

Fatigue and Notch Mechanics

Bruno Atzori, Giovanni Meneghetti and Mauro Ricotta

Abstract Linear Elastic Notch Mechanics (LENM) extends the concepts of the well known Linear Elastic Fracture Mechanics (LEFM) to notches having root radius different from zero and arbitrary notch opening angle. LENM is based on fundamental analytical results and definitions introduced by Williams [43] and Gross and Mendelson [17]. From the experimental point of view, it has been applied for the first time by Haibach [18] on pure phenomenological basis to analyse the fatigue strength of welded joints using the strain gauge technique. Subsequently, LENM was developed thanks to the progressively increasing use of the Finite Element Method (FEM). Nowadays, NM has been formalised and applied to structural strength assessment of components. Different application techniques exist, but the theoretical frame remains unchanged [6, 8, 12, 20, 21, 23, 24, 28, 31, 37, 38, 42]. The present paper, after recalling the classic notch fatigue design criterion and the LEFM, aims at illustrating the link between Notch Mechanics and those classic approaches. In particular the aim is two-fold: on one side the use of Notch Mechanics in notch fatigue design will be illustrated, on the other side it will be shown how it can be used to better analyse and explain in deep the fundamentals of the classic approaches mentioned previously.

1 Introduction

Classical methods of fatigue design of mechanical components are based on either stresses, elastically evaluated, or strains calculated in the elasto-plastic field. More recently, the use of different fatigue-relevant parameters has been put forward thanks to the development of numerical and experimental techniques: the finite element method on one side and the thermometric methods on the other side. As an example, the finite-volume strain energy density [22] has been extensively applied

B. Atzori (✉) · G. Meneghetti · M. Ricotta
Department of Industrial Engineering, University of Padova, via Venezia 1,
35131 Padua, Italy
e-mail: bruno.atzori@unipd.it

and has been recently reviewed on some publications [11, 34–36]; an experimental, energy-based approach to notch fatigue was proposed by one of the authors, taking the heat energy dissipated in a unit volume of material per cycle as a fatigue damage indicator [27, 29, 30]. However, these novel approaches have still a limited diffusion in practical applications.

Classical fatigue design approaches are based on material properties generated from fatigue testing of plain specimens under stress or strain control for the stress- or strain-based approaches mentioned previously, respectively. The use of such design curves when assessing mechanical components weakened by stress risers is critical due to two reasons:

1. the highly stressed volume at the notch tip which is smaller and smaller as the notch tip radius is reduced, so that the stress/strain peaks are no longer effective in correlating the fatigue strength;
2. as far as ductile engineering materials are concerned, the plastic redistribution which is present around the notch tip, that is significant particularly in the low cycle regime.

A limit case is represented by cracks that are characterised by a notch opening angle and a tip radius equal to zero, with a singular linear elastic stress distribution; therefore, the classical methods based on finite values of the peak stress/strain quantities are not applicable, but the Fracture Mechanics approach must be adopted. The assessment method of the latter approach is based on the complete local stress distribution, characterised by a stress singularity exponent equal to $1/2$, according to the Linear Elastic Fracture Mechanics. Fatigue characterisation of engineering materials is performed on cracked specimens and the adopted stress parameter is the Stress Intensity Factor that quantifies the intensity of the singular linear elastic stress distribution close to the crack tip. Relying on this theoretical background, the Notch Mechanics extends the stress field intensity approach to pointed V-shaped notches, that are characterised by a decreasing stress singularity exponent as the notch opening angle is increased [43]. If a small notch tip radius is introduced, the linear elastic stress distributions do not change significantly, apart from the finite linear elastic peak stress, causing a plateau in a log-log plot of the stress distribution ahead the notch [16].

2 Stress-Life Approach to Fatigue

Dealing with notch fatigue, the linear elastic stress concentration factor is considered, K_t , that is defined as the ratio between the peak stress and the nominal stress referred to the net-section or the gross section, $K_m = \frac{\sigma_{pe}}{\sigma_n}$ or $K_{tg} = \frac{\sigma_{pe}}{\sigma_g}$, respectively. If the fatigue strength were correlated by the range of the linear elastic peak stress,

$\Delta\sigma_{pe}$, then the fatigue curve relevant to notched specimens would be shifted downward by a factor equal to K_t as compared to the one generated from plain specimens. However, this does not occur, because of the two reasons aforementioned. Concerning the stress-gradient effect, the influence of the linear elastic peak stress is properly reduced by introducing a fatigue strength reduction factor K_f , that can be defined experimentally as the ratio between the fatigue strength of a plain specimen and that of the notched specimen for the same high number of fatigue cycles, N :

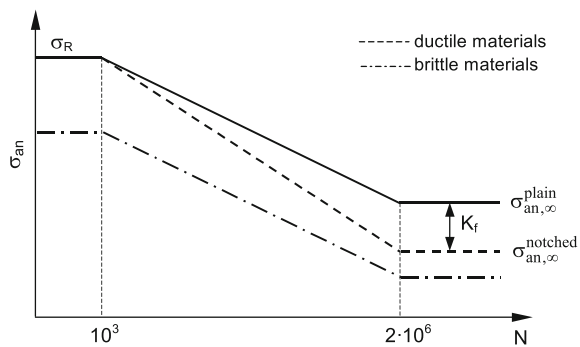
$$(K_f)_N = \left(\frac{\Delta\sigma_{plain}}{\Delta\sigma_{notched}} \right)_N = \left(\frac{\sigma_{eff}}{\sigma_{nom}} \right)_N \quad (1)$$

where, σ_{eff} is the effective stress. In practice, K_f is defined only at the fatigue limit, plastic stress re-distribution becoming more and more effective as the stress level is increased, as shown in Fig. 1 [2] for ductile and brittle materials. For brittle materials also a K'_f should be defined for the static behaviour, since plastic stress re-distribution is not possible or limited. In Fig. 1 it has been assumed a constant K_f , however K_f might change if the stress amplitude of the notched member is increased from the high cycle fatigue to the very low cycle fatigue regime. Concerning the influence of plasticity, which is significant in the low-rather than in the high-cycle fatigue, ductile metals are assumed to fail due to the fully plastic collapse of the net section in the fatigue range up to about 10^3 fatigue cycles. Therefore, the net-section stress amplitude is generally adopted to represent the fatigue design curves, as illustrated in Fig. 1.

The most widely adopted expression to estimate K_f starting from K_t was proposed by Peterson [33]:

$$(K_f - 1) = q(K_t - 1) \quad (2)$$

Fig. 1 Comparison of the fatigue curves for a plain and a notched specimen



However, other formulas are available in literature, many of which formulated subsequently in light of the knowledge progress [9, 10, 13, 19, 23, 25, 26, 32, 37–41].

3 Strain-Life Approach to Fatigue

In principle this approach eliminates the second problem mentioned above, because it is based on the elasto-plastic strain evaluated at the notch tip. Therefore, the strain-life curve derived experimentally on plain specimens, as depicted in Fig. 2, should be directly applicable in design of notched components. However, in case of severely notched specimens (sharp notches, characterised by a reduced notch tip radius), the elasto-plastic peak strain does not correlate the fatigue life and must be reduced or averaged inside a properly defined structural volume.

The elasto-plastic peak strain at the notch tip can be determined starting from a linear elastic stress analysis by using the Neuber's rule:

$$K_\sigma \cdot K_\epsilon = K_t^2 \quad (3)$$

combined with the stabilised cyclic stress-strain curve, as reported in Fig. 3. Alternative formulations are available in the literature (see as an example [14]). In case of severely notched components, K_t appearing in Eq. (3) is suggested to be substituted by K_f , but all uncertainties in K_f estimation still remains, as illustrated in the previous section dedicated to the stress-life approach.

Fig. 2 Manson-Coffin curves of heat treated ductile cast iron (adapted from [30])

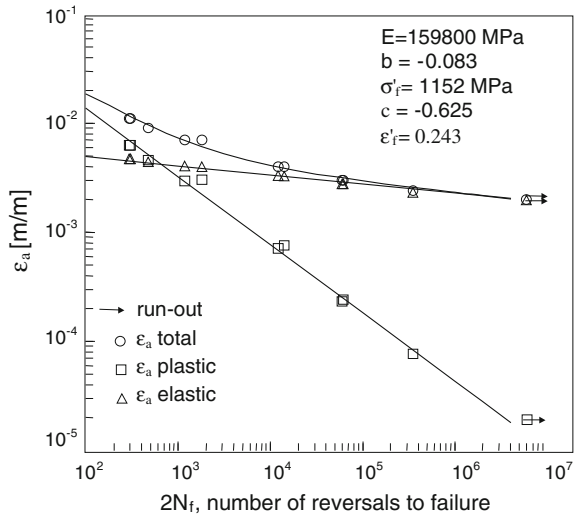
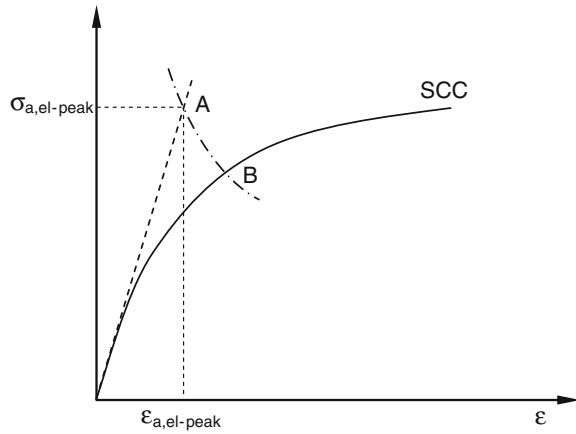


Fig. 3 Graphical representation of the Neuber's rule



4 Linear Elastic Fracture Mechanics (*LEFM*)

In high cycle fatigue, the *LEFM* is commonly adopted and therefore only this discipline will be analysed here. For the sake of simplicity, only the case of a crack having length equal to $2a$ in an infinitely wide plate subjected to mode I loading will be considered (Fig. 4). The σ_y local stress distribution along the x-axis is given by:

$$\sigma_y = \frac{K_I}{\sqrt{2\pi x}} \quad (4)$$

Fig. 4 A through crack centred in an infinitely wide plate

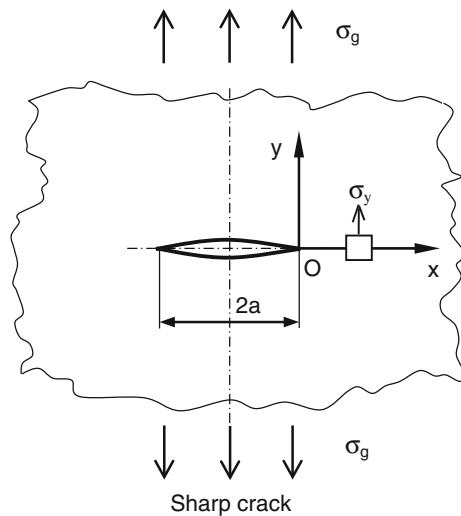
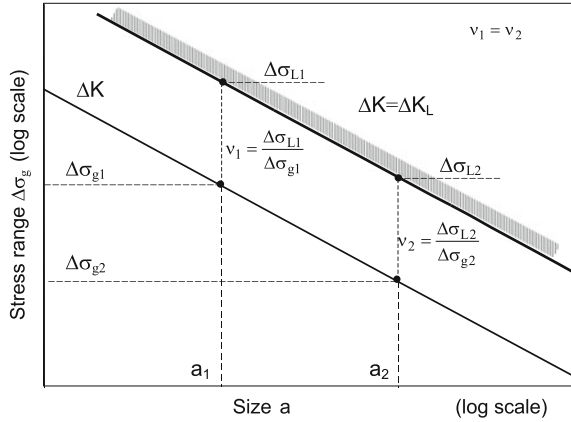


Fig. 5 Interpretation of the Fracture Mechanics Eq. (5) as a nominal stress-based strength criterion



where K_I is the Stress Intensity Factor (SIF), that can be calculated using the following expression:

$$K_I = \sigma_g \sqrt{\pi a} \quad (5)$$

where σ_g is the remotely applied nominal gross stress.

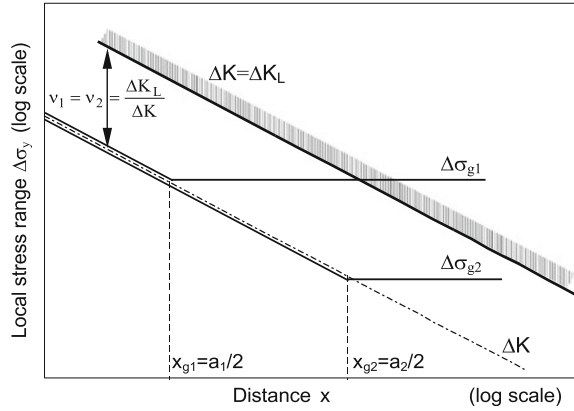
Equations (4) and (5) are often represented in log-log diagrams, as reported in Figs. 5 and 6, and they can be interpreted as an extension to crack problems of the classical approaches presented previously.

In particular, Eq. (5) shows that a nominal stress can be used in fatigue design, thus recalling the stress-based approach, as shown in Fig. 5. Each case analysed is represented in Fig. 5 by a single point, connecting the applied nominal stress to the dimension $2a$ of the crack. The scale effect for two cracks of different a_i but with the same ΔK is shown in this figure, evidencing also that the safety factor v does not change, changing a_i .

Alternately, Eq. (4) shows the local linear elastic stress distribution; because it can be immediately and easily re-converted into local strains due to the linear elastic hypothesis, then Eq. (3) recalls the strain-based approach. Each case analysed is represented in Fig. 6 by the full field of stress (local and nominal): two different crack lengths with the same ΔK give the same local stresses (and the same safety factor v) with two different nominal stresses $\Delta\sigma_g$ evidencing then the scale effect. Both Eqs. (4) and (5) are based on K_I , that is they are based on the asymptotic linear elastic stress distribution, and not on the peak stress; therefore they overcome the first problem of the classic approaches mentioned previously, related to the loss of significance of the linear elastic peak stress as the notch tip radius is reduced.

Concerning the second problem related to the development of plastic strains, as far as the high cycle fatigue behaviour of long cracks is concerned, small plastic zones develop only close to the crack tip, so that linear elastic stresses/strains can be adopted as fatigue relevant parameters. By extending the previous classical approaches to the fatigue crack problems analysed in the present section, Eq. (5)

Fig. 6 Interpretation of the Fracture Mechanics Eq. (4) as a local stress/strain-based strength criterion



will be referred to as stress approach, while Eq. (4) will be referred to as strain approach.

In fatigue design, expressions (4) and (5) along with the crack growth rate equation for long cracks known as Paris' law:

$$\frac{da}{dN} = C \cdot \Delta K^m \quad (6)$$

are adopted to estimate the number of cycles to spread a fatigue crack from an initial size a_i up to a final size a_f [14]:

$$N_f = \frac{a_f^{(1-\frac{m}{2})} - a_i^{(1-\frac{m}{2})}}{C(1-\frac{m}{2})(\Delta\sigma_g \cdot \sqrt{\pi})^m} \quad (7)$$

Concerning the determination of the fatigue limit of a cracked plate, Eq. (5) should be used for the stress approach as illustrated in Fig. 7, while Eq. (4) is valid for the strain approach as depicted in Fig. 8. The idealised trends reported in Figs. 7 and 8 show abrupt changes of the slope, while real trends are smooth, which can be described by proper analytical expressions [9, 13].

Concerning the static strength design, Eqs. (4) and (5) are still valid, provided that the threshold range of the SIF is substituted by the Fracture Toughness ΔK_c and the plain material fatigue limit $\Delta\sigma_0$ is substituted by the material tensile strength σ_R .

Figure 8 shows that the stress field approach according to the *LEFM* requires that the extension of the asymptotic stress field is sufficiently long, i.e. there will be a limitation of applicability as the crack size is reduced. The behaviour of short fatigue cracks is widely investigated in the literature; however, a simple and effective equation to estimate the threshold SIF of short cracks was proposed by El Haddad et al. [15] as follows (see Fig. 7):

Fig. 7 Fatigue limit estimation according to Eq. (5)

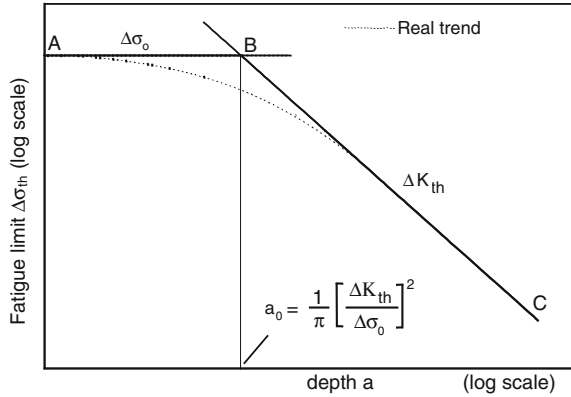
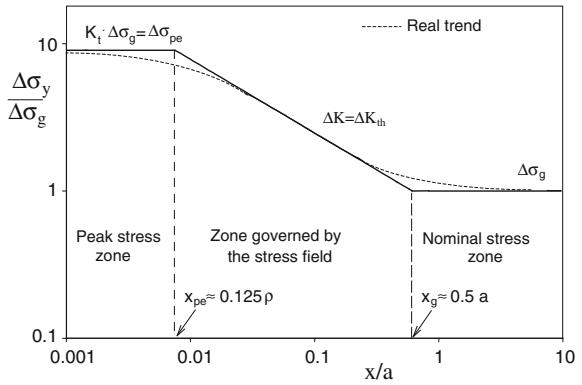


Fig. 8 Fatigue limits estimation according to different weakest link conditions



$$\Delta K_{th} = \Delta \sigma_{gth} \sqrt{\pi(a + a_0)} \quad (8)$$

Equation (8) is valid for cracks centred in an infinite plate. For real components, Eq. (8) should be re-arranged to account for a shape factor [9].

5 Linear Elastic Notch Mechanics

Linear Elastic Notch Mechanics is a non-conventional extension of the *LEFM* to include also pointed or rounded V-shaped notches, characterised by an arbitrary notch opening angle (*LEFM* considers only the crack case, i.e. the notch opening angle and the tip radius are equal to zero).

One of the main fields of application of the Notch Mechanics is the fatigue design of welded structures. About fifty years ago, Haibach proposed an experimental approach to evaluate the fatigue strength of welded structures, based on the

strain range evaluated at a fixed distance from the weld toe (supposed to be the crack initiation point) by means of a strain gauge having a well defined length [18]. By so doing, Haibach found that the fatigue strength of welded joints in structural steels tested in the as-welded conditions and failing from the weld toe could be synthesised by means of a single scatter band, despite the different plate thicknesses and joints' shape considered. Such an approach has been later applied to welded joints in aluminium alloys [3], but only after twenty years several finite element analyses demonstrated that the stress field close to the tip of sharp V-notches has a singularity exponent lower than $\frac{1}{2}$ and decreasing as the V-notch opening angle increases [1]. These stress analyses provided a theoretical justification to the phenomenological approach introduced by Haibach.

The research concerning the more general theory of notch stresses put in evidence that, several years before the works by Haibach, the stress singularity exponents for open V-notches had been provided by Williams [43], while later on a definition of Notch-Stress Intensity Factor, playing the same role of the *SIF* of the *LEFM*, was introduced by Gross and Mendelson for mode I loading [17]:

$$K_I^V = \sqrt{2\pi} \cdot \lim_{x \rightarrow 0} (\sigma_x \cdot x^\gamma) \quad (9)$$

γ being the stress singularity exponent, equal to $\frac{1}{2}$ for the crack case and decreasing to zero as the notch opening angle increases to 180° . On this basis, Lazzarin and Tovo formalised the equations of the linear elastic stress fields for open V-notches and by using the range of the *N-SIF* evaluated at the weld toe they were able to summarise the fatigue strength of welded joints in structural steels, tested in as-welded conditions and failing from the weld toe, as shown in Fig. 9 [20].

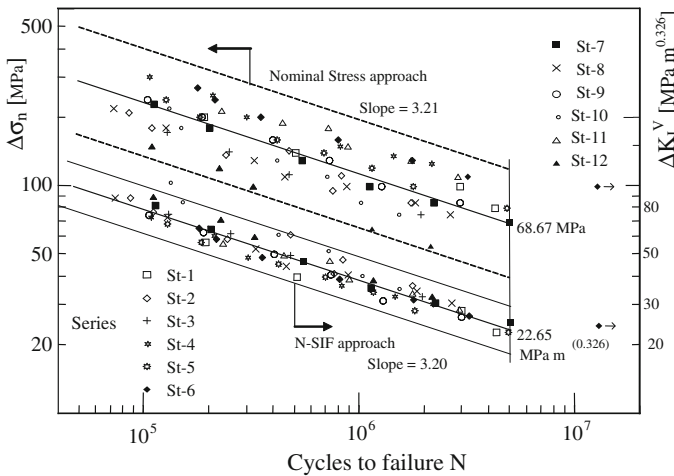


Fig. 9 Synthesis of the fatigue strength of welded joints in steel using the mode I N-SIF (from [21])

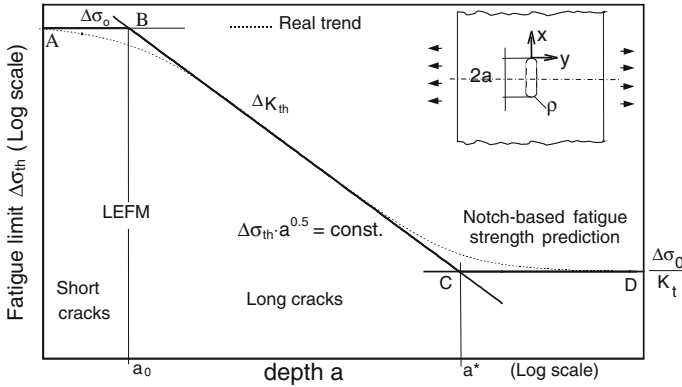


Fig. 10 Atzori-Lazzarin's diagram for U-shaped notches (from [5])

In the case of U-shaped notches having notch tip radius ρ different from zero, the notch acuity $\zeta = \frac{a}{\rho}$ is not infinite, as it occurs in the crack case. If the notch acuity (and then the K_t) is kept constant and the notch absolute dimensions are scaled in proportion, the threshold stress range $\Delta\sigma_{gth}$ (i.e. the fatigue limit) will vary according to Fig. 10, where the short crack/notch behaviour, the long crack/sharp notch behaviour and the blunt notch behaviour are highlighted [5]. The diagram reported in Fig. 10 has been validated using several experimental data, as shown in Fig. 11 [9].

Subsequently it has been extended and validated to include the V-notch case [10]; moreover, it has been demonstrated that the threshold values of the $N-SIF$ ΔK_{th}^V can be expressed as a function of the material fatigue properties ΔK_{th} of a crack and $\Delta\sigma_0$ of the plain material.

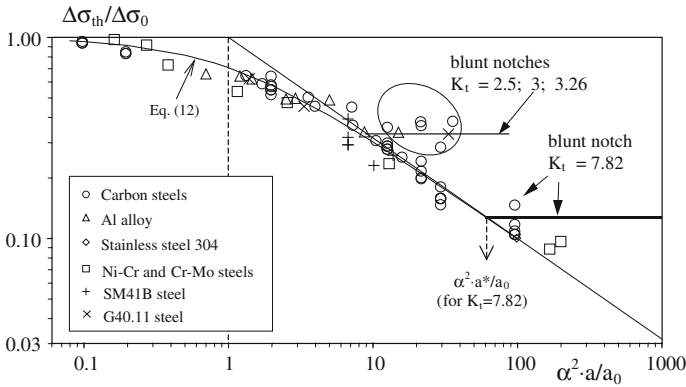


Fig. 11 Fatigue strength of specimens containing defects and notches [9]

6 Sharp Notches, Scale Effect and Woehler Curves

For the sake of brevity, only U-shaped sharp notches characterised by a (small) tip radius, a well defined acuity $\frac{a}{\rho}$, centred in an infinitely wide plate and subjected to pure mode I loading will be considered. Moreover a material characterised by a brittle behaviour will be analysed.

The fatigue model according to the Notch Mechanics is reported in Fig. 12 for the limit conditions corresponding to the fatigue limit ($\Delta K = \Delta K_{th}$) and to the static failure ($\Delta K = \Delta K_c$) [4, 5]. In the form presented in Fig. 12, $\Delta \sigma_g = f(a)$, the method recalls the classic stress-based approach. It is worth noting that Fig. 12 shows an idealised behaviour, the experimental trends generating smooth transitions between two intersecting straight lines.

To generate the Woehler curve of a notch having given acuity and size a_i , different constant amplitude fatigue tests should be performed corresponding to stress levels chosen in the range $\Delta \sigma_{gth}(a_i) - \Delta \sigma_R(a_i)$ (for the same initial notch size a_i). In each one of such tests, the notch-emanated fatigue crack would grow up to the final condition $\Delta K = \Delta K_c$ when $a = a_f$ as reported in Fig. 12. Depending on the size $2a_i$ of the tested notches, we can define different zones as indicated in the figure:

Zone I : $a \leq a_0$

Zone II : $a_0 < a \leq a_{0s}$

Zone III : $a_{0s} < a \leq a^*$

Zone IV : $a^* < a \leq a_s^*$

Zone V : $a > a_s^*$

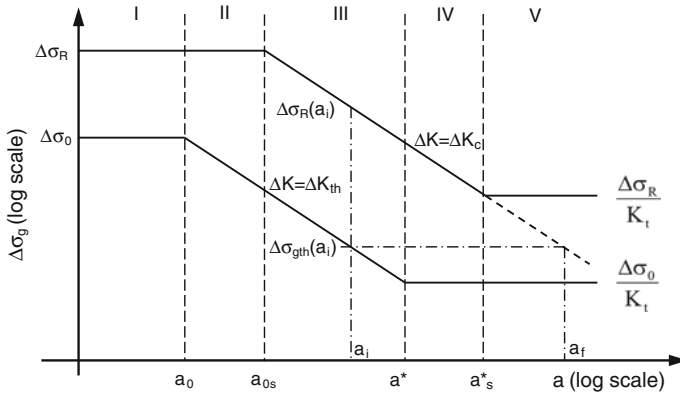


Fig. 12 Application of Linear Elastic Notch Mechanics to a notch having initial size a_i

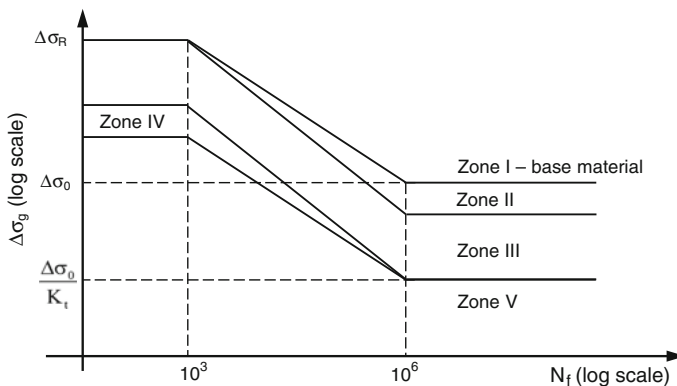


Fig. 13 Fatigue curves generated from testing notches having different initial sizes

A comparison between theoretical estimations and experimental results also for ductile materials has been reported elsewhere [7].

While increasing the notch size $2a_i$, the Woehler curve will modify according to Fig. 13. Starting from the base material fatigue behaviour (*zone I*), the sharp notch behaviour according to the Notch Mechanics is illustrated by *zone III* and finally the full notch sensitivity (in static as well as fatigue behaviour) is the one of *zone V*, where the fatigue curve of the base material is simply shifted downwards of a factor given by the stress concentration factor K_{tg} .

There are also two zones where the slope of the Woehler curve is different from that of the base material: the first is *zone II* where the inverse slope increases because the fatigue limit decreases, the static strength being unchanged; the second is *zone IV*, where the inverse slope decreases because the static strength decreases, the fatigue limit being constant. Figure 13 assumes that the high cycle and the low cycle knees are located at 10^3 and 2×10^6 cycles, respectively, according to the Classic Mechanics.

7 Conclusions

In the present work the fundamental concepts of the Notch Mechanics have been presented, along with its relation to the Fracture Mechanics and Classic Mechanics disciplines. For the sake of brevity, it has not been possible to present in deep all analytical formulations and experimental validations reported in the literature, which the reader is referred to.

Notch Mechanics is a non-conventional extension of the Fracture Mechanics concepts that are included as a particular case. All notch opening angles, notch tip radii and notch acuties are included in Notch Mechanics, thus largely extending the possibility to use Fracture Mechanics concepts in practical design situations. From a

theoretical point of view and considering a progressive reduction of the notch size, Notch Mechanics shows the limitations of applicability for fatigue design purposes of the linear elastic peak stress evaluated at a rounded notch tip and of the stress field evaluated at a sharp notch. The former can be applied as far as $a > a^*$ in the schematic Fig. 10, while the latter can be applied when a is between a_0 and a^* . The schematic diagram shown in Fig. 10 seems to suggest that a small notch having size $a \leq a_0$ behaves as a non-damaging defect. Indeed the experimental trend also reported in the same figure clarifies that there exists a so-called defect sensitivity according to which a fatigue strength decrease can be observed. Such reduction is approximately 40 % when $a = a_0$ and consequently cannot be neglected in practical fatigue design.

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