

Chapter 2

Wheel and Terrain Interaction

Nomenclature

m	Mass
v	Velocity
G	Linear momentum
F	Force
M	Moment
I	Mass inertia of moment
α	Angular acceleration
N	Reaction force component normal to the contact surface
N_t	Net traction force
R_r	Rolling resistance
l	Length of contact between the tire and surface
T	Torque applied to the tire
J_w	The moment of inertia of the wheel
ω_w	The angular velocity of the wheel, the overdot indicates differentiation with respect to time
R_w	The radius of the wheel
N_v	Vertical reaction force from the ground
T_e	The engine torque
T_b	Brake torque
F_t	Tractive force
F_w	Wheel ground friction
i, λ	Wheel slip
$\mu(\lambda)$	Adhesion coefficient
$k_x, k_y, \text{ and } k_z$	Tire stiffness in the x, y and z directions
Δz	Linear displacement of tire under normal load (vertical tire deformation)
Δx	Longitudinal tire deformation
Δy	Lateral tire deformation
U	Strain energy function

C_1, C_2	Are temperature-dependent material parameters
λ_1, λ_2 and λ_3	Principal stretch ratios
I_1, I_2	First and second strain invariants of the Green's deformation tensor
k_r	Dynamic stiffness of tire
c	Damping ratio of the tire
ω	Natural frequency
σ_z	z-oriented component of stress
τ_x	Longitudinal shear stress
τ_y	Lateral shear stress
$l_x(y)$	Half-length of footprint at y in the x -direction
$W_y(x)$	Half width at x in the y -direction
V^A	Constraint of body A
V^B	Constraint of body B
${}^t x_M^A$	Coordinate of the particle M on the surface ${}^t S^A$ of body A
${}^t x^B$	Coordinate of any desired particle on surface ${}^t S^B$
λ_N	Normal contact force
μ	Coefficient of friction
$\dot{\gamma}_\alpha$	Tangential slip rate in the tire/road contact plane
C_{sp} and N_b	Dimensionless road profile constants
$S_x(f)$	Power spectral density function
Ω	Spatial frequency
$S_x(\Omega)$	Power spectral density function
T	Period
u	Linear velocity of tire
K_s	Total longitudinal force per unit longitudinal slip ratio
K_β	Total lateral force per unit side-slip angle
V_s	Slip velocity

In wheel-terrain interaction, there are a diversity of influential factors that play a substantial role in the output of the system. Given that the attention of this book is more on the effect of terrain on vehicle performance and mobility index, the effect of environment (terrain) characteristics on wheel should be well covered since the wheels are the unique connections between the ground and the vehicle and nearly all forces and moments applied to the vehicle are transmitted through the wheels. The wheels are also responsible to support the vehicle and perform steering, handling and creating dynamic forces such as traction, braking and also they function as a part of vehicle suspension system. It should also be distinguished between the motion condition on paved road and terrain based off-road that provides different motion condition.

2.1 Identification of Wheel-Obstacle Collision

An important topic of specialized interest in wheel kinetics is impact force. The principles of impulse and momentum have an important use in describing the behavior of colliding bodies. Impact refers to the collision between two bodies and is characterized by the generation of relatively large contact forces which act over a very short interval of time. For the linear momentum, we may write the basic equation of motion by:

$$\sum F = m\dot{v} = \frac{d}{dt}(mv) \quad (2.1)$$

where the product of the mass and velocity is defined as the linear momentum $G = mv$ of the particle. Equation 2.1 can be written in the three scalar components as:

$$\sum F_x = \dot{G}_x \quad \sum F_y = \dot{G}_y \quad \sum F_z = \dot{G}_z \quad (2.2)$$

the effect of the resultant force ΣF on the linear momentum of the over a finite period of time is described as:

$$\int_{t_1}^{t_2} \sum F dt = G_2 - G_1 = \Delta G \quad (2.3)$$

Which can be rewritten as:

$$\begin{aligned} m(v_1)_x + \int_{t_1}^{t_2} \sum F_x dt &= m(v_2)_x \\ m(v_1)_y + \int_{t_1}^{t_2} \sum F_y dt &= m(v_2)_y \end{aligned} \quad (2.4)$$

the third direction is removed since the longitudinal and vertical forces are more significant during wheel traversing on surfaces with no slope that causes no lateral force generation. Different obstacle forms have been presented in Fig. 2.1.

However, velocity should be presented as a vector in both vertical and longitudinal directions:

$$\vec{v} = \dot{x}i + \dot{y}j \quad (2.5)$$

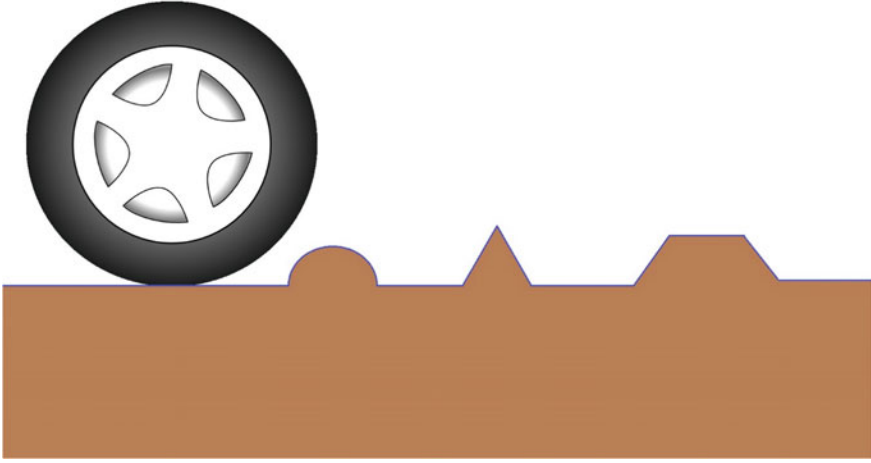


Fig. 2.1 Schematic representation of the tire traversing over different obstacles

The path of wheel is affected by the obstacle geometry as following:

$$y = \sin \frac{2\pi}{l} x \quad 0 < x < 2\pi \quad (2.6)$$

For the triangular shaped obstacle, the following equation is described:

$$y = \begin{cases} ax & x < \frac{l}{2} \\ -ax & x > \frac{l}{2} \end{cases} \quad (2.7)$$

l in this typical example is the obstacle length and a is the slope of obstacles at different determine heights, respectively. For a trapezoid type of obstacle Eq. 2.8 can be considered.

$$\text{trapezoid}(x; a, b, c, d) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{d-x}{d-c}, & c \leq x \leq d \\ 0, & d \leq x \end{cases} \quad (2.8)$$

The governing equation based on the Newtonian-Eulerian method can be presented for the problem of tire-obstacle collision as following (Fig. 2.2):

$$\begin{aligned} \sum F_x &= m_t a_x \\ \sum F_y &= m_t a_y \\ \sum M &= I \alpha = I \ddot{\theta} \end{aligned} \quad (2.9)$$

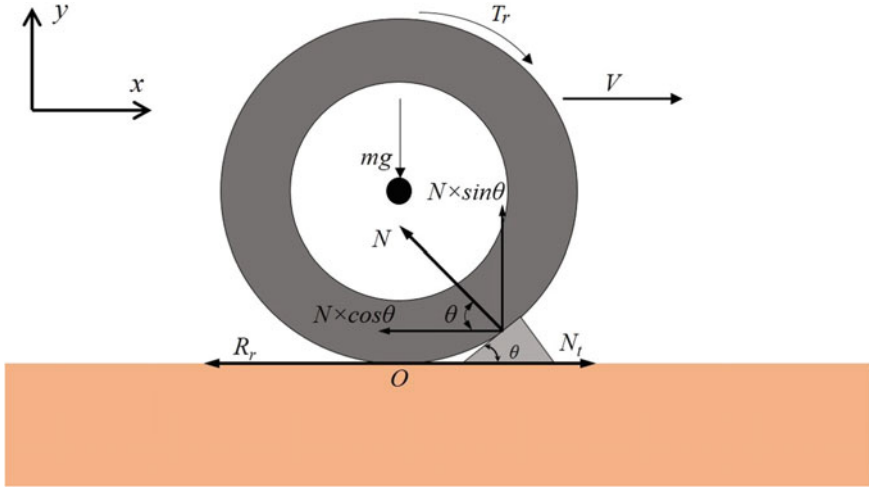


Fig. 2.2 The free-body diagram of the tire-obstacle impact

where m , M , I and α are tire mass, moment, mass inertia of moment and angular acceleration, respectfully. From Eq. 2.9, the following equation can be derived:

$$\begin{aligned} N_t - N \cos \theta - R_r &= ma_x \\ N \sin \theta - mg &= ma_y \\ N \cos \theta \times l \sin \theta + N \cos \theta \times l \cos \theta - T &= \bar{I} + m_t r^2 \end{aligned} \quad (2.10)$$

where N , N_t , R_r , l and T are the reaction force component normal to the contact surface, the net traction force, rolling resistance, length of contact between the tire and surface, and torque applied to the tire, respectively. In this condition, the longitudinal and lateral impact forces for the impact can be rewritten as following:

$$\begin{aligned} F_l &= N \cos \theta = N_t - ma_x - R_r \\ F_v &= N \sin \theta = m(a_y + g) \end{aligned} \quad (2.11)$$

However, it should be also noted that due to the acceleration vector components at x and y directions that should follow the obstacle shapes, the obtained variations of force changes in time domain.

Figure 2.3 shows the force variations in time domain while the tire collides and traverses over the Gaussian shaped obstacle in vertical direction at different obstacle heights. Also, validation process with experimental results can be seen too. As appreciated from Fig. 2.3, after the collision the force disturbance occurs until the shock absorbs and damps after a period of time. The peak values depend on the obstacle height in a manner that increased obstacle height results in the increment of impact force. Furthermore, the vertical force is greatly affected by the impact force

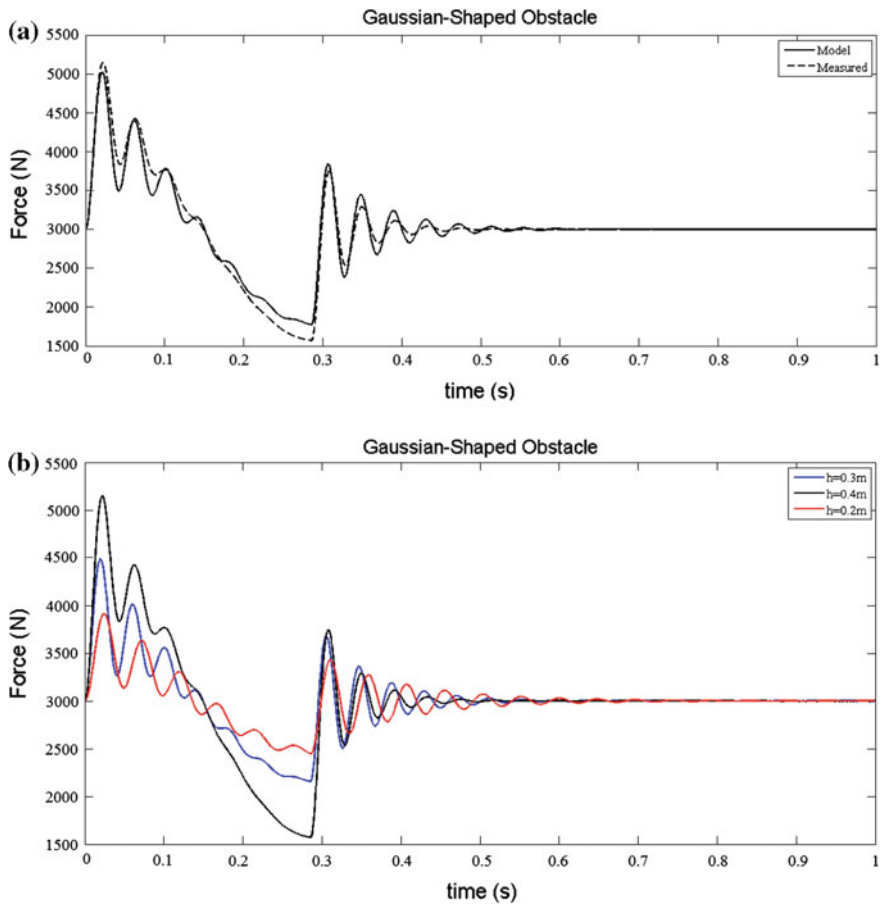


Fig. 2.3 Force variations in time domain while the tire collides and traverses over the Gaussian shaped obstacle in vertical direction at different obstacle heights; **a** model validation, **b** obtained results at different depths

with greater range of amplitude variation (compressive/extensive loads). In the interest of a balanced vehicle traversing, the accuracy and reliability of the complete vehicle model has to produce rational relation to the performance of the applied tire model. For the effect of obstacle height, it can be pointed out that due to the change of momentum in the vertical direction, a velocity change in the vertical direction of y (i.e. ΔV_y) is formed which results in the formation of linear impact in the same direction. Hence, an acceleration component at the same vertical direction of y is created owing to the aforesaid velocity change (i.e. ΔV_y) at the increased obstacle height leading to the increased vertically induced inertia forces. This process well describes the increase of vertical force with respect to the increase of obstacle height. Likewise, the increased obstacle height results in the reduction of the instantaneous velocity of wheel at the horizontal direction which in turn, results in a significant

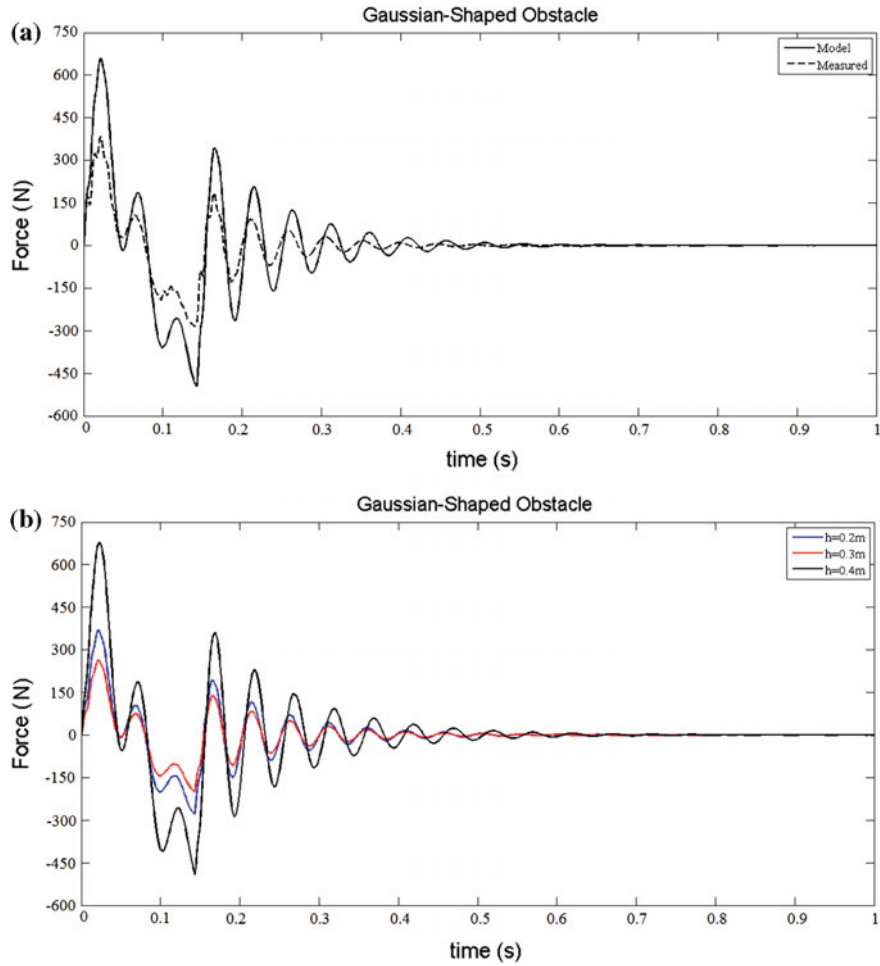


Fig. 2.4 Force variations in time domain while the tire collides and traverses over the Gaussian shaped obstacle in longitudinal direction at different obstacle heights; **a** model validation, **b** obtained results at different depths

change of the linear momentum and therefore greater linear impact in the horizontal direction is obtained. Figure 2.4 is dedicated to present the longitudinal force variations in time domain for both model validation and the variations of the force at different obstacle heights versus time. It is expectable, with abovementioned justifications that increased obstacle height results in the increment of impact force.

Figure 2.5 demonstrates the impact force variations in both directions addressing the trapezoidal shaped obstacles at different depths. Owing to the change of momentum in the vertical direction, a velocity change in the vertical direction of y (i.e. ΔV_y) is formed which results in the formation of linear impact in the same direction. Due to the decrease of the instant velocity of wheel at the longitudinal

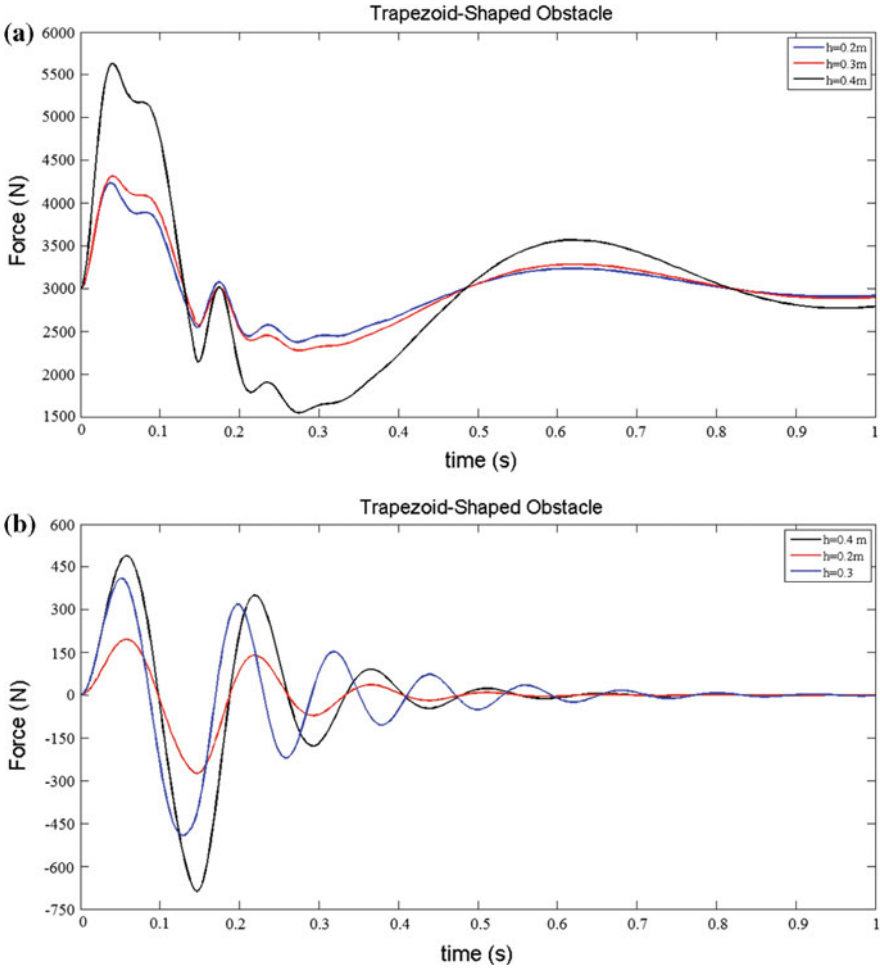


Fig. 2.5 Force variations in time domain while the tire collides and traverses over the trapezoidal shaped obstacle in **a** vertical and **b** longitudinal directions at different obstacle heights

direction results in variation of the linear momentum and greater longitudinal impact force. Figure 2.6 illustrates the impact force variations in both directions addressing the triangular shaped obstacles at different depths. The variations justifications are previously covered for the other obstacle geometries.

Based on the presented results, it is concluded that the trapezoidal shaped obstacles bring about the lowest values of longitudinally oriented impact force while the greatest values correspond to the triangular obstacles. While the greatest values of impact force in vertical direction correspond to the triangular obstacles, in contradictory to the longitudinal impact force, the lowest force corresponded to the Gaussian shaped obstacles. This can serve as an important step in vehicle suspension design and tire manufacturing industry.

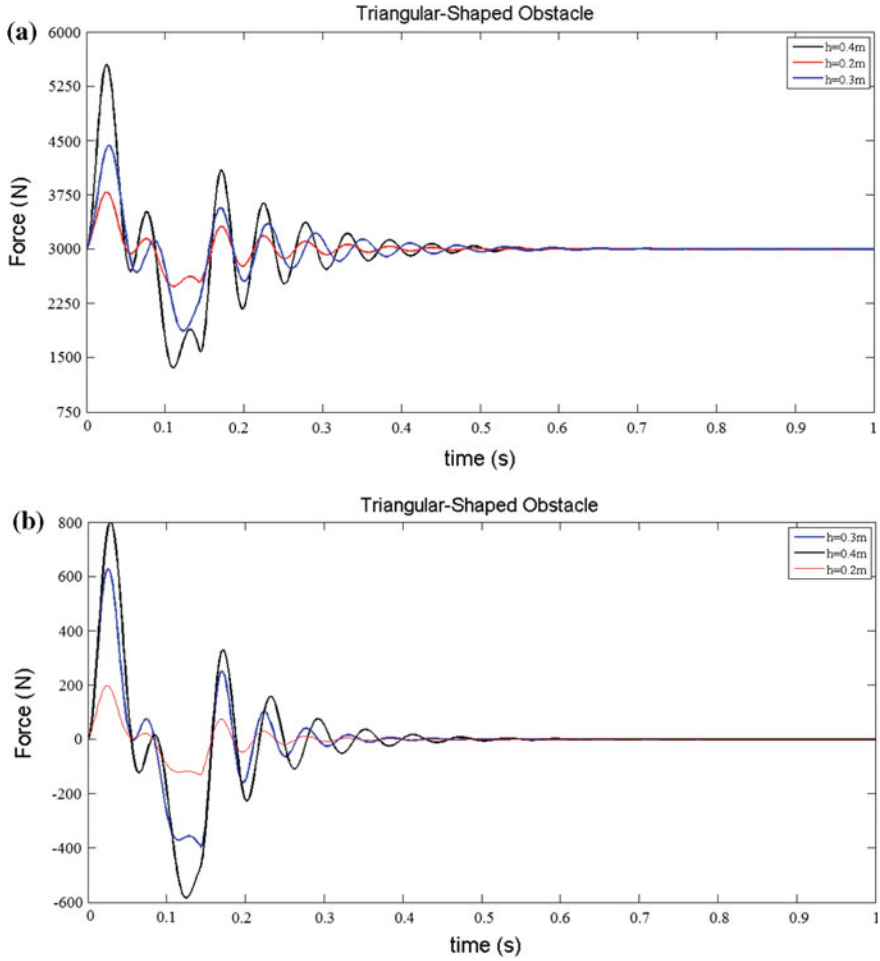


Fig. 2.6 Force variations in time domain while the tire collides and traverses over the Triangular shaped obstacle in **a** vertical and **b** longitudinal directions at different obstacle heights

2.2 Tire Modeling

The angular motion of the wheel can be developed with a dynamic equation of motion:

$$\dot{\omega}_w = \frac{[T_e - T_b - R_w F_t - R_w F_w]}{J_w} \quad (2.12)$$

where J_w is the moment of inertia of the wheel, ω_w is the angular velocity of the wheel, the overdot indicates differentiation with respect to time, R_w is the radius of

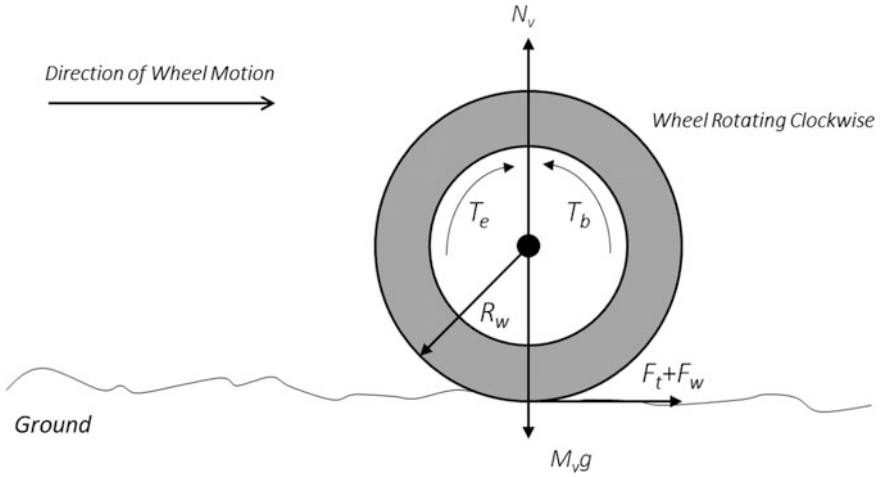


Fig. 2.7 Schematic description of wheel under the kinetics of motion

the wheel. N_v is vertical reaction force from the ground, T_e is the engine torque, T_b is Brake torque, F_t Tractive force and F_w wheel ground friction or so called rolling resistance [1]. The schematic figure of wheel under these kinetics of motion is depicted in Fig. 2.7.

The total torque acting on the wheel divided by the moment of inertia of the wheel equals the wheel angular acceleration (deceleration). The total torque consists of engine torque that acts in opposed direction with that of the brake torque and the torque components owing to the tire tractive force and the wheel ground friction force or rolling resistance force. The tire tractive (braking) force is given by:

$$F_t = \mu(\lambda)N_v \quad (2.13)$$

where the normal tire force (the reaction force from the ground to the tire), N_v , relies on vehicle parameters such as the mass of the vehicle, position of the vehicle center of gravity, and the steering and suspension dynamics. Applying a driving torque or a braking torque to a pneumatic tire produces tractive (braking) force at the tire-ground contact patch. The driving torque produces compression at the tire tread in front of and within the contact patch. Therefore, the tire travels a shorter distance than it would if it were free rolling. In the same way, when a braking torque is applied, it produces tension at the tire tread within the contact patch and at the front. Because of this tension, the tire travels a larger distance than it would if it were free rolling. This phenomenon is referred as the wheel slip or deformation slip. The adhesion coefficient, which is the ratio between the tractive (braking) force and the normal load, depends on the road-tire conditions and the value of the wheel slip λ while it can be presented as:

$$\lambda = \frac{(\omega_w - \omega_v)}{\omega}, \quad \omega \neq 0 \quad (2.14)$$

where $\omega_v = \frac{V}{R_w}$ is the vehicle angular velocity of the wheel which is defined as being equal to the linear vehicle velocity, v , divided by the radius of the wheel.

The variable ω is defined as:

$$\omega = \max(\omega_w, \omega_v) \quad (2.15)$$

which is the maximum of the vehicle angular velocity and wheel angular velocity. The adhesion coefficient $\mu(\lambda)$ is a function of wheel slip. For various road conditions $\mu(\lambda)$ curves have different peak values and slopes. In modeling the following function is used for a nominal

$$\text{curve: } \mu(\lambda) = \frac{2\mu_p \lambda_p \lambda}{\lambda_p^2 + \lambda^2} \quad (2.16)$$

where λ_p and μ_p are the peak values. For various road conditions, the curves have different peak values and slopes. The adhesion coefficient slip characteristics are also influenced by operational parameters such as speed and vertical load. The peak value for the adhesion coefficient usually has values between 0.1 (icy road) and 0.9 and for a terrain surface it is about 0.5. A typical adhesion coefficient versus wheel slip is demonstrated in Fig. 2.8.

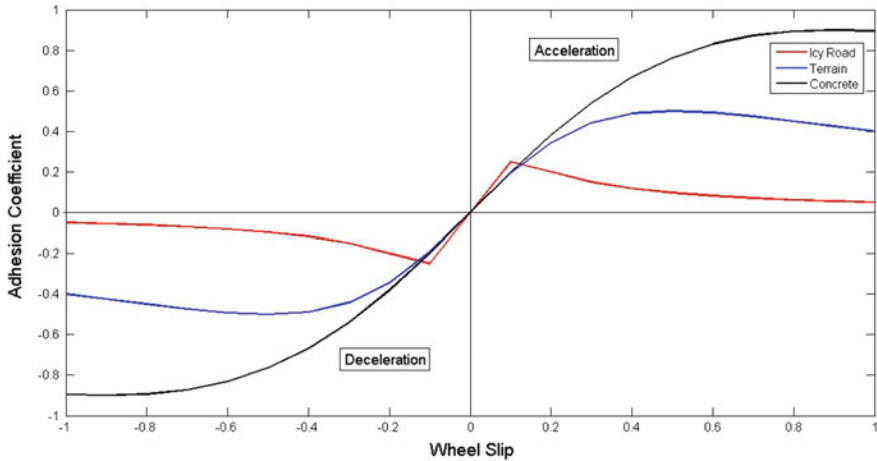


Fig. 2.8 Adhesion coefficient with respect to wheel slip in acceleration and deceleration regions for three road types

The contact interaction between tire and the road greatly influences the driving performance of vehicles. One dynamic field of study in interest for the engineer is of vehicle dynamics discipline is optimising the tire-road interaction so that the vehicle handles well and operates both safely and comfortably under any circumstance. To assess the effect of tire properties on the dynamic behaviour of vehicles, the researcher needs a precise description of the tire-road contact phenomena.

In order to determine the tire-road interaction and force system, a Cartesian coordinate system can be positioned at the center of the tireprint, as shown in Fig. 2.9. The tire plane and ground when intersects will form a line which can be considered as the x-axis of the coordinate system. In the condition that the tire is very narrow that it is seen as a plane, then the tire plane is formed. The z-axis acts normal to the x-axis in opposite direction to the gravitational acceleration g , and the y-axis makes the coordinate system a right-hand triplet [2].

Camber angle γ and sideslip angle α are used in order to demonstrate the tire orientation. The camber angle is the angle between the tire-plane and the vertical plane measured about the x-axis. The camber angle can be clearly identified in Fig. 2.9. The sideslip angle α is the angle between the velocity vector v and the x-axis measured about the z-axis (Fig. 2.9).

The force system that a tire receives from the ground is assumed to be located at the center of the tireprint and can be decomposed along x, y, and z axes. Therefore, the interaction of a tire with the road generates a 3D force system including three forces and three moments (Fig. 2.9).

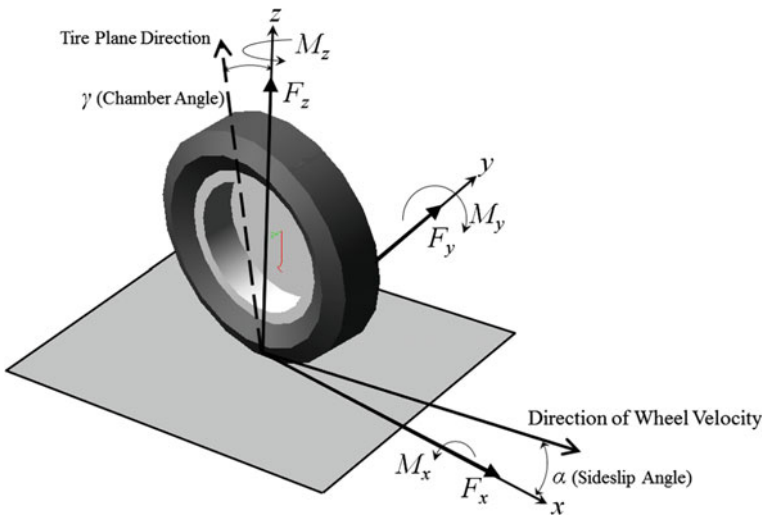


Fig. 2.9 3D force system including three forces and three moments

2.2.1 Forces and Moments

Longitudinal force F_x is a force that operates in the direction of the x-axis. In the case that the resultant longitudinal force $F_x > 0$, then the vehicle is accelerating, and if $F_x < 0$ then the vehicle is in deceleration (braking mode). Vertical force F_z is a vertically oriented force that acts normal to the ground plane. The resultant vertical force $F_z > 0$ in the case that it is upward and $F_z < 0$ when it acts downward. Lateral force F_y is a force that acts tangent to the ground and orthogonal to both F_x and F_z . The resultant lateral force $F_y > 0$ if it is in the y-direction and $F_y < 0$ when it acts apposite to the y-direction [2].

Roll moment M_x is a longitudinal moment about the x-axis while the resultant roll moment $M_x > 0$ when it is more likely to turn the tire about the x-axis. The roll moment is also called the bank moment, tilting torque, or overturning moment. Pitch moment M_y is a lateral moment about the y-axis while the resultant pitch moment $M_y > 0$ in the case that it more likely turn the tire about the y-axis and move forward. The pitch moment is also called rolling resistance torque. Yaw moment M_z is an upward moment about the z-axis. The resultant yaw moment $M_z > 0$ in the case that it is more likely to turn the tire about the z-axis. The yaw moment is so-called the aligning moment or self-aligning moment [2].

2.2.2 Tire Stiffness

It is clear-cut that at a certain value of tire normal deformation if the tire stiffness is available, the amount of applied normal load to the tire is predictable. This is in conformity with the tire elastic behavior, however, the stiffness characteristic determination is a significant step for understanding the tire behavior under different loading conditions. Based on Hooke's law, a principle of physics that states that the force F needed to extend or compress a spring (or flexible material) by some distance X is proportional to that distance, it can be approximated that:

$$F_x = k\Delta x \quad (2.17)$$

$$F_y = k\Delta y \quad (2.18)$$

$$F_z = k\Delta z \quad (2.19)$$

The coefficients k_x , k_y , and k_z represent the tire stiffness in the x, y and z directions, respectively and are also considered as longitudinal, lateral and normal stiffness characteristics.

Proof

Let's assume a linear displacement of tire under normal load being Δz , representing $z_2 - z_1$. Based on Taylor series one can develop the force about the static equilibrium (z_0) as following:

$$F_z(z_0 + \Delta z) = F_z(z_0) + \frac{\partial F_z}{\partial z} \Big|_{z=z_0} \Delta z + \frac{1}{2!} \frac{\partial^2 F_z}{\partial z^2} \Big|_{z=z_0} (\Delta z)^2 + \dots \quad (2.20)$$

Then it can be rewritten as:

$$F_z(z_0 + \Delta z) = F_z(z_0) + \Delta F_z \quad (2.21)$$

where ΔF_z represents the force variation due to tire deformation. ΔF_z can be presented as:

$$\Delta F_z = \frac{\partial F_z}{\partial z} \Big|_{z=z_0} \Delta z + \frac{1}{2} \frac{\partial^2 F_z}{\partial z^2} \Big|_{z=z_0} (\Delta z)^2 + \dots \quad (2.22)$$

If the displacement is negligible, the terms of Δz with higher degree of can be ignored. Therefore, it can be concluded that:

$$\Delta F_z = \frac{\partial F_z}{\partial z} \Big|_{z=z_0} \Delta z \quad (2.23)$$

$$\Delta F_z = k \Delta z \quad (2.24)$$

where k is a coefficient and is considered as stiffness coefficient as following:

$$k = \frac{\partial F_z}{\partial z} \Big|_{z=z_0} \quad (2.25)$$

It is generally known that the stiffness curve can be influenced by many parameters. The most effective one is the tire inflation pressure. Lateral and longitudinal force/deflection behavior is also determined experimentally by applying a force in the appropriate direction. The lateral and longitudinal forces are limited by the sliding force when the tire is vertically loaded.

It is appreciated from Fig. 2.10 that $k_x > k_z > k_y$ that the greatest and lowest tire stiffness are in longitudinal and lateral directions, respectively (Fig. 2.10). That is a greater force is needed to make a definite displacement in longitudinal and lateral directions. Vertically, longitudinally, and laterally tire deformations are also presented in Fig. 2.11 based on three forces of normal, longitudinal and lateral.

It is known that tires, being viscoelastic material, have the hysteresis effect that is the loading and unloading stiffness curves are not exactly the same. The area inside the loop in Fig. 2.12. represents the dissipated energy which is in accordance with a part of rolling resistance. Of course for off-road travelling condition, the residual

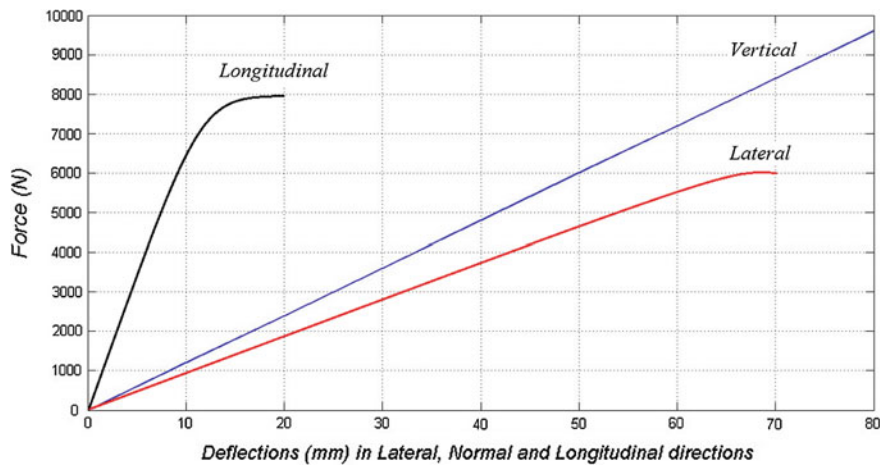


Fig. 2.10 Vertical, longitudinal, and lateral stiffness curves

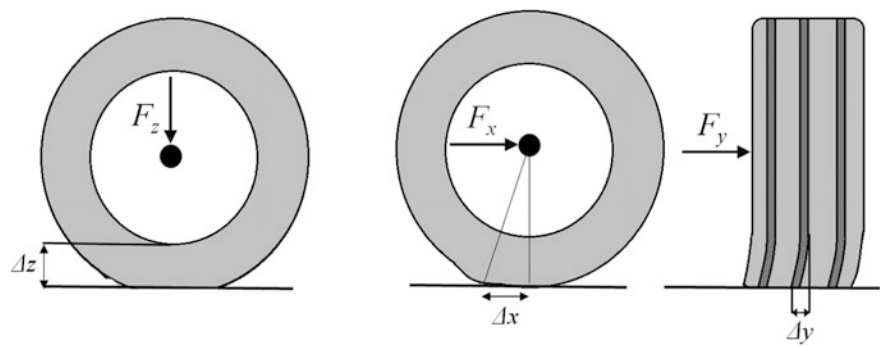
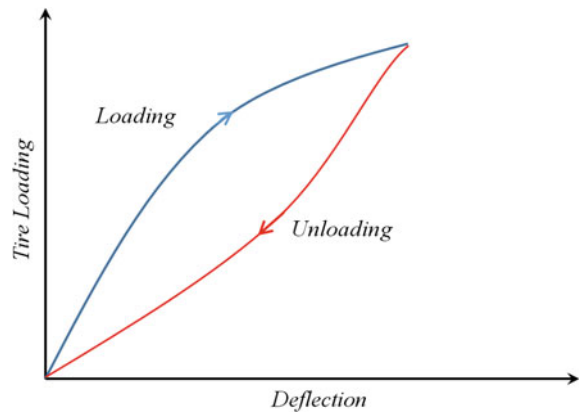


Fig. 2.11 Vertically, longitudinally, and laterally tire deformations

Fig. 2.12 Hysteresis loop in a vertically loading and unloading tire



section for rolling resistance is formed by soil sinkage. Tires are subject to cyclic loadings repeated cycles of deformation and recovery that the difference is energy loss released in heat form.

The hysteresis loop increases as a result of hysteresis damping forces that oppose tire deformation. For metallic structures it is clear that hysteresis forces are free from frequency, equal value to a portion of corresponding spring forces shifted in phase by 90° .

It should be pointed out that different experimental approaches have been designed for hyperelastic characterization. Mooney [3] made the first significant effort to establish the mathematical theory of large elastic deformations of rubber materials. The strain energy function was derived by Mooney based on the following:

$$U = C_1(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3) + C_2(\lambda_1^{-2} + \lambda_2^{-2} + \lambda_3^{-2} - 3) \quad (2.26)$$

where C_1 and C_2 are temperature-dependent material parameters, and λ_1 , λ_2 and λ_3 are the principal stretch ratios. Rivlin [4] considered the rubber material is homogeneous, isotropic and incompressible, and the strain energy function was derived as following:

$$U = \sum_{i+j=1}^{\infty} C_{ij}(I_1 - 3)^i(I_2 - 3)^j \quad (2.27)$$

where C_{ij} is material parameter, I_1 and I_2 are the first and second strain invariants of the Green's deformation tensor, respectively. If only two terms of the power series are taken, the strain energy function is reduced to

$$U = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \quad (2.28)$$

which is known as the Mooney-Rivlin equation. Yeoh [5] obtained the Rivlin's strain energy function by removing the dependence on the second invariant as I_2 has lower effect on the variations of strain energy function when compared to the first invariant I_1 . The function can be re-written as following:

$$U = \sum_{i=1}^3 C_{ij}(I_1 - 3)^i \quad (2.29)$$

Ogden developed strain energy model [6], and the strain energy function can be determined as following:

$$U \stackrel{\text{def}}{=} \sum_{i=1}^N \frac{2\mu_i}{\alpha_i^2} (\lambda_1^{-\alpha_i} + \lambda_2^{-\alpha_i} + \lambda_3^{-\alpha_i} - 3) \quad (2.30)$$

However, the nonlinear regression method for derivation of the Ogden strain-energy function, which is based on stress-strain data from one mode of deformation tests, is not able to predict behaviour in other deformation modes [7].

It is also important point to determine the dynamic tire characteristics. The dynamic stiffness of tires is one of the important factors influencing vehicle ride. To assess the influence of forward speed and obstacle geometric properties on dynamic stiffness of tires, a derivation method was proposed based on the transient dynamic responses to obtain the dynamic stiffness at resonance [8]. The peak of the mobility function was obtained by quantification of the dynamic stiffness at the natural frequency while the modulus of motion mobility $\dot{Y}/F$ can be found by the following term.

$$\left| \frac{\dot{Y}}{F} \right| = \frac{\omega}{\sqrt{(k_r - \omega^2 m)^2 + \omega^2 c^2}} \quad (2.31)$$

where k_r and c are the dynamic stiffness and damping ratio of the tire respectively, ω is natural frequency and m is the mass of the tire, and the peak of the modulus of motion mobility happens at:

$$\frac{d}{d\omega} \left| \frac{\dot{Y}}{F} \right| = \frac{d}{d\omega} \left(\frac{\omega}{\sqrt{(k_r - \omega^2 m)^2 + \omega^2 c^2}} \right) = 0 \quad (2.32)$$

The following term can be obtained as:

$$\frac{(k_r - \omega^2 m)(k_r + \omega^2 m)}{\sqrt{(k_r - \omega^2 m)^2 + \omega^2 c^2}((k_r - \omega^2 m)^2 + \omega^2 c^2)} = 0 \quad (2.33)$$

Therefore the dynamic stiffness of tire can be obtained as following:

$$k_r = \omega^2 m \quad (2.34)$$

One can conclude that for a tire with a constant mass, the dynamic stiffness is determined by the natural frequency. The natural frequency derived from the vertical spindle responses exhibit little sensitivity to the height of rectangular based obstacles that stands for the vertical dynamic stiffness has a negligible sensitivity to the height of obstacles. Furthermore, the natural frequency of spindle responses in longitudinal direction becomes lower with the increase of height of road obstacle which means that the dynamic stiffness in the longitudinal direction decreases [8].

2.2.3 Tire Footprint

The force applied on a tire in a contact patch that can be closely connected with tire stresses can be decomposed into a component vertical to the ground and a tangential component on the ground. It is the portion of a vehicle's tire that is in actual contact with ground and determines the share of the tire's tread that is in contact with the ground. The size and shape of the contact patch, as well as the pressure distribution within the contact patch are significant to the ride comfort, stability and handling parameters of the vehicle in motion.

The vertical stress applied to the tire in contact area is the z-oriented component of stress (σ_z) while the longitudinal and lateral shear stresses are represented by τ_x and τ_y . To consider the equilibrium theory in the steady-state loading condition, the following equations should be satisfied:

$$\int_A \tau_x(x, y) dA = 0, \quad \int_A \tau_y(x, y) dA = 0, \quad \int_A \sigma_z(x, y) dA = F_z \quad (2.35)$$

In order to achieve a reliable model of normal stress, it is essential to have a correct estimation of contact patch geometry shape and parameters. On a rigid surface, a better contact patch geometry may be better presented by an elliptic shape (Fig. 2.13):

$$\left(\frac{x}{a}\right)^k + \left(\frac{y}{b}\right)^k = 1, \quad k = 2n, n \in \mathbb{N} \quad (2.36)$$

where the number of power for the elliptic shape can be varied based on the tire type between 1 and 3. If the tire is a radial ply tire, and $n = 3$ is used, the following stress distribution function is used.

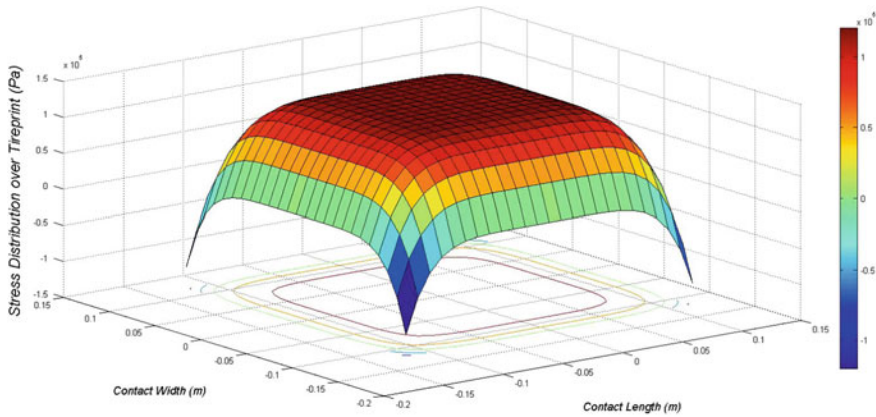


Fig. 2.13 Normal stress distribution with respect to contact length and contact width

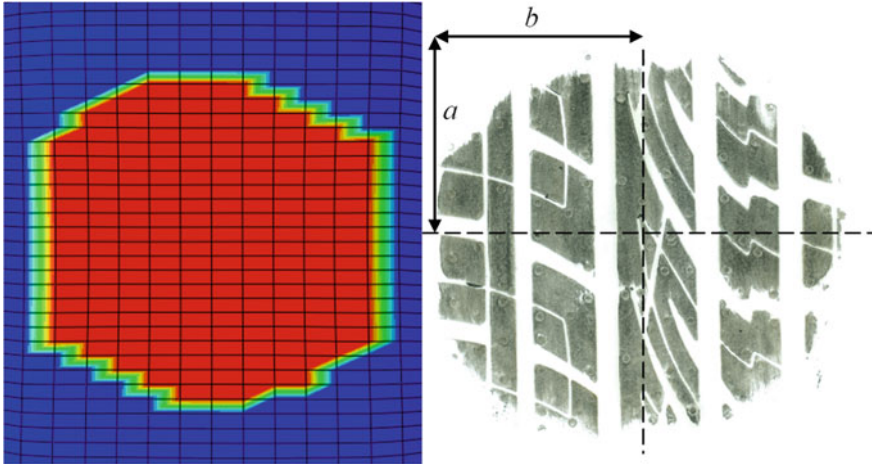


Fig. 2.14 A typical tire-ground contact area and its tire stress distribution [9]

The normal stress $\sigma_z(x, y)$ may be approximated by the function [2]:

$$\sigma_z(x, y) = \sigma_{z\max} \left(1 - \frac{x^6}{a^6} - \frac{y^6}{b^6} \right) \quad (2.37)$$

where a and b indicate the dimensions of the tire contact area (Fig. 2.14). In this manner, based on the equilibrium equation, the following function may be used:

$$F_z = \int_A \sigma_z(x, y) dA = \iint_A \sigma_{z\max} \left(1 - \frac{x^6}{a^6} - \frac{y^6}{b^6} \right) dx dy \quad (2.38)$$

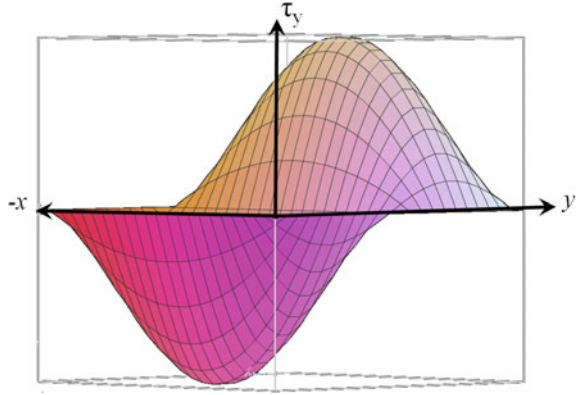
Based on equilibrium theory in the steady-state loading condition, the equations for tangential stresses should also be satisfied. The tangential stress, so-called shear stress, τ should be developed based on the contact patch area in both x and y directions.

The tangential stress on a tire is inward in x direction and outward in y direction. Hence, the tire tries to stretch the ground in the x -axis and compact the ground on the y -axis (Fig. 2.15).

It is also well known that the force distribution on the tireprint is not constant and is influenced by tire structure, load, inflation pressure, and environmental conditions. As available in the literature, the tire tangential stresses in x and y oriented directions can be presented as:

$$\tau_x(x, y) = -\tau_{x\max} \left(\frac{x^{2n+1}}{a^{2n+1}} \right) \sin^2 \left(\frac{x}{a} \pi \right) \cos \left(\frac{y}{2b} \pi \right) \quad n \in N \quad (2.39)$$

Fig. 2.15 Tangential tire stress distribution in x-direction and y-direction



$$\tau_y(x, y) = -\tau_{y\max} \left(\frac{x^{2n}}{a^{2n}} - 1 \right) \sin\left(\frac{y}{b}\pi\right) \quad n \in N \quad (2.40)$$

In this manner, the normal stresses for on road and soft soil (terrain) surfaces in the contact patch geometry are presented in Fig. 2.16. The difference between the trends of figures can be attributed to the soil profile sinkage (deformation) when compared with a rigid surface where the stress distribution in the contact area is more homogenous.

For a contact area of soil-tire interface, the periphery of the contact area may be better modeled by a super ellipse which, in an orthogonal coordinate system with center at the origin, is given as following.

$$\left| \frac{x}{a} \right|^n + \left| \frac{y}{b} \right|^n = 1 \quad (2.41)$$

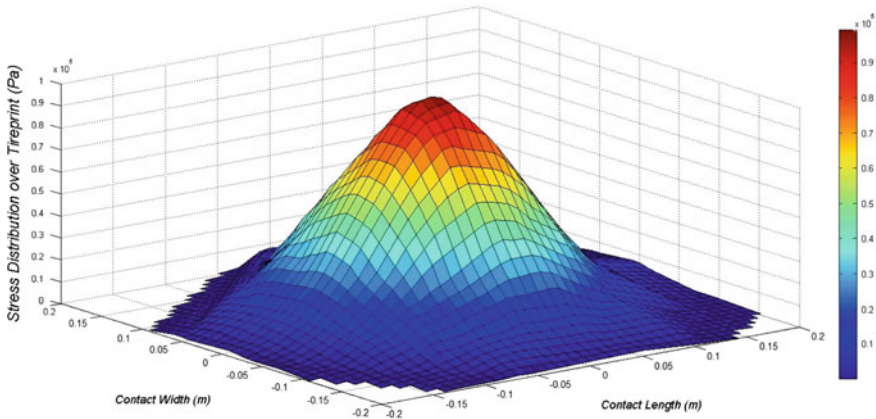
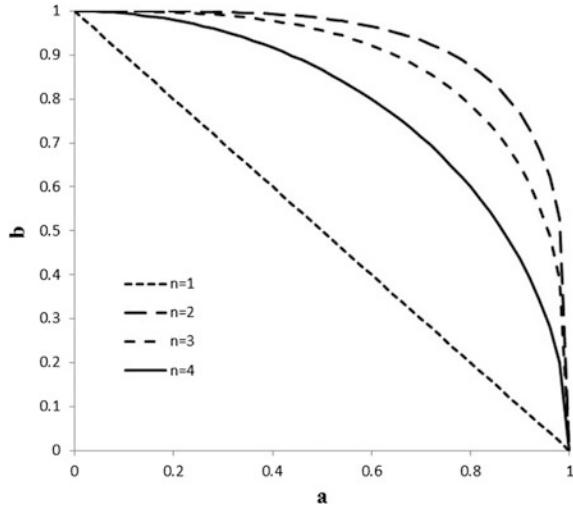


Fig. 2.16 Normal stresses for on soft soil (terrain)

Fig. 2.17 The shape of the curve in the first quadrant when $a = b = 1$ and $n = 1, 2, 3$ and 4 in order from the origin of coordinates



where the exponent n , is a positive real number that determines the shape, and parameters a and b determine the length of half the major axes and thus, the proportions of the surface.

Therefore, y is yielded as given by:

$$y = b \cdot \left(1 - \frac{x^n}{a^n}\right)^{\frac{1}{n}} \quad (2.42)$$

At a sample x range of between 0 and 0.6 and $a = b = 1$, the output y is schematically obtained as Fig. 2.17.

The aforementioned description was presented to shed light on some major principles of super ellipse shape and the effect of the exponent n . The area of one quadrant is given as [10]

$$A = b \int_0^a \left(1 - \frac{x^n}{a^n}\right)^{\frac{1}{n}} dx = kab \quad (2.43)$$

where k is a constant that is a function of n which can be found by numerical integration. As proposed by Keller [11], the parameter n was yielded as following.

$$n = 2.1(a \cdot b)^2 + 2 \quad (2.44)$$

where n is dimensionless and the width of the tire b , and length of contact a are in m. It is noteworthy that for $n = 2$, the curve is a pure ellipse whereas it grows towards a rectangle as $n \rightarrow \infty$.

Based on Frida Model developed by Per Schjønning et al. [12] the super elliptic assumption is again adopted:

$$\left|\frac{x}{a}\right|^n + \left|\frac{y}{b}\right|^n = 1 \quad (2.45)$$

Based on this model, Ω is proposed as following to represent the boundary and interior of the super ellipse.

$$\Omega = \{(x, y) | |x/a|^n + |y/b|^n \leq 1\} \quad (2.46)$$

The distribution of the vertical stress in the contact area may be modelled by the following Eq. 2.47, which is a combination of the equations suggested by Keller [11] except that we have normalised the equation to unit contact area width and to unit stress [12],

$$\begin{aligned} \sigma(x, y) &= F_{wheel} C(\alpha, \beta, a, b, n) f(x, y) g(x, y) \\ &\text{for } (x, y) \in \Omega \text{ and } 0 \text{ otherwise,} \end{aligned} \quad (2.47)$$

$$\begin{aligned} f(x, y) &= \left\{ 1 - \left| \frac{x}{l_x(y)} \right|^\alpha \right\} \\ g(x, y) &= \left\{ \left(1 - \left| \frac{y}{W_y(x)} \right| \right) (1/g_{\max}) \exp \left(-\beta \left(1 - \left| \frac{y}{W_y(x)} \right| \right) \right) \right\} \end{aligned} \quad (2.48)$$

where $g_{\max}$ is the maximum value of g in the range $(0 < y < W_y(x))$ expressed in terms of β :

$$\begin{aligned} \beta \leq 1: g_{\max} &= \exp(-\beta), \\ \beta > 1: g_{\max} &= \exp(-1)/\beta \end{aligned} \quad (2.49)$$

F_{wheel} is the wheel load in kN, $C(\alpha, \beta, a, b, n)$ is a function of the parameters an integration constant ensuring that when integrating $s(x, y)$ over the contact area Ω the total load is F_{wheel} .

Additionally, $l_x(y)$ is the half length of footprint at y in the x -direction, and $W_y(x)$ the half width at x in the y -direction. The f function determines the form of stress distribution in the driving direction, that is the relative stress as a function of relative contact area half-length, while the g function shows the form of stress distribution in the direction vertical to the driving direction (across the wheel). Figure 2.18 represents the f function and exhibits the stress distribution in the driving direction for some certain amounts of α . Great values of a show a satisfactory ability to distribute the stress in the driving direction, while the stress peak in the middle of the contact area (below the axle) gets increasingly steep with decreasing α [12].

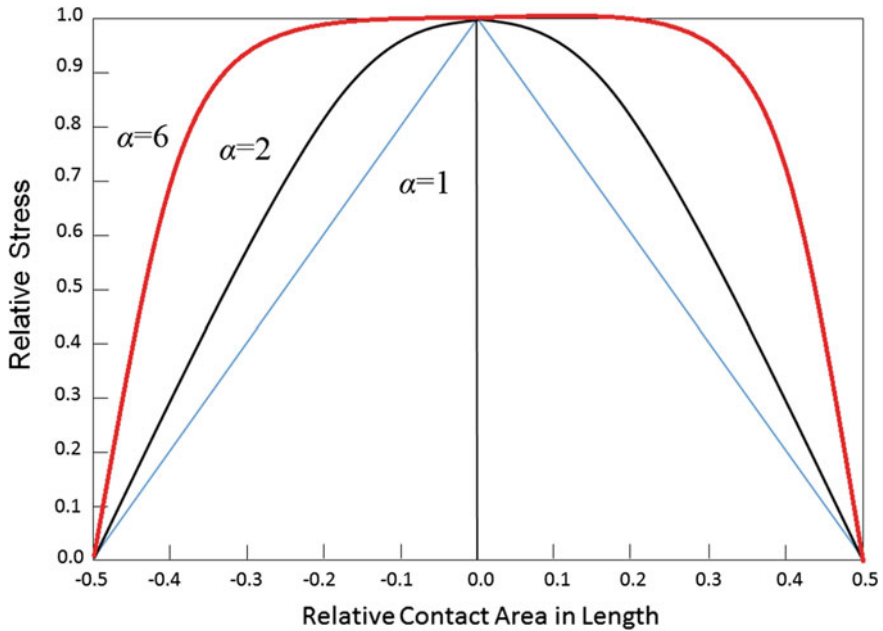


Fig. 2.18 Relative stress versus relative contact area length

The condition gets more complex for stress distribution across the wheel while the stress will peak in the center of the tire for $\beta \leq 1$ and the peak will be increased while b decreases (Fig. 2.19). For $\beta > 1$ the stress shows two peaks that converge to the edges of tire for increasing levels of β . Consequently, the model is able to describe parabolic-like as well as U-shaped stress distributions which are required to deal with the monitored stresses [12].

2.2.4 Tire Road Modeling

In order to limit the motion of the two bodies that are in contact, a normal contact condition is created (Fig. 2.20). The constraints V^A of body A and V^B of body B of the system are created so that the bodies in contact cannot penetrate in each other while are in motion [13]. In the tire and ground contact theoretical model, C^A and C^B represent the tire and road respectively.

Let's consider that ${}^t x_M^A$ denotes the coordinate of the particle M on the surface ${}^t S^A$ of body A at time t , the distance l between the particle and the nearest particle $N({}^t x^B)$ on the surface ${}^t S^B$ Fig. 2.19 can be presented as following:

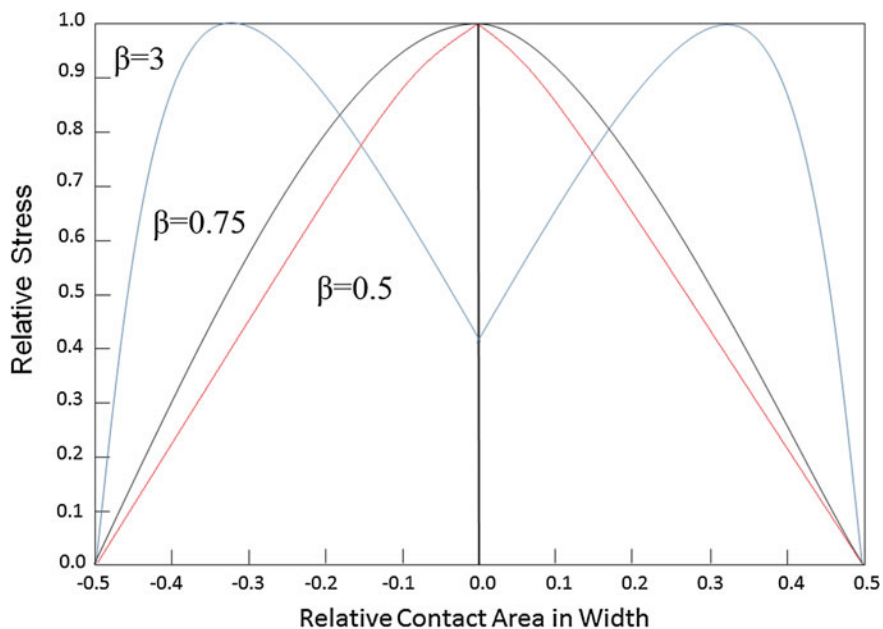


Fig. 2.19 Relative stress versus relative contact area width

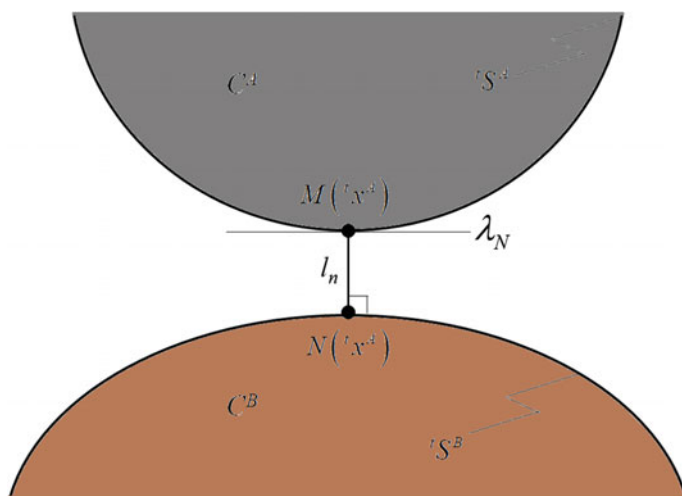


Fig. 2.20 Schematic representation of two body's contact

$${}^t l = l({}^t x_M^A, t) = |{}^t x_M^A - {}^t x_N^B| = \min |{}^t x_M^A - {}^t x^B| \quad (2.50)$$

where ${}^t x^B$ denotes the coordinate of any desired particle on surface ${}^t S^B$ and orientation of l has to be equal to ${}^t \mathbf{n}^B$ that represents the unit vector normal to the contact plane. Therefore, the vector ${}^t l$ can be expressed as:

$${}^t \mathbf{l} = \mathbf{l}({}^t x_M^A, t) = ({}^t x_M^A - {}^t x_N^B) \cdot {}^t \mathbf{n}_N^B \quad (2.51)$$

In order to meet the requirement of impenetrability for the two bodies (the tire and the road), we use the following constraint to define any particle P on the surface ${}^t S^B$

$${}^t l_n = \mathbf{l}({}^t x_M^A, t) = ({}^t x_M^A - {}^t x_N^B) \cdot {}^t \mathbf{n}_N^B \geq 0 \quad (2.52)$$

in which ${}^t l_n > 0$ means a gap exists between the particle M and the surface ${}^t S^B$, while ${}^t l_n = 0$ means the particle M has a contact with ${}^t S^B$. As the above function is effective for any particle on surface ${}^t S^A$ and ${}^t S^B$, the function can be described by

$${}^t l_n = l({}^t x^A, t) = ({}^t x^A - {}^t x^B) \cdot {}^t \mathbf{n}^B \geq 0 \quad (2.53)$$

In addition, since the equivalent normal contact force is provided by the pressure, the condition of normal contact force λ_N (Fig. 2.20) is denoted by:

$$\lambda_N \geq 0 \quad (2.54)$$

In the tangential direction of rolling contact analysis, Coulomb model of friction is adopted to define the tangential contact conditions. In engineering analysis, coulomb model of friction is widely used because of its characteristics of simplicity and applicability [13]. The definition in the Coulomb friction model is that slip occurs when the tangential contact force (frictional resistance) $\tau_{eq} = \sqrt{\tau_1^2 + \tau_2^2}$ equals to the critical stress $\tau_{crit} = \mu \lambda_N \geq 0$, which means that the friction force cannot be allowed to exceed the critical stress, which is

$$\tau_{eq} = \sqrt{\tau_1^2 + \tau_2^2} \leq \mu \lambda_N \quad (2.55)$$

in which μ is the coefficient of friction, τ_1 and τ_2 are shear stresses in t_1 direction and t_2 direction respectively, and λ_N represents the normal contact force. On the other hand, no relative motion between the tire and the rigid road surface occurs when $\tau_{eq} < \tau_{crit}$. The stiff viscous behaviour is adopted to approximately simulate the non-relative-motion condition,

$$\tau_\alpha = \kappa \dot{\gamma}_\alpha \quad (\alpha = 1, 2) \quad (2.56)$$

in which τ_α represents the shear stress in the tangential direction, $\dot{\gamma}_\alpha$ is the tangential slip rate in the tire/road contact plane, $\kappa = \mu\lambda_N/2\Delta\omega R$ is the stick viscosity, ω is the rotation angular velocity and R is the rolling radius of the tire, and Δ is the slip tolerance [7].

2.2.5 Tire Rolling Resistance

One of important tire-road force products is the rolling resistance that is basically the parasitic energy dissipated by tire and the main source for loss of energy is the constant deformations of the tire while traversing and the soil beneath wheel. The phenomenon is quite complicated, and almost all operating conditions can influence the final product. In off-road vehicles during wheel motion the tire interacts with soil (depending on contact area) and a phenomenon occur known as rolling resistance wherein mechanical energy is changed to heat which is the consequence of rubber and soil deformation. The converted energy equals with the required energy to deform the rubber (temporary or elastic deformation) or the soil beneath the tire (constant or plastic deformation) in motion. Rolling resistance performs as a resistive force applied to wheel against the direction of traversing. In this manner, the energy loss owing to the irregular road is denoted by the work done by the longitudinal force opposing motion:

$$E_d = \sum |F_x(i)| \cdot U_x(i) \quad (2.57)$$

In the condition that the tire rolls at a constant amount of velocity v on the uneven road and the tire lose the free rolling condition, the state of leads to a negative longitudinal force, while the state of $\omega \cdot r > v$ leads to a positive longitudinal force. However, in any form of being negative or positive longitudinal forces, the state of interrupting of tire from free rolling mode occur that are an indication of energy loss in the direction of motion and are considered rolling resistance.

Rolling resistance can be determined as the dissipated energy in unit of traveled distance that can be denoted as following:

$$F_R = D_E/L = D_E/\sum U_x(i) \quad (2.58)$$

where F_R is rolling resistance force, L is length of motion, and D_E is the dissipated energy. As an important factor in vehicle fuel consumption, the rolling resistance has been paid more and more attention by researchers, since the energy dissipation due to tire rolling resistance is a major component of vehicle energy loss. Rolling resistance is the energy that is lost when the tire is rolling and the main reason for loss of energy is the constant deformation of the tire. Rolling resistance is principally created by non-elastic effects; i.e. not all the energy needed for deformation

(or movement) of the wheel is recovered when the pressure is removed. There are two forms of energy loss that are hysteresis losses, and permanent (plastic) deformation of terrain. Another cause of rolling resistance is regarded as the slip-page between the wheel and the surface, which dissipates energy. Hence, the condition of road surface and terrain properties plays a significant role in determination of the total rolling resistance force.

In the investigations regarding vehicle ride performance, the road unevenness was considered as sinusoidal harmonic wavy roads, triangular shapes and step functions. However, this easy shape of road profiles may not serve as functional foundation for the investigations about vehicle dynamic performance. To put forward a more realistic type of road profile, a random function may provide a better analysis platform to obtained accurate results about vehicle dynamics products. In the condition that the road profile is identified as a random function, it can be characterized by a power spectral density function while it has been found that the road unevenness related to the power spectral density for the road profiles can be approximated by [14]

$$S_x(\Omega) = C_{sp}\Omega^{-N_b} \quad (2.59)$$

where Ω is the spatial frequency, $S_x(\Omega)$ ($m^3/cycle$) is the power spectral density function of the elevation of the road profile, C_{sp} and N_b are dimensionless road profile constants to determine the road unevenness. In order to carry out time domain conversion for road profiles, the power spectral density is expressed in terms of time-frequency, which is given by

$$S_x(f) = C_{sp}v^{N_b-1}f^{-N_b} \quad (2.60)$$

where v is the travelling velocity of the vehicle, f is the time frequency, and the unit of the power spectral density function $S_x(f)$ is m^2/Hz . Different values of N and C_{sp} for different PSD functions for various road surfaces is provided in Table 2.1.

The objective of Discrete Fourier Transform (DFT) is to build the relationship between the time sequence $x(n)$ and the frequency sequence $X(k)$ [15], which can be given by

$$\text{DFT } X(k) = DFT[x(n)] = \sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}nk} \quad k = 0, 1, 2, \dots, N-1 \quad (2.61)$$

Table 2.1 Values of N and C_{sp} for different PSD functions for various road surfaces [14]

Road surfaces	N_b	C_{sp}
Smooth runway	3.8	4.3×10^{-11}
Rough runway	2.1	8.1×10^{-6}
Smooth highway	2.1	4.8×10^{-7}
Highway with gravel	2.1	4.4×10^{-6}

$$\text{IDFT } x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{\frac{j2\pi nk}{N}} \quad k = 0, 1, 2, \dots, N-1 \quad (2.62)$$

in which N represents the length of the two sequences.

Generally, the definition of power spectral density for road file is limited in $[0, +\infty)$ in the DFT process, the one-sided PSD $S_x(f)$ needs to be transformed to the two-sided PSD $G_x(f)$. Based on the characteristics of real even function $G_x(f)$, the power spectral density, the relationship between $S_x(f)$ and $G_x(f)$ can be given by [7]:

$$S_x(f) = \begin{cases} 2G_x(f) & f \geq 0 \\ 0 & f < 0 \end{cases} \quad (2.63)$$

and according to the definition of power spectral density, $S_x(f)$ can be expressed by

$$S_x(f) = \lim_{T \rightarrow \infty} \frac{2}{T} |X(f)|^2 = \lim_{T \rightarrow \infty} \frac{2}{T} \left| \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt \right|^2 \quad (2.64)$$

in which t is the simulation time, T is the period, $x(t)$ is the time sequence, and $X(f)$ is the corresponding sequence in terms of frequency. When $t > 0$ and the period T is limited, the Eq. (2.64) can be written as [7]

$$S_x(f) = \frac{2}{T} |X(f)|^2 = \frac{2}{T} \left| \int_0^T x(t) e^{-j2\pi ft} dt \right|^2 \quad (2.65)$$

By applying DFT, the power spectral density can be presented as

$$S_x(f_k) = \frac{2}{N\Delta t} \left| \sum_{n=0}^{N-1} x(n) e^{-j2\pi f_k n \Delta t} \Delta t \right|^2 = \frac{2\Delta t}{N} |X(k)|^2 = \frac{2T}{N^2} |X(k)|^2 = \frac{2}{\Delta f N^2} |X(k)|^2 \quad (2.66)$$

where Δt is the time increment, $N = T/\Delta t$, $\Delta f = 1/N\Delta t$ and $f_k = k\Delta f$. Hence, based on the relationship between power spectral density and the time sequence, the frequency sequence can be achieved as

$$\begin{aligned} |X(k)| &= |DFT[x(n)]| = N\sqrt{S_x(f_k)\Delta f/2} \\ &= N\sqrt{S_x(f = k\Delta f)\Delta f/2} \quad (k = 0, 1, \dots, N_r - 1) \end{aligned} \quad (2.67)$$

The time sequence can be obtained by taking the inverse discrete Fourier transform function on the frequency sequence. Based on the above method, the road surface elevations for different kinds of road surfaces are obtained.

As pointed out, the calculation length is used for the derivation of effective rolling resistance. However, the starting point for rolling resistance calculation is the beginning of the uneven road, which consists of the transition length plus the calculation length. As described in Eq. 6.6, for a tire rolling over a distance L of the uneven road, the rolling resistance for this part of the uneven road is calculated in terms of [7]

$$F_R|_{0 \rightarrow L} = \frac{D_E}{L} = \frac{D_E}{\sum U_x(i)} \quad (2.68)$$

in which the $F_R|_{0 \rightarrow L}$ represents the average rolling resistance for the tire traversing a distance L of the uneven road.

The effective rolling resistance is obtained by extracting the rolling resistance in the distance covered in three revolutions of the tire and calculating the average value of the rolling resistance for this distance, the function of the effective rolling resistance $F_R|_e$ is defined by [7]

$$F_R|_e = \frac{F_{R1} + F_{R2} + F_{R3}}{3} \quad (2.69)$$

where F_{Ri} , $i = 1, 2, 3$ represent the rolling resistance for revolution 1, revolution 2 and revolution 3, respectively.

2.2.6 Acceleration and Deceleration Characteristics Effects of Tire

The longitudinally oriented forces acting on a tire are functional on traction and braking performance and acceleration/deceleration characteristics. The model can be developed to analyze the braking/traction forces if the tire rubber is well fitted with wheel rim. The tread rubber consists of numerous independent springs around the tire circumference which are so-called as the brush model. The tire model provided is adopted from Bernard et al. [16] and Abe studies [17].

During time Δt , the displacement x is made according to:

$$x = u\Delta t \quad (2.70)$$

Given that the linear velocity of tire equals with the angular velocity multiplied by tire radius ($u = R_0\omega$), the x -coordinate of point P' from point O' is (Fig. 2.21):

$$x' = R_0\omega\Delta t \quad (2.71)$$

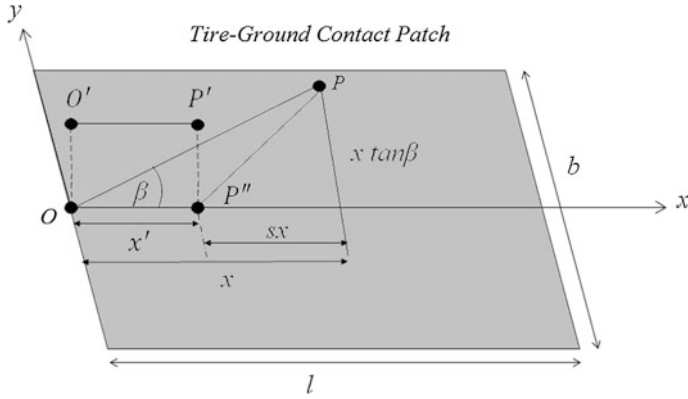


Fig. 2.21 Tire deformation in contact plane

The difference between the two before terms (x and x') which represent the tread rubber deformation can be denoted as:

$$\Delta x = x - x' = \frac{u - R_0 \omega}{u} u \Delta t \quad (2.72)$$

If the tire longitudinal slip is considered as:

$$s = \frac{u - R_0 \omega}{u} \quad (2.73)$$

Then

$$\Delta x = xs \quad (2.74)$$

The distance in y-direction from O to P can be found by:

$$y = x \tan \beta = \frac{\tan \beta}{1 - s} x' \quad (2.75)$$

Since there is no displacement of point P' in the y-direction, the above is the deformation of tread rubber in the y-direction. Therefore, the forces per unit length and width, acting on point P, in the x-direction and y-direction, respectively, are σ_x and σ_y

$$\sigma_x = -K_x(x - x') = -K_x \frac{s}{1 - s} x' \quad (2.76)$$

$$\sigma_y = -K_y y = -K_y \frac{\tan \beta}{1 - s} x' \quad (2.77)$$

The sign of these forces is taken as opposite to the axes direction. Moreover, the resultant force magnitude is,

$$\sigma = \left(\sigma_x^2 + \sigma_y^2 \right)^{\frac{1}{2}} = \left(K_x^2 s^2 + K_y^2 \tan^2 \beta \right)^{\frac{1}{2}} \frac{x'}{1-s} \quad (2.78)$$

where K_x and K_y are the longitudinal and lateral tread rubber stiffnesses per unit width and unit length. When the tire longitudinal slip ratio and side slip angle are produced, tire deformation occurs. As a result, a distribution of the contact surface force, proportional to x' , is created at the tire contact surface. Assuming a tire pressure distribution that is the same as in Sect. 2.2.3,

$$p = \frac{6F_z x'}{bl} \left(1 - \frac{x'}{l} \right) \quad (2.79)$$

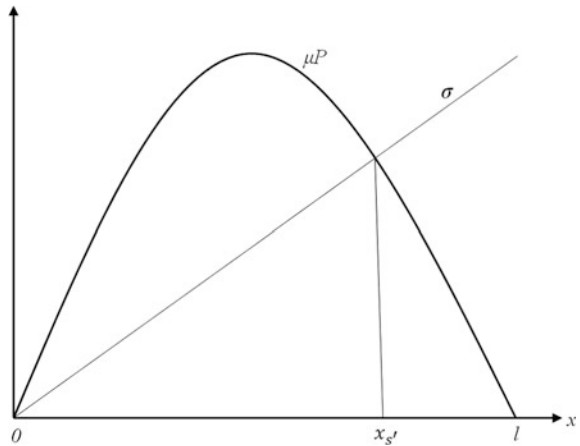
the tire contact surface force, as given by Eq. (2.78), is denoted by $0 \leq x' \leq x'_s$ in the adhesive region, and $x \geq x''$ in the slip region where the tire contact surface force is given by μp .

In the adhesive region, the forces acting at the contact surface, in x- and y-directions are σ_x and σ_y ; respectively. In the slip region, the forces are $\mu p \cos \theta$ and $\mu p \sin \theta$. Here, θ determines the direction of the tire slip. Force distributions in contact plane is provided in Fig. 2.22.

By replacing $\sigma = \mu p$ to find x'_s and introducing a dimensionless variable can be provided as:

$$\xi_s = \frac{x'_s}{l} = 1 - \frac{K_s}{3\mu F_z} \frac{\lambda}{1-s} \quad (2.80)$$

Fig. 2.22 Force distributions in contact plane`



where

$$\lambda = \left(s^2 + \left(\frac{K_\beta}{K_s} \right)^2 \tan^2 \beta \right)^{\frac{1}{2}} \quad (2.81)$$

$$K_s = \frac{bl^2}{2} K_x, \quad K_\beta = \frac{bl^2}{2} K_y \quad (2.82)$$

From the above, the overall forces acting on the whole tire contact surface, in x- and y-directions, are expressed as follows.

When $\xi_s > 0$, for a contact surface composed of adhesive and slip regions:

$$F_x = b \left(\int_0^{x'_s} \sigma_x dx' + \int_{x'_s}^l -\mu p \cos \theta dx' \right) \quad (2.83)$$

$$F_y = b \left(\int_0^{x'_s} \sigma_y dx' + \int_{x'_s}^l -\mu p \sin \theta dx' \right) \quad (2.84)$$

When $\xi_s > 0$, for a contact surface composed of slip regions only:

$$F_x = b \int_0^l -\mu p \cos \theta dx' \quad (2.85)$$

$$F_y = b \int_0^l -\mu p \sin \theta dx' \quad (2.86)$$

By replacing Eqs. (2.78)–(2.79) in Eqs. (2.85)–(2.86) denotes F_x , F_y in as following:

If

$$\xi_s = 1 - \frac{K_s}{3\mu F_z} \frac{\lambda}{1-s} > 0 \quad (2.87)$$

Then,

$$F_x = -\frac{K_s s}{1-s} \xi_s^2 - 6\mu F_z \cos \theta \left(\frac{1}{6} - \frac{1}{2} \xi_s^2 + \frac{1}{3} \xi_s^3 \right) \quad (2.88)$$

$$F_y = -\frac{K_\beta \tan \beta}{1-s} \xi_s^2 - 6\mu F_z \sin \theta \left(\frac{1}{6} - \frac{1}{2} \xi_s^2 + \frac{1}{3} \xi_s^3 \right) \quad (2.89)$$

If,

$$\xi_s = 1 - \frac{K_s}{3\mu F_z} \frac{\lambda}{1-s} < 0 \quad (2.90)$$

then

$$F_x = -\mu F_z \cos \theta \quad (2.91)$$

$$F_y = -\mu F_z \sin \theta \quad (2.92)$$

The direction of the slip force, θ ; is approximated by the slip direction at the slip start point.

$$\tan \theta = \frac{K_y \frac{\tan \beta}{1-s} x'}{K_x \frac{s}{1-s} x'} = \frac{K_\beta \tan \beta}{K_s s} \quad (2.93)$$

Therefore,

$$\cos \theta = \frac{s}{\lambda} \quad (2.94)$$

$$\sin \theta = \frac{K_\beta \tan \beta}{K_s \lambda} \quad (2.95)$$

It can be stated that

$$\begin{cases} K_s = 0 & \text{at } s \rightarrow 0 & \text{at } \beta = 0 \\ K_\beta = 0 & \text{at } \beta \rightarrow 0 & \text{at } s = 0 \end{cases} \quad (2.96)$$

In this condition, K_s represents the total longitudinal force per unit longitudinal slip ratio and K_β represents the total lateral force per unit side-slip angle.

F_z , slip velocity, V_s , and friction coefficient μ are closely interconnected and V_s is defined as following [17]:

$$V_s = \left((u - R_0 \omega)^2 + (\mu^2 \tan^2 \beta) \right)^{\frac{1}{2}} = u(s^2 + \tan^2 \beta)^{\frac{1}{2}} \quad (2.97)$$

It can be thus inferred that the longitudinal slip ratio, slip angle, tire load and tire traveling speed affect tire longitudinal and lateral forces.

In the case of acceleration, the slip ratio is denoted by:

$$s = \frac{u - R_0\omega}{R_0\omega} \quad (2.98)$$

Therefore, the x-oriented and y-oriented stresses can be presented as following:

$$\begin{aligned} \sigma_x &= -K_x s x' \\ \sigma_y &= -K_y (1 + s) \tan \beta x' \end{aligned} \quad (2.99)$$

Since

$$\sigma = \left(\sigma_x^2 + \sigma_y^2 \right)^{\frac{1}{2}} \quad (2.100)$$

Therefore;

$$\sigma = \left(K_x^2 s^2 + K_y^2 (1 + s)^2 \tan^2 \beta x' \right)^{\frac{1}{2}} \quad (2.101)$$

According to the theory for braking condition regarding the point the slip changes from the adhesive region to the slip region, the following can be developed:

$$\xi_s = 1 - \frac{K_s}{3\mu F_z} \lambda \quad (2.102)$$

where

$$\lambda = \left(s^2 + \left(\frac{K_\beta}{K_s} \right)^2 (1 + s^2) \tan^2 \beta \right)^{\frac{1}{2}} \quad (2.103)$$

There are two conditions when $\xi_s > 0$ and $\xi_s < 0$.

If $\xi_s > 0$, then:

$$F_x = -K_s s \xi_s^2 + 6\mu F_z \cos \theta \left(\frac{1}{6} - \frac{1}{2} \xi_s^2 + \frac{1}{3} \xi_s^3 \right) \quad (2.104)$$

$$F_y = -K_\beta (1 + s) \tan \beta \xi_s^2 - 6\mu F_z \sin \theta \left(\frac{1}{6} - \frac{1}{2} \xi_s^2 + \frac{1}{3} \xi_s^3 \right) \quad (2.105)$$

If $\xi_s < 0$, then:

$$\begin{aligned} F_x &= -\mu F_z \cos \theta \\ F_y &= -\mu F_z \sin \theta \end{aligned} \quad (2.106)$$

where,

$$\tan \theta = \frac{K_y(1+s) \tan \beta x'}{K_x s x'} = \frac{K_\beta \tan \beta(1+s)}{K_s s} \quad (2.107)$$

$$\cos \theta = \frac{s}{\lambda} \quad (2.108)$$

$$\sin \theta = \frac{K_\beta \tan \beta(1+s)}{K_s \lambda} \quad (2.109)$$

and the slip velocity is obtained at:

$$V_s = u \left(\frac{s^2}{(1+s)^2} + \tan^2 \beta \right)^{\frac{1}{2}} \quad (2.110)$$

It is finally concluded that from the theoretical analysis that braking and acceleration affect the tire cornering characteristics.

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