# 2 Seismic Waves

## 2.1 The Fundamental Conditions

An application of an external force, on part of a medium (elastic medium), leads to creation of internal opposing forces which intend to resist the deformations caused by that external force. Typical forms of the resulting deformations are changes in volume and/or in shape which are created at the affected location. In consequence, the medium will return to its original condition after the external force is removed. This property of resisting of changes in volume and in shape and return to original conditions after removal of the external force is called (elasticity). Provided that the changes are small, rock media in nature are considered to be perfectly elastic in nature.

As a result of the elasticity property of media, the changes (volume and shape changes) oscillate about their neutral positions and, at the same time, propagate away from the energy source-location. Energy transfer in this manner (motion that leaves out no permanent distortions) is commonly referred to as (wave motion).

The fundamental condition for the creation and propagation of seismic waves (seismic field) is a source of mechanical energy of impulsive type which is initiated within an elastic medium. The energy source may be natural (as in earthquake-generated waves) or artificial (as in firing of a dynamite charge). In both cases wave motion of elastic waves are generated and can be recorded by the appropriate detection instruments. These are normally referred to as the (seismic waves).

## 2.2 Theory of Elasticity

As it is stated above, the fundamental conditions for a seismic field to be created is that the medium must possess the elasticity property. Two main concepts are governing the propagation of seismic waves in an elastic medium: the (stress) and the (strain). Stress represents the external force applied to the elastic medium, and strain is the resulting changes in volume and in shape. The relation between stress and strain, for a particular medium (perfectly elastic medium), gives evaluation expressions for the elasticity property of that medium. The stress-strain proportionality constants are the elastic coefficients which serve as measures of the elasticity of a particular medium.

The principal types of changes experienced by a medium due to passage of a seismic wave are re-distribution of the internal forces (stress changes) and modification of the volume and geometrical shape (strain changes). The theory of elasticity deals with analysis of these principal effects and the related physical changes.



H.N. Alsadi, *Seismic Hydrocarbon Exploration*, Advances in Oil and Gas Exploration & Production, DOI 10.1007/978-3-319-40436-3_2

### 2.2.1 Stress

In the broad sense, stress is represented by a force (called traction) which is acting on a finite area occupying an arbitrary position within the medium. However, for more precise definition, the stress (**T**) is defined to be a limiting value of the ratio of a force (**F**) acting on an elementary area (**ΔA**) which is diminishing to zero. That is:

$$\mathbf{T} = \lim_{\Delta \mathbf{A} \to 0} (\mathbf{F}/\Delta \mathbf{A})$$

In general, the stress (**T**) is a vector that can be resolved into components parallel and perpendicular to the area (**ΔA**). The normal component ($\mathbf{T_z}$) is called the normal stress or dilatational or pressure stress as it is sometimes called. The other two components ($\mathbf{T_x}$ and $\mathbf{T_y}$) which are in the plane of the elementary area, are called tangential or shearing stresses (Fig. 2.1).

The stress system within a body is completely defined if, at each point in that body, the normal stress and the two shearing stresses are all determined for three mutually perpendicular plane areas.

It follows, therefore, that nine stress components are needed to completely define the stress at a given point. This nine-component set constitutes what is known as the (stress tensor) at that point. Once the nine components are defined at a certain point (with respect to a given area-set) it is possible, through suitable mathematical transformation, to determine the stress with respect to any other area-set defined for that point.

The three mutually perpendicular elementary areas, called an (area set), and the nine components of the stress tensor is shown in Fig. 2.2.

A stress component may be represented by ($T_{ab}$) where a (=x, y, z) stands for the area-set an b (=x, y, z) for the direction of the component. Using this convention, the nine components of the stress tensor (shown in Fig. 2.2) may be written as in Table 2.1.

For a stressed body which is in equilibrium (i.e. experiencing no rotation), it can be shown (see Bullen 1965, p. 10) that, due to the symmetry of a stress tensor acting on a body in a state of equilibrium, we have $T_{ab} = T_{ba}$. Applying this property to this stress system, we get reduction in the number of the components to a total of six components which are independent of each other. That is Table 2.2:

These six components are sufficient to define the stress system at a point within a stressed body. Further simplification is possible if the three areas of an area-set are chosen such that the normals to these areas are coincident with the directions of the normal stresses. With this kind of set-up the shearing components will all reduce to zero leaving only the components ($\mathbf{T_{xx}}$, $\mathbf{T_{yy}}$, $\mathbf{T_{zz}}$) for the definition of the stress system. In such a case, when the shear components become all equal to zero, the components ($\mathbf{T_{xx}}$, $\mathbf{T_{yy}}$, $\mathbf{T_{zz}}$) are referred to as the principal stresses.

For the simple one-dimensional case, when a force (**F**) is acting uniformly at the cross-sectional area (**A**) of a bar or a metal wire, stress is defined as the force per unit area of the cross sectional area, (F/A). This stress is called tensile stress when the force is a pull-force and it is called compressive stress when it is a push-force.

Units of stress is the same as those used in measuring pressure (force per unit area), and in the SI unit system the unit is Newton per square

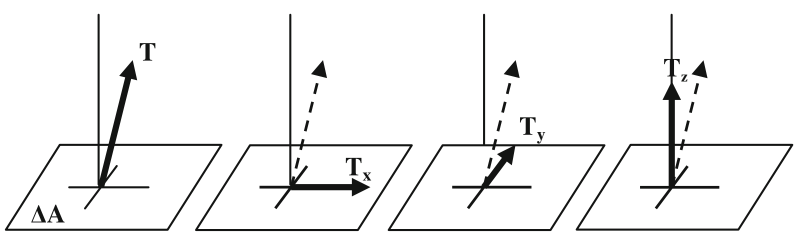



**Fig. 2.1** Stress ($T$) and its components ($T_x$ ,$T_y$, $T_z$) acting on the elementary area ($\Delta A$)

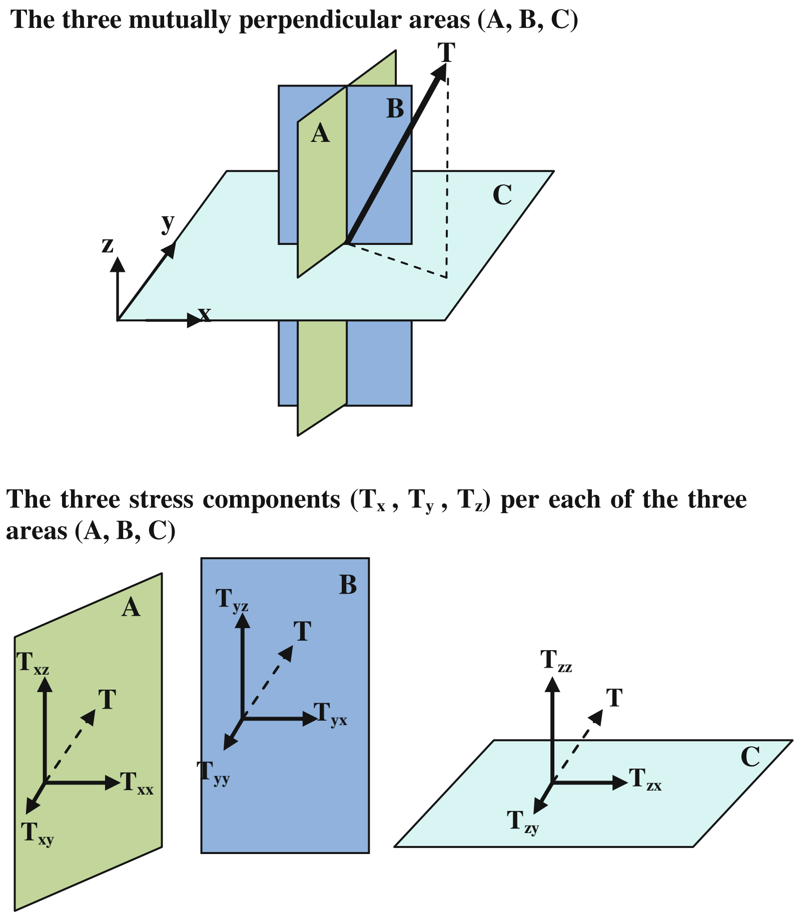



**Fig. 2.2** The stress components. The mutually perpendicular planes (*A*, *B*, *C*) making up the area-set and the nine components of the involved stress tensor

**Table 2.1** The complete nine components of the stress tensor

| Stress component along the: | Area (a) perpendicular to: x-axis | Area (b) perpendicular to: y-axis | Area (c) perpendicular to: z-axis |
|---|---|---|---|
| (a) x-axis | $T_{xx}$ | $T_{yx}$ | $T_{zx}$ |
| (b) y-axis | $T_{xy}$ | $T_{yy}$ | $T_{zy}$ |
| (c) z-axis | $T_{xz}$ | $T_{yz}$ | $T_{zz}$ |

**Table 2.2** The six independent components of the stress tensor

| Stress component along the: | Area (a) perpendicular to: x-axis | Area (b) perpendicular to: y-axis | Area (c) perpendicular to: z-axis |
|---|---|---|---|
| (a) x-axis | $T_{xx}$ | | |
| (b) y-axis | $T_{xy}$ | $T_{yy}$ | |
| (c) z-axis | $T_{xz}$ | $T_{yz}$ | $T_{zz}$ |

meter (N/m$^2$) which is called Pascal, where one Pascal is equal to 1 N/m$^2$.

### 2.2.2 Strain

In reference to Fig. 2.3, let us consider the two points (**$P_1$** and **$P_2$**) located within an unstressed body, where the first point, **$P_1$** is located at (**x, y, z**) and the second point (**$P_2$**) at (**x + dx, y + dy, z + dz**). Now, we let this body to deform as a result of a stress system created within it. If the two points (**$P_1$** and **$P_2$**) were displaced from their original positions by equal displacements (**D**, say), then it is considered that there is no strain taking place. Strain occurs only when there is variation of displacement of any point, within that medium, with respect to the others. In the language of mathematics, we say that strain depends on the derivatives of the displacement-components with respect to the chosen coordinates (**x, y, z**). The concept is clarified in Fig. 2.3.

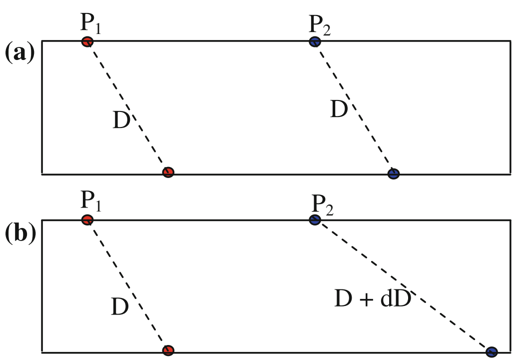



**Fig. 2.3** Displacement of two adjacent points in a medium under stress. **a** Case of equal point-displacements giving no-strain state. **b** Case of different point-displacements giving the strain state

In general, when a body is subjected to elastic stress, both of its size and shape will change. As it is mentioned above, the resulting changes represent elastic strains when each point of the stressed body experiences a displacement of its own which is different from the displacements experienced by the other points of the body. This implies that there are two types of strains, namely the "volume strain" and the "shape strain" (Fig. 2.4).

### 2.2.3 Common Types of Strain

Mathematical analyses of strain show that the total strain of a three dimensional body, depends on only six different derivatives of displacements. These strain components (**$e_{ab}$**), can be written down as follows (Richter 1958, p. 236):

$$\boxed{\begin{aligned}
e_{xx} &= \partial D_x / \partial x \\
e_{yy} &= \partial D_y / \partial y \\
e_{zz} &= \partial D_z / \partial z \\
e_{xy} &= (\partial D_x / \partial y + \partial D_y / \partial x) / 2 \\
e_{xz} &= (\partial D_x / \partial z + \partial D_z / \partial x) / 2 \\
e_{yz} &= (\partial D_y / \partial z + \partial D_z / \partial y) / 2
\end{aligned}}$$

These equations represent two groups of strain components of a strained elastic body. The first group (**$e_{xx}$, $e_{yy}$, $e_{zz}$**) involve purely translational displacement resulting in compressional or dilatational strain. The second group (**$e_{xy}$, $e_{xz}$, $e_{yz}$**) involve purely rotational deformation resulting in shear strain. As it is stated in our discussion of stress, the compressional (or dilatational) strains are called (principal strains) when the shearing strains are all of zero values. Common types of strains are cases of compression, bulk contraction, tension, and shear strains.

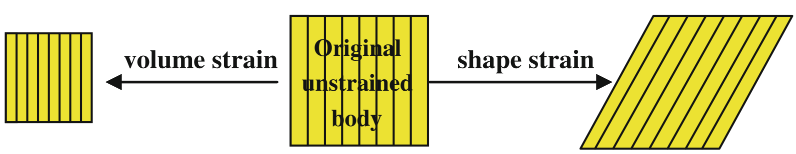



**Fig. 2.4** The two types of strain; "volume" and "shape" strains

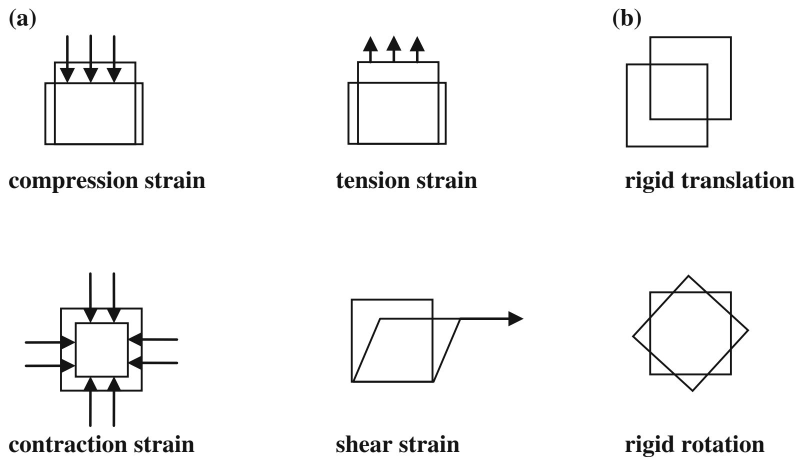



**Fig. 2.5** Types of simple strains (**a**) and no-strain changes (**b**)

Rigid body-translation and rotation represent cases of no strain, since no volume and no shape deformation are involved. In Fig. 2.5, an elementary cube (shown here in plan) is used to show simple types of elastic deformation (strain) and no-strain changes.

In general, an elastic body under stress can experience two types of distortions; changes in volume and changes of shape. These changes, which occur as result of stress, are expressions of the physical properties of the stressed body. In its simple form, elastic strain can be divided into two main types. These are: the volume-changing strain (leading to body compression or dilatation) and the shape-changing strain (leading to body shape distortion).

### 2.2.4 The Volume-Changing Strain

The familiar example on this type of strain is the longitudinal strain of a body under an extensional (or compressional) stress. The longitudinal strain (**e**) is defined to be the change in length in a certain dimension, of a body under stress relative to its original length. For a rectangular lamina of dimensions (**Δx** by **Δy**), the longitudinal strains ($\mathbf{e_x}$ and $\mathbf{e_y}$) in the **x** and **y** directions are defined as (Fig. 2.6):

The longitudinal strains can be extensional (tensile strain) or compressional (contraction strain). Longitudinal strains ($\mathbf{e_x}$ and $\mathbf{e_y}$) are defined as:

$$\mathbf{e_x = D_x/\Delta x}$$
$$\mathbf{e_y = -D_y/\Delta y}$$

where ($\mathbf{D_x}$ and $\mathbf{D_y}$) are the changes in length in **x** and **y** directions respectively.

The minus sign that appeared in the $\mathbf{e_y}$ expression is entered to denote that the change ($\mathbf{D_y}$) is compression which is in opposite direction to the dilatation change ($\mathbf{D_x}$), in the **x**-direction.

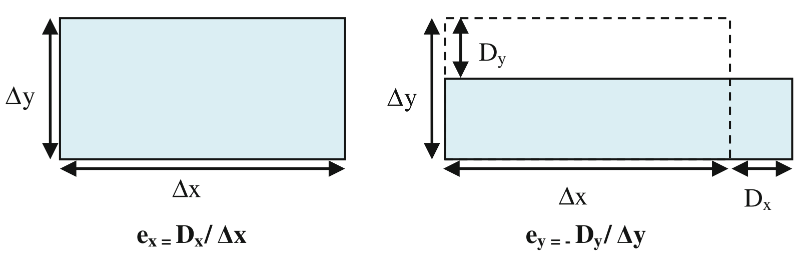



**Fig. 2.6** Definition of the longitudinal strain as applied for a rectangular lamina of dimensions (*Δx* by *Δy*)

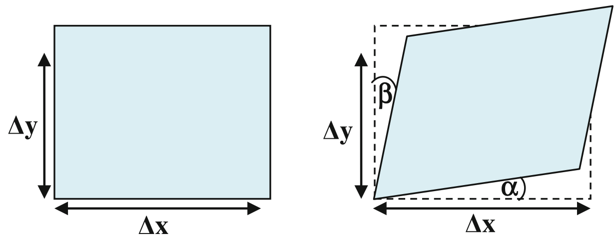



**Fig. 2.7** Concept of shear or angular strain, $e_{xy}$ where: $(\mathbf{e_{xy}} = (\boldsymbol{\alpha} + \boldsymbol{\beta})/2$ $= (\mathbf{D_x}/\boldsymbol{\Delta}\mathbf{y} + \mathbf{D_y}/\boldsymbol{\Delta}\mathbf{x})/2$

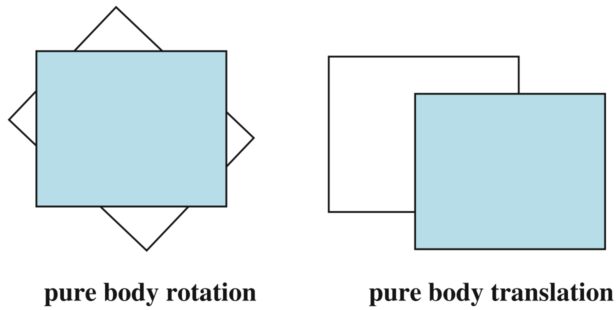



**Fig. 2.8** Pure rotation and pure translation changes of a body, are not considered to be elastic strains

### 2.2.5 The Shape-Changing Strain

As longitudinal strain gives expression for the volume changes resulting from stress application, the shear strain gives the corresponding measure for the shape deformation. Using the example above (**Δx** by **Δy** rectangular lamina). The shear strain (also called angular strain) is considered to be the average of the two angles by which two neighboring sides rotate as a result of the shearing stress. Thus, the shear strain ($\mathbf{e_{xy}}$) is defined as:

$$\mathbf{e_{xy}} = (\propto + \boldsymbol{\beta})/\mathbf{2}$$

where ($\propto$ and $\boldsymbol{\beta}$) represent the angles of rotation of the two sides (**Δx** and **Δy**) brought about by the shear stress (Fig. 2.7).

Since these two angles are very small (usually so, in seismic-field conditions), they can be represented by their corresponding tangents, giving

$$\mathbf{e_{xy}} = (\boldsymbol{\alpha} + \boldsymbol{\beta})/\mathbf{2} = (\mathbf{D_x}/\boldsymbol{\Delta}\mathbf{y} + \mathbf{D_y}/\boldsymbol{\Delta}\mathbf{x})/\mathbf{2}$$

where the angles ($\boldsymbol{\alpha}$ & $\boldsymbol{\beta}$) are in radians.

It should be emphasized that strain occurs only if the body particles experience unequal displacements. When the displacements are equal a body may experience pure translation (rigid body-translation) or pure rotation (rigid body-rotation), as shown in Fig. 2.8.

Rigid-body changes which do not involve volume or shape changes, such as these, are not considered to be elastic strains.

### 2.2.6 The Cubical Dilatation

A parameter, closely related to longitudinal strain and of special importance in the theory of elasticity is the Cubical Dilatation ($\boldsymbol{\theta}$). At a certain point within a strained medium, this is defined as the fractional change in a unit volume surrounding that point. Thus, for a three dimensional body with longitudinal strains ($\mathbf{e_{xx}}, \mathbf{e_{yy}}, \mathbf{e_{zz}}$), the cubical dilatation can be computed as follows:

$$\boldsymbol{\theta} = (\mathbf{1} + \mathbf{e_{xx}})(\mathbf{1} + \mathbf{e_{yy}})(\mathbf{1} + \mathbf{e_{zz}}) - \mathbf{1}$$

For small strains $\mathbf{e_{xx}}, \mathbf{e_{yy}}, \mathbf{e_{zz}}$ (which is the case in seismic-field conditions), the products of these terms may be neglected giving the result:

$$\theta = e_{xx} + e_{yy} + e_{zz} = D_x/\Delta x + D_y/\Delta y + D_z/\Delta z$$

The sign convention of ($\theta$) is negative for compression and positive for expansion strains.

### 2.2.7 Stress-Strain Relationship

It is a common experience that a body under stress undergoes deformation of a form and value depending on the applied load and on the physical properties of that body. Bodies of the type which, under stress, exhibit a proportional strain are called elastic bodies. When the proportionality is linear, these are called perfectly elastic bodies.

Normally, bodies, under increasing stress, exhibit linear stress-strain behavior up to a certain stress-limit, beyond which the material may still be elastic but with no more linear relationship. Usually there is a point (the elastic limit) after which the deformation becomes irrecoverable and in this case the body behavior is described to be plastic. An increase of stress beyond the elastic limit produces large increase in strain, and it does so even with decreasing stress. With further increase of an extensional stress (tensile loading) for example, a point is reached where the body can no longer sustain the applied stress. At this point (called the rupture point) the body breaks up. Behavior of a ductile solid-body under an increasing extensional stress is shown in Fig. 2.9.

For an isotropic body (physical properties are independent of direction) and for an elastic body, under small strain, strain varies linearly with the applied stress. This linear stress-strain relationship is governed by a well-known mathematical equation. It is the Hooke's law.

### 2.2.8 Hooke's Law for Isotropic Media

In its simple form, Hooke's law states that the strain-stress relationship is linear. It is applicable to the behavior of stressed bodies when stresses are sufficiently small. If several stresses are acting on a body, the net strain produced is the sum of the individual strains. This is one of the important outcomes of the linearity property of the stress-strain relationship of isotropic media under small strains. A medium under stress condition, in which Hooke's law holds, is called Hookean medium.

When the stressed bodies are isotropic (their physical properties do not change with direction) the linear stress-strain relationship becomes relatively simple linear function (Sheriff and Geldart 1995, p. 37). The linear equation that connects stress to strain of isotropic media is commonly found in the geophysical literature, as in Bullen (1965, p. 20), McQuillin et al. (1984, p. 11), Sheriff and Gildart (1995, p. 37).

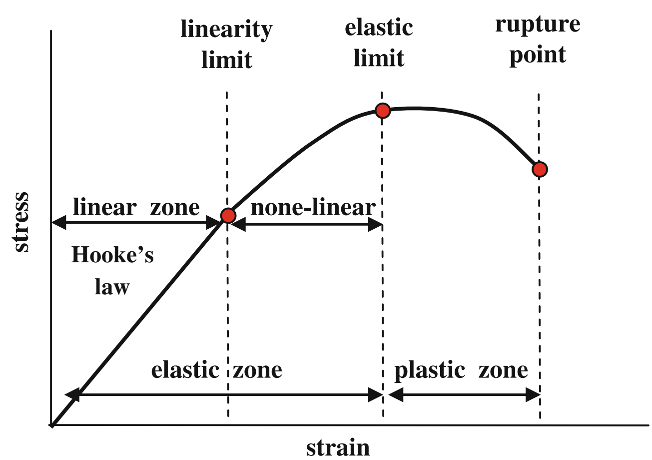



**Fig. 2.9** Elastic and plastic zones shown by a solid ductile body under an increasing tensile stress

The linearity property governed by Hooke's law means that there is a proportionality-constant for the linear stress-strain relation for any particular body under stress. Mathematical studies showed that, for an isotropic body, two elastic coefficients are sufficient (Richter 1958, p. 238). These are the Lame's coefficients (**λ & μ**), which are sufficient in characterizing the elastic properties of a medium.

By use of Lame's coefficients (**λ & μ**), Hooke's law can be presented in the following compact form:

$$\boxed{T_{ij} = \lambda\,\theta\,\delta_{ij} + 2\mu\, e_{ij}}$$

where the symbols (**i** & **j**) take the values x, y, and z, and the term $\delta_{ij} = 1$ when ($i = j$), and $\delta_{ij} = 0$ when ($i \neq j$). **T** and **e** are the stress and strain respectively.

This compact form of the Hooke's law can be presented in the following explicit equations:

$$\boxed{\begin{aligned} T_{xx} &= \lambda\,\theta + 2\mu\, e_{xx} \\ T_{yy} &= \lambda\,\theta + 2\mu\, e_{yy} \\ T_{zz} &= \lambda\,\theta + 2\mu\, e_{zz} \end{aligned}}$$

For pure shear strain (that is with no change in volume, for $\theta = 0$), the Law expresses the relations for purely shearing strain, that is:

$$\boxed{\begin{aligned} T_{xx} &= 2\mu\, e_{xx} \\ T_{yy} &= 2\mu\, e_{yy} \\ T_{zz} &= 2\mu\, e_{zz} \end{aligned}}$$

From this equation, it is evident that, the stress is consisting of the sum of two parts; the first part ($\lambda\ \theta$), involving the elastic coefficient ($\lambda$) multiplied by the volume change (the dilatation, $\theta$) and the second part ($2\mu\ e_{ij}$) which is involving the second elastic coefficient ($\mu$) multiplied by the longitudinal strain ($e_{ij}$). The coefficients ($\lambda$ and $\mu$) are called Lame's constants. These two constants (and other related constants) which are representing proportionality constant between stress and strain, are normally referred to as the elastic coefficients, or elastic moduli.

### 2.2.9 The Elastic Moduli

The elastic modulus of a body is the proportionality constant of the stress-strain linear relationship. It expresses an important physical property which is the extent of resistance of that body to the applied stresses. Moduli of important practical applications are Young's Modulus, bulk modulus, and shear modulus. These are defined in the following discussions.

(i) **Young's Modulus and Poisson's Ratio**

Let a simple tensile stress ($T_x$) be applied to an isotropic bar placed along the x-axis. This will cause the bar to experience a longitudinal extension ($e_x$) in the x-direction and, at the same time, it experiences lateral contractions along y- and z-directions. Being an isotropic body, the contractions in the y- and z-directions ($e_y$, & $e_z$) are equal. These changes (expressed by the strains $e_x$, $e_y$, & $e_z$) are governed by the elastic coefficients of the stressed body. The coefficients which govern the stress-strain relation, in the presence of the tensile stress ($T_x$), are Young's modulus (**Y**) and Poisson's ratio ($\sigma$).

For a one-dimensional stress acting on a body obeying Hooke's law, Young's modulus (**Y**) is the proportionality constant in the linear relation that connects stress ($T_x$) with strain ($e_x$). The relationship is:

$$T_x = Ye_x$$

In the case of a rectangular rod of length (**L**), cross-sectional area (**ΔA**) stretched by (**ΔL**) due to force (**F**), Young's modulus (**Y**) is given by Fig. 2.10:

$$Y = T_x/e_x = (F/\Delta A)/(\Delta L/L)$$

Young's modulus is measured by pressure units (as psi, dyne/cm$^2$ or N/m$^2$).

The Poisson's ratio ($\sigma$), on the other hand, is defined as the ratio of transverse strain ($e_y$ or $e_z$) to longitudinal strain ($e_x$). For an isotropic body, this is given by:

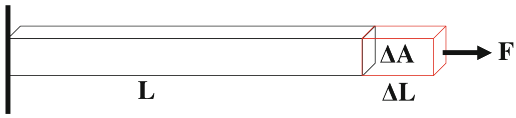



**Fig. 2.10** An elastic rectangular rod under extension force

$$\sigma = -e_y/e_x = -e_z/e_x$$

The minus sign is used to indicate that ($e_y$) and ($e_z$) are contractions for elongation ($e_x$).

(ii) **The Bulk Modulus**

The Bulk modulus (**B**) is defined to be the ratio of change in hydrostatic pressure (**ΔP**), acting on a solid body of volume (**V**), to the relative decrease in its volume (**ΔV/V**). For a cube of volume (**V**) under hydrostatic pressure-change (**ΔP**), the bulk modulus (**B**) is given by (Fig. 2.11):

$$B = -\Delta P/(\Delta V/V) = -V \cdot (\Delta P/\Delta V) = -\Delta P/\theta$$

The minus sign is entered to denote volume decrease for increase in compression and ($\theta$) is the cubical dilatation:

The Bulk Modulus (**B**) is a measure for the body resistance to uniform compression. An equivalent expression for the bulk modulus can be given in terms of density change ($\Delta\rho$) instead of the volume change. Thus, the definition becomes:

$$B = +\rho(\Delta P/\Delta\rho)$$

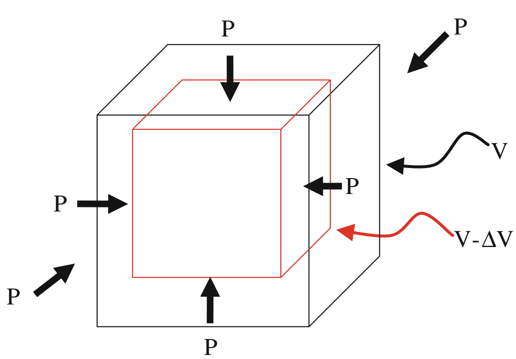



**Fig. 2.11** An elastic cube under hydrostatic compression forces

The plus sign is entered to denote density increase for increase in compression.

It is sometimes called (Incompressibility) and its inverse (**1/B**) is called (Compressibility). Its SI unit is the pressure measuring unit (the pascal).

(iii) **The Shear Modulus**

The shear modulus, ($\mu$), which expresses the relationship between shearing stress and shearing strain, is defined as the ratio of the shear stress ($T_{xy}$) and the shearing strain ($e_{xy}$) represented by the resulting angular change. For tangential force (F) acting on the face of a rectangular block of area (**ΔA**), the shear modulus ($\mu$) is defined as follows (Fig. 2.12):

$$\mu = T_{xy}/e_{xy} = (F/\Delta A)/(\Delta x/h)$$

The strain ($e_{xy}$) in this case is tangent of the angle of shear (**Ø**), or the angle in radians for small value of the angle (**Ø**). That is,

$$\mu = T_{xy}/Ø$$

The angle (**Ø**) is normally called angle of shear and the coefficient ($\mu$) is the shear modulus or rigidity modulus as it is sometimes called. Measurement unit of the shear modulus is pressure units as in the case of Young's modulus.

It is to be noted here that ($\mu$) serves as measure for the resistance of an elastic solid body to shearing deformation (i.e. to shape changes) and that is why it is called rigidity modulus. For this reason, it is equal to zero for a fluid medium as it has zero-resistance to shape-changes.

(iv) **Lame's Elastic Coefficients**

Lame's coefficients (also called Lame's parameters) are two parameters ($\lambda$ & $\mu$) which are used

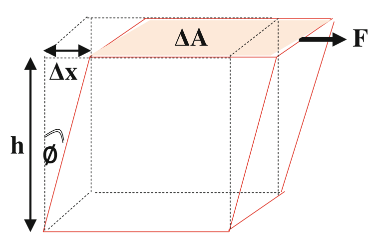



**Fig. 2.12** An elastic rectangular block under shearing force (*F*) acting on area (*ΔA*)

**Table 2.3** The mathematical interrelationships of elastic moduli, for an elastic isotropic body (Sheriff 1973, pp. 69–70)

| Modulus | Relation-1 | Relation-2 |
|---|---|---|
| Young's modulus (Y) | $Y = \mu(3\lambda + 2\mu)/(\lambda + \mu)$ | $Y = 9B\mu/(3B + \mu)$ |
| Bulk modulus (B) | $B = (3\lambda + 2\mu)/3$ | $B = Y/3(1 - 2\sigma)$ |
| Shear modulus (μ) | $\mu = 3(B - \lambda)/2$ | $\mu = Y/2(1 + \sigma)$ |
| Lame's modulus (λ) | $\lambda = (3B - 2\mu)/3$ | $\lambda = \sigma Y/(1 + \sigma)(1 - 2\sigma)$ |
| Poisson's modulus (σ) | $\sigma = \lambda/2(\lambda + \mu)$ | $\sigma = (3B - 2\mu)/(6B + 2\mu)$ |

in characterizing the elastic properties of an isotropic medium. These two coefficients give complete elastic characterization of homogenous and isotropic media. They serve as the proportionality constants in the stress-strain linear relationship which is mathematically expressed by Hooke's law.

To understand the physical implication of the first coefficient (**λ**), let us assume a solid cube being stretched by a tensile stress ($\mathbf{T_{zz}}$) resulting in a corresponding tensile strain ($\mathbf{e_{zz}}$). The lateral tensile stress ($\mathbf{T_{xx}}$) needed to prevent lateral contraction is, according to (Sheriff 1969), given by ($\mathbf{T_{xx}} = \boldsymbol{\lambda}\ \mathbf{e_{zz}}$). This relation furnishes the formal definition of the Lame's coefficient (**λ**). The second coefficient (**μ**) is the shear (or rigidity) modulus.

Both of Lame's coefficients (**λ** & **μ**) are functions of other elastic constants. For instance, they are functions of Young's modulus (**Y**) and Poisson's ratio (**σ**). The relations are:

$$\boldsymbol{\lambda} = \boldsymbol{\sigma}\mathbf{Y}/(\mathbf{1} - \mathbf{2}\boldsymbol{\sigma}) \cdot (\mathbf{1} + \boldsymbol{\sigma})$$

$$\boldsymbol{\mu} = \mathbf{Y}/\mathbf{2}(\mathbf{1} + \boldsymbol{\sigma})$$

Other relations are presented in Table 2.3.

### 2.2.10 The Elastic Moduli Interrelationships

For a homogeneous and isotropic medium under stress, the stress-strain relationship is linear within the elastic (Hookean) state. The proportionality constants are the elastic moduli or elastic constants, as we mentioned above. In addition to the two Lame's coefficients (**λ & μ**), the other moduli: Young's modulus (**Y**), Bulk modulus (**B**), and Poisson's ratio (**σ**) can be used in characterizing the elastic properties of an isotropic body. Any of these moduli can be expressed in terms of two other moduli as it is summarized in Table 2.3:

It is evident from this table that any one of the five constants can be expressed in terms of any two of the remaining constants. This implies that any two of these three constants can be used to define the elastic properties of a homogeneous and isotropic medium.

For most rocks, values of the moduli (**Y**, **B**, & **μ**) lie in the range ($2 \times 10^{10} - 12 \times 10^{10}$) N/m$^2$, with (**Y**) being the largest and (**μ**) the smallest of these three (Sheriff and Geldart 1995, p. 38). Table of values of elastic moduli of rocks have been published by Birch (1966).

## 2.3 Wave Motion Equation

If two neighboring points in a stressed medium experience the same stress, no motion of one of them will occur with respect to the other. However, relative motion will take place when there is a stress difference. In other words, motion occurs when there is a stress gradient. This reminds us of an analogous case we met in the creation of strain (see Sect. 2.2). The two cases may be expressed as follows: Displacement gradient is required to create strain and stress gradient is required to cause motion.

### 2.3.1 One-Dimensional Scalar Wave Equation

In this section, we shall deal with the wave motion equation which expresses the motion of a disturbance in one dimension. The disturbance in this particular case is the scalar quantity, the cubical dilatation (**θ**).

Let us consider an elementary parallelepiped (of dimensions: δx, δy, Δz) located inside an elastic isotropic medium (Fig. 2.13).

At each face of this elementary body, when it is under elastic stress, there exist three stress components: one is normal and two are tangential to the particular face. The three stress-components ($\mathbf{T_{xx}}$, $\mathbf{T_{yx}}$, and $\mathbf{T_{zx}}$) are acting on the face perpendicular to the x-axis. Under elastic stress-strain conditions, each of these components will have a gradient in the x-direction ($\partial\mathbf{T_{xx}}/\partial\mathbf{x}$, $\partial\mathbf{T_{xy}}/\partial\mathbf{x}$, and $\partial\mathbf{T_{xz}}/\partial\mathbf{x}$). For a complete three dimensional state, additional similar gradients occur in the other two directions (y-direction and z-direction).

In order to simplify the mathematical derivation of the equation of seismic wave motion, let a plane compressional seismic wave to be advancing in the x-direction. In this case the three stress-components are reduced to only one component ($\mathbf{T_{xx}}$) creating the corresponding strain ($\mathbf{e_{xx}}$). When a seismic plane wave propagates in the x-direction, the two faces (perpendicular to the x-axis) of the parallelepiped will be unequally displaced, and hence, it is subjected to an elastic strain ($\mathbf{e_{xx}}$) which is, by definition, given by the displacement gradient ($\partial\mathbf{D_x}/\partial\mathbf{x}$). This strain ($\mathbf{e_{xx}} = \partial\mathbf{D_x}/\partial\mathbf{x}$) is produced by the corresponding stress gradient ($\partial\mathbf{T_{xx}}/\partial\mathbf{x}$).

By making use of the fact that the net force acting on any face is given by the stress acting on that face times the face area, we get the resultant force ($\mathbf{F_x}$) in the x-direction due to the stress change ($\partial\mathbf{T_{xx}}/\partial\mathbf{x}$) · $\mathbf{\delta x}$ that occurred across the distance ($\mathbf{\delta x}$). This is computed as follows:

$$\mathbf{F_x} = (\partial\mathbf{T_{xx}}/\partial\mathbf{x}) \cdot \mathbf{\delta x} \cdot \mathbf{\delta y} \cdot \mathbf{\delta z}$$

By applying Newton‘s second law of motion we can express ($\mathbf{F_x}$) in terms of mass of the parallelepiped ($\mathbf{\delta x} \cdot \mathbf{\delta y} \cdot \mathbf{\delta z}$ times density $\boldsymbol{\rho}$) multiplied by acceleration ($\partial^2\mathbf{D_x}/\partial\mathbf{t}^2$) in the **x**-direction giving:

$$\mathbf{\delta x} \cdot \mathbf{\delta y} \cdot \mathbf{\delta z} \cdot \boldsymbol{\rho}\left(\partial^2\mathbf{D_x}/\partial\mathbf{t}^2\right) = (\partial\mathbf{T_{xx}}/\partial\mathbf{x}) \cdot \mathbf{\delta x} \cdot \mathbf{\delta y} \cdot \mathbf{\delta z}$$

or:

$$\boldsymbol{\rho}\left(\partial^2\mathbf{D_x}/\partial\mathbf{t}^2\right) = (\partial\mathbf{T_{xx}}/\partial\mathbf{x})$$

This i*s* the one-dimensional (dimension, **x** in this example) wave motion equation which describes particle motion (displacement, $\mathbf{D_x}$) in terms of the applied stress ($\mathbf{T_{xx}}$). However, the motion can be expressed in terms of displacement only. This is done by using the stress-strain linear relationship expressed by Hooke’s law equation for isotropic media ($\mathbf{T_{xx}} = \boldsymbol{\lambda}\ \boldsymbol{\theta} + 2\boldsymbol{\mu}\ \mathbf{e_{xx}}$). Substituting for ($\mathbf{T_{xx}}$), the previously-derived wave equation becomes:

$$\boldsymbol{\rho}\left(\partial^2\mathbf{D_x}/\partial\mathbf{t}^2\right) = \boldsymbol{\lambda}(\partial\boldsymbol{\theta}/\partial\mathbf{x}) + 2\boldsymbol{\mu}(\partial\mathbf{e_{xx}}/\partial\mathbf{x})$$

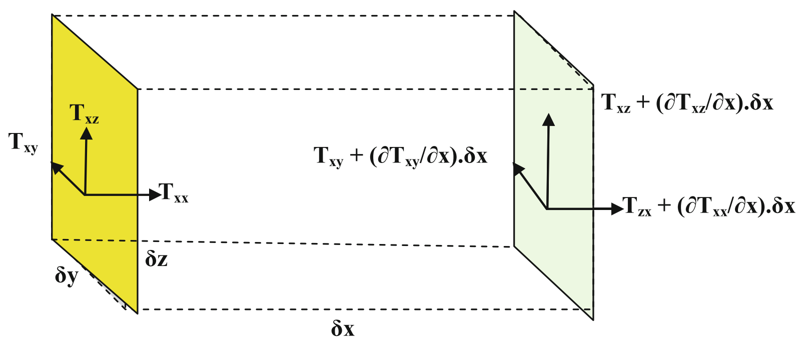



**Fig. 2.13** A small parallelepiped element of volume under elastic stress

Since, by definition, ($\mathbf{\theta = e_{xx} + e_{yy} + e_{zz}}$) and ($\mathbf{e_{yy} = e_{zz} = 0}$) in this case (case of restricting the disturbance to be displacement in the x-direction, with no lateral contraction), we can readily write:

$$\boldsymbol{\rho}\left(\partial^2 \mathbf{D_x}/\partial \mathbf{t^2}\right) = (\boldsymbol{\lambda} + \mathbf{2}\boldsymbol{\mu})(\partial \mathbf{e_{xx}}/\partial \mathbf{x})$$

or,

$$\left(\partial^2 \mathbf{D_x}/\partial \mathbf{t^2}\right) = [(\boldsymbol{\lambda} + \mathbf{2}\boldsymbol{\mu}/)\boldsymbol{\rho}]\left(\partial^2 \mathbf{D_x}/\partial \mathbf{x^2}\right)$$

This is a partial differential equation of the form ($\partial^2\mathbf{y}/\partial\mathbf{t^2} = \mathbf{v^2}\ \partial^2\mathbf{y}/\partial\mathbf{x^2}$), which has the general solution, $\mathbf{y(x, t) = f(x - vt) + g(x + vt)}$. In analogy to this standard form of partial differential equation, we can write the solution of the one-dimensional wave equation as:

$$\mathbf{D_x}(\mathbf{x}, \mathbf{t}) = \mathbf{f}(\mathbf{x} - \mathbf{vt}) + \mathbf{g}(\mathbf{x} + \mathbf{vt}),$$
$$\text{where, } \mathbf{v} = [(\boldsymbol{\lambda} + \mathbf{2}\boldsymbol{\mu})/\boldsymbol{\rho}]^{\mathbf{1/2}}.$$

This solution represents one dimensional wave equation, which is a disturbance (particle displacement, $\mathbf{D_x}$) moving with speed of ($\mathbf{v}$) in the positive x-direction as expressed by the first term, $\mathbf{f(x - vt)}$. The second term, $\mathbf{g(x + vt)}$ represents a wave moving in the negative x-direction.

## 2.3.2 The Scalar and Vector 3D Wave Equations

Solution of the one-dimensional wave equation, $\mathbf{q(x, t)}$ expresses the variation of the disturbance ($\mathbf{q}$) along the travel distance ($\mathbf{x}$) at any time ($\mathbf{t}$). In the three-dimensional case, we have the dependant variable; $\mathbf{q(x, y, z, t)}$ which possesses, at any time ($\mathbf{t}$), a defined value at any point in the surrounding space ($\mathbf{x, y, z}$).

The standard wave equation describes the strain-changes as function of space and time as it propagates (with constant velocity) through a perfectly elastic medium. It can be shown (see for example Richter 1958, pp. 657–658; Sheriff and Geldart 1995, pp. 39–40) that the standard three-dimensional wave-motion equation is a linear second order partial differential equation. The general wave equation of a disturbance, $\mathbf{q(x, y, z, t)}$ moving in space with velocity ($\mathbf{v}$), is given by:

$$\partial^2\mathbf{q}/\partial\mathbf{t^2} = \mathbf{v^2}[\partial^2\mathbf{q}/\partial\mathbf{x^2} + \partial^2\mathbf{q}/\partial\mathbf{y^2} + \partial^2\mathbf{q}/\partial\mathbf{z^2}].$$

or (using Laplacian operator ($\nabla^2\ \mathbf{q} = \partial^2\mathbf{q}/\partial\mathbf{x^2} + \partial^2 q/\partial\mathbf{y^2} + \partial^2\mathbf{q}/\partial\mathbf{z^2}$), this can be written as:

$$\partial^2\mathbf{q}/\partial\mathbf{t^2} = \mathbf{v^2}\nabla^2\mathbf{q}$$

In seismic waves there are two types of disturbance ($\mathbf{q}$) which propagate through the Earth materials. These are the two forms of elastic strains which represent the scalar "volume" changes (expressed by the cubic dilatation, $\boldsymbol{\theta}$) and the vector "shape" changes (expressed by the shear strain, $\boldsymbol{\psi}$).

(i) **The Scalar quantity ($\boldsymbol{\theta}$)**
This is the scalar cubic dilatation ($\boldsymbol{\theta}$). It represents disturbance (volume-changes) that moves in space with velocity ($\mathbf{v}$), where $\mathbf{v} = [(\boldsymbol{\lambda} + \mathbf{2}\boldsymbol{\mu})/\boldsymbol{\rho}]^{\mathbf{1/2}}$. According to the wave motion equation (see for example, Richter 1958, p. 658):

$$\boldsymbol{\rho}\,\partial^2\theta/\partial\mathbf{t^2} = (\boldsymbol{\lambda} + \mathbf{2}\boldsymbol{\mu})\nabla^2\boldsymbol{\theta}$$

By definition, the cubic dilatation ($\boldsymbol{\theta}$) is related to displacement $\mathbf{D}$ (vector quantity of components $\mathbf{D_x, D_y, D_z}$) by the formula ($\boldsymbol{\theta} = \partial\mathbf{D_x}/\partial\mathbf{x} + \partial\mathbf{D_y}/\partial\mathbf{y} + \partial\mathbf{D_z}/\partial\mathbf{z}$). Using vector notation, the scalar quantity ($\boldsymbol{\theta}$) is, therefore, the divergence of the vector ($\mathbf{D}$). That is:

$$\boldsymbol{\theta} = \mathbf{div\,D} = \nabla\cdot\mathbf{D}$$

(ii) **The Vector quantity ($\boldsymbol{\psi}$)**
The second type of moving disturbance is the vector quantity (shear strain, $\boldsymbol{\psi}$). It represents (shape-changes) which moves with velocity, $\mathbf{v} = [\boldsymbol{\mu}/\boldsymbol{\rho}]^{\mathbf{1/2}}$. The three components of the vector ($\boldsymbol{\psi}$) are ($\boldsymbol{\psi_x}, \boldsymbol{\psi_y}, \boldsymbol{\psi_z}$), defined by (Richter 1958, p. 658):

$$\psi_x = (\partial D_z/\partial y - \partial D_y/\partial z)$$
$$\psi_y = (\partial D_x/\partial z - \partial D_z/\partial x)$$
$$\psi_z = (\partial D_y/\partial x - \partial D_x/\partial y)$$

Each of these three components moves with velocity (**v**) in accordance to the following wave-motion equations.

$$\rho\partial^2\psi_x/\partial t^2 = \mu\nabla^2\psi_x$$
$$\rho\partial^2\psi_y/\partial t^2 = \mu\nabla^2\psi_y$$
$$\rho\partial^2\psi_z/\partial t^2 = \mu\nabla^2\psi_z$$

It is clear from the definitions of the components of the vector quantity ($\psi$) that each of the components ($\psi_x$, $\psi_y$, $\psi_z$) is the **curl** of the corresponding displacement-components ($D_x$, $D_y$, $D_z$) as expressed above. That is:

$$\psi_x = \mathrm{curl}_x D = \nabla \times D$$

Similarly, for the other two components ($\psi_y$, $\psi_z$).

### 2.3.3 Plane Waves

A plane wave is defined as that wave for which the moving disturbance is constant at all points of any plane perpendicular to the propagation direction (Fig. 2.14).

For the seismic plane waves, the elastic disturbances, $\theta(x, t)$, or $\psi(x, t)$ are functions of the travelled distance **(x)** only. In either of these two types of disturbances, therefore, the wave front is represented by a plane normal to the x-axis. Further, when the moving disturbance **f(x − vt)** is in the form of sinusoidal function the moving disturbance is referred to as (plane harmonic wave). Such a function is:

$$f(x,t) = a\cos k(x - vt),$$

where (**a** & **k**) are constants.

According to Fourier Theorem, a moving pulse of an arbitrary shape can be transformed into its harmonic components by superposing many sinusoidal functions, each of which is of the form:

$$f(x,t) = a\cos k(x - vt),$$

or,

$$f(x,t) = a\cos 2\pi(x/\lambda - t/\tau)$$

This equation is showing that **f(x, t)** is a periodic function of (**x** & **t**) which is oscillating with wavelength ($\lambda$) and period ($\tau$) in respect to distance and time respectively, where **k** (=$2\pi/\lambda$) is called the wave number. The factor (**a**) represents the amplitude of the particular harmonic component. The two forms of these two equations are equivalent since $v = \lambda/\tau$.

Periodicity of the harmonic plane wave is expressed by two parameters. These are the spatial frequency in cycle per meter ($f_x$) and the temporal frequency in cycle per second ($f_t$). These are related to the wavelength ($\lambda$) and to the wave period ($\tau$) by the relations ($f_x = 1/\lambda$) and ($f_t = 1/\tau$) respectively.

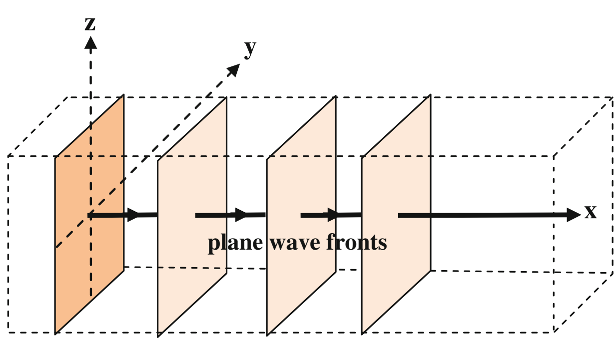



**Fig. 2.14** Wave fronts of a plane wave advancing in x-direction

### 2.3.4 The P- and S-Waves

As it is presented above, there are two types of disturbance that can move in accordance with the standard wave motion equation. These are the scalar cubic dilatation ($\boldsymbol{\theta}$) and the vector shear strain ($\boldsymbol{\psi}$).

From the wave equation it can be shown that the disturbance ($\boldsymbol{\theta}$) moves faster than the other disturbance ($\boldsymbol{\psi}$). Thus, when the two disturbances are generated by a certain source, the (**$\boldsymbol{\theta}$-wave**) arrives earlier than the (**$\boldsymbol{\psi}$-wave**). For this reason the two waves are called Primary (**P-wave**) and Secondary (**S-wave**) respectively.

It is to be noted that the ratio of the P-wave velocity ($\mathbf{v_p} = [(\lambda + 2\mu)/\rho]^{1/2)}$ to the S-wave velocity ($\mathbf{v_s} = [\mu/\rho]^{1/2}$) is equal to $[(\lambda + 2\mu)/\mu]^{1/2}$. Using the relationship connecting ($\lambda$) and ($\mu$) in which $\lambda/\mu = 2\sigma/(1 - 2\sigma)$, we can write:

$$\mathbf{v_p}/\mathbf{v_s} = [(\mathbf{2} - \mathbf{2}\sigma)/(\mathbf{1} - \mathbf{2}\sigma)]^{1/2}$$

This formula clearly shows that the ratio of the P-wave velocity ($\mathbf{v_p}$) to the S-wave velocity ($\mathbf{v_s}$) is function of Poisson's ratio ($\boldsymbol{\sigma}$) only.

According to (Dobrin 1960, p. 18), Poisson's ratio ($\boldsymbol{\sigma}$) generally ranges from 0.05 to 0.40, averaging about 0.25 for hard rocks. With this value ($\boldsymbol{\sigma} = \mathbf{1/4}$), the velocity ratio ($\mathbf{v_p/v_s}$) becomes $\mathbf{3^{1/2}}$ (=**1.732**). This means that P-wave moves with velocity which is about 1.7 times as fast as the S-wave moving in the same medium. It is useful to note that P-wave velocity is 330 m/s in air, 1450 m/s in water, and (2000–6000) m/s in rocks.

A solid medium having its Poisson's ratio equal to (**1/4**) is called Poisson's solid (Sheriff 2002, p. 266).

## 2.4 Classification of Common Elastic Waves

From analyses of stress and strain, we have seen that strain is, in general, made up of two types of elastic disturbance; the cubic dilatation and the shear strain. Solution of the equation of motion showed that each of these types of deformation travels through the medium with its own velocity. The first type of disturbance represents the moving "volume" strain and the second type involves the "shape" strain. The first type is called Longitudinal, Compressional, or Primary wave (or just P-wave) which travels faster than the second type which is called Transverse, Shear, or Secondary wave (or just S-wave).

These two types of waves (P- and S-waves) belong to a class of waves (called body waves) because they can propagate through the interior of the earth body. This group of waves is called so to differentiate them from another class of waves which move on and near the free surface of the medium, called (surface waves) which include Rayleigh- and Love-waves. Classification of the common elastic (seismic) waves is shown in Fig. 2.15.

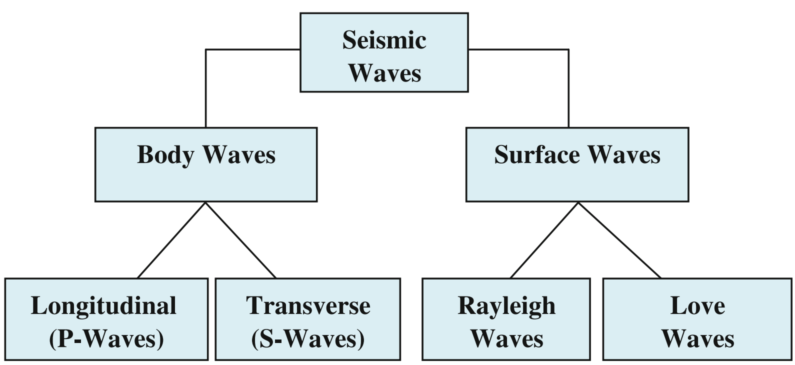



**Fig. 2.15** Classification of the common seismic waves

### 2.4.1 Body Waves

Body waves are waves that can travel through an elastic materialistic medium in any direction. As they move, the waves may experience changes in their energy level and in their travel-path geometry subject to the physical properties of the medium. There are two sub-types of these waves; the longitudinal and the transverse waves (Fig. 2.16).

(i) **Longitudinal Waves**

This type of waves is also known as compressional, Primary, or just P-wave. The travelling disturbance in this case is "volume" deformation expressed by the cubical dilatation (**θ**) as defined above.

The particles of the medium, traversed by a plane P-wave, vibrate about their neutral positions in the direction of the wave propagation. The travel path consists of a sequence of alternating zones of compressions and rarefactions (Fig. 2.16a). This is the type of waves which is commonly employed in seismic reflection and refraction exploration work.

P-wave is the fastest wave for a given medium and, therefore, its arrival at a certain observation point is the earliest among the seismic wave-types. This is a common observation of seismologists working on analysis of earthquake seismograms. Propagation velocity ($\mathbf{v_p}$) of P-wave depends on the medium density (**ρ**) and elastic properties (**λ** & **μ**) and it is given by the expression $\mathbf{v_p = [(\lambda + 2\mu)/\rho]^{1/2}}$.

(ii) **Transverse Waves**

The travelling disturbance in this case is the shear strain or "shape" deformation. The medium which is traversed by this type of waves experiences no volume changes. A consequence of the shear strain (rotation of part of the medium) is the transverse displacement of the path particles relative to the propagation direction. They are also called (shear waves) or (Secondary, or just S-waves).

A horizontally moving S-wave, which is so polarized that the particle motion is confined to vertical plane, is known as SV-wave (Fig. 2.16 b). When the polarization plane is horizontal, it is

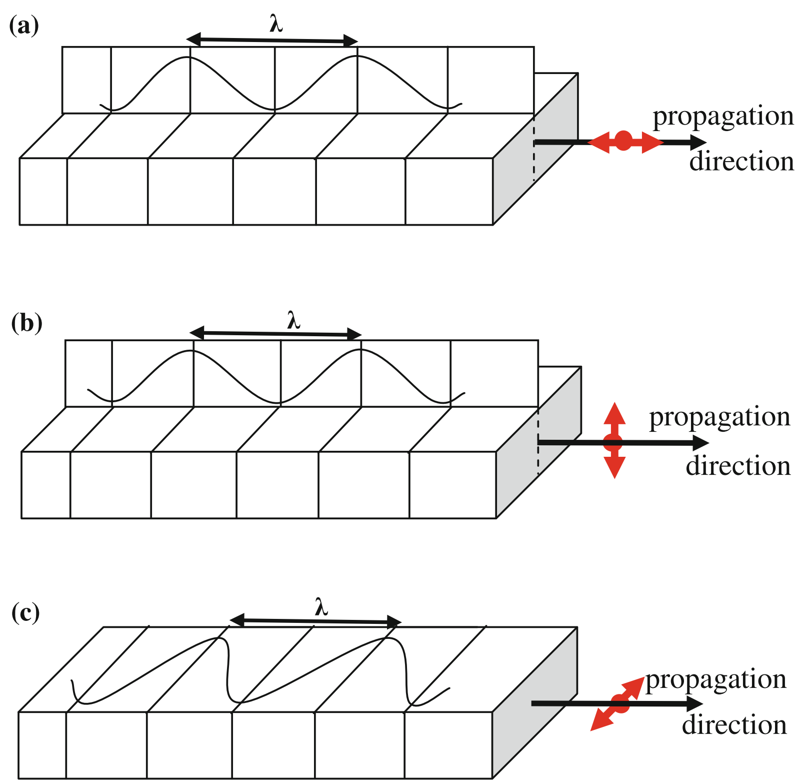



**Fig. 2.16** Particle displacement-mode of a medium traversed by plane body-waves, $\lambda$ is wavelength. **a** P-wave, **b** SV-wave, **c** SH-wave

called SH-wave (Fig. 2.16c). The velocity of S-waves, $\mathbf{v_s}$ is given by $\mathbf{v_s} = [\boldsymbol{\mu}/\boldsymbol{\rho}]^{1/2}$. In liquid-media, where ($\boldsymbol{\mu} = \mathbf{0}$), S-waves do not propagate.

### 2.4.2 Surface Waves

As it is implied by its name, surface waves are waves that move on the free surface of the earth. The main features common among all surface waves, observed on earthquake seismograms, are their relatively large amplitudes (high energy content) and low frequencies when compared with the body waves. In addition to that, they move with velocity which is generally slower than body waves moving in the same medium. It is a common observation that the dispersion phenomena are more prominent in surface waves due to dependence of the velocity on the frequency of individual harmonic component.

The main sub-types of surface waves are Rayleigh waves and Love waves (Fig. 2.17).

(i) **Rayleigh Waves**
Rayleigh waves, which were discovered by an English scientist, Lord Rayleigh in 1885, are usually developing at the free surface of a semi-infinite solid medium. Its wave amplitude decays rapidly with increasing depth. The travelling disturbance in this case is a sort of combination of particle-motions of both P- and SV-waves. The particle motion, which has a retrograde elliptical orbit, takes place in a vertical plane parallel to propagation direction (Fig. 2.17a). The minor axis of the elliptical orbit is parallel to wave motion direction and it is equal to two-thirds of its major axis. Rayleigh waves travel on the surface of a solid medium with velocity of 0.92 of the velocity of S-waves moving in that medium (Bullen 1965, p. 90). In a sense, Rayleigh waves are similar to the familiar water waves, with a fundamental difference, and

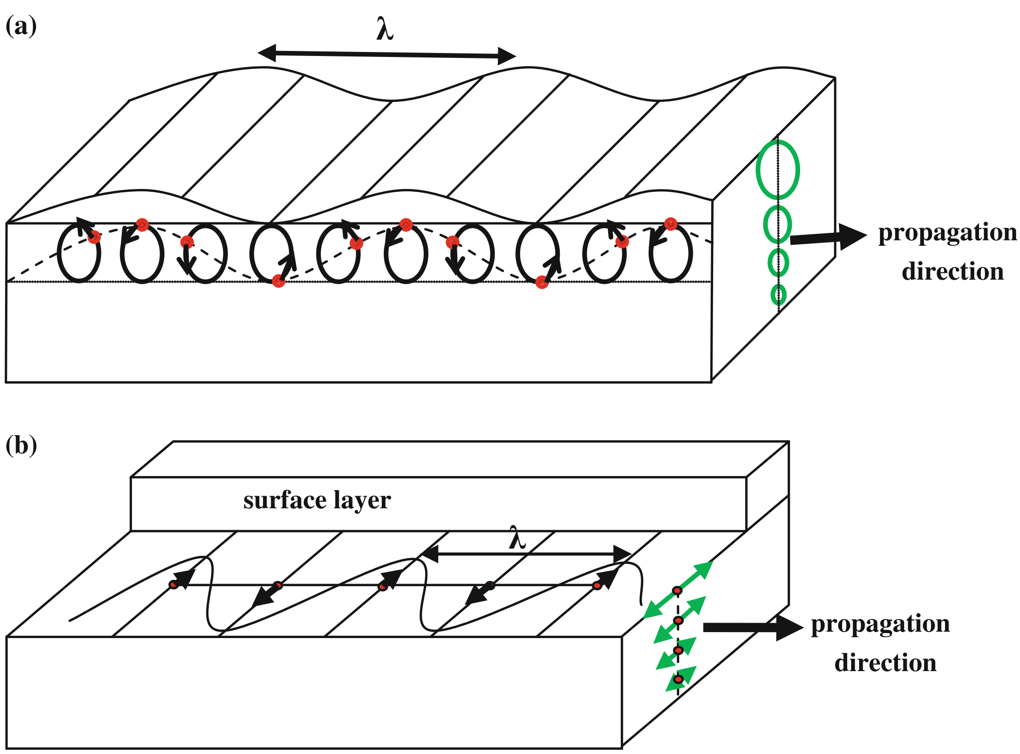



**Fig. 2.17** Particle displacement-mode of a medium traversed by plane surface-waves, ($\lambda$) is wavelength. **a** Rayleigh Wave, **b** Love Wave

that is the particle motion in case of Rayleigh waves describe an elliptical path whereas the particle-motion path in case of water waves are circular in shape.

In the case where the semi-infinite medium is overlain by a low-velocity surface layer, Rayleigh waves exhibit a phenomenon known as (dispersion). Harmonic components of longer periods (lower frequencies) travel faster. Consequently, the Rayleigh wave seismograms would, in general, show decrease in period along the wave-train. Components of too-long wavelengths (too long compared with the thickness of the surface layer) penetrate deeper and travel with velocity of about 0.9 times the S-wave velocity in the subsurface material. The short wavelengths travel mainly in the surface layer with velocity of about 0.9 times the S-wave velocity in the surface layer.

Surface waves, normally seen on shot records, obtained in seismic reflection surveys, are commonly called (ground roll) and these are identified to be of Rayleigh-wave type. Sometimes, these are called pseudo-Rayleigh waves (Sheriff 2002). Ground-roll waves are considered to be unwelcomed noise and efforts are usually made to get rid of them or at least minimize their masking effect caused to the seismic reflection signal.

(ii) **Love Waves**

This is the second sub-type of surface waves which was discovered, in 1911, by another English geophysicist named A.E.H. Love (1863–1940). It develops only in cases where a solid elastic semi-infinite medium is overlain by a horizontal low-velocity layer. Like SH-wave vibration mode, the particle movement is transverse and is confined to the horizontal plane (Fig. 2.17b). Love waves travel by multiple reflections between the top and bottom boundary-planes of the surface layer. The propagation velocity approaches S-wave velocity in the subsurface medium for very long wavelengths and to that of the surface layer for short wavelengths (Dobrin 1960, p. 23). Love waves always exhibit dispersion. As in the case of Rayleigh waves, Love waves propagation-velocity increases with the period of the harmonic component. Again, the vibration amplitude decays exponentially with depth in the lower medium.

Since they possess no vertical component, Love waves are not detected by the geophone or by any such-like vertical-component sensing instrument.

### 2.4.3 Seismic Noise

Broadly speaking, the term (noise) used in seismology, is applied to all types of disturbance which may interfere with (and impose masking effects to) the seismic signal of interest. In this way, the concept of seismic noise bears a relative implication. Thus, when the interest is focused on reflected body waves, surface waves and other non-reflection waves (as direct and refraction arrivals) are considered to be the unwanted troublesome noise. If the interest is in the refraction arrivals, reflection arrivals become the unwanted noise. In the strict sense, however, the ambient seismic disturbances (usually of random energy distribution which form the background of a distinct travelling signal) are considered to be the seismic noise.

Seismic noise has destructive effects on the seismic signals of interest. A signal recorded amid a background of noise is distorted and weakened because of the interfering noise. Signal resolution is badly affected with noise development. A measure for the signal resolution, called the signal-to-noise ratio (S/N ratio) is usually applied. It is defined to be the ratio between signal amplitude detectable amid a background seismic noise.

In exploration seismology seismic noise is divided into two main types; coherent and incoherent noise (Fig. 2.18).

(i) **Coherent Noise**

Coherent noise is a seismic event characterized by a distinct apparent velocity and well-defined onset. In reflection seismology, coherent noise which appear on shot records, are source-generated seismic events. They are made up mainly of surface waves (ground roll) and air-waves which are of fairly narrow bandwidth with low frequency range. Frequency content of this type of noise is typically below 20 Hz (Fig. 2.18).

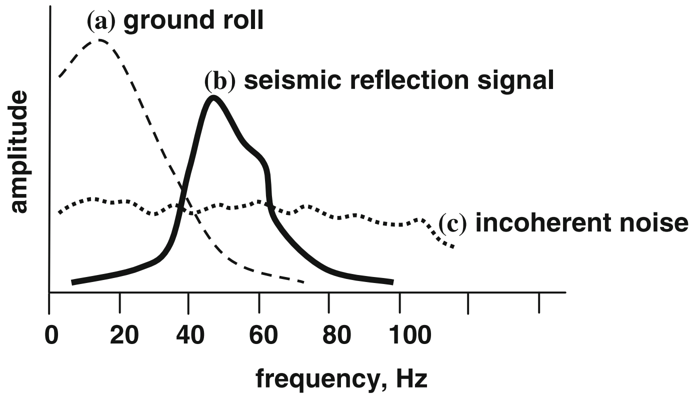



**Fig. 2.18** Sketch showing amplitude spectra of common coherent noise, ground roll (**a**), and incoherent noise, random noise (**c**), in relation to that of the reflection signal (**b**)

(ii) **Incoherent Noise**

Unlike coherent noise, the incoherent noise consists of seismic events with unpredictable amplitude and onset. This type of noise, which is basically of random nature, forms the seismic-energy background of any seismic shot-record. In earthquake seismology it is commonly known as (microseisms), and in prospecting seismology it is called (incoherent background noise), or (ambient noise) as it is sometimes referred to. In addition to the randomness nature, the incoherent noise is characterized by a broad amplitude spectrum that covers a wide range of frequencies compared with the nearly limited bandwidth of reflection signals or coherent noises (Fig. 2.18). In the geophysical literature we sometimes meet terms like (white noise) indicating wide-band noise, and (red noise) for low-frequency random noise.

Intensive research work has been undertaken, directed towards a greater understanding of the source and characteristics of the incoherent noise. It is now generally accepted that it is generated as a result of external energy sources like wind movements, sea-waves collisions with sea coasts, in addition to other various natural and artificial man-made activities.

Because of seismic noise which are unavoidable seismic events which get recorded alongside the objective signal, the signal-to-noise-ratio (S/N) becomes an important parameter in signal detection studies. The S/N ratio is used as measure for the signal quality-level. Signal clarity (S/N enhancement) is a central objective, aimed at, in seismic data acquisition. Several ways and means are followed in the field-acquisition stage or in the following data-processing stage to get enhanced S/N ratio. Suitable measures are applied to the parameters of the seismic source and detectors as well as those measures applied in processing work, in order to attenuate these noises and enhance the S/N ratio.

## 2.5 Propagation of Seismic Waves

Seismic waves are generated from a sudden change in the internal strain occurring inside an elastic medium. The generating source may be natural as in the case of earthquakes or artificial, like exploding a charge of dynamite, as normally done in seismic exploration. All parameters of an advancing seismic wave (waveform, speed, and travel-path geometry) may change during the wave propagation. Form and magnitude of these changes depend on the physical properties of the host medium. Whether the source is natural or artificial, a seismic field is created when a sudden pressure pulse is initiated. The generated seismic energy moves away from the source zone in a form of a wave motion propagation. Under these conditions (seismic energy source within an elastic medium), the seismic wave spreads out from the source zone in every possible direction. A travel ray-path, in a particular medium, is defined once the locations of both of the source-point and detector-point are defined.

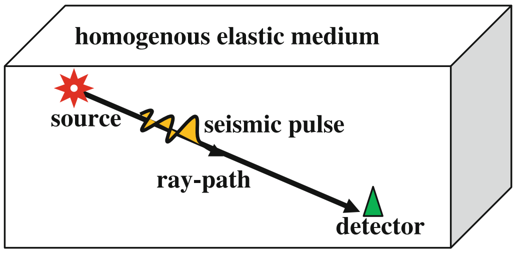



**Fig. 2.19** Elements of the seismic field, shown for an idealized homogenous and elastic medium

### 2.5.1 Elements of the Seismic Field

In case of an idealized simple medium (one which is homogeneous, isotropic, and perfectly elastic medium) the wave-motion propagation is expected to be along straight ray-paths, with constant velocity. A seismic field is created when a mechanical energy within an elastic medium generates a seismic pulse that propagates through that medium. The fundamental elements of a seismic field are, thus, a source of mechanical energy, elastic medium, and detector.

Except for the geometrical spreading effect, the wave moves through such an idealized medium, with no changes taking place on ray-path direction or on the waveform of the travelling seismic pulse (Fig. 2.19).

### 2.5.2 Concepts of Wave-Fronts and Rays

From a mechanical energy-source, (such as a mechanical pressure pulse), a seismic wave spreads out into the three dimensional space of the host medium. If the medium is homogeneous, the seismic energy would advance in every possible direction with constant velocity. This means that after any given travel-time the energy would have reached points of equal distances from the source. These points fall on a spherical surface which is marking the (**wave-front**). For a harmonic seismic wave, the wave front is defined to be the locus of points having the same phase of particle vibration.

At any point in the wave-field, the line which is perpendicular to the wave front at a certain instant represents a (**ray**). The ray is an imaginary line normal to the wave front at a certain point which indicates the motion-direction of the advancing wave at that point. Near the source point, the wave fronts of seismic waves travelling through a homogeneous medium are of spherical shapes and thus the rays are straight lines radiating in all directions from the source point. At very large distances, the wave fonts are approximating to planes and the rays, in this case, become parallel straight lines perpendicular to the plane wave-fronts.

The familiar example is the wave which develops on the surface of water when a small solid object (a pebble, say) is dropped vertically into it. The crests and troughs of the generated wave spread out from the source-point in the form of concentric circles. In fact these circles are depicting the surface expression of the spherical wave-fronts which are advancing through the three-dimensional space of the water medium. By definition, the ray at any point on

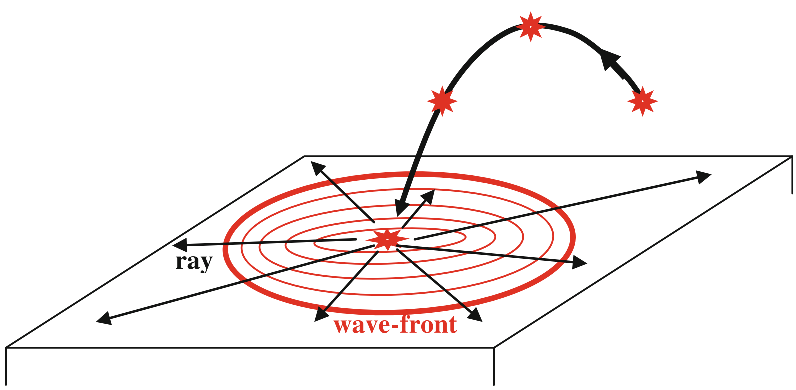



**Fig. 2.20** Concepts of the wave fronts and rays as seen when a water wave is created from dropping a pebble into a still pond

the wave front is a line drawn normal to the spherical wave-front (circles on the surface plane) at that point. Concepts of the wave-front and rays are shown in (Fig. 2.20) for a case of dropping a pebble into a still pond.

### 2.5.3 Huygens' Principle

Huygens' Principle states that each point on a wave-front acts as a source of a new wave which, in a homogeneous medium, generates a secondary spherical wave-front, the envelope of which defines the position of a wave generated at some later time.

Huygens' model of wave propagation requires that the secondary wave-fronts are active only at the points where the envelope touches their surfaces. The wave energy is spreading out from the primary source-points in all directions, but their mutual interactions make the resultant disturbance zero everywhere except at the points where they touch the common envelope. Applying the principle on plane-wave propagation in a homogeneous, and in an inhomogeneous medium, is shown in Fig. 2.21.

### 2.5.4 The Concept of the Interface

The Interface is that boundary-surface separating two different media. As far as the changes (changes in spectral structure and propagation direction) of seismic waves are concerned, two media are considered to be different if both of the wave propagation velocity and the medium bulk density are different. Since velocity is function of elastic coefficients, it can be said that density and elastic properties are the factors which control the specific characters of the media. The parameter which expresses the combined effect of velocity and density is called (acoustic impedance) which is defined to be the product of velocity by the density.

To clarify the concept of the interface and the roll of the acoustic impedance waves hitting an interface let us consider a two-layer model which consists of two adjacent media ($M_1$ & $M_2$) of velocities and densities ($v_1$ & $\rho_1$) for medium ($M_1$) & ($\mathbf{v_2}$ & $\rho_2$) for medium ($M_2$). The acoustic impedances ($z_1$ & $z_2$) for the two layers are ($z_1 = \rho_1 v_1$) and ($z_2 = \rho_2 v_2$) as shown in Fig. 2.22.

In analogy to the role of electrical impedance in the flow of electrical current, the acoustic impedance expresses the extent of resistance the

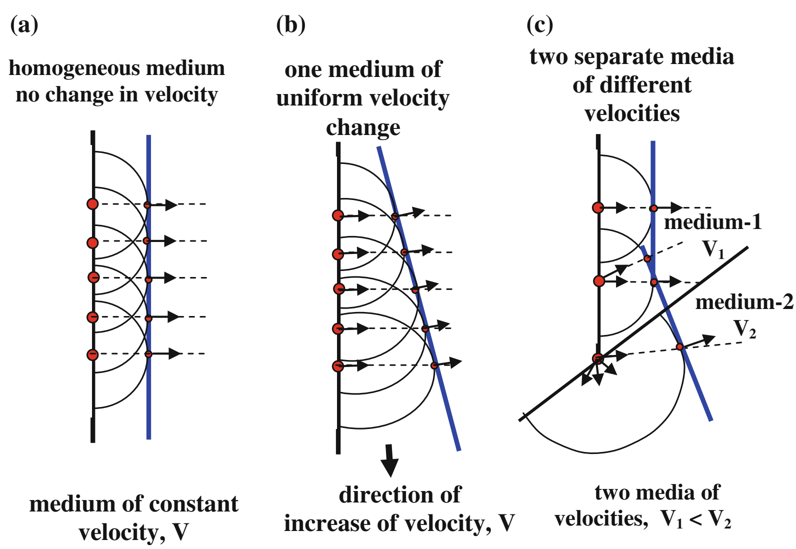



**Fig. 2.21** Plane-wave propagation according to Huygens' Principle. **a** Through a homogeneous medium where velocity is constant. **b** Inhomogeneous medium of velocity which is uniformly changing across the propagation direction. **c** Two media of different velocities

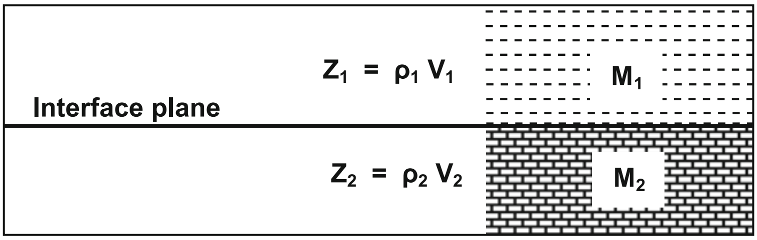



**Fig. 2.22** Concept of the Interface and definition of the Acoustic Impedance, z (=ρv)

seismic energy meets when traversing a medium. The higher the acoustic impedance, the lower the particle vibration-velocity will be, and vice versa. Acoustic impedance is measured by (kg $s^{-1}$ $m^2$) or by the equivalent (Ns $m^3$) units.

At an interface, an incident seismic wave (normally a P-wave in seismic exploration work) would, under certain geometrical conditions, give rise to wave conversion in addition to reflection, refraction, and diffraction. These cases shall be dealt with in some details in the following discussions.

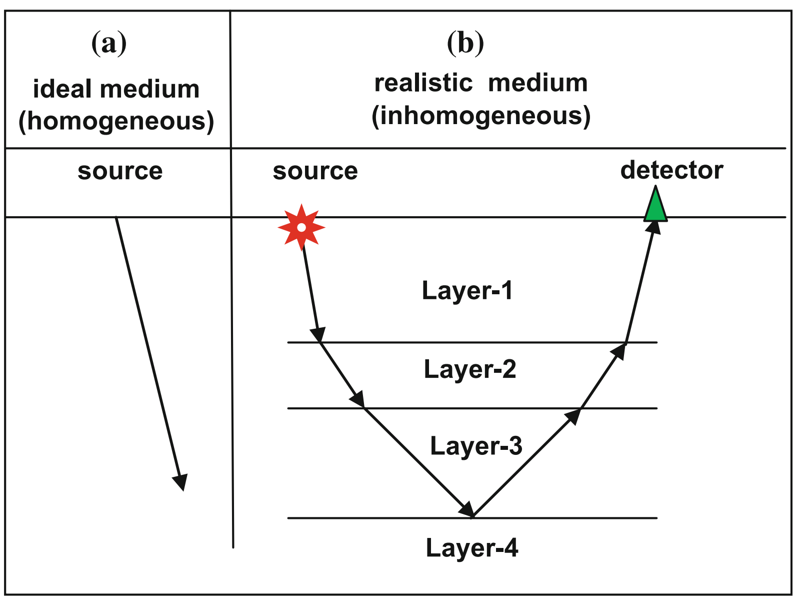
### 2.5.5 Changes of Propagation Direction at Interfaces

In an idealized homogenous and elastic medium, a seismic wave propagates with no changes taking place on ray-path direction or on the waveform of the travelling seismic pulse. In nature, however, the medium is far from this idealized form. In the solid crust of the Earth, it is commonly made up of rock layers of varying physical properties and varying geometrical forms and sizes.

In such inhomogeneous environments a moving seismic wave would suffer from a number of changes whenever it meets an interface across which there is change in the properties of the medium. In particular, changes in energy content, waveform (spectral structure), propagation velocity, direction of motion, and new wave generation. These changes, are generated at the interface planes defining the layer bounding surfaces (Fig. 2.23).

The common changes in ray-path direction, which are of significance to exploration seismology, are: reflection, refracted transmission

**Fig. 2.23** **a** Infinite, elastic homogeneous medium showing straight ray-path. **b** Inhomogeneous (layered) medium showing changes in ray-path direction



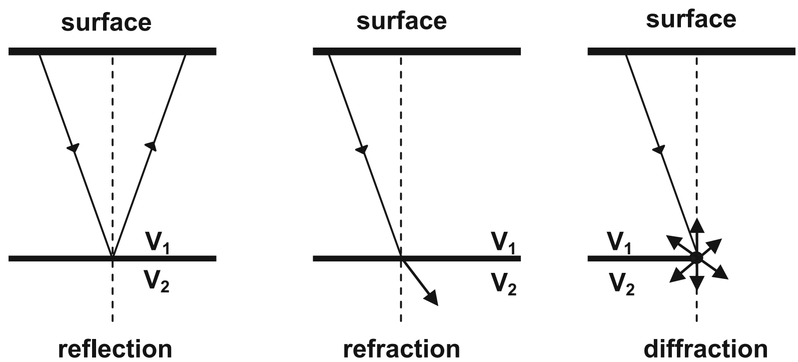



**Fig. 2.24** Ray-path geometry of the three most common wave propagation ray-paths: reflection, refraction, and diffraction

(refraction), and diffraction. These shapes of the moving wave ray-path occur at the boundaries of media having different seismic propagation velocities (Fig. 2.24).

### 2.5.6 Wave Conversion at Interfaces

When a seismic wave impinges on an interface separating two media, which differ in acoustic impedances, the incident seismic energy is partly reflected and partly transmitted with certain waveform changes. When the ray-path of an incident seismic wave is oblique, that is inclined with respect to an interface, new waves are generated. If, for example, the incident wave is P-wave (or SV-wave) separating two solid media of different density and elastic properties (different acoustic impedances), four new wave phases are generated; reflected and refracted P- and SV-waves. If, however the incident is SH-wave, the generated waves are only reflected and refracted SH-wave. The SV-waves, generated from an incident P-wave, (or P-waves generated from an incident SV-wave) are called (converted waves) (Fig. 2.25).

An incident seismic wave onto an interface will be partly reflected and partly transmitted across the interface. In general, the interface will bring about wave conversion, reflection, transmission, and diffraction. It should be noted here that refraction is a special case of transmission. Refraction (ray-path bending) occurs only in the case of inclined incidence.

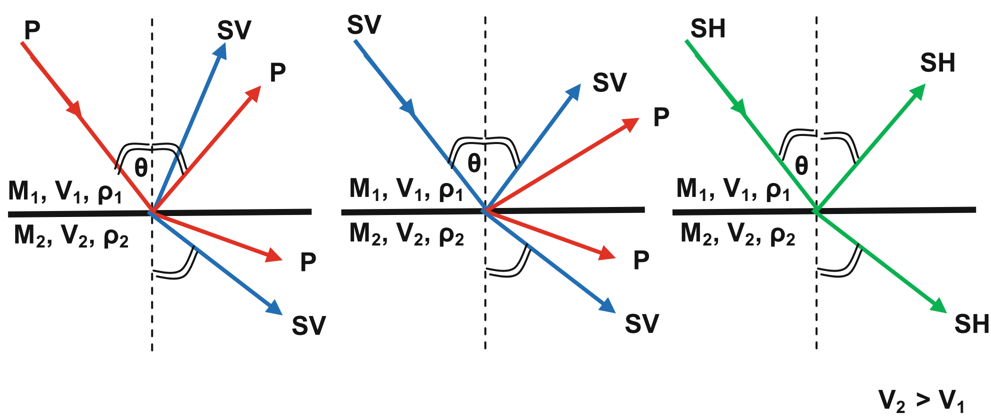



**Fig. 2.25** Wave conversion at an interface for three types of incident waves (*P*, *SV*, and *SH* waves). The symbol ($\theta$) denotes angle of incidence

### 2.5.7 Energy Partitioning and Zoeppritz Equations

The mathematical expressions describing the energy partitioning of an obliquely incident wave among the created converted waves, were derived first by Knot (1899) and later on by Zoeppritz (1907), but not published until (1919). Using an approach (based on displacement computations), Zoeppritz has derived the equations (commonly known as Zoeppritz equations) which express the relative energy partitioning as function of angle of incidence and acoustic impedances of the media separated by the involved interface.

In the geophysical literature, these equations are presented in the form of curves for certain two-layer models with defined density and elasticity properties (see for example, Grant and West 1965, p. 54, Telford et al. 1990, p. 157, Sheriff 2002, p. 401). A complete coverage of various types of incident waves, with different types of media, is found in (Ewing et al. 1957).

Because of the numerous possible parameter-values required to define the behavior of energy-distribution as function of incidence angle, many curves are required for the various cases. These cases represent selected types of incident waves (P, SV, SH) and impedances elements (velocity and density) for each of these wave-types. A typical set of Zoeppritz curves for the case of an obliquely incident P-wave is shown in Fig. 2.26.

Referring to Fig. 2.26, it can be seen that, for normal incidence (angle of incidence = 0), no S-wave is generated and thus all the energy is shared by the reflected and transmitted (refracted) P-wave. At a small angle of incidence, the converted S-waves are of small energy level. As this angle increases the generated S-waves grow stronger at the expense of reflected and refracted P-waves. At the critical angle of the incident P-wave, the transmitted P-wave energy falls to zero, and at the same time, both of reflected P-wave and reflected S-wave grow large. Build–up of energy of reflected P-wave, as the critical angle is approached, is referred to as (wide-angle reflection). This phenomenon (increase of reflection coefficient near the critical angle) is sometimes made use of in seismic reflection exploration (Sheriff 1973, p. 241).

Further, as the angle of incidence approaches grazing incidence (angle of incidence = 90), energy of the reflected P-wave increases and, at grazing incidence (where there is no vertical component for the incident P-wave), the S-waves disappear and no transmission process occurs

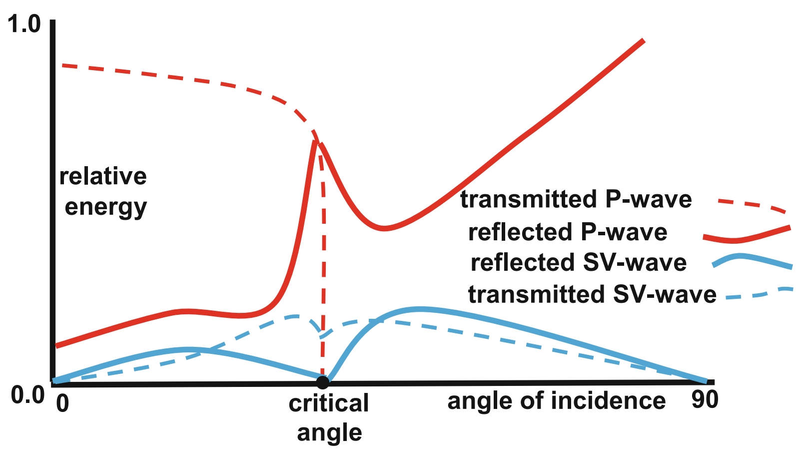



**Fig. 2.26** Typical Zoeppritz curves of energy partitioning as function of angle of incidence. The curves are for the converted waves created by an oblique incident P-wave at an interface separating two media of specified properties (sketched, based on Dobrin and Savit 1988, p. 43)

and consequently all the incident energy is confined to the reflected P-wave.

### 2.5.8 Amplitude Variation with Angle of Incidence

For non-normal incidence, an incident P-wave leads to wave conversion in which both reflected and transmitted P- and S-waves are sobtained. The obliquely-incident wave energy is distributed among all the converted waves in such a way depending on the properties of the involved media on both sides of the interface. According to Zoeppritz equations, the reflection coefficient is function of rock properties (density and elastic properties) in addition to the angle of incidence. For a given reflector, the amplitude variation with angle of incidence (AVA) of a reflected seismic wave is found to be dependent on Poisson's ratio as well as on impedance contrast across the reflection interface. In this way, the parameter (AVA) possesses the same information contained in a combined P- and S-waves data.

It is important to note that Zoeppritz equations give direct relation of amplitude variation with angle of incidence (AVA) and not amplitude variation with offset (AVO). However, offset is proportional to angle of incidence, in case of reflections from a given horizontal reflector. There are, however, situations where the angle of incidence does not vary with the offset. Thus, in a multi-reflector case, the angle of incidence (which is equal to angle of reflection for the same wave-type), varies with reflector depth for a fixed offset. Also, in certain cases, it is possible to get different offsets for a fixed value of reflection angle. These two cases which occur in multi-reflector situation are shown in Fig. 2.27.

As expressed by Zoeppritz equations, the reflection coefficient shows variation with increasing angle of incidence (or with increasing offset). Depending on the distribution of the acoustic impedance on both sides of the interface, the reflection coefficient can vary from large-negative to large-positive values. This behavior (variation of reflection coefficient with angle of incidence) can therefore be used as an indicator to predict lithological changes or type of fluid deposits.

## 2.6 Effect of the Medium on Wave Energy

Due to the earth filtering effect and other causes, the wavelet generated by the source energy, is changed from its initial high-energy, impulsive

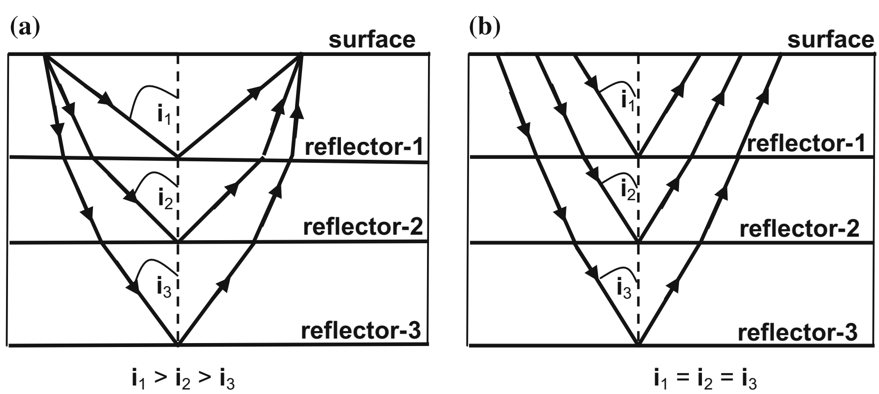



**Fig. 2.27** Variation of reflection angle with reflector depth for a fixed offset and variation of receiver offset for a fixed reflection angle. **a** Angle of incidence (i) decreases with increase of reflector depth for fixed offset, and **b** angle of incidence is constant for varying offset

**Table 2.4** Factors contributing to energy changes of a travelling seismic signal

| 1. In the source zone | 2. In the path zone | 3. In the detector zone |
|---|---|---|
| Energy-Source parameters | Reflection coefficient | Detector response |
| Source coupling | Geometrical spreading | Detector coupling |
| Near-source geology | Inelastic attenuation | Near-detector geology |
| Source-generated noise | Wave conversion | Surface noises |
| Noises and interferences | Noises and interferences | Noises and interferences |

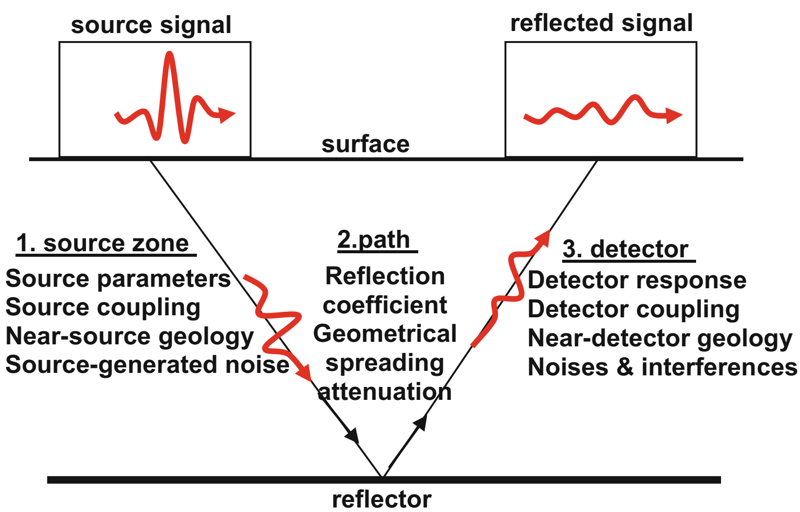



**Fig. 2.28** Reflection–signal playground. The source impulse getting weaker and broader as it progresses along its reflection travel-path. Factors affecting the signal prevailing in the three zones: source-, path-, and detector-zone

form into a lower energy and stretched-form wavelet when observed at the end of its travel-path. Taking the case of reflection of a seismic signal, the complete signal play-ground and the main factors, contributing to the signal energy changes, are summarized in Table 2.4 and Fig. 2.28.

We have already discussed the energy changes (expressed by the reflection and transmission coefficients) due to incidence of a seismic wave onto an interface. Two other important types of energy changes due to the medium through which the wave is propagating are to be discussed. These are the geometrical spreading and the inelastic attenuation effects.

### 2.6.1 Geometrical Spreading

In case of a homogeneous medium, seismic energy generated at the source, spreads out as spherical wave fronts concentric at the source point. Due to expansion of the advancing wave-front, wave energy is distributed over increasing wave-front surfaces. Mechanism of reduction of the wave energy level with travel-distance can be presented with the aid of (Fig. 2.29).

Referring to Fig. 2.29, let a source energy (**E**) be generated at the source point, then, after travelling distances ($\mathbf{r_1}$) and ($\mathbf{r_2}$) the corresponding energy-density of the spherical wave fronts will be ($\mathbf{e_1}$) and ($\mathbf{e_2}$) respectively. The same

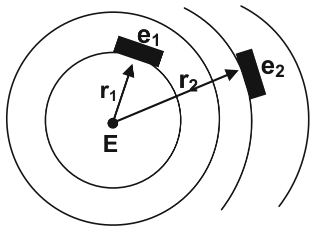



**Fig. 2.29** Wave-fronts generated by a point source are spreading out as spherical wave-fronts in a homogenous medium

energy quantity (**E**) is distributed over the wave-fronts of radii ($\mathbf{r_1}$ and $\mathbf{r_2}$), hence:

$$\mathbf{E} = 4\pi(\mathbf{r_1})^2 \cdot \mathbf{e_1} = 4\pi(\mathbf{r_2})^2 \cdot \mathbf{e_2}$$

giving,

$$\mathbf{e_1}/\mathbf{e_2} = (\mathbf{r_2})^2/(\mathbf{r_1})^2$$

or, (since energy is function of the square of amplitude, **a**),

$$\mathbf{a_1}/\mathbf{a_2} = \mathbf{r_2}/\mathbf{r_1}$$

This result has shown that amplitude attenuation due to spreading of the wave-fronts (called geometrical spreading) is proportional to the travelled distance. In isotropic media, the energy spreads out in the form of advancing spherical surfaces. For this reason the phenomenon is sometimes called (spherical divergence).

The amplitude value is related to the travelled distance (**r**) according to inverse relation. Thus, (**a**) is proportional to (1/**r**), or to ($1/\mathbf{v}(\mathbf{t}) \cdot \mathbf{t}$), where $\mathbf{v}(\mathbf{t})$ is the velocity expressed as function of travel-time **(t).** For a medium made up of parallel layers, it was shown by Newman (1973) that geometrical spreading depends on ($1/\mathbf{v}^2(\mathbf{t}) \cdot \mathbf{t}$) and not on ($1/\mathbf{v}(\mathbf{t}) \cdot \mathbf{t}$), that was derived for homogeneous media.

It is important to be aware that geometrical spreading is independent of frequency.

### 2.6.2 Inelastic Attenuation

Due to friction between the vibrating particles of a medium traversed by a propagating seismic wave, some of the vibration energy is lost as a result of being converted into heat. The amount of loss increases with the increase of distance (**r**) from source point. Experimentally, this is found to take an exponential function of the form:

$$\mathbf{a}(\mathbf{r}) = \mathbf{a_o}\mathbf{e}^{-\alpha\mathbf{r}},$$

or,

$$\mathbf{a}(\mathbf{t}) = \mathbf{a_o}\mathbf{e}^{-\alpha.\mathbf{v}(\mathbf{t}).\mathbf{t}}$$

where, $\mathbf{a}(\mathbf{r})$ is the amplitude at distance (**r**), ($\mathbf{a_0}$) is the initial amplitude, ($\alpha$) is the attenuation coefficient (expressing amplitude reduction due to absorption), (**v**) propagation velocity, (**t**) travel-time, and (**e**) is base of natural logarithm (=2.71828).

Earthquake seismologists often express the attenuation function in a different form (see for example Båth 1974, p. 275),

$$\mathbf{a}(\mathbf{t}) = \mathbf{e}^{-\omega\mathbf{t}/2\mathbf{Q}},$$

Comparing this form with the form involving the absorption coefficient ($\alpha$), the $\alpha$-**Q** relationship is obtained. That is:

$$\alpha = \pi\mathbf{f}/\mathbf{Q}\mathbf{v}$$

This means that the absorption coefficient ($\alpha$) is linearly dependant on frequency, implying that higher frequencies are attenuated faster with increasing distance (or with time). This is supported by experimental evidences, which are proving that the earth is acting as a high-cut (or low-pass) filter to travelling seismic waves. That is why frequencies decrease with depth, as it is commonly observed on raw reflection-records.

The parameter (**Q**), called the quality factor, expresses the absorption-capability of the medium. It is dimensionless quantity and independent of frequency. The quality factor is inversely proportional to the attenuation factor (**α**). The term **1/Q** is called the specific dissipation.

### 2.6.3 Seismic Wave Energy Measurement and the DB Unit

We are all familiar with the units with which physical quantities are measured. Common examples are: gram for measuring mass, meter for lengths, and seconds for time. Ratios, on the other hand, have no units as such. The decibel unit (or db), which is one-tenth of the bell unit, is introduced for measuring values of ratios in just the same way as in measuring masses, lengths, and other physical quantities. The db unit seems to have been developed in connection with measuring energy- or power-ratios of sound-wave intensity expressed by its wave energy or by its wave amplitude. Likewise, the db-unit is usually used in measuring seismic wave energy.

The decibel is defined to be the unit of measuring a power (energy) ratio (**E**), expressed in logarithmic domain to the base 10, hence,

$$[\mathbf{E}]_{\mathbf{db}} = \mathbf{10\,log_{10}\,E} = \mathbf{20\,log_{10}\,A}$$

where the power quantity (**E**) is related to the square of amplitude (**A**).

From this definition, it is apparent that the ratio expressed in decibels is positive when **E** > 1, and negative for **E** < 1, and it is zero when **E** = 1. Another useful note is that ratios (in the db-domain) are added or subtracted corresponding to multiplication or division of the original ratios. For example the ratio of (2/1) in db units, is 20log(2) which is equal to (6 db), and that of the ratio (1/2), it is (−6 db).

### 2.6.4 The Logarithmic Decrement

There is an attenuation parameter, called the logarithmic decrement (**δ**), closely associated with the inelastic attenuation coefficient (**α**). This parameter is defined to be the natural logarithm of the ratio of two neighboring amplitudes of a gradually fading wave-train. This is customarily measured by the ratio of two amplitudes separated by one wavelength (Fig. 2.30).

By definition, the logarithmic decrement (**δ**) is given by:

$$\boldsymbol{\delta} = \mathbf{ln(a_1/a_2) = ln\,e^{-\alpha r}/e^{-\alpha(r+\lambda)} = ln\,e^{\alpha\lambda}}$$

hence,

$$\boldsymbol{\delta = \alpha\lambda}$$

From this result the mathematical relations connecting (**α, Q,** and **δ**) can be readily obtained. Thus,

$$\boldsymbol{\alpha = \delta/\lambda = \pi f/Q\mathbf{v}},$$

Due to these natural attenuation effects, the reflection arrivals from deep reflectors are much weaker than those coming from shallow reflectors. If a raw seismic trace is displayed, the reflection-events from deep reflectors are so weak that they can be barely noticeable. However, when the attenuation due to spherical divergence (geometrical spreading) and due to absorption, is compensated, all events (shallow and deep events) can all be clearly seen (Fig. 2.31).

### 2.6.5 Wave Dispersion

Wave dispersion is a phenomenon that occurs to the propagating wave for which velocity is

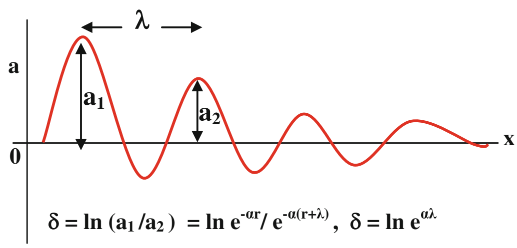



**Fig. 2.30** Definition of the logarithmic decrement ($\delta$)

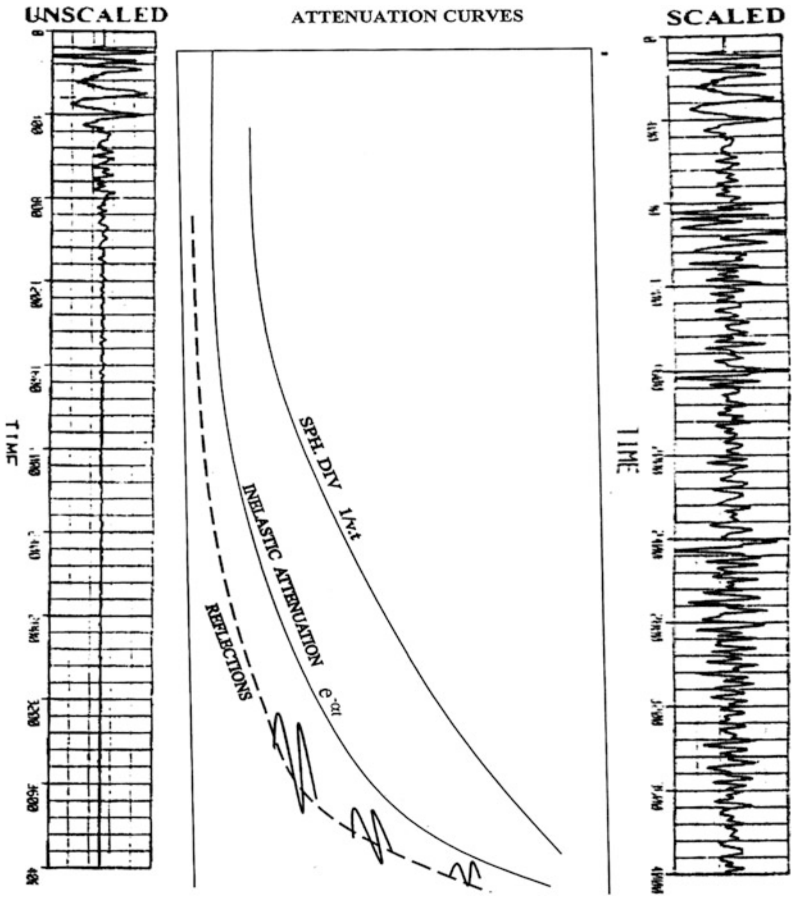



**Fig. 2.31** Seismic trace before application (unscaled) and after application (scaled) of the spherical divergence and inelastic attenuation corrections

function of the frequency component of the travelling wave. Dependence of velocity on frequency means that each frequency component of a seismic signal moves with its own velocity. Thus a wave, composed of several frequency-components will experience component-separation, and hence, change-of-form that occurs during travel. Distortion of the wave-form due to dependence of the velocity on individual frequency-components is called (wave-dispersion).

The dispersion phenomenon leads to changing of the shape of the wave train with travelled distance. Each frequency component (that is, each wave-phase) moves with its own individual velocity (the phase velocity, V). This is the velocity with which a given point, marked on the travelling wave, is moving. The wave-train or the energy package (expressed by the envelope of the wave train) is travelling with different velocity called (group velocity, U) as shown in Fig. 2.32.

The group velocity (**U**) is mathematically related to the phase velocity (**V**) and wavelength (**λ**) of the frequency component, by:

$$\mathbf{U} = \mathbf{V} - \boldsymbol{\lambda}(\mathbf{dV}/\mathbf{d\lambda})$$

where, **V**, **λ**, and **dV/dλ** are average values for the range of frequencies making up the principal part of the pulse (Telford et al. 1990, p. 154).

When the phase velocity (**V**) increases with increase of the component period, it is termed as (normal dispersion), and in this case the group

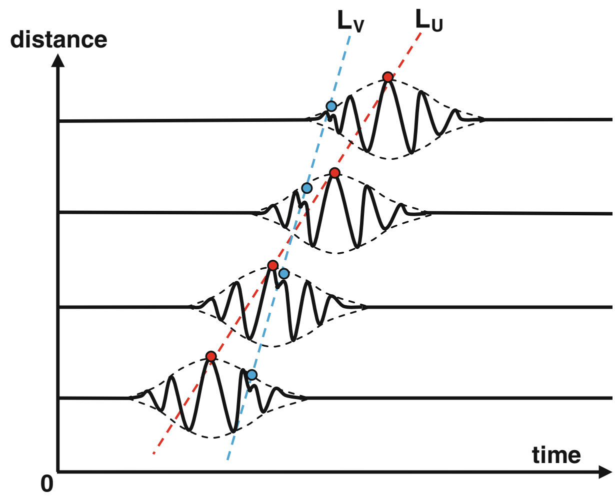



**Fig. 2.32** A wave train showing normal dispersion as it is propagating. The group velocity (U) and phase velocity (v) are given by the slopes of the lines $L_U$ and $L_V$ respectively

velocity is less than the phase velocity (**U** < **V**). For the opposite case (inverse dispersion), it is when phase velocity decreases with period we get (**U** > **V**). In the absence of dispersion, the two velocities are equal (**U** = **V**) and no distortion to the wav-form occurs.

Dispersion phenomenon occurs in a dispersive medium, as when surface waves are travelling through a semi-infinite medium which is overlain by a low velocity surface layer. Dispersion of seismic body waves (P- and S-waves) are too small to be detected in practice.

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Seismic Hydrocarbon Exploration
2D and 3D Techniques
Alsadi, H.N.
2017, XV, 331 p. 350 illus., 284 illus. in color.,
Hardcover
ISBN: 978-3-319-40435-6