Chapter 2
Background

The chapter begins with a description of MIMO system under consideration and introduces the concepts of MIMO detection as well as all the notations used in the book. A brief description of the fundamental algorithmic choices for MIMO detection is also addressed in the subsequent parts of the chapter.

2.1 MIMO System Model

Let us consider a MIMO system with N_{R} transmit antenna and N_{R} receiving antenna. In this book, N_{R} is considered to be equal to or greater than N_{T}. At time n, a complex vector, $s^{\mathrm{c}}(n) = \left[s_1(n), s_2(n), \ldots s_{N_{\mathrm{T}}}(n)\right]^T$ is transmitted through N_{T} parallel streams. Each element $s_i(n)$ is taken from a complex constellation, $\mathcal{O}$ such as rectangular quadrature amplitude modulation (QAM) which consists of $M = |\mathcal{O}| = 2^{M_{\mathrm{c}}}$ distinct points. It means that every M_{c} consecutive bit is mapped to one complex constellation point. The transmission rate of the respective MIMO in spatial multiplexing (SM) mode is equal to $r = N_{\mathrm{T}} \log_2 M = N_{\mathrm{T}} M_{\mathrm{c}}$ bits per channel. The signal vector, s^{c} is normalized before transmission so that the average transmitted power is one, i.e., $E\left\{\|s\|^2\right\} = 1$. Hence, the MIMO system can be presented as:

$$y^{\mathrm{c}} = H^{\mathrm{c}} s^{\mathrm{c}} + n^{\mathrm{c}}, \tag{2.1}$$

where $y^{\mathrm{c}} = \left[y_1, y_2, \ldots y_{N_{\mathrm{R}}}\right]^T$ is the N_{R} dimensional complex-received symbol vector transmitted, H^{c} is $N_{\mathrm{R}} \times N_{\mathrm{T}}$ dimensional complex channel matrix. H^{c} denotes the channel gain between each transmit and receive antenna. Noise vector, $n^{\mathrm{c}} = \left[n_1, n_2, \ldots n_{N_{\mathrm{R}}}\right]^T$ is a N_{R} dimensional circularly symmetric complex zero-mean Gaussian noise vector with variance, σ^2. The signal to noise ratio (SNR) is defined as the ratio between the total normalized transmitted power to the variance of thermal noise. Hence, $\mathrm{SNR} = 1/\sigma^2$. A MIMO system model can be shown as (Fig. 2.1):

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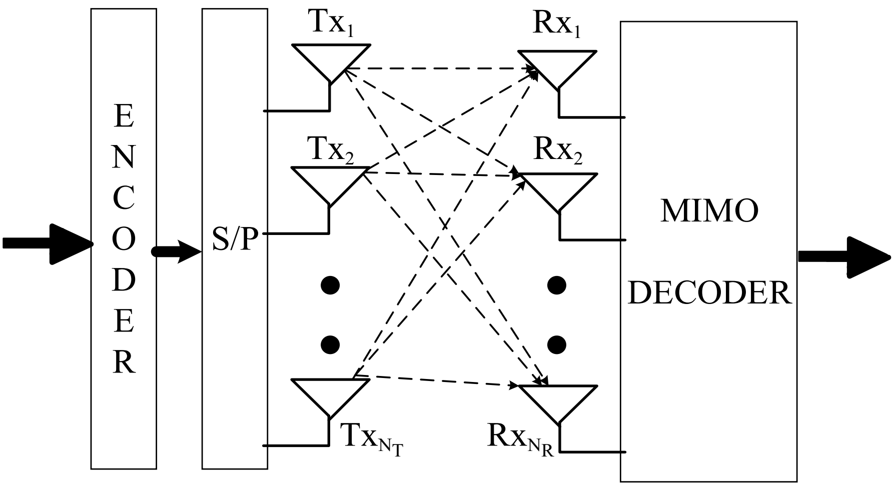

Fig. 2.1 A MIMO system model

The main objective of MIMO detector is to obtain the best possible estimate of the transmitted vector, s^c from the Euclidean distance, i.e.,

$$\hat{s}^c = \arg_{s^c \in \mathcal{O}^{N_T}} \min \| y^c - H^c s^c \|^2 . \tag{2.2}$$

Here, $\hat{s}^c$ is estimated as complex vector and ||.|| denotes the 2-norm. The channel estimator at the receiver end provides the estimate of current channel status based on previously known transmitted pilot symbols. However, we have considered a perfectly known channel in this book. The corresponding real signal mode following [20, 21] is:

$$\begin{bmatrix} \Re[y^c] \\ \Im[y^c] \end{bmatrix} = \begin{bmatrix} \Re[H^c] & -\Im[H^c] \\ \Im[H^c] & \Re[H^c] \end{bmatrix} \begin{bmatrix} \Re[s^c] \\ \Im[s^c] \end{bmatrix} + \begin{bmatrix} \Re[n^c] \\ \Im[n^c] \end{bmatrix},$$

$$y = Hs + n, \tag{2.3}$$

where $s = \left[s_1, s_2, \ldots s_{2N_T} \right]^T$, $y = \left[y_1, y_2, \ldots y_{2N_R} \right]^T$ and $n = \left[n_1, n_2, \ldots n_{2N_R} \right]^T$. The real and imaginary parts of a complex number are denoted by R(·) and I(·), respectively. ML detector solves for the transmitted signal by calculating:

$$\hat{s} = \arg_{s \in S^{2N_T}} \min \| y - Hs \|^2 . \tag{2.4}$$

Here ||·|| denotes the 2-norm and $S^{2N_T} = |\mathcal{O}|^{N_T}$ which means that a complex $N_R \times N_T$ MIMO system can be modeled as a real $2N_R \times 2N_T$ MIMO system. S is the set of all possible real entries in the constellation for in-phase and quadrature parts as follows:

$$s_i \in S = \left\{ \frac{\left(-\sqrt{M}+1\right)}{E_s}, \ldots, \frac{-1}{E_s}, \frac{+1}{E_s}, \ldots, \frac{\left(\sqrt{M}-1\right)}{E_s} \right\}, \tag{2.5}$$

where $E_s = 2(M-1)/3$ is the average symbol energy for an M-QAM constellation.

2.2 MIMO Detection Schemes

As aforementioned, the objective of MIMO detector is to resolve the transmitted vector from the received signal. There are various algorithms proposed so far in order to perform this task trading off between complexity and performance. Generally, there are two classes of MIMO detectors: hard decision-based and soft-decision-based detector. For hard decision, data symbols are decided based on the confidence of the detection with no extra estimation or information. Hence, it is useful for uncoded transmission. A soft-decision-based detector calculates the log likelihood ratio (LLR) of each bit using error correction coding scheme (ECE) and performs the bit correction based on the estimation. Hence, a soft information is being exchanged between detector and decoding modules required by both iterative detection and decoding scheme. This kind of detector is called soft input soft output (SISO) detector, which is suitable for subsequent iterative decoding [13, 14]. In this book, we will focus on both hard and soft decision-based decoder.

As shown in Fig. 2.2, the MIMO detection scheme can be classified into three groups based on their relative detection accuracy: optimal, suboptimal, and near-optimal methods. All of these schemes lead to specific approaches of MIMO detection trading off between BER performance and complexity. The focus of this book is the K-Best decoder, highlighted with a gray box in Fig. 2.2.

2.2.1 Optimal MIMO Detection

The most popular optimal MIMO detector is Maximum-Likelihood (ML) detector achieving the lowest BER performance. With the presence of additive white Gaussian noise (AWGN), ML detector searches for all the possible lattice points, s in the constellation $\mathcal{O}$ and reaches closest to the received point, y in the lattice. Hence, if the size of the scalar complex constellation transmitted from each antenna is M, this scheme needs to search over M^{N_T} vectors, where N_T is the

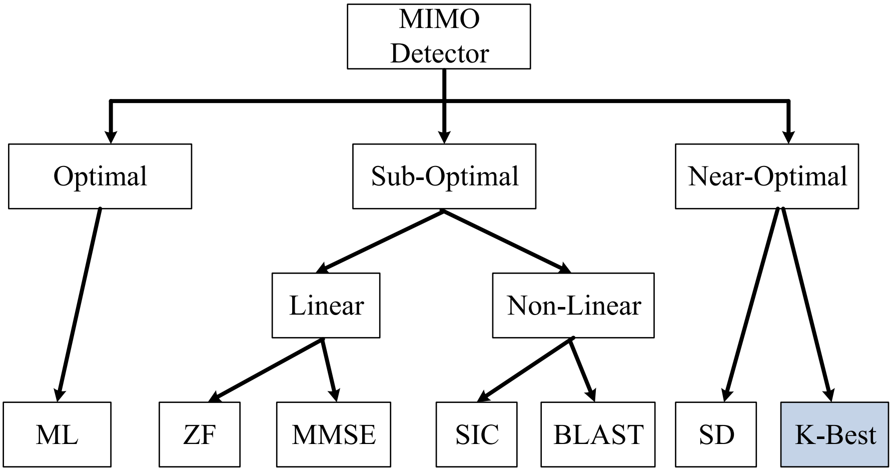

Fig. 2.2 The taxonomy of MIMO detection schemes

number of transmit antenna. Therefore, the complexity of ML detector grows exponentially with the increasing number of transmitting antenna and constellation size. Due to its characteristics of being an exhaustive search, it is not considered practical for implementation in MIMO receivers [22]. Instead, it is used as a reference in simulation for the performance analysis with other MIMO detection schemes.

2.2.2 *Suboptimal MIMO Detection*

Suboptimal MIMO detectors can be divided into two groups: linear and nonlinear suboptimal detectors. Zero-forcing (ZF), Minimum-mean-square-error (MMSE), etc. are considered as linear suboptimal detectors due to its linear complexity; where Successive-interference-cancelation (SIC), Bell-labs-layered-space-time (BLAST) detectors are the examples of nonlinear suboptimal detectors.

2.2.2.1 Linear Detectors

Linear MIMO detector is based on the linear estimation of the MIMO detection problem with the aim of reversing the effect of channel. It processes the parallel streams of data all at once without taking into consideration of the order, thereby leading to low computational complexity. Hence, they can only achieve the diversity order of $N_R - N_T + 1$ [23], resulting poor performance especially for the symmetric MIMO system where $N_T = N_R$ at high SNR. The linear detectors, ZF and MMSE detectors, are described as follows:

Zero-Forcing Detector

Zero-Forcing (ZF) detector solves the problem according to the method of least squares, which inverts the frequency response of the channel [24]. Multiplying y by H^H in (2.3), we get:

$$\check{y} \cong H^H y = Rs + \check{n} , \tag{2.6}$$

where $R = H^H H$ is a $N_{Tx} \times N_{Tx}$ square matrix. Now, multiplying $\check{y}$ by the inverse of R, s, is recovered as:

$$\hat{s} = R^{-1} H^H y = s + n_{ZF}, \tag{2.7}$$

where $n_{ZF} = R^{-1}\check{n}$. When $n = 0$, $\hat{s} = s$. This is zero forcing solution of linear MIMO detector. Hence, although ZF detector removes the interference between parallel streams, power of the noise increases which thereby leads to poor performance.

MMSE Detector

The problem of noise enhancement of ZF detector is addressed by MMSE detector, which tries to minimize the overall expected error considering the channel noise [25]. It tries to find the minimum mean squared errors between the actual transmitted signal and the output of the linear detector. First step is to determine the coefficient matrix, A, such that the estimate of s can minimize the norm of the error vector, ε.

Here, $\varepsilon = E\left[|e|^2\right] \cong E\left[|Ay - s|^H\right]$ and estimate of s can be represented by $\hat{s}_{MMSE} = Ay$. A can be determined using orthogonal principle:

$$E\left[ey^H\right] = E\left[(Ay - s)y^H\right] = AE\left[yy^H\right] - E\left[sy^H\right] = 0. \tag{2.8}$$

Thus, A satisfies the following:

$$A = \left(E\left[sy^H\right]\right)\left(E\left[yy^H\right]\right)^{-1} \cong H^H\left(HH^H + \sigma^2 I\right)^{-1}, \tag{2.9}$$

where we assume $E\left[ss^H\right] = I, E\left[ns^H\right] = 0$. Finally, $\hat{s}_{MMSE}$ became:

$$\hat{s}_{\mathrm{MMSE}} = AH^{\mathrm{H}}y = H^{\mathrm{H}}\left(HH^{\mathrm{H}} + \sigma^2 I\right)^{-1} y. \tag{2.10}$$

When SNR goes to infinity, MMSE receiver converges to as ZF receiver:

$$\lim_{\sigma^2 \to 0}\left\{H^{\mathrm{H}}\left(HH^{\mathrm{H}} + \sigma^2 I\right)^{-1}\right\} = \left(H^{\mathrm{H}}H\right)^{-1} H \tag{2.10}$$

Although it provides better performance compared to ZF detector, the performance is poor compared to ML one.

2.2.2.2 Nonlinear Detectors

Nonlinear suboptimal detector depends on detecting the symbols in an order, from strongest to weakest symbol. It uses the previous decision for earlier symbols to choose the later symbols. Two examples of nonlinear detectors are as follows:

SIC Detector

In Successive Interference Cancellation (SIC) detector, the symbols of the parallel data streams are considered one after another and their contribution is removed from the received vector before detecting the next stream. Hence, SIC achieves an increase in diversity with each iteration. The diversity of the first stream will be in the order of $N_{\mathrm{R}} - N_{\mathrm{T}} + 1$, the second stream will attain $N_{\mathrm{R}} - N_{\mathrm{T}} + 2$ and so on. However, BER performance depends on the detection order as shown in [26].

In SIC detector, the most important step is to cancel the effect of the strongest interfering signal before detecting the weaker signals. Therefore, the specific symbol detection ordering, designed based on several criteria, is quite critical for the SIC detector's performance. The method performs well when there is a substantial difference in the received signal strength of the multiple simultaneously transmitted symbols. However, it is sensitive to decision error propagation. Therefore, the SIC detector is well-suited for multiple-access systems suffering from the near-far problem.

BLAST Detector

Bell Labs Layered Space-Time (BLAST) detector is based on the principle of both SIC and zero nulling [27, 28]. It detects the symbols consecutively one after another. Hence, the detection order of the symbols significantly affects the BER performance of BLAST detector. It has the complexity in the order of $O(N_{\mathrm{T}}^2)$ and the complexity increases when the channel coherence time decreases.

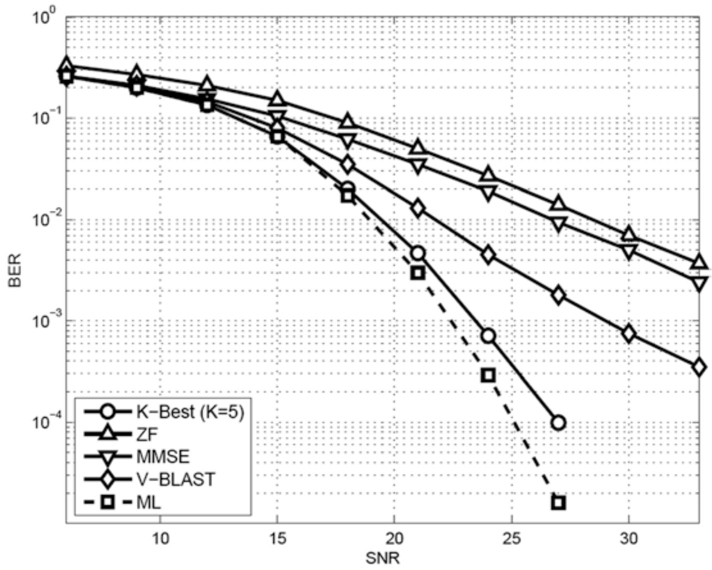

Fig. 2.3 The comparison of multiple detectors with ML detector for 4×4 MIMO with 16 QAM modulation scheme [12]

BLAST detector outperforms the linear detectors, although there remains a considerable performance gap from ML detector. Hence, near-optimal detectors such as K-Best decoder, Sphere decoder (SD), etc. are introduced with better performance compared to linear detectors, as shown in Fig. 2.3.

2.2.3 Near-Optimal MIMO Detection

Near-optimal detectors are capable of achieving near ML performance with less complexity compared to ML. MIMO detection problem can be considered as the closest point problem for a given lattice L(H) [29]. If the lattice bases are orthogonal, this search becomes easier. The complexity of closest point problem can be considered as NP-hard problem, since the lattice basis are built with channel matrix and are completely arbitrary. It can also be restated as a tree-search problem, with the leaves of the tree presenting the set of all potential solutions. To form the tree structure, first QR decomposition is performed on H matrix, i.e., $H = QR$, where Q is a unitary matrix and R becomes an upper triangular matrix. Hence, (2.3) becomes:

$$\hat{y} = Q^H y = Rs + Q^H n. \tag{2.11}$$

The original detection problem in (2.3) can be remodeled as shown in (2.12). Since R is a triangular matrix, the partial distance of ith QAM symbol (s^i) becomes a function of consecutive QAM symbols ($s^{i+1}, s^{i+2}, \ldots, s^M$).

$$d(s) = \left\| \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \hat{y}_3 \\ \hat{y}_4 \end{bmatrix} - \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ 0 & R_{22} & R_{21} & R_{24} \\ 0 & 0 & R_{33} & R_{34} \\ 0 & 0 & 0 & R_{44} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}^2 \right\| \tag{2.12}$$

Figure 2.4 demonstrates a tree for three transmit antennae with binary phase shift keying (BPSK) modulation, where each level of the tree corresponds to a transmit antenna. The goal of the tree search is to find the smallest branch from the root to the last layer of the tree (node).

ML detector considers all the leaves to find the optimum node. Thus, it provides optimal solution with exponential complexity. However, the search can be reduced with the method of tree pruning, which is eliminating the subtree leading to unlikely solutions based on pre-defined performance matric (generally partially Euclidean distance (PED)). Figure 2.5 demonstrates the effect of tree pruning with initial distance set to ∞. Once a leaf node with less PED is found, it is chosen for further expansion. And the one with greater weights are then pruned (shown in the shaded box).

Tree searching methods can be classified into two major categories: depth-first search and breadth-first search. The details of the two methods are given below:

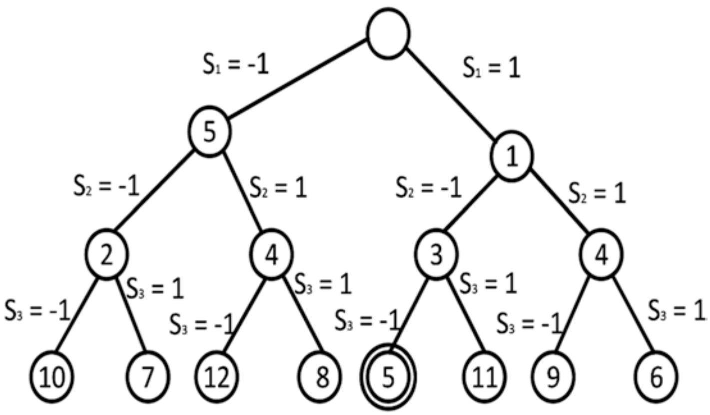

Fig. 2.4 An example of BPSK with 3 transmit antennae

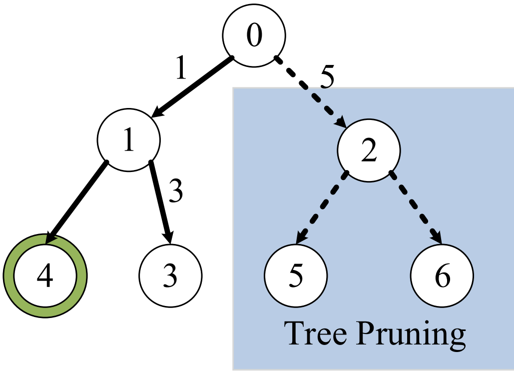

Fig. 2.5 An example of tree pruning

2.2.3.1 Depth-First Tree Search

Depth-first tree search is a recursive method, which starts from the root and traverses in both forward and backward direction along the tree. Sphere decoding (SD) [30] is the most common depth-first approach. It is also called depth-first search least sphere decoder (DPS-LSD). In order to reduce the number of candidate nodes, the search is constrained to only those who lie within a hypersphere with radius r around the receiver symbol y. Hence, the corresponding inequality can be given by:

$$\| y - Hs \|^2 < r^2. \tag{2.13}$$

Here, r is considered as radius constraint. In the beginning, it is important to have an initial guess of r to start with. Choice of r affects the performance of the algorithm significantly. If r is chosen to be a large number, it will take a long time to get the solution. However, no solution may fit in if r is too small. Therefore, throughput of this algorithm is not fixed.

2.2.3.2 Breadth-First Tree Search

Breath-first tree search explores all the children of a parent node before visiting the admissible siblings of that parent node. Initially, it tries to find the admissible child based on PED. If it exists, it is chosen as future parent node to be expanded. Otherwise, it returns to the parent of the current node to consider the remaining children. It is a non-recursive scheme and it traverses only in the forward direction. Among the breadth-first approach, K-Best algorithm is the most well-known scheme [31]. K-Best algorithm guarantees a fixed throughput independent of SNR with performance close to ML. In this book, we focus on the K-Best algorithm, which will be discussed in Chaps. 3–7. The list of different decoders with computational complexity and BER performance is given in Table 2.1.

Table 2.1 List of MIMO detectors

Detector type	BER	Complexity
Optimal detectors	Optimal	Exponential
• Maximum Likelihood (ML)		
Suboptimal detectors	Poor	Low (linear)
• Zero Forcing (ZF)		
• Minimum Mean Square Error (MMSE)		
• Successive Interference		
• Cancellation		
Near-ML detectors	Near-optimal	Moderate (polynomial)
• Sphere Decoder (SD)		
• K-Best Decoder		

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