

## Chapter 2

# Unconfined Aquifers

The chapter considers analytical solutions and analytical methods used in pumping tests in an unconfined aquifer with infinite lateral extent. The construction of groundwater flow equations for unconfined aquifers, semi-infinite or bounded in the horizontal plane, is briefly described in Sect. 2.2. Section 2.3 focuses on pumping tests in unconfined aquifers with sloped bottoms.

The basic assumptions and conditions (Fig. 2.1) are:

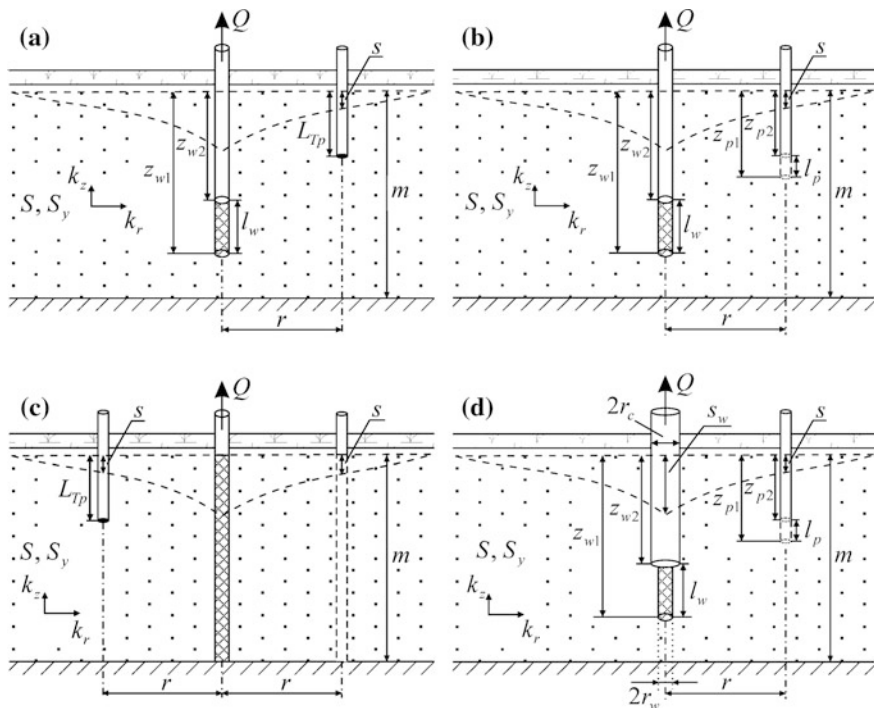
- the aquifer is unconfined, homogeneous, isotropic or vertically anisotropic, infinite in the horizontal plane, and underlain by an aquiclude; the case of leakage from an underlying layer and pumping in a confined–unconfined aquifer is considered (Fig. 2.2);
- the initial saturated aquifer thickness is constant;
- the pumping well is fully or partially penetrating;
- wellbore storage, wellbore skin, and delayed response of observation piezometer can be taken into account in the evaluation of the drawdown.

The drawdown is determined in a fully or partially penetrating observation well or a piezometer at any point of the aquifer.

Typical plots of drawdown in the observation well are given in Figs. 12.17 and 12.18. For the effect of the storage coefficient and specific yield on the drawdown in an unconfined aquifer, see Fig. 12.20.

### 2.1 Aquifer of Infinite Lateral Extent

This section gives transient and quasi-steady-state analytical solutions for calculating the drawdown in an unconfined aquifer infinite in the horizontal plane. The hydraulic characteristics to be determined are listed for each case, depending on the chosen solution.



**Fig. 2.1** Vertically anisotropic unconfined aquifer. **a** Partially penetrating pumping well and piezometer; **b** partially penetrating pumping and observation wells; **c** fully penetrating pumping well with a piezometer and fully penetrating observation well; **d** an example of unconfined aquifer scheme for Moench solution

## Basic Analytical Relationships

### Transient Flow Equations

1. Neuman solution (see Appendix 7.6) for the average drawdown in an observation well and the drawdown in a piezometer for a fully or partially penetrating pumping well in an anisotropic aquifer (Neuman 1972–1975) is:

$$s = \frac{Q}{4\pi k_r m} \int_0^\infty 4\tau J_0\left(\tau \chi \frac{r}{m}\right) \left[ u_0(\tau) + \sum_{n=1}^\infty u_n(\tau) \right] d\tau, \quad (2.1)$$

where  $s$  is the drawdown in the observation well (or piezometer),  $m$ ;  $Q$  is the discharge rate,  $m^3/d$ ;  $\chi = \sqrt{k_z/k_r}$  is anisotropy factor;  $k_r$ ,  $k_z$  are the horizontal and

vertical hydraulic conductivities, respectively,  $m/d$ ;  $m$  is the initial saturated thickness of an unconfined aquifer (see Fig. 2.1),  $m$ ;  $r$  is the radial distance from the pumping to the observation well (or to a piezometer),  $m$ ;  $J_0(\cdot)$  is Bessel function of the first kind of the zero order (see Appendix 7.13).

Depending on the degree of penetration (full or partial) of the pumping and observation wells, the functional expressions for  $u_0(\tau)$  and  $u_n(\tau)$  can be written as follows.

1.1. For a partially penetrating pumping well and piezometer (Neuman 1974) (see Fig. 2.1a):

$$u_0(\tau) = \beta_0 \cosh\left(\gamma_0 \frac{m - L_{Tp}}{m}\right) \frac{\sinh\left(\gamma_0 \frac{m - z_{w2}}{m}\right) - \sinh\left(\gamma_0 \frac{m - z_{w1}}{m}\right)}{\frac{l_w}{m} \sinh \gamma_0}, \quad (2.2)$$

$$u_n(\tau) = \beta_n \cos\left(\gamma_n \frac{m - L_{Tp}}{m}\right) \frac{\sin\left(\gamma_n \frac{m - z_{w2}}{m}\right) - \sin\left(\gamma_n \frac{m - z_{w1}}{m}\right)}{\frac{l_w}{m} \sin \gamma_n}, \quad (2.3)$$

where  $z_{w1}$  and  $z_{w2}$  are the vertical distances from the initial water table to the bottom and the top of the pumping well screen, respectively (Fig. 2.1a, b),  $m$ ;  $L_{Tp}$  is the vertical distance from the initial water table to the open part of the piezometer,  $m$ ;  $l_w$  is pumping-well screen length,  $m$ .

1.2. For partially penetrating pumping and observation wells (Neuman 1974) (see Fig. 2.1b):

$$u_0(\tau) = \beta_0 \left[ \sinh\left(\gamma_0 \frac{m - z_{p2}}{m}\right) - \sinh\left(\gamma_0 \frac{m - z_{p1}}{m}\right) \right] \frac{\sinh\left(\gamma_0 \frac{m - z_{w2}}{m}\right) - \sinh\left(\gamma_0 \frac{m - z_{w1}}{m}\right)}{\frac{l_w l_p}{m m} \gamma_0 \sinh \gamma_0}, \quad (2.4)$$

$$u_n(\tau) = \beta_n \left[ \sin\left(\gamma_n \frac{m - z_{p2}}{m}\right) - \sin\left(\gamma_n \frac{m - z_{p1}}{m}\right) \right] \frac{\sin\left(\gamma_n \frac{m - z_{w2}}{m}\right) - \sin\left(\gamma_n \frac{m - z_{w1}}{m}\right)}{\frac{l_w l_p}{m m} \gamma_n \sin \gamma_n}, \quad (2.5)$$

where  $z_{p1}$  and  $z_{p2}$  are the vertical distances from the initial water table to the bottom and top of the observation well screen, respectively (Fig. 2.1b),  $m$ ;  $l_p$  is the observation-well screen length,  $m$ .

1.3. For a fully penetrating pumping well and piezometer (Neuman 1972, 1973) (see Fig. 2.1c):

$$u_0(\tau) = \beta_0 \cosh\left(\gamma_0 \frac{m - L_{Tp}}{m}\right), \quad (2.6)$$

$$u_n(\tau) = \beta_n \cos\left(\gamma_n \frac{m - L_{Tp}}{m}\right). \quad (2.7)$$

1.4. For fully penetrating pumping and observation wells (Neuman 1973, 1975) (see Fig. 2.1c):

$$u_0(\tau) = \beta_0 \frac{\sinh \gamma_0}{\gamma_0}, \quad (2.8)$$

$$u_n(\tau) = \beta_n \frac{\sin \gamma_n}{\gamma_n}. \quad (2.9)$$

In expressions (Eqs. 2.2–2.9),  $\gamma_0$  and  $\gamma_n$  are the roots of the equations

$$\sigma \gamma_0 \sinh \gamma_0 = (\tau^2 - \gamma_0^2) \cosh \gamma_0, \quad \gamma_0^2 < \tau^2;$$

$$\sigma \gamma_n \sin \gamma_n = -(\tau^2 + \gamma_n^2) \cos \gamma_n, \quad (2n - 1) \frac{\pi}{2} < \gamma_n < n\pi, \quad n \geq 1;$$

$$u = \frac{k_z t}{Sm}, \quad \sigma = S/S_y,$$

$$\beta_0 = \frac{1 - \exp(-u(\tau^2 - \gamma_0^2))}{\tau^2 + (1 + \sigma)\gamma_0^2 - (\tau^2 - \gamma_0^2)^2/\sigma} \times \frac{1}{\cosh \gamma_0},$$

$$\beta_n = \frac{1 - \exp(-u(\tau^2 + \gamma_n^2))}{\tau^2 - (1 + \sigma)\gamma_n^2 - (\tau^2 + \gamma_n^2)^2/\sigma} \times \frac{1}{\cos \gamma_n},$$

where  $S$  is the storage coefficient, dimensionless;  $S_y$  is the specific yield, dimensionless;  $t$  is the time elapsed from the start of pumping, d.

Equation 2.1 is solved by using the algorithm of DELAY2 code (see Appendix 5.1). The Neuman solution (Eq. 2.1) is used to determine the hydraulic conductivities in the vertical and horizontal directions ( $k_r$ ,  $k_z$ ), storage coefficient ( $S$ ) and specific yield ( $S_y$ ) of an unconfined aquifer.

2. For the Boulton solutions (Boulton 1963) for the drawdown in a fully penetrating observation well in an isotropic aquifer (the pumping well is fully penetrating), two such solutions are used, yielding nearly the same drawdown:

$$s = \frac{Q}{2\pi km} \int_0^\infty \frac{1}{\tau} J_0\left(r \sqrt{\frac{\alpha S_y}{km}} \tau\right) \left\{ \frac{1}{\tau^2 + 1} \exp\left(-\frac{\alpha t}{\tau^2 + 1} \tau^2\right) - \frac{\tau^2}{\tau^2 + 1} \exp\left[-\alpha \frac{S + S_y}{S} t (\tau^2 + 1)\right] \right\} d\tau, \quad (2.10)$$

$$s = \frac{Q}{2\pi km} \int_0^\infty \frac{1}{\tau} \left\{ \frac{1 - \exp(-\mu_1)}{\cosh \mu_2 + \frac{\alpha t \eta (1 - \tau^2)}{2\mu_2} \sinh \mu_2} \right\} J_0 \left( r \sqrt{\frac{\alpha(S + S_y)}{T}} \tau \right) d\tau, \quad (2.11)$$

where

$$\mu_1 = \frac{\alpha t \eta (1 + \tau^2)}{2}, \quad (2.12)$$

$$\mu_2 = \frac{\alpha t \sqrt{\eta^2 (1 + \tau^2)^2 - 4\eta \tau^2}}{2}, \quad (2.13)$$

$$\eta = \frac{S + S_y}{S}, \quad (2.14)$$

$k$  is the hydraulic conductivity of an isotropic unconfined aquifer, m/d.

The empirical parameter  $\alpha$  (so-called reciprocal of Boulton's delay index) (1/d) can be defined as

$$\alpha = \frac{3k}{S_y m}. \quad (2.15)$$

For a vertically anisotropic aquifer,  $k = k_z$  in formula (2.15). Neuman (1975, 1979) proposed another relationship for  $\alpha$ :

$$\alpha = \frac{k_z}{S_y m} \left[ 3.063 - 0.567 \lg \left( \frac{\chi r}{m} \right)^2 \right], \quad (2.16)$$

which suggests that the further the observation well is from the pumping well, the less the value of  $\alpha$ .

Boulton solutions (Eqs. 2.10 and 2.11) are used to determine the hydraulic conductivity, the storage coefficient, and specific yield ( $k$ ,  $S$ ,  $S_y$ ) of an unconfined aquifer.

3. The Boulton solution (Boulton 1954) for the drawdown of the water table (corresponds to the gravity-drainage period) in a fully penetrating observation well in an isotropic aquifer (the pumping well is fully penetrating) is:

$$s = \frac{Q}{2\pi km} F_B \left( \frac{kt}{S_y m}, \frac{r}{m} \right), \quad (2.17)$$

$$F_B(u, \beta) = \int_0^\infty \frac{1}{\tau} J_0(\beta \tau) [1 - \exp(-u \tau \tanh \tau)] d\tau, \quad (2.18)$$

where  $F_B(u, \beta)$  is the Boulton function (see Appendix 7.5).

The Boulton solution (Eq. 2.17) with drawdown values corresponding to a gravity-drainage period (see Fig. 12.17) is used to determine the hydraulic conductivity ( $k$ ) and the specific yield ( $S_y$ ) of the unconfined aquifer.

4. The Moench solution (Moench 1993, 1996) for the drawdown in a fully or partially penetrating observation well or a piezometer is:

$$s = \frac{Q}{4\pi k_r m} f(t, r, m, l_w, l_p, L_{Tw}, L_{Tp}, k_r, k_z, S, S_y), \quad (2.19)$$

where  $L_{Tp}$  is the vertical distance from the initial water table to the open part of the piezometer or the center of observation well screen (for the observation well,  $L_{Tp} = z_{p2} + l_p/2$ ), m;  $L_{Tw} = z_{w2} + l_w/2$  is the vertical distance from the initial water table to the center of the pumping well screen, m.

The functional relationship (Eq. 2.19) is treated using the algorithm of WTAQ2 code (see Appendix 5.2). The Moench solution (Eq. 2.19) is used to evaluate the vertical and horizontal hydraulic conductivities ( $k_r$ ,  $k_z$ ), as well as the storage coefficient ( $S$ ) and specific yield ( $S_y$ ) of an unconfined aquifer.

5. The Moench solution (Moench 1997) for the drawdown in a fully or partially penetrating observation well or a piezometer with the storages of the pumping and observation wells taken into account is:

$$s = \frac{Q}{4\pi k_r m} f(t, r, r_w, r_c, r_p, m, l_w, l_p, L_{Tw}, L_{Tp}, k_r, k_z, S, S_y, k_{skin}, m_{skin}), \quad (2.20)$$

where  $r_w$ ,  $r_c$ ,  $r_p$  are the radiuses of the pumping well, its casing, and the observation well, m;  $k_{skin}$ ,  $m_{skin}$  are the hydraulic conductivity (m/d) and the thickness (m) of the wellbore skin (see Appendix 2).

The functional relationship (Eq. 2.20) takes into account the wellbore storage, the delayed response of the observation piezometer, and the wellbore skin. The algorithm of WTAQ3 code is used for its treatment (see Appendix 5.3). The Moench solution (Eq. 2.20) is used to determine the horizontal and vertical hydraulic conductivities ( $k_r$ ,  $k_z$ ), the storage coefficient ( $S$ ), the specific yield ( $S_y$ ) of the unconfined aquifer and, additionally, to evaluate the hydraulic conductivity and the thickness of the wellbore skin ( $k_{skin}$ ,  $m_{skin}$ ).

In addition, the functional relationships (Eqs. 2.20 and 2.21) enable one to take into account the effect of the delayed drainage from the unsaturated zone on the drawdown in the wells during pumping. In this case, the code WTAQ version 2 (see Appendix 5.4) is used for calculations (Barlow and Moench 2011).

6. Moench's solution (Moench 1997) for the drawdown in a fully or partially penetrating pumping well:

$$s_w = \frac{Q}{4\pi k_r m} f(t, r_w, r_c, m, l_w, L_{Tw}, k_r, k_z, S, S_y, k_{skin}, m_{skin}). \quad (2.21)$$

The functional relationship (Eq. 2.21) takes into account the wellbore storage and the wellbore skin. The algorithm of WTAQ3 code is used for its calculation (see Appendix 5.3). The parameters being determined are similar to the Moench relationship (Eq. 2.20). Here, as well as in (Eq. 2.20), the effect of the capillary fringe is evaluated.

7. A simplified solution for the drawdown during the gravity-drainage period (see Fig. 12.17) in a fully penetrating observation well in an isotropic unconfined aquifer (the pumping well is fully penetrating) (Jacob 1963) is:

$$s = m - \sqrt{m^2 - \frac{Q}{2\pi k} W\left(\frac{r^2}{4at}\right)}, \quad (2.22)$$

where  $a$  is the hydraulic diffusivity of the unconfined aquifer,  $m^2/d$ ;  $W(u)$  is a well-function (see Appendix 7.1).

Solution (Eq. 2.22) is used to evaluate the hydraulic conductivity ( $k$ ) and the hydraulic diffusivity ( $a = km/(S + S_y) \approx km/S_y$ ) of the unconfined aquifer.

8. A simplified solution for the drawdown in a fully penetrating observation well in an isotropic unconfined aquifer (the pumping well is fully penetrating) during gravity-drainage period with the leakage through aquifer bottom taken into account (Fig. 2.2a) is:

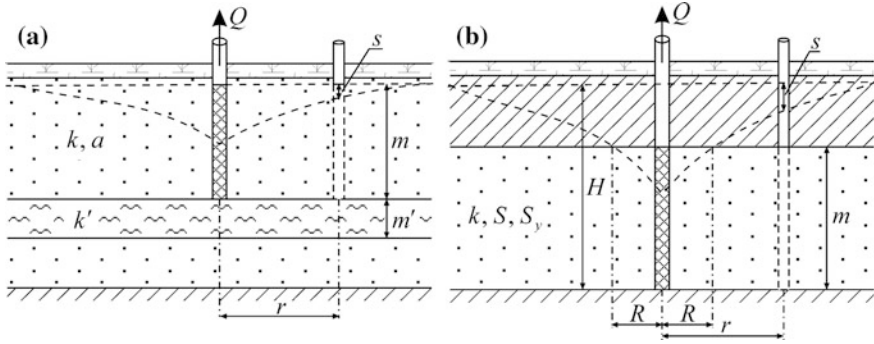
$$s = m - \sqrt{m^2 - \frac{Q}{2\pi k} W\left(\frac{r^2}{4at}, \frac{r}{B}\right)}, \quad (2.23)$$

$$B = \sqrt{\frac{k\bar{m}m'}{k'}}, \quad (2.24)$$

where  $B$  is the leakage factor,  $m$ ;  $\bar{m} = m$  is the initial saturated thickness of the unconfined aquifer,  $m$ ;  $k'$ ,  $m'$  are the hydraulic conductivity ( $m/d$ ) and thickness ( $m$ ) of the aquitard;  $W(u, \beta)$  is the well-function for leaky aquifers (see Appendix 7.2).

The solution (Eq. 2.23) is used to evaluate the hydraulic conductivity ( $k$ ) and hydraulic diffusivity ( $a = km/S_y$ ) of the unconfined aquifer, as well as the leakage factor ( $B$ ).

9. The Moench–Prickett solution (Moench and Prickett 1972) for a confined–unconfined aquifer (Fig. 2.2b) is:



**Fig. 2.2** **a** Unconfined leaky aquifer and **b** confined-unconfined aquifer

$$\begin{cases} s = \frac{Q}{4\pi km} \left[ W\left(\frac{r^2 S_y}{4kmt}\right) - W\left(\frac{R^2 S_y}{4kmt}\right) \right] + H - m & \rightarrow r < R \\ s = \frac{Q}{4\pi km} \exp\left(-\frac{R^2 (S_y - S)}{4kmt}\right) W\left(\frac{r^2 S}{4kmt}\right) & \rightarrow r > R, \end{cases} \quad (2.25)$$

where  $H$  is the initial head,  $m$ ;  $m$  is confined-aquifer thickness,  $m$ ;  $R$  is the horizontal distance from the pumping well to the point where the confined aquifer becomes unconfined (Fig. 2.2b).

The distance  $R$  is calculated from a transcendent equation for any moment  $t$ :

$$\frac{Q}{4\pi km(H - m)} \exp\left(-\frac{R^2 S_y}{4kmt}\right) - \exp\left(-\frac{R^2 S}{4kmt}\right) / W\left(\frac{R^2 S}{4kmt}\right) = 0. \quad (2.26)$$

Depending on the radius, a solution from the system of Eq. 2.25 is chosen, in which the top equation accounts for the drawdown for the periods when the observation well is located in the unconfined flow zone, and the bottom equation accounts for the same for the confined zone. When the initial head is below the top of the aquifer, the system of Eq. 2.25 transforms into an equation for gravity-drainage conditions

$$s = \frac{Q}{4\pi km} W\left(\frac{r^2 S_y}{4kmt}\right), \quad (2.27)$$

which is an analog of the solution (Eq. 2.22). The Moench–Prickett solution (Eq. 2.25) is used to determine the hydraulic conductivity of the aquifer ( $k$ ), the storage coefficient of the confined flow zone ( $S$ ), and the specific yield for the zone of unconfined flow ( $S_y$ ).

When evaluating drawdown in an unconfined aquifer with the use of transient flow relationships, one should keep in mind that: (1) the screen length of the observation well ( $l_p$ ) is eliminated in solutions (Eq. 2.19) and (Eq. 2.20) in the calculation of the drawdown in the piezometer; and (2) the drawdown in the



observation well derived from solutions (Eqs. 2.1, 2.19, and 2.20) is averaged over its screen length.

The solutions (Eqs. 2.1, 2.10, 2.11, 2.17, and 2.19–2.21) imply a slight decline in the water table in the course of testing, relative to the initial saturated thickness of the aquifer. Otherwise, it is recommended to introduce a correction (Jacob 1963) to the drawdown evaluated by these relationships, i.e.

$$s_c = m - \sqrt{m^2 - 2ms}, \quad (2.28)$$

where  $s_c$  is the corrected drawdown,  $m$ ;  $s$  is the drawdown derived from formulas without correction,  $m$ .

The need to introduce corrections to the drawdown (see Fig. 12.19) in the solutions given above, depending on the degree of penetration (full or partial) of the pumping well and the magnitude of drawdown, is classified in Table 2.1.

*Quasi-Steady-State Flow Equation* (corresponds to the gravity-drainage period)

$$s(2m - s) = \frac{Q}{2\pi k} \ln \frac{2.25kmt}{r^2 S_y} = \frac{0.366Q}{k} \lg \frac{2.25kmt}{r^2 S_y} \quad (2.29)$$

or, in terms of hydraulic diffusivity:

$$s(2m - s) = \frac{Q}{2\pi k} \ln \frac{2.25at}{r^2} = \frac{0.366Q}{k} \lg \frac{2.25at}{r^2}. \quad (2.30)$$

The equation for quasi-steady-state period implies the full penetration of the pumping and observation wells and the isotropy of the unconfined aquifer.

**Table 2.1** Application of analytical solutions for different degrees of penetration of the pumping well and the magnitude of drawdown

Pumping well penetration	Condition	Solution	Correction
Full	Small drawdown <sup>a</sup>	2.1, 2.10, 2.11, 2.17, 2.19–2.21	Not required
Full	Large drawdown <sup>a</sup>	2.1, 2.10, 2.11, 2.17, 2.19–2.21	Obligatory
Partial	Water table lies above the screen top <sup>b</sup>	2.1, 2.19–2.21	Not applied
Partial	Water table lies below the screen top <sup>c</sup>	The solutions are inapplicable	Not applied

<sup>a</sup>The drawdown can be considered small when it is less than 20 % of the initial saturated thickness of an unconfined aquifer, i.e.,  $s < 0.2m$  (Borevskiy et al. 1973)

<sup>b</sup>The screen of the pumping well during the pumping test remains fully within the saturated zone

<sup>c</sup>Water table can drop below the top of the pumping well screen during the test: the length of the part of the screen within the saturated zone varies during the pumping test

### Graphic-Analytical Processing

The relationships given in Table 2.2 have been derived from simplified solutions (Eqs. 2.22 and 2.30), which assume the pumping and observation wells are fully penetrating. The graphic-analytical processing involves only drawdown values corresponding to the gravity-drainage period in an isotropic aquifer.

In Table 2.2, the values of the hydraulic conductivity and hydraulic diffusivity are determined independently. Given these characteristics, the specific yield of the unconfined aquifer can be readily evaluated:  $S_y = km/a$ . In addition, the hydraulic diffusivity and the specific yield can be evaluated based on the intercept of the straight line on the abscissa (Table 2.3).

**Table 2.2** Graphic-analytical parameter evaluation

Plot	Method	Relationship
$s(2m - s) - \lg t$	Straight line	$k = \frac{0.366Q}{C}$ , $\lg a = \frac{A}{C} + \lg \frac{r^2}{2.25}$
$s(2m - s) - \lg r$	The same	$k = \frac{0.732Q}{C}$ , $\lg a = 2\frac{A}{C} - \lg(2.25 \cdot t)$
$s(2m - s) - \lg \frac{t}{r^2}$	The same	$k = \frac{0.366Q}{C}$ , $\lg a = \frac{A}{C} - \lg 2.25$
$\lg[s(2m - s)] - \lg t$	Type curve: $\lg W(u) - \lg \frac{1}{u}$	$k = \frac{Q}{2\pi \cdot 10^D}$ , $a = \frac{r^2 \cdot 10^E}{4}$
$\lg[s(2m - s)] - \lg \frac{t}{r^2}$	The same	$k = \frac{Q}{2\pi \cdot 10^D}$ , $a = \frac{10^E}{4}$
$\left[ \begin{matrix} s_1(2m - s_1) - \\ -s_2(2m - s_2) \end{matrix} \right] - \lg t$	Horizontal straight line	$k = \frac{Q}{\pi \cdot A \ln \frac{r_2}{r_1}}$

$A$  is the intercept on the ordinate axis (see Sects. 12.1.1 and 12.1.2);  $C$  is the slope of the straight line (see Sect. 12.1.1);  $D, E$  are the shifts of the plot of the actual and reference curves (see Sect. 12.1.3) in the vertical ( $D$ ) and horizontal ( $E$ ) directions, respectively;

$s_1, s_2, r_1, r_2$  is the drawdown ( $s$ ) in and the distance to the pumping well ( $r$ ) for the first and second observation wells, respectively. In the case of vertical anisotropy, horizontal hydraulic conductivity is determined:  $k = k_r$

**Table 2.3** Graphic-analytical parameter evaluation

Plot	Method	Relationship
$s(2m - s) - \lg t$	Straight line	$a = \frac{r^2}{2.25t_x}$ , $S_y = \frac{2.25kmt_x}{r^2}$
$s(2m - s) - \lg r$	The same	$a = \frac{r_x^2}{2.25t}$ , $S_y = \frac{2.25kmt}{r_x^2}$
$s(2m - s) - \lg \frac{t}{r^2}$	The same	$a = \frac{1}{2.25(t/r^2)_x}$ , $S_y = 2.25km(t/r^2)_x$

$t_x, r_x$  and  $(t/r^2)_x$  are the intercepts on the abscissas of appropriate plots (see Fig. 12.1)

## 2.2 Semi-infinite and Bounded Unconfined Aquifers

To solve the boundary problem for unconfined aquifers, one may use the same approach as for confined aquifers (see Sects. 1.1.2–1.1.5). The solutions for the drawdown are derived from basic solutions (Eqs. 2.1, 2.10, 2.11, 2.17, and 2.19) with the effect of image wells taken into account through the superposition principle.

For example, the Boulton solution (Eq. 2.17) for a semi-infinite aquifer with a constant-head boundary can be written as:

$$s = \frac{Q}{2\pi km} \left[ F_B\left(\frac{kt}{S_y m}, \frac{r}{m}\right) - F_B\left(\frac{kt}{S_y m}, \frac{\rho}{m}\right) \right], \quad (2.31)$$

and the solution (Eq. 2.22) for an impermeable boundary as:

$$s = m - \sqrt{m^2 - \frac{Q}{2\pi k} \left[ W\left(\frac{r^2}{4at}\right) + W\left(\frac{\rho^2}{4at}\right) \right]}, \quad (2.32)$$

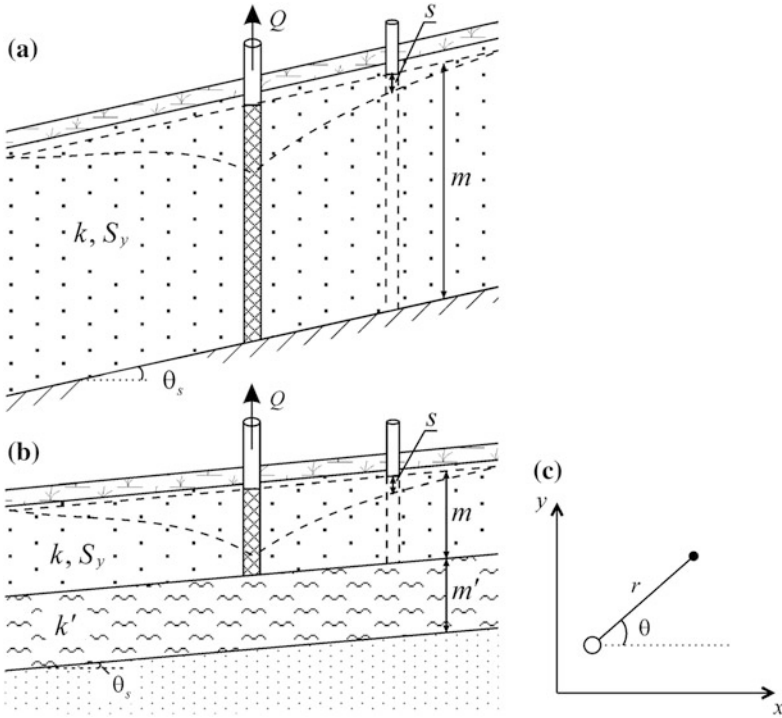
where  $\rho$  is the horizontal distance from the observation to the image well (see Fig. A3.2 and Eq. A3.1),  $m$ .

## 2.3 Sloping Unconfined Aquifer

The basic assumptions and conditions (Figs. 2.3 and 2.4) are:

- the aquifer is unconfined, isotropic, sloped, and underlain by an aquiclude (Fig. 2.3a) or an aquitard (Fig. 2.3b), through which water leaks during the test;
- the initial saturated thickness of the aquifer does not change over the space;
- the slopes of the aquiclude and groundwater table are the same; the presented solutions are applicable to aquicludes with a slope less than 0.2;
- the drawdown in the pumping well must not exceed half the initial saturated thickness of the main aquifer;
- in the case of a leaky aquifer, the storage of the aquiclude is neglected;
- the aquifer is infinite in the horizontal plane (Fig. 2.3) or semi-infinite with a constant-head boundary (Fig. 2.4).

The drawdown is determined in the aquifer at any distance from the pumping well for the gravity-drainage period. The drawdown for sloped aquifers depends on both the distance to the pumping well and the angle  $\theta$  (Fig. 2.3c), as well as the relative positions of the observation and pumping wells: whether the former lies upstream or downstream of the latter. Ideally, the drawdown in the observation well upstream of the pumping well is less than that downstream of it.



**Fig. 2.3** Sloped unconfined aquifer of infinite lateral extent: **a** nonleaky aquifer; **b** leaky aquifer. **c** Planar view

The analytical relationships are used to determine the hydraulic conductivity ( $k$ ) and specific yield ( $S_y$ ) of the unconfined aquifer. In the case where leakage is taken into account, the leakage factor ( $B$ ) is also evaluated.

### Basic Analytic Relationships

#### Transient Flow Equations

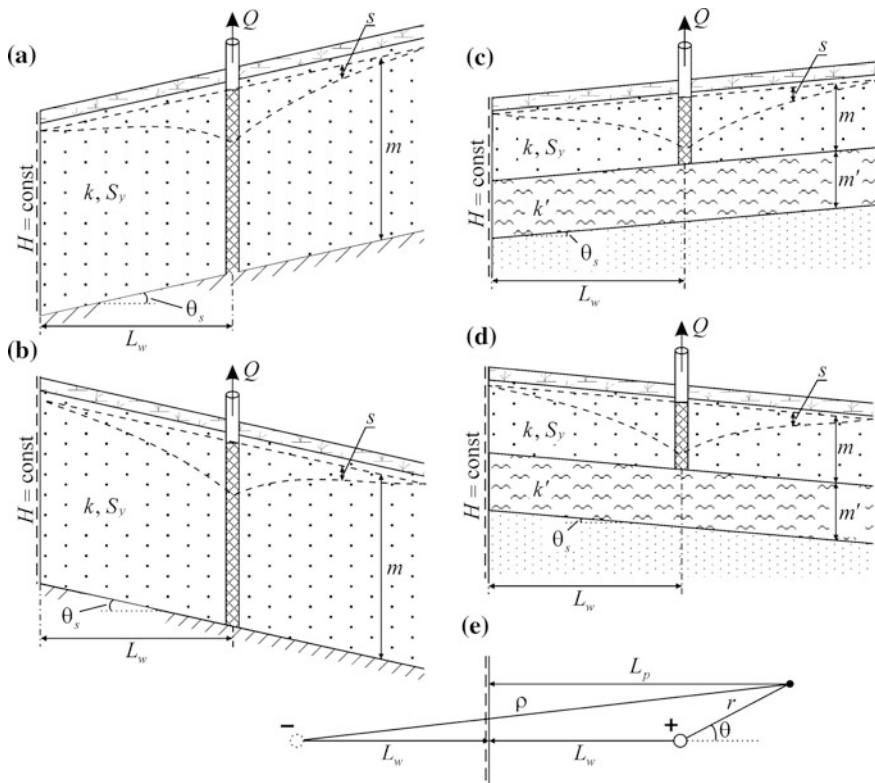
##### 1. Solutions for unconfined nonleaky aquifer (Hantush 1962)

##### 1.1. Aquifer of infinite lateral extent (Fig. 2.3a):

$$s = m - \sqrt{m^2 - \frac{Q}{2\pi k} \exp\left(-\frac{r}{\gamma} \cos \theta\right) W\left(\frac{r^2 S_y}{4kmt}, \frac{r}{\gamma}\right)}, \quad (2.33)$$

$$\gamma \approx (1.75 \div 2) \frac{m}{\tan \theta_s}, \quad (2.34)$$

where  $m$  is the initial saturated thickness of the sloped unconfined aquifer, m;  $\theta_s$  is the slope of the bottom of the aquifer, degree;  $\theta$  is the angle between the  $x$  axis and



**Fig. 2.4** Sloped unconfined aquifer semi-infinite in the horizontal plane: **a, b** nonleaky aquifers; **c, d** leaky aquifers. **e** A planar view with the distances to the boundary and an image well (the signs of actual and image flow rates are given near the wells); **a, c** wells are located upstream of the boundary; **b, d** wells are located downstream of the boundary.  $L_w$ ,  $L_p$  are the distances from the pumping and observation wells to the boundary, respectively

the line connecting the pumping and observation wells (see Figs. 2.3c and 2.4e); the cosine of the angle in degrees can be expressed in terms of the distances (Eq. 1.119).

1.2. Semi-infinite aquifer (Fig. 2.4a, b):

$$s = m - \sqrt{m^2 - \frac{Q}{2\pi k} \exp\left(n \frac{r}{\gamma} \cos \theta\right) \left[ W\left(\frac{r^2 S_y}{4kmt}, \frac{r}{\gamma}\right) - W\left(\frac{\rho^2 S_y}{4kmt}, \frac{\rho}{\gamma}\right) \right]}, \quad (2.35)$$

where  $n = -1$  for a well upstream from the boundary (Fig. 2.4a, c) and  $n = 1$  for a well downstream from the boundary (Fig. 2.4b, d);  $\rho$  is the horizontal distance from the observation well to the image well (Eq. A3.1),  $m$ .

2. Solutions for unconfined leaky aquifer (Hantush 1964)

2.1. Aquifer infinite in the horizontal plane (Fig. 2.3b):

$$s = m - \sqrt{m^2 - \frac{Q}{2\pi k} \exp\left(-\frac{r}{\gamma} \cos \theta\right) W\left(\frac{r^2 S_y}{4kmt}, r\gamma'\right)}, \quad (2.36)$$

$$\gamma' = \sqrt{\frac{1}{\gamma^2} + \frac{1}{B^2}}, \quad (2.37)$$

where  $B$  is the leakage factor (Eq. 2.24),  $m$ .

2.2. Aquifer semi-infinite in the horizontal plane (Fig. 2.4c, d):

$$s = m - \sqrt{m^2 - \frac{Q}{2\pi k} \exp\left(n \frac{r}{\gamma} \cos \theta\right) \left[W\left(\frac{r^2 S_y}{4kmt}, r\gamma'\right) - W\left(\frac{\rho^2 S_y}{4kmt}, \rho\gamma'\right)\right]}. \quad (2.38)$$

### Steady-State Flow Equations

1. Solutions for an unconfined nonleaky aquifer (Hantush 1962)

1.1. Aquifer infinite in the horizontal plane (Fig. 2.3a):

$$s = m - \sqrt{m^2 - \frac{Q}{\pi k} \exp\left(-\frac{r}{\gamma} \cos \theta\right) K_0\left(\frac{r}{\gamma}\right)}, \quad (2.39)$$

where  $K_0(\cdot)$  is a modified Bessel function of the second kind of the zero order (see Appendix 7.13).

1.2. Semi-infinite aquifer (Fig. 2.4a, b):

$$s = m - \sqrt{m^2 - \frac{Q}{\pi k} \exp\left(n \frac{r}{\gamma} \cos \theta\right) \left[K_0\left(\frac{r}{\gamma}\right) - K_0\left(\frac{\rho}{\gamma}\right)\right]}. \quad (2.40)$$

2. Solutions for unconfined leaky aquifer (Hantush 1964)

2.1. Aquifer infinite in the horizontal plane (Fig. 2.3b):

$$s = m - \sqrt{m^2 - \frac{Q}{\pi k} \exp\left(-\frac{r}{\gamma} \cos \theta\right) K_0(r\gamma')}. \quad (2.41)$$

2.2. Aquifer semi-infinite in the horizontal plane (Fig. 2.4c, d):

$$s = m - \sqrt{m^2 - \frac{Q}{\pi k} \exp\left(n \frac{r}{\gamma} \cos \theta\right) [K_0(r\gamma') - K_0(\rho\gamma')]}. \quad (2.42)$$

## References

- Barlow PM, Moench AF (2011) WTAQ version 2—a computer program for analysis of aquifer tests in confined and water-table aquifers with alternative representations of drainage from the unsaturated zone. U.S. Geological Survey. Techniques and Methods 3-B9
- Borevskiy BV, Samsonov BG, Yazvin LS (1973) Methods for aquifer parameters estimation from pumping test data. Nedra, Moscow (In Russian)
- Boulton NS (1954) The drawdown of the water-table under non-steady conditions near a pumped well in an unconfined formation. *P I Civil Eng* 3(4):564–579
- Boulton NS (1963) Analysis of data from non-equilibrium pumping tests allowing for delayed yield from storage. *P I Civil Eng* 26(3):469–482
- Hantush MS (1962) Hydraulics of gravity wells in sloping sands. *J Hydr Eng Div-ASCE* 88 (HY4):1–15
- Hantush MS (1964) Hydraulics of wells. In: Te Chow Ven (ed) *Advances in hydroscience*, vol 1. Academic Press, New York and London, pp 281–432
- Jacob CE (1963) Determining the permeability of water-table aquifer. In: Bentall R (ed) *Methods of determining permeability, transmissibility and drawdown*. Geological Survey Water-Supply, paper 1536-I, Washington, pp 245–271
- Moench AF (1993) Computation of type curves for flow to partially penetrating wells in water-table aquifers. *Ground Water* 31(6):966–971
- Moench AF (1996) Flow to a well in a water-table aquifer: an improved Laplace transform solution. *Ground Water* 34(4):593–596
- Moench AF (1997) Flow to a well of finite diameter in a homogeneous, anisotropic water table aquifer. *Water Resour Res* 33(6):1397–1407
- Moench AF, Prickett TA (1972) Radial flow in an infinite aquifer undergoing conversion from artesian to water table conditions. *Water Resour Res* 8(2):494–499
- Neuman SP (1972) Theory of flow in unconfined aquifers considering delayed gravity response. *Water Resour Res* 8(4):1031–1045
- Neuman SP (1973) Supplementary comments on theory of flow in unconfined aquifers considering delayed gravity response. *Water Resour Res* 9(4):1102–1103
- Neuman SP (1974) Effect of partial penetration on flow in unconfined aquifers considering delayed gravity response. *Water Resour Res* 10(2):303–312
- Neuman SP (1975) Analysis of pumping test data from anisotropic unconfined aquifers. *Water Resour Res* 11(2):329–345
- Neuman SP (1979) Perspective on delayed yield. *Water Resour Res* 15(4):899–908

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