray water on two of the beams while keeping the other two dry. Keep your concrete beams indoors, as the concrete may freeze and stop the hydration process. After 2 days, test two of your test beams (one kept dry and one kept wet) by placing loads on them, as shown in Figure SE 2.2.

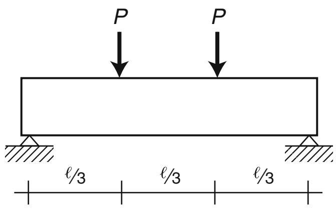


Figure SE 2.2 Plain concrete beam test.

Increase the loads until the beams fail. Record the loads at which the two specimens fail.

After seven days, repeat the tests with the remaining two beams and record the loads at which they fail.

Experiment 3

In this experiment, you will learn about the importance of reinforcing steel in concrete beams and compare the results with those of Experiment 2.

When you pour the four plain concrete beams for Experiment 2, build two reinforced concrete beams with the same dimensions as those of the plain concrete beams. You can use steel wires for the reinforcement (depending on your beam size). Place these wires on only one side of the beam (singly-reinforced beam).

After 2 days, place one of the beams on two supports and apply loads as shown in Figure SE 2.3a. Increase the load, and record your observations.

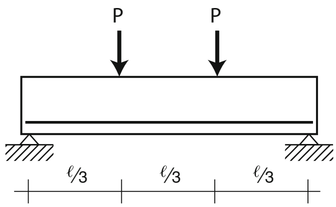


Figure SE 2.3a Reinforced concrete beam test 1

Repeat this test for the remaining reinforced concrete beam after seven days. (Perform these tests at the same time as Experiment 2.) DO NOT TRY TO FAIL THE REINFORCED CONCRETE BEAMS! Turn the beams upside down (Figure SE 2.3b) and repeat the tests. Add loads until the beams fail. Record your observations.

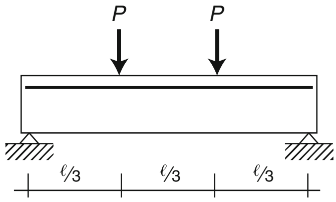


Figure SE 2.3b Reinforced concrete beam test 2

Answer the following questions regarding Experiments 2 and 3:

1. Which of the samples (dry or wet) had more strength? Why?
2. Was the 7-day-old sample stronger than the 2-day-old one? Why?
3. Find the modulus of rupture for the 7-day-old plain concrete beams.
4. How did the reinforcement affect the concrete beam strength?
5. What happened when you turned the beam upside down and tested it?

Experiment 4

This experiment demonstrates the behavior of one-way and two-way slabs, and the reinforcing of one-way slabs.

Test 1

Use two Styrofoam pieces to represent one-way and two-way slabs. For the two-way slab, cut the Styrofoam into a square piece, and for the one-way slab make it such that length/width ≥ 2. Place the square Styrofoam on two parallel supports and apply a load as shown in Figure SE 2.4a. Support the same model on four edges and repeat the test as shown Figure SE 2.4b. Make notes on how the two models deform and their differences.

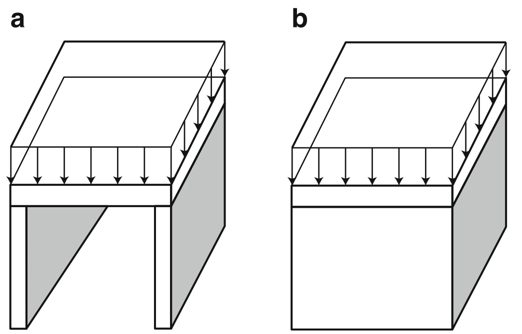


Figure SE 2.4 Slabs under loads: (**a**) two parallel supports; (**b**) supports along all edges

Test 2

Repeat Test 1 using the one-way slab model. Record your observations.

Experiment 5

This experiment deals with the reinforcement in slabs.

Cast two slab models with a thickness of approximately 1 in. and a width of at least 12 in. Make one from plain concrete and the other from concrete reinforced with a grid of thin wires (provide about ¼ in. cover).

One week after making the samples, compare the two slabs in terms of crack formation. Which one has more surface cracks?

http://www.springer.com/978-3-319-24113-5

Concrete Structures
Setareh, M.; Darvas, R.
2017, XVIII, 680 p. 428 illus., Hardcover
ISBN: 978-3-319-24113-5