
Preface

For decades, statisticians have enjoyed the use of “statistical packages” which read in a (potentially) large data set, calculate numerical summaries such as the sample mean and sample variance, calculate more sophisticated statistical quantities such as confidence interval bounds and p -values associated with hypothesis tests, and generate statistical graphics such as histograms, box plots, and multidimensional plots. But pity the poor probabilist, who through all those decades had only paper and pencil for symbolic calculations. The purpose of this monograph is to address the plight of the probabilist by providing algorithms to perform calculations associated with random variables. We refer to a collection of data structures and algorithms that automate probability calculations as “computational probability.” The data structures and algorithms introduced here have been implemented in a language known as APPL (A Probability Programming Language). Several illustrations of problems from the mathematical sciences that can be solved by implementing these algorithms in a computer algebra system are presented in the final chapters of this monograph.

The algorithms for manipulating random variables (e.g., adding, multiplying, transforming, ordering) symbolically result in an entire class of new problems that can now be addressed. APPL is able to perform exact probability calculations for problems that would otherwise be deemed intractable. The work is quite distinct from traditional probability analysis in that a computer algebra system, in this case Maple, is used as a computing platform.

The use of a computer algebra system to solve problems in operations research and probability is increasing. Other researchers also sense the benefits of incorporating a computer algebra system into fields with probabilistic applications, for example, Parlar’s *Interactive Operations Research with Maple* [98], Karian and Tanis’s second edition of *Probability and Statistics: Explorations with Maple* [58], Rose and Smith’s *Mathematical Statistics and Mathematica* [105], and Hasting’s second edition of *Introduction to the Mathematics of Operations Research with Mathematica* [46].

This monograph significantly differs from the four titles listed above in two ways. First, the four titles listed above are all textbooks, rather than research monographs. They contain exercises and examples geared toward students, rather than researchers. Second, the emphasis in most of these texts is much broader than the emphasis being proposed here. For example, Parlar and Hasting consider all of OR/MS, rather than the probabilistic side of OR/MS proposed here in much more depth. Also, Karian and Tanis emphasize Monte Carlo solutions to probability and statistics problems, as opposed to the exact solutions given in APPL.

The monograph begins with an introductory chapter that contains short examples involving the elementary use of APPL. Chapter 2 reviews the Maple data structures and functions necessary to implement APPL. This is followed by a discussion of the development of the data structures and algorithms (Chaps. 3–6 for continuous random variables and Chaps. 7–9 for discrete random variables) used in APPL. The monograph concludes with Chaps. 10–15 introducing a sampling of various applications in the mathematical sciences. The two most likely audiences for the monograph are researchers in the mathematical sciences with an interest in applied probability and instructors using the monograph for a special topics course in computational probability taught in a mathematics, statistics, operations research, management science, or industrial engineering department. The intended audience for this monograph includes researchers, MS students, PhD students, and advanced practitioners in stochastic operations research, management science, and applied probability.

An indication of the utility of APPL is that the research efforts of the authors and other colleagues have produced many related refereed journal publications, many conference presentations, the ICS Computing Prize with INFORMS, a government patent, the INFORMS Undergraduate Research Prize, a GEM Scholarship based on APPL research, and multiple improvements to pedagogical methods in numerous colleges and universities around the world. We believe that the potential of this field of computational probability in research and education is unlimited. It is our hope that this monograph encourages people to join us in attaining future accomplishments in this field.

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