

# Preface

In this book, we focus on deterministic forecasting based on governing dynamical equations (typically in the form of differential equations). These equations require specification of a control vector for their solution (initial conditions, boundary conditions, physical and/or empirical parameters). Defining forecast error is central to our study as the book title implies. An all-inclusive definition of forecast error is difficult to formulate. Therefore, we find it best to define this error categorically: (1) error due to incorrectly specified terms in the governing equations (or the absence of important terms in these equations), (2) inexact numerical approximations to the analytic form of the dynamical equations including artificial amplification/damping of solutions in the numerical integration process, and (3) uncertainty in the elements of control. That is, error in prediction results from incorrect dynamical laws, numerical inexactitude, and uncertainty in control. And the true test of forecast goodness rests on comparing the forecast with accurate observations. Ofttimes, we are unable to definitively determine the source(s) of error. But given this error, we ask the question: Can we improve the forecast by altering the control vector or empirically correcting the dynamical law? And just as important a question, can we determine the relative impact of the various elements of control on the forecast of interest?

Immediately we see that there is a desire to determine sensitivity of forecast to elements of control—labeled forecast uncertainty. This uncertainty is then used to find optimal elements of control—values of the elements that minimize the sum of squared differences between the forecast and observations and ideal placement of observations in space-time. Clearly, this path blends sensitivity with least squares fit of model to observations. A methodology that considers all of the factors mentioned above is called the forecast sensitivity method (FSM), a relatively new form of dynamic data assimilation that is the centerpiece of this book.

There is a rich history of work in both sensitivity analysis (SA) and dynamic data assimilation (DDA). Sensitivity analysis has been a mainstay of both dynamical systems and biostatistical systems. The works of Gregor Mendel and Ronald Fisher are excellent examples of sensitivity in the field of population genetics—necessarily in the form of statistics for hybridization of peas in Mendel’s case and statistics of

crop production in Fisher's case. Henri Poincaré was the pioneer in the uncertainty of dynamical forecasts with respect to elements of control. Even in the absence of computational power in the late nineteenth century, he clearly understood the extreme sensitivity of the three-body problem's solution to slight changes in initial conditions (presented as an exercise in this book). One of the most frequently quoted sentences in his studies was "La prediction deviant impossible" [The prediction becomes impossible]. Lorenz (1995) took this to mean that Poincaré was close to discovering the theory of chaos. These issues in deterministic forecasting have led to the investigation of predictability limits, that point in time when the forecast is no better than "climatology" (the average state of affairs in the system). In engineering and control theory, Hendrik Bode, a research mathematician at Bell Laboratories, developed sensitivity analysis in service to feedback control that he studied during WWII when he addressed problems of gunnery control. His classic work is titled *Network Analysis and Feedback Amplifier Design* (Bode et al. 1945). The two-volume treatise on *Sensitivity and Uncertainty Analysis* by Cacuci (2003) and Cacuci et al. (2005) deals extensively with the discussion of adjoint sensitivity and its applications to geosciences. For other applications of sensitivity analysis to engineering systems, refer to Cruz (1982), Deif (2012), Eslami (2013), Fiacco (1984), Frank (1978), Kokotovic and Rutman (1965), Ronen (1988), Rozenwasser and Yusupov (1999), Saltelli et al. (2000, 2004, 2008), and Sobral (1968).

In regard to DDA, celebrated mathematician Carl Gauss did his fundamental work on least squares fitting of observations to models in the first decade of the nineteenth century (Gauss 1809) (a discussion of his problem is found in this book). This method has never fallen out of use. Gauss's work was expanded from particle dynamics to continuous media through the work of Alfred Clebsch (Clebsch 1857). In the 1950s, Japanese meteorologist Yoshi Sasaki used Gauss' fundamental idea in combination with the Clebsch transformation to develop DDA for numerical weather prediction (NWP). This methodology has come to be called variational analysis in meteorology and in abbreviated form 4D-VAR (Sasaki 1958). Lewis et al. (2006) review variational analysis in the context of NWP. Also refer to Lewis and Lakshmivarahan (2008) for more details. A resource for numerous papers in sensitivity analysis can be found in Rabitz et al. (1983), Nago (1971) and Tomović and Vukobratović (1972).

As might be expected, there is equivalence between classic Gaussian least squares methodology (variational analysis) and FSM. We carefully examine this connection in the book and offer several problems (and demonstrations) that explore this connection along with advantages/disadvantages of these DDA methods.

The book is partitioned into two sections: Part I, a general theory of FSM with a variety of practical problems that give substance to the theory, and Part II, an in-depth analysis of FSM applied to well-known geophysical dynamics problems—the dynamics of shallow water waves and air-sea interaction under the condition of a convective boundary layer. Recently, FSM has been applied to solve estimation problems in ecology, hydrology, and interdependent security analysis. For lack of space, we could not include these interesting applications.

A good working knowledge at the BS level of the standard calculus sequence, differential equations, linear algebra, and a good facility with programming constitute adequate prerequisites for a course based on this book. We have used this book as a secondary text along with parts of our earlier book (Lewis et al. 2006) in a senior/first year-graduate-level course devoted to solving static and dynamic deterministic inverse problems at the University of Oklahoma, Norman, Oklahoma, and at the University of Nevada, Reno, Nevada, USA.

We have strived to eliminate typographical errors, and we would very much appreciate hearing from readers who identify remaining errors.

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