Preface

This book has been ten odd years in the making. Its first inception was a lecture which I was asked to give at a refresher course for mathematics teachers. The text evolved into a little booklet for the Dutch Zebra series (Epsilon publishers)[1]. This is a series of 60-page booklets aimed at high school pupils preparing for their final exams. As the old saying goes, "Du choc des idées jaillit la lumière", discussions with the editors led to a text which had more potential than the allowed 60 pages. What you have before you is a greatly expanded version of that booklet.

This book is not intended as a history of mathematics, nor is it a complete overview of the problems at hand, the duplication of the cube, the trisection of an angle and the squaring of a circle. Anyone looking for new historical insights should consult other books. We only put forward our mostly educational view, sometimes new, of constructions which have been published numerous times before. The history (and sometimes mythology) of mathematics is used as an introductory story to raise students' interest in the problem.

On the other hand, neither is this a mathematics manual that observes a deductively oriented organisation. It is a well-known fact that deductive organisation is the last step in any given discovery process. Getting acquainted with a historical problem will show students that the pathway to a solution is not linear but curved, sometimes even going backwards, full of mistakes due to lack of appropriate mathematical language and symbolism. Indeed, if anything can be said about the classical Greek construction problems, it is that they are a prime example of this non-linear development. They have captured the imagination of mathematicians for thousands of years, they have led them astray, they have led them into new realms of mathematics until it turned out they were unsolvable with the restraints placed on them.

The following pages are excursions, at undergraduate student level, into these three famous problems. A historian of mathematics faces a dilemma: citing and subsequently explaining an ancient text or translating the text into modern mathematical terminology. The first approach alienates the modern reader, the second betrays the ideas of the ancient author. The author of an educational exposé does not face that dilemma. He is concerned more with heritage than with history per se. Here you will find ancient problems written in modern terminology, making them accessible to the mathematical community at large. By making the point that the ancient mathematicians had fewer tools at their disposal the reader will acquire even greater admiration for their accomplishments, possibly even finding him/herself drawn to ancient texts and perhaps to exploring them in greater depth.

The history of mathematics is important for teachers of mathematics. It gives them the opportunity to let their students see the motivation behind the introduction of certain concepts, it makes clear that mathematics is a non-linear endeavour and students get a glimpse of the people who created mathematics in all their human

[1] Meskens and Tytgat (2015).

aspects. It also makes students aware that they often come up against the same difficulties in seeking to grasp certain concepts as the inventors did.

We hope that many teachers will use these chapters as enrichment material in their classes and enable their students to gain a deeper understanding of geometry. The use of dynamic geometry software packages, enables them to explore geometric relationships in an educational, enquiry-based fashion. Relationships which most likely would have remained hidden in a classic pencil and paper approach.

By focusing on constructions and the use of Interactive Geometry Software or IGS for short, the reader is confronted with the same problems that ancient mathematicians once faced. The neusis construction is of particular interest to be explored with IGS, as it lets readers discover a class of interesting curves. Readers get to retrace the footsteps of Euclid, Viète and Cusanus amongst others and then, by experimenting, discover geometric relationships that far exceed their accomplishments. Over 140 exercises guide readers through methods which were developed to try and solve the problems. The exercises are at undergraduate student level and only require a command of elementary Euclidean geometry and pre-calculus algebra. These exercises are especially well-suited for students who are thinking of becoming a mathematics teacher.

It may be argued that the constructions performed in IGS are not real constructions because IGS uses coordinate geometry. This is of course true, more or less. Any construction, be it on paper, or using a computer program, constitutes a particular case. IGS has the advantage that, using the Move tool, a multitude of particular cases can be checked. While a geometric assertion may not be proved using IGS it may be disproved, by finding a counterexample. IGS will not replace the deductive approach, but it adds a feature to doing geometry: experimentation. For some students, an animation showing that the medians of a triangle always intersect in one point may be more convincing than the proof itself. For the more mathematically-minded students, it is an invitation to search for that proof, showing that what IGS suggests is indeed always true. IGS is a didactic tool of which we are only seeing the beginnings of its possibilities as an incentive for engaging in proper mathematics.

Writing a book like this is not possible without the help of many people and institutions. I have the pleasure of thanking Epsilon Uitgaven and NVvW (Dutch Association of Mathematics Teachers) for their permission to reuse parts of our booklet in the Zebra series. I also wish to thank the editors of the Zebra series for the useful feedback and the many valuable suggestions. Their comments made me turn a classic text into an enquiry-based and experiment-oriented exposé. Thanks are also due to VVWL (Flemish Association of Mathematics Teachers) for the use of parts of articles which appeared in their journal *Wiskunde en Onderwijs*.

Jean Paul van Bendegem, Hiram Bollaert and Joost Bambust read through the English manuscript and also gave many valuable suggestions. I could count on Stephen Hargreaves for useful advice on English terminology.

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Rttaalkantoor corrected the English.

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Finally, I would like to thank my wife Nicole whose support, for over thirty years, has been a continuing labour of love.

In one apocryphal story, King Ptolemy wanted Euclid to teach him mathematics, but he got bored very quickly with the theorems and proofs. He asked Euclid whether there was a faster way of learning mathematics, to which Euclid replied "Sire, in mathematics there is no Royal Road." A similar story is told about Alexander the Great and Menaechmus. You, reader, will not walk a regal road, but when you have finished the book you will feel you have been given a royal gift.

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