

Chapter 2

Creativity, Imagination, and Early Mathematics Education

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Abstract In this chapter, we draw heavily on a new typological model of creativity and show its consequences for early maths education. According to this model, creativity is made up of three interrelated components: creative abilities (mainly creative imagination and divergent thinking), openness to experiences, and independence. This model is our starting point for the description of the importance and organization of the Mathematical Creative Problem Solving Model. We describe the assumptions, aims, and elements of this model, as well as demonstrate the practical and methodological aspects of supporting the development of mathematics. We also focus on the role played by visual and creative imagination and on new ways of enhancing mathematical creativity using heuristic rhymes.

Keywords Typological model of creativity • Creativity • Visual and creative imagination • Mathematical creative problem solving • Mathematical heuristic rhymes

2.1 Introduction

In knowledge-based society, creativity is perceived as a source of innovation and progress (Sawyer 2006). Concurrently, innovativeness is frequently equaled with mathematical thinking when it comes to engineering and invention (Wang and Shang 2014), but also with regard to teaching mathematics (see inventive mathematical thinking; Harskamp 2014, p. 371). The psychology of creativity proposes a search for connections not just between innovation and creativity but also between

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those two and imagination – the Imagination–Creativity–Innovation (ICI) model (see Beghetto 2014). As creativity and imagination are interdisciplinary constructs (Gillson and Shaley 2004; Glăveanu 2010), they have many connotations that are sometimes contradictory (Kaufman 2009). This is why we start this chapter by defining them with reference to the typological model of creativity (Karwowski 2010; Karwowski and Lebuda 2013) and the conjunctural model of creative imagination (Dziedziewicz and Karwowski 2015). Those definitional solutions will be the basis for the analysis of relations between the constructs we are interested in this chapter and will serve as reference points in the description of the role of heuristic rhymes in Mathematics Creative Problem Solving.

2.2 The Typological Model of Creativity

The history of psychological research on creativity is usually divided into two periods (see Sawyer 2006): before and after 1950, which is when Joy Paul Guilford delivered his breakthrough address during the Convention of the American Psychological Association (Guilford 1950; Kaufman 2009). As is widely known, Guilford perceived divergent thinking as the intellectual operation responsible for creative thinking, with several important characteristics, namely: (1) fluency, understood as the ability to come up with many ideas; (2) flexibility, or the ability to create solutions that are qualitatively diverse; (3) originality, responsible for producing rare and untypical ideas; and (4) elaboration – the ability to develop ideas (Guilford 1967). The number of empirical studies grew after Guilford's address, and the understanding of creativity as an egalitarian characteristic also became widespread. For example, humanist psychologists (Fromm 1959; Maslow 1959; Rogers 1970) considered it not only as a characteristic of eminent creators but also – to a greater or lesser extent – as a trait commonly found in the entire population.

Attempts undertaken by researchers and theoreticians to define creativity most frequently came down to two characteristics of its product – newness, associated with originality (Cropley 2001; Boden 2004), and value (utility) (Cropley 1999; Runco 2009). Thus, creativity is defined as activity that leads to the emergence of new (original) and useful products (Amabile 1983). With time, creativity began to be identified with a compound of personal traits. Aside from divergent thinking, the mechanisms considered by researchers to be key for creating include creative imagination (Khatena 1975; LeBoutillier and Marks 2003) as well as personality characteristics: primarily openness to experiences (Dollinger and Clancy 1993; Feist 1998; Perrine and Broderson 2005) and independence (Batey and Furnham 2006; Eysenck 1994; Nickerson 1999; Stravidou and Furnham 1996). Numerous studies of this kind made it possible to more thoroughly determine the conceptual range of creativity, but they also resulted in the emergence of a *sui generis* “hybrid of creativity” – a system of cooperating elements (traits) related to creative behaviors, which reveals the complexity and multilayer character of this phenomenon. The proposed typological model of creativity (Karwowski 2010; Karwowski and Lebuda 2013) is an

attempt to systematize the relations between and among these traits. According to this model, the following hypothetical dimensions determine creativity: (1) creative abilities (cognitive dispositions that determine the effectiveness of generating, developing, and implementing solutions characterized, among other things, by a high level of originality and value, divergent thinking, and imaginative abilities); (2) openness (appreciation of intellect, willingness to meet new people and cultures, as well as learning); and (3) independence (a personality dimension marked by non-conformism and low agreeableness as well as readiness to oppose the situationally evoked influence of the group and external factors). The model implies the special importance of four creativity types, labeled and defined as follows: complex creativity (a combination of creative abilities, openness, and independence), subordinate creativity (a combination of creative abilities and openness with low independence), rebellious creativity (a combination of creative abilities and independence with low openness), and self-actualizing creativity (a combination of openness and independence with low creative abilities). Initial empirical analyses (Karwowski 2010) indicate their specific determinants (different parental attitudes as well as social and economic status), school functioning patterns (grades and satisfaction with learning), creativity styles, creative self-efficacy beliefs, and perceptions of the climate for creativity.

2.3 The Conjunctural Model of Creative Imagination

Long before the Guilford address, Francis Galton conducted the first documented study into imagination and analyzed individual differences in the clarity of representations produced by scientists (Galton 1880; Holt 1964). Almost concurrently, Théodule A. Ribot (1906) coined the concept of creative imagination. Soon after, Lev S. Vygotsky (1930/2004, 1931/1998) proposed the combinatorial (creative) imagination theory. The 1960s saw the emergence of further holistic conceptualizations of creative imagination (e.g., Rozet 1977/1982; Ward 1994). As research and theories developed, similarly as in the case of creativity, attention was drawn to the complexity of creative imagination. Its constitutive factors (properties) were indicated: the vividness (clarity) of images (“The weirdness of visions lies in their sudden appearance in their vividness while present, and in their sudden departure” Galton 1883, p. 121), the ability to manipulate the resulting images (“People can assign novel interpretations to ambiguous images which have been constructed out of parts or mentally transformed” Finke et al. 1989, p. 51), as well as the originality (newness) and value of those images (“Activity that results not in the reproduction of previously experienced impressions or actions but in the creation of new images or actions is an example of [...] creative or combinatorial behavior” Vygotsky 1930/2004, p. 9). These dimensions contributed to the development of the conjunctural model of creative imagination (Dziedziejewicz and Karwowski 2015), whereby creative imagination was defined as the ability to create and transform mental

representations based on the material of past observations, but significantly transcending them.

In this model, the hypothetical dimensions of creative imagination are: vividness – the ability to create expressive and highly complex images, originality – ability to create unique images, and transformative ability – the ability to transform images. The model is conjunctural – that is, the combination of its three dimensions allows the typological analysis focusing on the four basic types of imaginative creative abilities: (1) creative imaging ability (high vividness of imagery, high originality, and high transformative ability), (2) pro-creative imaging ability (high originality and high transformative ability), (3) passive imaging ability (high vividness of imagery and high originality), and (4) vivid imaginative abilities (high vividness and high transformative ability).

2.4 Creativity and Imagination

Implicit and lay theories of creativity define divergent thinking and imaginativeness as traits of creative individuals (e.g., Montgomery et al. 1993). The 1960s mark the point when first correlational studies appeared. They measured the strength and direction of the relation between imaginativeness (visual and creative) and creativity (Schmeidler 1965). Researchers mainly focused on the relation between imaginativeness and creative abilities primarily via divergent thinking (e.g., Gonzales et al. 1997). Much less frequently did they analyze the relation of imagination with personality factors, such as openness and independence (Khatena 1975; Schmeidler 1965, among others). The results of these analyses reveal the existence of a relation between creative imagination and creativity, yet the strength of this relation depends on the examined domain. The combination of imaginativeness with elements of creative attitude (openness, independence) is evidently weaker than its relation with divergent thinking, especially in the domains of vividness and originality (see Dziedziewicz et al. 2013; Schmeidler 1965). On the one hand, this confirms the legitimacy of including creative imagination in the creative abilities factor in the typological model of creativity. On the other hand, though, this relation is so weak ($r=.2-.4$) that it is justifiable to consider these traits separately as relatively independent facets of creativity.

Further in this chapter, imaginative abilities (creating original images and transforming them) will be analyzed in conjunction with divergent thinking as creative abilities. This will render it possible to conduct detailed and systematized analysis of the role of creativity in Mathematics Creative Problem Solving in the domain of cognitive (creative imagination and divergent thinking) and personality (openness, independence) components.

2.5 Creativity in Mathematics or Mathematical Creativity?

The important question discussed among creativity scholars (e.g., Baer 1998; Chen et al. 2006; Kaufman and Baer 2005; Plucker 1998) is whether a general *c* factor – analogous to the *g* factor (Jensen 1998) associated with creativity in multiple and diverse domains, including mathematics (see Kaufman 2009, p. 57) – does indeed exist. Creativity is associated with a particular domain in the situation when researchers focus on a creative product (Plucker 2004). Consolidating the domain-specific and domain-general perspectives, Kaufman and Baer (2005) proposed the Amusement Park Theoretical Model (APT), which was meant to be the “Aristotelean golden mean.”

The APT model inspired us to create profiles of mathematical creativity on the basis of the typological model of creativity. After Kaufman and Baer, we defined the general thematic framework as the “problem-solving domain,” whereby mathematics became the chosen field and solving word problems became the microdomain. We claim that solving problems reflects the nature of mathematical thinking (see Silver 1994). Moreover, word problems are used in mathematics education – which is why we decided to refer to them as well. Furthermore, the analysis of their role in mathematics education frequently emphasizes the creative use and performance of particular mathematical operations.

In the early stages of mathematical education, word problems are commonly of a practical character. Generally, they are simple stories referring to childhood experiences that end with a question one needs to find the only correct answer to (closed questions), after analyzing the information that a given story contains, the data, the unknown, and the relations between them. The stories resemble brain-teasers, which are known to have a solution, and the only task at hand is to find it. If we assumed that creativity is about producing original (new) and useful solutions, it would be difficult to speak about creative solutions to word problems because they are known to mathematicians and even more so to their authors. Hence, it is the way of working towards the solution – that is, defining the problematic situation presented, competences associated with hypothesizing, and planning ways to test the hypotheses – that will provide evidence of the creativity of children solving this type of exercises. On each of the listed stages of mathematical creative problem solving, creative abilities as well as personality factors (openness and independence) will play a significant role (Table. 2.1).

Importantly, the way children solve word problems is significantly influenced by the way they formulate a particular mathematical problem. In the early stages of education, word problems are frequently built using simple and “round” numbers. The predictable form of such problems raises the (fully legitimate!) temptation to guess the result.

Example

Dorothy and Alex have 12 chocolate bars in total. Dorothy has two more bars than Alex. How many bars does Alex have?

Table 2.1 Examples of manifestations of mathematical creativity in the process of solving word problems

Mathematical creative problem solving	Mathematical creativity		
	Creative abilities	Openness	Independence
Understanding mathematical problems	The ability to define the mathematical problem illustrated in the task from multiple perspectives	Tolerance to information that is incomplete, poorly defined, or polysemous	Constructing one's own internal language, where mathematical concepts indispensable for solving the problem are set out and explained
	The ability to clearly visualize the situation presented in the task as well as vividly capture dependency relationships across data	Recognition of the potential value resulting from becoming acquainted with ways other than one's own of perceiving and describing the mathematical problem illustrated in the task at hand	Separating the meanings of mathematical concepts from the meanings of everyday language
Generating possible solutions	The ability to formulate multiple and frequently atypical hypotheses referring to the possible solutions to the mathematical problem illustrated in the task at hand	Cognitive curiosity that results in readiness to become acquainted with possible ways of solving the problem	Courage in questioning commonly accepted rules and principles in order to find new and/or atypical ways of solving the mathematical problem
	The ability to create original images that render it possible to break away from typical solutions to the mathematical problem and use analogies in order to find new ones	Ease in analyzing new information and ways of solving the problematic situation presented in the task at hand	Autonomy and perseverance in searching for possible solutions to the problematic situation
Planning for action	Flexibility in applying various strategies of solving the problem	Openness to the verification of all possible solutions to the problem	Strong belief in the success of the undertaken activities aimed at solving the problem
	The ability to transform images of possible solutions to the problematic situation illustrated in the task at hand	The acceptance of variability in applying the various problem-solving strategies	The ability to critically assess attempts – one's own and other people's – to solve the problem

Instead of counting, many pupils confronted with the above problem will respond that Alex has 5 bars and Dorothy has 7, without even being aware that they have guessed the result by making an intuitive attempt to come up with a single number, because the problem is structured in such a way that the number of potential solutions is significantly reduced. In this situation, it is necessary to reflect on whether guessing at the answer can be considered as a manifestation of creative ability. Another question to consider is this: what is the value (usefulness) – even the subjective value, for the pupils themselves – of solving the problem with the trial-and-error method, which disregards the way towards the solution and, instead, focuses solely on the solution itself? When solving word problems with this method, students frequently do not consider the relationships between the given and the unknown. They attempt to quickly reach the goal (the solution) and usually act thoughtlessly. The heuristic solution pattern anticipates the understanding of the essence of the mathematical problem illustrated by the task. It also anticipates finding the way to use the solution again in an analogous problematic situation.

Let us consider what would happen if the problem was formulated as follows:

Example

Dorothy and Alex have 5 and $2/5$ of a chocolate bar in total. Dorothy has 1 and $1/3$ of a bar more than Alex. How many bars does Alex have?

As it appears, in this case the method of guessing fails entirely even though we are dealing with exactly the same mathematical problem – the only difference is that numbers are no longer easy to calculate mentally. As a result of adopting the tactics of guessing at solutions to simple problems, those students who have learned to thoughtlessly follow this approach, regarding it as verified and effective, fail to understand the sense and purpose of general methods of solving a particular type of problems. Still less motivated are they to develop such methods.

What can be done to encourage children to solve problems in accordance with the heuristic scheme, which aims at the creative pursuit and discovery of ways to solve problems? We propose a “tablet of changes” – an instructional method whose aim is to create an educational situation conducive to making sense of certain mathematical concepts and operations independently. A simple task that should not pose a problem for any child at a particular stage of development is the starting point in this table. The purpose of solving this task is to strengthen self-efficacy and thereby to encourage children to attempt to solve further, more and more difficult problems. However, the most important aim is to devise a practical illustration of the relations between various branches and aspects of mathematics that will render it possible to consolidate the previously learned concepts and computational techniques. Realizing what the solution to the first problem of each row is makes it possible to apply the same method of solving the mathematical problem in the case of the remaining problems (Table 2.2).

Table 2.2 Tablet of changes – dividing fractions

Six chocolate bars were divided equally among three people. How much chocolate did each person receive?	Six chocolate bars were divided equally among four people. How much chocolate did each person receive?	Two and a half chocolate bars were divided equally among five people. How much chocolate did each person receive?	Two and a half chocolate bars were divided equally among seven people. How much chocolate did each person receive?
Solution:	Solution:	Solution:	Solution:
A group of four people have eight chocolate bars. How much chocolate will each person get?	A group of six people have eight chocolate bars. How much chocolate will each person get?	A group of three people have one and a half chocolate bar. How much chocolate will each person get?	A group of three people have two and a half chocolate bars. How much chocolate will each person get?
Solution:	Solution:	Solution:	Solution:
Fifteen chocolate bars were divided into three equal portions. How much chocolate is there in each portion?	Fifteen chocolate bars were divided into four equal portions. How much chocolate is there in each portion?	One and two thirds of a chocolate bar was divided into five equal portions. How much chocolate is there in each portion?	One third of a chocolate bar was divided equally into three fourths of a portion. How much chocolate is there in each portion?
Solution:	Solution:	Solution:	Solution:
A single portion is made up of two chocolate bars. How many such portions can be made from five chocolate bars?	A single portion is made up of two chocolate bars. How many such portions can be made from five chocolate bars?	A single portion is made up of one and one third of a chocolate bar. How many such portions can be made from five chocolate bars?	A single portion is made up of one and a half chocolate bars. How many such portions can be made from three fifths of a chocolate bar?
Solution:	Solution:	Solution:	Solution:

The problems in the tablet of changes are placed in four columns and four rows. Four is a number that everyone recognizes without making calculations. Therefore, sixteen problems in a 4×4 columnar format do not make an impression of being a large number that is hard to grasp, but that number is sufficient to conduct competence profile assessment of children within the range of problems they face solving. It makes it possible to assess the stage at which difficulties may begin to occur with regard to problem interpretation, the way of coding the solution, making calculations, or possibly even a combination of various types of complexity.

Moving along the tablet of changes to the right, along the rows, we encounter problems characterized by the same extent of conceptual difficulty but more and more complex when it comes to calculations. Moving downwards along the columns, we encounter problems with a similar degree of calculative complexity but more and more difficult when it comes to the concepts whose understanding they require. Problems in rows are usually characterized by similar wording and refer to the same objects in order not to distract children towards insignificant aspects but to keep them focused on each described mathematical problem, on the presented data, on the question posed, and on response interpretation. Such a form also renders it possible to explain to children that the purpose is to solve sixteen different problems

because there are different versions of the same problem or problems. This may prevent premature discouragement from making an effort.

Tables of examples may be used at various stages of education because it is not necessary for children to solve all the problems in a given table right away. The exercise may be limited to one or two rows/columns, depending on students' skills, and then resumed after some time. Thanks to the possibility of using the same tablet of changes in a group of children with diverse levels of mathematical competence, this method enables the individualization of math classes.

2.6 Mathematical Heuristic Rhymes

Rhythm accompanies people throughout their lives. It is a constant and obvious element of nature, revealing itself in the cyclical character of astronomical phenomena, for example in the circadian rhythm that results from the Earth revolving around its own axis and determines the timing of human activity and rest. It is therefore not only a natural component of the course of human life but also an important way of perceiving the world. Colloquial language uses the evocative concept of “being thrown off balance,” which refers to undesirable disturbance or destruction of the rhythmic pattern of an activity. After all, rhythm gives a sense of order, predictability, and security. Already in prenatal life we feel and remember the rhythm of our mothers' hearts and that is why newborns calm down when they are placed on their mothers' chest. Rhythm is also present in many basic forms of human activity. We breathe, walk, and run rhythmically. The language we use also has a particular rhythm and melody (Patel and Daniele 2003). It is hard to imagine the effective performance of these activities without proper rhythm. However, we rarely realize the omnipresence of rhythm and most frequently associate it with dance, music, and poetry. Similarly to mathematics, it is not associated with creativity (Kaufman and Baer 2004), and conversely: mathematics is rarely associated with rhythm. Yet, this domain is replete with rhythms, for instance decimal rhythm in the positional system, the alteration of even and odd numbers, or number multiples. This is why the reference to students' natural sense of rhythm and the use of rhythmical rhymes in early mathematics education has multiple positive functions. Counting itself stems from rhythmical indication of objects, so it is hard to find better justification for combining structures that operate on the principle of rhythm, such as rhymes, with learning mathematics. The idea of combining a rhyme with mathematical concepts appeared as early as the nineteenth century in the stories of the famed Mother Goose (Bellos 2010):

As I was going to St Ives,
I met a man with seven wives,
Every wife had seven sacks,
Every sack had seven cats,
Every cat had seven kits.
Kits, cats, sacks, wives,
How many were going to St Ives?

Rhymes perform many functions in early mathematics education. In a natural way, they draw attention to the content about to be revealed and provoke children to anticipate or at least expect such words in consecutive lines that will rhythmically fit into the pattern and rhyme with those the children have already heard. This is why, frequently, there is no need to read the final word of a rhyme to children: they are able to deduce it from the previously heard content and finish saying it, especially that rhyme is an additional indicator. This develops in children the sense of rhythm and order that will be important in shaping their mathematical abilities in the future. It also fuels the sense of satisfaction, positively influences self-assessment and intrinsic motivation, and thus activates further active listening. Using rhymes in teaching mathematics also helps practice memorization and encourages imaginative creation of images, which makes it easier, for example, to assimilate abstract mathematical concepts. Moreover, when revealing a mathematical problem in many situational contexts, rhymes enhance its comprehension and teach thinking flexibility.

In early mathematical education, rhythm is helpful not just in combination with proficiency in counting. The ability to identify rhythm, associated with the way of measuring time and calendric calculations, is also important. Equally important is noticing geometric regularity on all dimensions: linear (e.g., the repetition of a sequence of items positioned in a series), surface (e.g., system design on a ball), or spatial (e.g., the regularity of architectural elements). The development of students' active sense of rhythm is one of the most important tasks for early mathematics education. The fact that rhythm is omnipresent in children's lives does not mean that they are able to give proper rhythm to the activities they perform. The ability to sing rhythmically and make appropriate use of pause, whose length results directly from the song's rhythm, may serve as an example. Many individuals encounter a problem with deciding when to begin singing the next phrase because they are unable to reproduce its rhythm by themselves when it is not chimed or accentuated for them. Reading or repeating good rhymes (including mathematical ones) promotes the development of an active sense of rhythm in children, even though this task is not always easy. Unfortunately, it happens that the authors of rhymes do not make this task any simpler because they frequently fail to observe the elementary principles of balanced and predictable distribution of accents or proper number of syllables in each line. This is why it seems crucial to select correct educational rhymes that should, among other things, (1) be made up of lines with the same number of syllables or repetitions in accordance with an easily comprehensible key, (2) be readable in the sense that accentuated syllables should make up a recognizable rhythmical pattern (somewhat resembling a melody), (3) include an intelligible idea, story, anecdote, or humor that will be clear to children, use word play, or include a surprising punch-line, as well as (4) be concise, but interesting or amusing to children even when the rhymes are read repeatedly (Szwajkowski 2011).

In order to stimulate students' mathematical creativity, it is a good idea to combine rhymes with solving diverse word problems. A good rhyme written into the content of a word problem may make it easier for children to remember and elicit information that is crucial for solving that problem. Additionally, thanks to the attractive form, it may also encourage students to face the presented mathematical

problem. If, additionally, the problem will be constructed in a way that renders multiple solutions possible, the problem itself may become even more contextually interesting and motivating.

Below is an example of a mathematical heuristic rhyme (The rhyme is followed by a picture of hamster and cookies in four colors: 2 green - apple, 7 orange - orange, 4 red - cherry, 6 blue - "berry"):

HAMSTER COOKIES

Hamster George who was so cute
Got some cookies made of fruit:
He bought apple, orange, cherry,
And one more that ends with 'berry.'
Never ate so much before -
He had just eleven more!
Just two flavors he left behind
How many ate he and of what kind?

The heuristic scheme of this problem fosters active learning. The rhyme makes it possible for children to independently discover data. It may also interestingly illustrate a problematic situation when mathematical data needed to solve the problem are provided after or during the reading of its content. Finally, children actively and creatively seek possible problem solutions.

Let us analyze a model way of seeking possible solutions. It is not a simple one, as when the problem is approached on a heuristic level. A number of solutions exist and they can be reached through a number of stages of elimination and inference.

Data: The hamster bought 2 apple cookies, 7 orange ones, 4 cherry ones, and 6 berry ones

Looking for potential solutions: The hamster bought a total of 19 cookies ($2+7+4+6=19$). Since he had 11 left, he must have eaten 8 ($19-11=8$). He could not have any apple cookies left because they do not add up to 11 when taken together with any of the remaining flavors. Consequently, he surely ate 2 apple cookies. He could not have any cherry or berry cookies left because there are not enough of them in total ($4+6=10$). He may have had orange and cherry cookies left ($7+4=11$), which would mean that he ate 2 apple cookies, 4 cherry ones, and any two of all orange and berry ones, which leaves 3 more possibilities.

The hamster may have had orange and cherry cookies left ($7+4=11$), which would mean that he ate 2 apple and 6 berry cookies. It is also possible that orange and cherry cookies were left, which would mean that the hamster ate 2 apple cookies, 4 cherry ones, and any two of orange and berry ones, which leaves 3 more possibilities.

Observing children when they are solving such a problem opens many possibilities for analyzing various aspects of mathematical creativity, such as flexibility in applying various strategies of dealing with an open mathematical problem (e.g., searching for a possible solution using a functional method – that is, by manipulating objects – or searching for solutions with the use of a drawing or symbolic calculative method) or the ability to formulate original hypotheses that refer to the probability of one of those solutions to occur (e.g., response to the question of why the hamster ate cookies of only two flavors, namely apple and orange, when he had the opportunity to try all four?). Based on a class conducted using a mathematical heuristic rhyme, the teacher can indicate the strengths and weaknesses of creatively solving open mathematical problems; such a class also allows the teacher to infer her or his students' mathematical creativity profile. Tasks of this type also foster students' integration with their teacher because they create many opportunities to follow students' reasoning as well as support this process by means of asking additional questions and providing guidelines.

The result of observing a group of 9- and 10-year-old children solving this problem indicates that solving the first part of the problematic situation, namely answering the question of how many cookies the hamster ate, is quite simple. The possibility of providing the answer to this question relatively quickly encourages children to perform further inquiries into the problem and to seek to answer the second part of the mathematical problem that refers to the kind of cookies the hamster ate, namely to the stage of analyzing various solutions and drawing conclusions. The conclusion that the hamster could not have any apple cookies left because they do not add up to 11 with any of the remaining flavors, so he surely ate 2 apple cookies, is not so certain, but the children reached this conclusion by modelling various situations with the aid of disks and a specially prepared board that featured the hamster while performing simple calculations. These calculations did not constitute an end in itself, but were an activity that supported making conclusions.

2.7 Mathematical Creative Problem Solving

At the end of this chapter, let us present a mathematics class interaction using a tablet of changes and a mathematical rhyme. We have prepared this interaction with early mathematical education in mind, and this is why we used the so-called balance beam – an original teaching aid that makes it easier for children to move from the level of concrete things (counting particular objects) to the symbolic level (numerical record of calculations).

A balance beam is a small device that resembles scales with weights in form of colorful disks. The disks are identical in dimensions and weight. They have holes in the middle thanks to which it is possible to easily place them on the scales' pins in defined positions. As Fig. 2.1 shows, the distance between the pins and the center of the scales is a multiple of the disk's diameter.

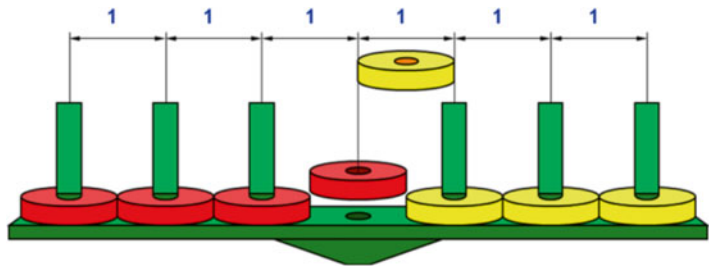


Fig. 2.1 Balance beam

<p>“This is to believe in : scales are really, truly even when the sides on both those ends weigh the same and when none bends.</p> <p>Disks give weight, but which, my mate, weigh much heavier: in the middle, or away? Now, that’s a riddle!</p> <p>How to measure weight transitions in these very cool positions?”</p>	
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Fig. 2.2 Mathematical rhyme about the balance beam

Sample interactions with the use of the balance beam are carried out according to consecutive steps of mathematical creative problem solving: (1) understanding mathematical problem; (2) generating possible solutions; (3) planning for action. In the beginning, thanks to a short rhyme, children become acquainted with the general principles of balance beam’s operation and in this way they familiarize themselves with the mathematical problem that relates to the equilibrium condition (Fig. 2.2).

Our experiences with the use of the balance beam show that children do not experience problems in applying the number of disks. They also notice that the farther away the disk is from the middle, the more weight it applies to a particular side of the scales.

While practicing with the balance beam, children have an opportunity to experiment and test their hypotheses in practice. Solving simple problems provided by the teacher (see Fig. 2.3), they have a chance to independently comprehend the equilibrium condition and verify its correctness by arranging the disks in a way that renders it possible to make a calculation.

Finding a solution to a mathematical problem, such as the equilibrium condition on the balance beam, is an interesting challenge for children. It is an excellent exercise in mathematical creativity, because solving it requires generating a new amount that is a product of the number of disks and their distance from the middle of the scales.

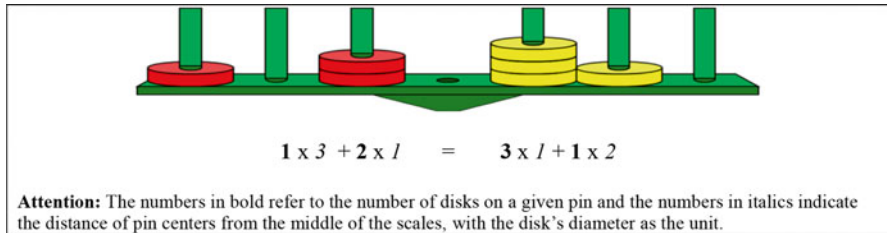


Fig. 2.3 Equilibrium condition as exemplified by the balance beam

2.8 Conclusion

Creativity is usually analyzed within the domain of art and is not often considered to be an important part of mathematical thinking or math education (Sriraman 2004, 2005). Math, by contrast, is highly algorithmic and is often perceived by teachers and students as not allowing much space for heuristics or creative thinking. In this chapter, we intended to show the possibilities of using creative thinking while solving mathematical problems and the possibilities of using creative tasks to enhance children's mathematical abilities.

The methods described in this chapter were deduced from the theoretical models of creativity and imagination we started with. We have especially focused on using heuristic rhymes and the process of mathematical problem solving as illustrations of simultaneous engagement of creative abilities, openness, independence, and different aspects of creative imagination, especially vividness, originality, and transformativeness. Both heuristic rhymes and creative problem solving of mathematical problems are being intensively introduced within classes taught by teachers we cooperate with; the effects are promising. We do hope that the methods described above as well as similar attempts at developing children's creativity in the domain of mathematics will enhance both their creative abilities and imagination as well as improve their school achievement. Future studies will assess the effectiveness of these methods.

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