The Description of Motion: Kinematics

2

Part I

2.1 Definition of Motion. Frames of Reference

Motion refers to a change in the location of an object with time, as seen from a fixed, rigid frame, the "frame of reference". This supplementary specification is quite essential. We can see this from a randomly-chosen example: A bicyclist looks down at her feet and sees them moving in circular paths with the pedals. An observer standing on the sidewalk sees a very different picture of the motion of the bicyclist's feet; for her or him, the feet follow a wavelike path, namely the cycloids which are sketched in Fig. 2.1.

The rigid solid body which is our frame of reference for the description of motion in the rest of this chapter is the earth or the floor of the room where we are located. We leave the daily rotation of the earth out of consideration. (In reality, we are practicing physics on a large carousel. The earth is also not really rigid, but instead is deformable.)

Later, we will occasionally change the standpoint of our observations, i.e. our frame of reference. We will take the earth's rotation into account in some discussions, and sometimes also the deformation of the earth. This will always be mentioned explicitly. Otherwise we would have an endless confusion, especially when we treat rotational motions.

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For the description of all motions, also called *kinematics*, we require the concepts of *velocity* and *acceleration*. We will begin with them.

Figure 2.1 The path followed by the pedals of a bicycle as seen by a stationary observer

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2.2 Definition of Velocity. Example of a Velocity Measurement

Suppose that an object moves through a distance Δl within the time interval Δt. Then we define

$$u_{\mathrm{m}} = \frac{\text{Distance moved } \Delta l}{\text{Time interval } \Delta t} \tag{2.1}$$

as the *mean* velocity along the direction of the distance Δl. This quotient *changes* in general if one successively decreases the distance Δl. However, the changes gradually decrease to below the precision of the measurements. The value of u_{m} which is then measured, which depends only on the starting point, is denoted as the velocity u at the starting point. Mathematically, one thus finds the velocity u as the limiting value of u_{m} by taking the limit $\Delta t \to 0$. The symbol Δ is conventionally replaced by d, giving for the definition of the velocity

$$u = \frac{\mathrm{d}l}{\mathrm{d}t} \tag{2.2}$$

i.e. the differential quotient of the distance travelled divided by the time interval.

This definition in many cases requires the measurement of rather short times. As an example, we consider the measurement of the *muzzle velocity* of a bullet from a pistol.

Figure 2.2 shows a suitable setup for this measurement. The distance interval Δl is fixed by two thin cardboard disks; its length could be for example 22.5 cm. The time measurement is performed in a straightforward manner by referring to the basis of all time measurements,

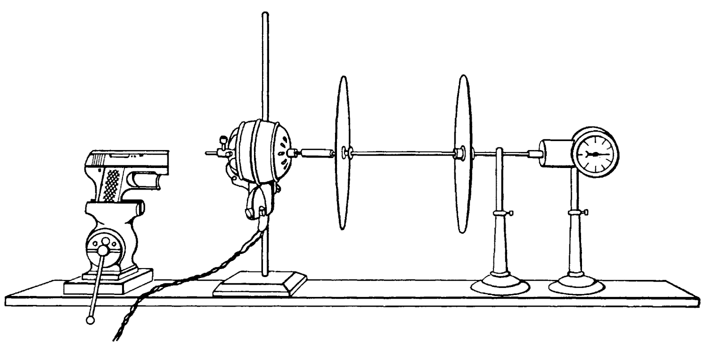


Figure 2.2 Measurement of the velocity of a bullet from a pistol with a simple "time recorder". On the right is the tachometer which registers the rotational frequency[C2.1].

C2.1. For this velocity measurement, the defining equation (2.2) or (2.1) is used directly. This principle is also applied e.g. to the measurement of the velocities of molecules in a molecular beam. For macroscopic objects, however, a much more elegant method is the one described in Sect. 5.9 (Fig. 5.19), which makes use of momentum conservation.

a uniform rotation. The time markers are registered automatically. For this purpose, an electric motor causes the two disks on a common shaft to rotate at a uniform, rapid rate. Their rotational frequency ν, i.e. the quotient of the number N of rotations/time t, is determined by a tachometer which measures rotational frequencies[1], for example $\nu = 50\,\mathrm{Hz}$.

The bullet first passes through the left disk, and the bullet hole that it leaves is our first time marker. While it travels over the distance of 22.5 cm to the second cardboard disk, a certain time elapses, and the bullet hole or time marker in the second disk is shifted relative to the first marker by a certain angle, corresponding to the rotation of the shaft during this elapsed time. After stopping the rotation, we measure an angle of ca. 18° or 1/20 of the circumference of the disks.

By pushing a pin through the two bullet holes, we make the angular shift visible for distant observers in the silhouette of the apparatus.

The time delay Δt was thus $\frac{1}{20} \cdot \frac{1}{50}\,\mathrm{s} = 10^{-3}\,\mathrm{s}$. From this, we find the velocity

$$u = \frac{22.5\,\mathrm{cm}}{10^{-3}\,\mathrm{s}} = \frac{0.225\,\mathrm{m}}{10^{-3}\,\mathrm{s}} = 225\frac{\mathrm{m}}{\mathrm{s}}.$$

We then repeat the experiment with a smaller distance Δl between the disks of only 15 cm. The end result is the same. Thus, the distance chosen in the first experiment was already short enough. It permitted us to measure the true muzzle velocity and not the smaller mean value over a longer trajectory[C2.2].

C2.2. The word 'trajectory' originally meant 'flight path'. It is now used in a more general sense to mean the path (location *vs*. time or geometrical form) followed by *any* moving object. When this path is circular (like the path of a satellite around a planet), it is called an 'orbit'. We will use the simpler term 'path' here in most cases.

Only in cases with a constant or uniform velocity can we choose the quantities Δl (measured distance) and Δt (elapsed time) freely to allow the most convenient measurement. In such cases, one can write the velocity in abbreviated form as $u = l/t$.

"One should early on adopt the habit of always writing the units after every numerical value of a physical quantity. It belongs to good physics practice!"

One should early on adopt the habit of always writing the units after every numerical value of a physical quantity. It belongs to good physics practice! This spares the reader from having to deduce the units meant from the physical context, and spares oneself from committing frequent computational errors. When different units are used, the *numerical values* of the measured quantities also change. Their recalculation is automatically reliable if the quantitative results are always given as numerical values *and* units[C2.3].

C2.3. POHL indicates here the advantage of using *physical-quantity equations*, which he consistently employs. In a preliminary remark on writing physical equations (included in the volume on 'Mechanics' since the 12th edition), he writes among other things, "For each symbol, we write both the numerical value *and* the unit. The choice of units is free. Those mentioned under some equations are simply examples" (see also Sect. 2.6).

Example

The velocity $u = 225\,\mathrm{m/s}$ is to be recalculated in terms of kilometers per hour. We have $1\,\mathrm{m} = 10^{-3}\,\mathrm{km}$ and $1\,\mathrm{s} = (1/3600)$ hour; then

$$u = 225\frac{10^{-3}\,\mathrm{km}}{(1/3600)\,\mathrm{hour}} = 810\,\mathrm{km/h}.$$

[1] If no tachometer is available, one can make use of a simple reducing gear: a pulley of circumference d is mounted on the motor shaft. An endless belt made of string with length $L \gg d$ drives another pulley a few meters away; its knot serves as a marker. The number N' of revolutions of the string in a time t can be counted using this marker. Then $\nu = \frac{N'}{t} \cdot \frac{L}{d}$ is the rotational frequency.

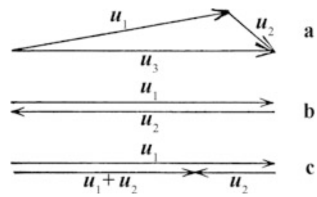


Figure 2.3 The geometrical addition of vectors, e.g. of two velocities

Well-formulated units can often be seen as a compact form of measurement instructions. – We will encounter this in many places in this book.

In everyday life, we often make do with the *magnitude* of a velocity, e.g. 10 m/s. In physics, however, the magnitude is only *one* of the two determining quantities for a velocity. The second is its *direction*. In physics, velocities are always directed quantities, i.e. mathematically, they are *vectors* (represented graphically as an arrow). This can be most clearly seen in the process, well known also to non-physicists, of *addition of two velocities*.

In Fig. 2.3a, the high velocity $\boldsymbol{u}_1$ (e.g. the velocity of an aircraft relative to the surrounding air) and the much lower velocity $\boldsymbol{u}_2$ (e.g. the local wind velocity), which generally points in a different direction, are vectorially combined to yield the "resultant" velocity $\boldsymbol{u}_3$ (the groundspeed of the aircraft).

Vectors which point in opposite directions differ in their signs; for example, Fig. 2.3b is described by the equation $\boldsymbol{u}_1 = -\boldsymbol{u}_2$ or $\boldsymbol{u}_1 + \boldsymbol{u}_2 = 0$. – Therefore, $\boldsymbol{u}_1 + \boldsymbol{u}_2$ in Fig. 2.3c refers to the geometrical addition or combination of the two oppositely-directed vectors $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$. The resultant vector has a magnitude (length of the arrow) of $|\boldsymbol{u}_1 + \boldsymbol{u}_2| = |\boldsymbol{u}_1| - |\boldsymbol{u}_2|$. Magnitudes are denoted by vertical bars on both sides of the symbol[C2.4].

C2.4. The rules for vector addition are explained here in an intuitive manner using the example of combining velocities. Vector quantities are denoted by printing their symbols in boldface, while their magnitudes are represented by normal type, dispensing with the vertical bars for simplicity.

2.3 Definition of Acceleration: The Two Limiting Cases

Motions with a constant velocity are rare. In general, the magnitude and the direction of the velocity change along the path of the motion.

In Fig. 2.4, the vector $\boldsymbol{u}_1$ indicates the velocity of a body *at the beginning* of a time interval Δt. *During* the time interval, the body is supposed to gain an additional velocity $\Delta\boldsymbol{u}$ in an arbitrary direction, represented by the short second arrow. *At the end* of the time interval Δt, the body has the velocity $\boldsymbol{u}_2$. It is determined graphically in Fig. 2.4 as the arrow $\boldsymbol{u}_2$.

Then we define

$$\boldsymbol{a}_\mathrm{m} = \frac{\text{Velocity increase } \Delta\boldsymbol{u}}{\text{Time interval } \Delta t}$$

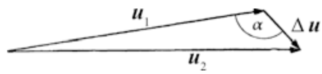


Figure 2.4 The general definition of the acceleration

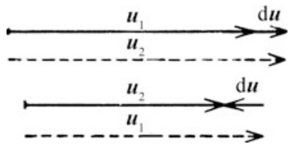


Figure 2.5 The definition of the path acceleration

as the *mean* acceleration. The time interval Δt is chosen so that the quotient no longer changes measurably when Δt is further decreased. Mathematically, one carries out the limit $\Delta t \to 0$, replaces the symbol Δ by d, and thus obtains for the *acceleration*:

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t}\,. \tag{2.3}$$

Just like the velocity, the acceleration is also a vector. The *direction* of this vector is the same as that of the increase in the velocity $\Delta \boldsymbol{u}$ (Fig. 2.4).

In Fig. 2.4, the angle α between the increase of the velocity $\Delta \boldsymbol{u}$ and the initial velocity $\boldsymbol{u}_1$ was arbitrary. We now consider *two limiting cases:*

1. $\alpha = 0$ or $\alpha = 180°$, Fig. 2.5. The velocity increase lies along the same direction as the original velocity. Then only the *magnitude*, not the direction of the velocity changes. In this case the acceleration is referred to as a *path acceleration* with the magnitude

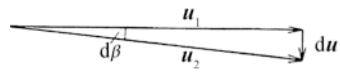
$$a = \frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\mathrm{d}^2 l}{\mathrm{d}t^2}. \tag{2.4}$$

2. $\alpha = 90°$, Fig. 2.6. The velocity increase points in a direction perpendicular to the original velocity $\boldsymbol{u}$. Now, only the *direction* and not the magnitude of the velocity changes; within the time interval dt through the angle dβ. In this case, one refers to d$\boldsymbol{u}$/dt as the *transverse acceleration* or *radial acceleration* $\boldsymbol{a}_\mathrm{r}$. From Fig. 2.6, we see immediately that the following relation[2] holds:

$$\mathrm{d}\beta = \frac{\mathrm{d}u}{u} \quad \text{or} \quad \mathrm{d}u = u \cdot \mathrm{d}\beta\,.$$

[2] *Example:* $\mathrm{d}\beta = 4.5°$, $° = 0.0175$, $\mathrm{d}t = 0.1\,\mathrm{s}$, $\omega = \frac{\mathrm{d}\beta}{\mathrm{d}t} = \frac{4.5 \cdot 0.0175}{0.1\,\mathrm{s}} = 0.79/\mathrm{s}\,.$

Figure 2.6 The definition of the radial acceleration



The quotient

$$\frac{d\beta}{dt} = \omega \tag{2.5}$$

is called the angular velocity[C2.5], and the radial acceleration becomes

C2.5. The angular velocity is also a vector. It points in the direction of the axis of rotation: if we look along this direction, we see the rotation as clockwise. Then Eq. (2.6) has the general form: $\boldsymbol{a}_r = \boldsymbol{\omega} \times \boldsymbol{u}$ (for the definition of the vector product, see Chap. 6). In all cases considered here, with $\omega \perp u$, the magnitude equation is sufficient.

$$a_r = \omega \cdot u\,. \tag{2.6}$$

The word *acceleration*, according to the above definitions, is used in physics in a quite different sense than in everyday language. First of all, in everyday life an accelerated motion usually means a motion at a higher speed, e.g. the accelerated circulation of a document or file. – Secondly, the word acceleration in everyday language leaves the direction completely out of consideration.

For the majority of motions, path accelerations $\boldsymbol{a}$ and transverse accelerations $\boldsymbol{a}_r$ are both present at the same time; along the path of the motion, both the magnitude and the direction of the velocity are changing during the motion. Nevertheless, we will limit ourselves for the moment to the limiting cases of pure path acceleration (straight-line motion) or pure radial acceleration (circular orbit).

2.4 Path Acceleration and Linear Motion

(G. GALILEI, 1564–1642.) The path acceleration changes only the magnitude, not the direction of the velocity. As a result, the motion follows a straight-line path; its trajectory is linear.

A path acceleration is in principle easy to measure: We determine the velocity at two times with the time interval Δt; these velocities are u_1 and u_2. Then we compute $\Delta u = (u_2 - u_1)$ (positive or negative) and form the quotient $\Delta u/\Delta t = a$[C2.6].

C2.6. Here, we see among other things that POHL is concerned not only with the definition of a quantity, but rather he always gives a compact instruction for its measurement; an aspect which is not emphasized in many textbooks. It is in this sense that the detailed descriptions of measurements of velocities and accelerations given here are to be understood.

Δt, as we have already pointed out, must be chosen to be sufficiently small. The result of the measurement should not change on a further decrease of Δt. Practically, this requirement usually means that rather short time intervals Δt must be used. The latter are measured with some sort of *registration procedure*. That is, the course of the motion is first recorded automatically and the record is then evaluated after the motion is complete. But there is also a much simpler procedure. For example, time marks can be imprinted on the moving

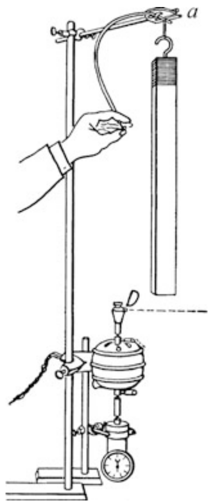


Figure 2.7 Measurement of the acceleration of a freely-falling body **(Video 2.1)**

Video 2.1:
"Free fall"
http://tiny.cc/jpqujy

object by a clock. Of course, the imprinting process must not disturb the motion itself. We give a practical example: The acceleration of a freely-falling wooden bar is to be determined. Fig. 2.7 shows a suitable arrangement. It can also be used for many other types of acceleration measurements[C2.7].

The essential element is a fine ink jet which is rotating in the horizontal plane. The jet is sprayed out of a nozzle D on the side of a rotating ink container (Fig. 2.8) (electric motor with its shaft vertical). The frequency, e.g. $\nu = 50\,\mathrm{Hz}$, is determined by a frequency meter. Here again, the time measurement is referred to a uniform rotation.

The wooden bar is wrapped in a jacket of white paper and hung at the point a. A cable trigger lets it fall at the desired time. The bar then falls through the rotating ink jet and on to the floor. – Figure 2.9 shows the result: a clean sequence of time marks at time intervals of 1/50 of a second.

C2.7. For example, with modern apparatus, the accelerated motion of a freely-falling body can be conveniently recorded electronically using photoelectric triggers. Beams of light which are directed at photocells are interrupted briefly by the passing object, and the resulting signals control an electronic stopwatch.

The object continues falling while the ink jet swishes past; this causes the curvature of the time marks.

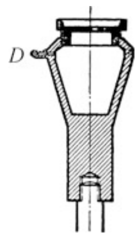
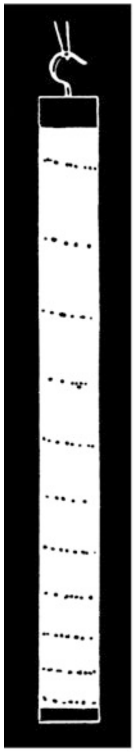


Figure 2.8 The ink jet used in Fig. 2.7, at half its actual size



Velocity $u = \frac{\Delta l}{\Delta t}$ (cm/s)	Velocity change Δu in $\Delta t = \frac{1}{50}$ s (cm/s)	Acceleration $a = \frac{\Delta u}{\Delta t}$ (m/s^2)
285.50		
	22.50	11.25
263.00		
	17.50	8.75
245.50		
	18.00	9.00
227.50		
	21.25	10.63
206.25		
	21.25	10.63
185.00		
	18.50	9.25
166.50		
	19.00	9.50
147.50		
	18.00	9.00
129.50		
	19.50	9.75
110.00		
Mean values:	19.50 cm/s	9.8 m/s^2

Figure 2.9 Falling body with time marks and their evaluation, with the usual experimental and readout errors. This experiment also shows that the measurement of a second-order differential quotient is in general an awkward matter. (**Video 2.1**)

Video 2.1:
"Free fall"
http://tiny.cc/jpqujy

We can see with the unaided eye that the motion is accelerated: The spacing of the time marks, i.e. the distance Δl through which the object falls in the time $\Delta t = (1/50)$ s, increases continuously. The computed values of the velocity $u = \Delta l/\Delta t$ are written beside the marks; the velocity increases on the average within each (1/50) s by the amount 19.5 cm/s. Here, we ignore the inevitable errors in the individual values (scatter). This case of *free fall* is one of the rare examples of a *constant or uniform path acceleration*. For the magnitude of this constant acceleration, we find

$$a = 9.8\,\text{m/s}^2.$$

Repetition of the experiment with an object made of a different material, for example a brass tube instead of the wooden bar, yields the same value for the acceleration. *The constant acceleration a for free fall is the same for all falling bodies. It is denoted by g.* A more precise value is $g = 9.81\,\text{m/s}^2$. It is called the *acceleration of gravity*

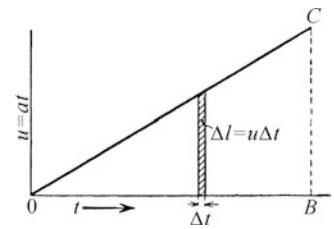


Figure 2.10 The velocity u as a function of time for constant path acceleration

or *apparent gravity*[3]. These are incidental experimental facts at this point. Their great significance will become apparent later.

Our practical example of a measurement led us to the special case of a *constant* path acceleration. This is an important case.

Constant acceleration means that the increase in the velocity Δu is the same in equal time intervals Δt. The velocity u increases as shown in Fig. 2.10 linearly with time t. In each time interval Δt, the object travels through the distance Δl. Therefore, we have $\Delta l = u\Delta t$. Here, u is the average value of the velocity within each time interval Δt. Such an interval is shown in Fig. 2.10 as a shaded area. The entire triangular area *0BC* is the sum of all of the distance segments Δl traversed in the time t. Thus for the case of constant path acceleration, the overall distance l traversed in the total time t is given by the equation[C2.8]

$$l = \tfrac{1}{2}at^2\,, \tag{2.7}$$

i.e. the distance increases as the square of the time during which the acceleration acts.

C2.8. The formula explained graphically here (2.7) follows mathematically from the defining equations $u = \mathrm{d}l/\mathrm{d}t$ (2.2) and $a = \mathrm{d}u/\mathrm{d}t$ (2.3). For $a = $ const and from the rules of integral calculus, we obtain:
$l = \int u\,\mathrm{d}t = \int at\,\mathrm{d}t$
$= a\int t\,\mathrm{d}t = \tfrac{1}{2}at^2\,.$

If the body already had an initial velocity u_0 before the beginning of the acceleration, then instead of Eq. (2.7), the equation

$$l = u_0 t + \tfrac{1}{2}at^2 \tag{2.8}$$

would apply.

The origin of the constant path acceleration is completely irrelevant. It could be for example an electrical force instead of a mechanical force.

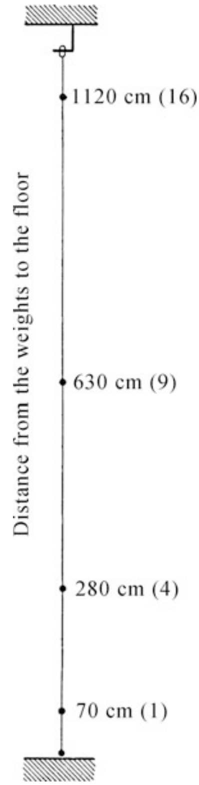
Usually, for verifying Eq. (2.7), one makes use of the constant acceleration $a = g$ which acts on a freely falling body. As an example, we mention the well-known falling rope experiment.

This experiment consists of a thin rope, hung perpendicularly, with a series of lead weights attached to it; cf. Fig. 2.11. The lowest weight nearly

[3] This numerical value holds near the surface of the earth; for most purposes, g can be considered to be constant. A more precise consideration shows that g depends weakly on the geographical latitude of the location (Sect. 7.6). Furthermore, it also depends on local variations in the surface features of the earth (e.g. deposits of heavy ores under the ground), and, although only weakly, on the altitude of the location where the observations are made.

Part I

Figure 2.11 Falling rope



touches the floor. The spacing of the other weights is arranged to correspond to the squares of whole numbers. When the upper end of the rope is released, the weights hit the floor one after another. The experimenter hears their impacts at equal time intervals.

Strictly speaking, observations of free fall should be carried out in a vacuum. Only then can perturbations due to air resistance be eliminated. In a highly evacuated glass tube, all bodies indeed fall at exactly the same speed[C2.9]. A lead ball and a fluffy feather fall at the same speed in a vacuum, and strike the ground at the same time. In room air, in contrast, the feather falls much more slowly, as we well know. But heavy bodies with a relatively small surface area are only weakly slowed by air resistance (cf. Fig. 5.20).

C2.9. Free fall can be used for carrying out experiments under conditions of weightlessness, e.g. in the evacuated drop tower at University of Bremen, which is over 120 m high (cf. Comment C7.2).

2.5 Constant Radial Acceleration and Circular Orbits

(C. HUYGHENS, 1629–1695.) The radial acceleration $\boldsymbol{a}_\mathrm{r}$ does not change the magnitude of a velocity $\boldsymbol{u}$, but only its direction. Let the radial acceleration $\boldsymbol{a}_\mathrm{r}$ be constant and let us assume that no other accelerations are present. Then the direction of $\boldsymbol{u}$ changes by the same angular increment $d\beta$ in equal time intervals dt. The resulting orbit is a circle. It is traversed with the constant angular velocity $\omega = d\beta/dt$.

The time required for a complete revolution was termed the (rotational) *period* T in Sect. 1.8, and its inverse was called the rotational (or mechanical) *frequency* ν; thus $\nu = 1/T$. Then for the orbital velocity u, we have

$$u = \frac{\text{Circumference}}{\text{Period}} = \frac{2\pi r}{T} = 2\pi r \nu \tag{2.9}$$

and the angular velocity is

$$\omega = \frac{2\pi \cdot \text{Angle}}{\text{Period}} = 2\pi\nu \tag{2.10}$$

so that[C2.10]

$$u = \omega r\,. \tag{2.11}$$

C2.10. In general, Eq. (2.11) is written as a vector equation: $\boldsymbol{u} = \boldsymbol{\omega} \times \boldsymbol{r}$.

When a circular orbit is traversed with a constant velocity, the angular frequency ω is thus a factor of 2π greater than the mechanical frequency ν, that is $\omega = 2\pi\nu$; therefore, ω is often called the *circular frequency* (unit usually 1/s). This definition and these relations hold quite generally for periodic processes (e.g. the rotation of an electric motor shaft).

We combine Eqns. (2.6) and (2.11) and obtain the result[C2.11]

$$a_\mathrm{r} = \omega^2 r = \frac{u^2}{r}\,. \tag{2.12}$$

C2.11. Using the vector formulations of Eqns. (2.6) and (2.11), we obtain for the first part of Eq. (2.12) likewise a vectorial form: $\boldsymbol{a}_r = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$ or, for $\boldsymbol{\omega} \perp \boldsymbol{r}$: $\boldsymbol{a}_r = \boldsymbol{\omega}^2 \boldsymbol{r}$. The radius vector $\boldsymbol{r}$ points from the center of the circular orbit outwards, while the acceleration vector points in the opposite direction towards the center.

This radial acceleration a_r must be present so that a body can traverse a circular orbit of radius r with the constant angular velocity (circular frequency) ω or the constant orbital velocity u.

Intuitively, the constant radial acceleration required for a circular orbit has the following interpretation (Fig. 2.12):

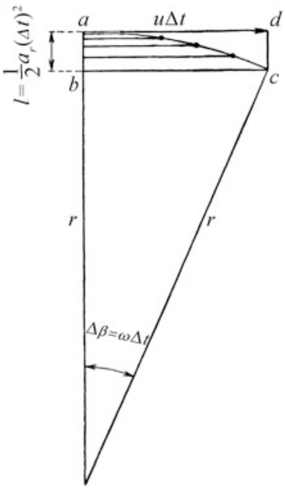
Suppose that a body traverses a circular segment *ac* within the time interval Δt. Imagine this orbit to be composed of two steps that occur one after the other, namely:

1. A motion along a (tangential) path *perpendicular* to the radius with the constant velocity u, $ad = u\Delta t$;

2. an accelerated motion along a (radial) path (anti-parallel to the radius), $l = \frac{1}{2} a_\mathrm{r} (\Delta t)^2$. The thin horizontal lines (time markers) in the figure permit us to see that the motion along l is accelerated, so that we can apply Eq. (2.7).

A numerical example can be useful. The earth's *moon* moves during the time $\Delta t = 1\,\mathrm{s}$ along the direction *ad*, that is *perpendicular* to

Figure 2.12 The explanation of radial acceleration



its orbital radius, by 1 km, thereby "increasing" its distance from the earth slightly. At the same time, it "approaches" the earth along the orbital radius in an accelerated motion, traversing the distance $l = \frac{1}{2}a_{\mathrm{r}}(\Delta t)^2 = 1.35\,\mathrm{mm}$. Thus, the net effect is that the radius remains unchanged, and the orbit is circular. The radial acceleration of the moon is found to be $a_{\mathrm{r}} = 2.70\,\mathrm{mm/s^2}$.

2.6 Distinguishing Physical Quantities and Their Numerical Values

C2.12. The following two sections do not belong directly to the content of the rest of this chapter. POHL however added them here, after introducing the first important concepts, from the 10th edition on; possibly because of his experience with examinations. They deal with simple and indeed self-evident topics, which are however to some extent not consistently treated in most textbooks even today.

C2.13. Such "false definitions" can still be found in many textbooks!

In commerce[C2.12], the price of every item is a "quantity", i.e. the product of a *numerical value* and a *unit*. For example, a hat might cost 10 \$ and a pencil 10 cents. No one would consider these two prices to be the same. The ratio of the two prices is rather

$$\frac{10\,\$}{10\,\mathrm{cent}} = \frac{10 \cdot 100\,\mathrm{cent}}{10\,\mathrm{cent}} = 100.$$

The same principle holds in physics: Distance l, time t, velocity u, acceleration a, frequency ν etc. are measured as *physical quantities*, i.e. as products of a *numerical value* with a *unit*. A velocity of $u = 7$ is meaningless. It becomes meaningful only when expressed as for example $u = 7\,\mathrm{m/s}$. The confusion of physical *quantities* (e.g. distance $l = 5\,\mathrm{km}$ and velocity $u = 5\,\mathrm{km/h}$) with their *numerical values* (in the example, the value of the distance is 5 and the value of the velocity is 5) gives rise to widespread but *incorrect* definitions, such as for example "the velocity is the distance traversed in a unit time"[C2.13]. The velocity is not a *distance*, but rather the quotient of distance/time. – Or still worse: "The frequency is the number of oscillations in one second". First of all, a frequency is not a number, but rather the quotient number/time; e.g. the pulse frequency of a person is ca. 70/minute. Secondly, a physical quantity which is supposed to be *generally applicable* cannot be defined in terms of a particular *unit* such as the second.

2.7 Base Quantities and Derived Quantities

Some few physical quantities are termed *base quantities* and are measured in units defined especially for those quantities, the *base units*; for example, *time* with the unit *second*, or *temperature* with the unit *kelvin*. If one wishes to introduce a base quantity, it can be defined only in terms of axioms which are founded on extensive experiments or observations, and not on equations.

Most physical quantities are defined as *derived quantities*. This means that they and their units can be defined not in terms of axioms, but instead by means of equations which contain other quantities and their units. We recall the example of the velocity, $u = \mathrm{d}l/\mathrm{d}t$ and its units, chosen for example to be meters/second or kilometers/hour, etc.

The possibility of defining quantities and their units in terms of equations is the only point in which derived *quantities differ from the* base *quantities employed.*

No physical quantity is *in its essence* a base quantity; one could introduce many different quantities as base quantities. The number and type of the base quantities should be chosen insofar as possible so that no two derived quantities have the same defining equation. – In distinguishing between base quantities and derived quantities, one should in no case envision a hierarchy; base quantities are not imbued with a special aura, nor should limiting them to a certain number (e.g. three) be raised to the status of a dogma.

The currently agreed-upon base quantities in the international unit system (SI) and their base units are[C2.14]:

- Length (or distance), unit *meter* (m),
- Time, unit *second* (s),
- Mass, unit *kilogram* (kg),
- (Thermodynamic) temperature, unit *kelvin* (K),
- Electric current, unit *ampere* (A),
- Luminous intensity, unit *candela* (cd); and
- Amount of substance, unit *mole* (mol).

C2.14. In these books (Vol. 1 and Vol. 2), we use mainly (but not exclusively) the units defined within the SI. See for example http://physics.nist.gov/cuu/index.html or http://www.bipm.org/en/measurement-units/.

Note that the names of *units* and their abbreviations are printed in Roman type (m, s, kg, …), while *physical quantities* are printed in *italics* (*distance* = l or d, *mass* = M or m, etc.). Avoid confusing them; there are not enough letters in the Roman and Greek alphabets to allow us to use different symbols for all the quantities and units that we require[C2.15].

C2.15. Recommendations for writing physical-quantity expressions and equations and for naming or abbreviating units are given at http://physics.nist.gov/cuu/pdf/checklist.pdf.

Exercises

2.1 A person wants to row her boat along a straight-line path from A to B. A and B are points opposite each other on either side of the mouth of a river, which is 600 m wide; B is also 300 m upriver from A. The person rows at the same velocity as the upriver flow rate of the tide which is coming in. In which direction should she row? (Sect. 2.2)

2.2 Calculate the distance l which is traversed during the fourth second by an object in free fall which began falling from a resting position at the time $t = 0$. The acceleration of gravity is $g = 9.81\,\mathrm{m/s^2}$. (Sect. 2.4)

2.3 A parachute jumper initially falls freely over a distance of 50 m with the acceleration of gravity $g = 9.81\,\mathrm{m/s^2}$ after jumping (we may neglect air friction). Then his parachute opens, delaying his fall (i.e. it produces a negative acceleration, upwards) by $2\,\mathrm{m/s^2}$, so that he finally lands on the ground with a velocity of 3 m/s. How long (time t) was he in the air, and from what height h did he jump? (Sect. 2.4)

2.4 An object of mass $m = 20\,\mathrm{kg}$ increases its velocity from 15 m/s to 18 m/s, covering a distance of 20 m in the process. What is the magnitude of its constant acceleration and the corresponding force? (Sect. 2.4 and 3.2)

2.5 A particle begins at rest at the point P and moves along a straight-line path to the point O, a distance of 3 m away. There, its velocity is 6 m/s. Plot its velocity *vs.* position for (a) constant acceleration, and (b) a sinusoidal motion back and forth through the point O. How long does the motion from P to O take in each case? (Sect. 2.4 and 4.3)

2.6 How large is the radial acceleration a_r of a person who is at 51.5° north latitude (e.g. in London)? The radius of the earth is 6378 km. (Sect. 2.5)

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