Preface

Mechanics is the science of motion: It predicts the motion we see with our eyes. Motion is involved in most scientific activities and in most engineering work. The importance of this topic has resulted in an axiomatization of mechanics and to have mathematics together with experiments to be tools widely used.

There is a large variety of motions we see with our eyes: the motion of the floor of a room, the motion of the grains of a sand pile, the motion of a debris flow, the motion of galaxies, etc. The elements which are used by engineers and scientists to describe them may seem different.

But these theories have in common the concepts to describe the motion. The equations of motion are either simple or very sophisticated. The sophistication may result from the need to define and quantify precisely together the shape of the system which is considered and the evolution of this shape, i.e., the shape change or the deformation of the system, and the velocity of deformation.

A tool to introduce the mechanical effects of the evolution of the shape of a system is the principle of virtual work. It has a status which is perhaps too theoretical even if it is widely used in numerics with the so-called variational formulations [1]. We show it is actually related to observation and experiments. Its utilization is flexible and may be adapted to produce predictive theories of numerous phenomena.

Part I is devoted to relate the virtual work principle to what we see with our eyes.

Part II shows how flexible it is. A large number of examples are given.

The principle is applied in Part III to predict the motion of solids with large deformations. The principle requires the description of the deformations: the way the shape of solids changes. We know that there are a large variety of possibilities. The choice has to be as simple as possible, but it has to cope with the every day life actions. It results from observations that third-order derivatives with respect to space of the displacement are needed to have a coherent description of large deformations.

Once the principle has defined the internal forces and given the equations of motion, we have to face the derivation of the constitutive laws which describe how a material behaves. Equations of motion are general. Constitutive laws are peculiar to each material. Theory and observation intervene in the derivation of the constitutive laws. For what concerns theory, the Clausius–Duhem inequality is the useful tool. For what concerns observation, experiments guide the choice of the free energy and the choice of the pseudo-potential of dissipation.

Following the examples of Parts I and II, we identify an internal constraint on the elongation matrix velocity. Following the way Lagrange takes into account an internal constraint, une liaison parfaite in French and un vincolo perfetto in Italian, we introduce a reaction [2]. As usual in Lagrangian mechanics, this reaction is given by both the constitutive laws and the equations of motion. It is impossible to derive entirely the value of the reaction with a constitutive law. It depends on the whole solid and on the external actions.

These problems have been investigated at the Università degli Studi di Roma "Tor Vergata" in the Dipartimento di Ingegneria Civile e Ingegneria Informatica and in the framework of the Laboratorio Lagrange, bringing together Italian and French scientists. Some of the topics have been taught in lectures given at the Scuola di Ingegneria of the Università.

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