

# Preface to the Second Edition

This revised monograph still aims at a unified geometric foundation of gauge theories of elementary particle physics and gravity. The underlying geometric structure is unfolded in a coordinate-free manner via the modern mathematical notions of fiber bundles, exterior differential forms, and their Clifford-algebra-valued generalizations. In the first part, Maxwell theory is treated as the simplest example, with an emphasis on the more recent measurement of the vector potential 1-form  $A$  via electron interference.

By transferring these concepts to local spacetime symmetries, affine generalizations of Einstein's theory of gravity arise in a Riemann–Cartan space with curvature and torsion. In this context, recent accounts on the Einstein–Cartan theory, teleparallelism, as well Yang's gauge approach to gravity are treated in more detail, with emphasis on gravitational instantons. Duality projections of curvature squared models, with their Einsteinian macroscopic “nucleus,” are analyzed with respect to the issue of quantization, or at least asymptotic safeness. The Cartan-type geometric structure of BRST quantization with nonpropagating topological ghosts is developed in some detail.

In order to obtain more insight into the open issue of quantizing gravity, Chern–Simons-induced topological three-dimensional gravity, like the Mielke–Baekler model, is analyzed, in which torsion provides a kind of linearization of the vacuum field equations. Moreover, the peculiar feature of Dirac fields in curved 3D space is geometrically related to flexural modes of new materials such as graphene.

Quantized Dirac fields suffer from nonconservation of the axial current, leading to chiral and trace anomalies also in Riemann–Cartan space.

Since the discovery of the Higgs boson, concepts of spontaneous symmetry-breaking in gravity have come again into focus: departing from a topological de Sitter-type gauge theory, some new progress in the constrained BF model with a primordial  $SL(5, \mathbb{R})$  gauge group is presented. After a tiny symmetry-breaking and the spontaneous generation of the metric, Einstein's standard general relativity with cosmological constant again emerges as the classical background.

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Since the days of Riemann, scientific work in natural philosophy has concentrated on answering questions “... concerning the intrinsic (physical) basis for the metrical relations in space ...”<sup>1</sup> thus striving for a unified geometric description of fundamental physical interactions. The present study reports on recent achievements in this endeavor.

It will begin with a coordinate-free presentation of the underlying geometric structure of electromagnetic fields and their nonabelian generalizations by utilizing rather modern mathematical concepts, such as those of fiber bundles and Lie-algebra-valued differential forms. Such nonabelian theories of Yang–Mills type are founded on Weyl’s basic principle of gauge invariance and appear to be the most appropriate framework in which to describe phenomena so as the weak and strong interactions in particle physics. In particular, the unification of weak and electromagnetic forces within the Weinberg–Salam model has gained much empirical evidence during the last two decades.

As for macroscopic gravity, it is taken for granted that Einstein’s theory of general relativity is the geometric theory that is empirically the most successful. However, concerning attempts of quantization, it is flawed by the conceptual disadvantage that it cannot be molded completely into the scheme that is put forward by Maxwell’s theory. Following, however, the suggestions of not only Weyl but Einstein himself, theories of gravity can be worked out that not only incorporate general relativity, but are also invariant with respect to local translations and local Lorentz transformations. Such Poincaré gauge theories are to be located in a Riemann–Cartan spacetime with curvature and torsion, and consequently can be coupled not only to the mass but also canonically to the spin of fundamental particles. The resulting gravitational field equations are of a formal structure that is

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<sup>1</sup>“... die Frage nach dem inneren Grunde der Massverhältnisse des Raumes ...” (Bernhard RIEMANN, 10 Juni 1854).

analogous to that of the “spontaneously broken” Yang–Mills–Higgs model. For this reason, they can be solved by means of appropriate duality Ansätze as in the case of the instanton solutions of nonabelian gauge theories. It turns out that the metrical background, concerning macroscopic length measurements, is represented by Einstein spaces. Deviations are to be expected only within microscopic regions. This is especially true for interactions with fundamental spinor fields.

Not considering questions concerning the global topology, it has to be realized that the contortion of spacetime induces principally a nonlinearity into the Dirac equation. This self-interaction of the axial vector-type is equivalent to what was suggested by Heisenberg in his unified field theory of elementary particles. The soliton solutions, which are occurring in such nonlinear models, are investigated with respect to their semiclassical and quantum meaning.

The ultimate goal of all these unifications is to build up a theoretical superstructure, an all-encompassing grand synthesis of all physical interactions. Within the limits of the present study, theoretical models are dealt with that are of unequivocal geometric character. These include the Rainich geometrization of the Einstein–Maxwell system, the nonabelian generalizations of the five-dimensional theory of relativity by Kaluza and Klein, and last but not least, the tensor dominance model of Salam et al. It is above all the Kaluza–Klein model being coupled to Dirac fields that, in contrast to the generally covariant Yang–Mills theory, promises a far deeper understanding of the parity violations occurring in the decay of certain metastable mesons.

Most of today’s outstanding theories of fundamental interactions postulate - following Gell-Mann - the so-called quarks as hypothetical building blocks of matter. In order to guarantee the permanent confinement of these enigmatic archaic forms in the observable hadrons, a geometrodynamical mechanism of confinement common to all geometric models of unification is proposed in a speculative prospect.

The bulk of the present study is a slightly revised and amended version of the author’s habilitation thesis, which was submitted to the Faculty of Science of the Christian-Albrechts-University of Kiel in April 1982.

Apart from Prof. J.A. Wheeler, it is especially Prof. F.W. Hehl to whom I owe innumerable direct and indirect suggestive ideas that have helped me to shape structurally important physicomathematical concepts that have entered this work during the stages of its genesis. I would also like to express my sincere gratitude for the hospitality and the highly stimulating working atmosphere at the International Centre for Theoretical Physics, Trieste, which were extremely helpful and for which I should like to thank Prof. Abdus Salam, the International Atomic Energy Agency, and UNESCO.

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