

Chapter 2

Massive MIMO for Interference Suppression: Cell-Edge Aware Zero Forcing

Abstract Ubiquitous high-speed coverage and seamless user experience are among the main targets of next generation wireless systems, and large antenna arrays have been identified as a technology candidate to achieve them. By exploiting the excess spatial degree of freedom from the large number of base station (BS) antennas, we propose a new scheme termed *cell-edge-aware* (CEA) zero forcing (ZF) precoder for coordinated beamforming in massive MIMO cellular network, which suppresses inter-cell interference at the most vulnerable user equipments (UEs). In this work, we combine the tools from random matrix theory and stochastic geometry to develop a framework that enables us to quantify the performance of CEA-ZF and compare that with a conventional *cell-edge-unaware* (CEU) ZF precoder in a network of random topology. Our analysis and simulations show that the proposed CEA-ZF precoder outperforms CEU-ZF precoding in terms of (i) increased aggregate per-cell data rate, (ii) higher coverage probability, and (iii) significantly larger 95 %-likely rate, the latter being the worst data rate that a UE can reasonably expect to receive when in range of the network. Results from our framework also reveal the importance of scheduling the optimal number of UEs per BS, and confirm the necessity to control the amount of pilot contamination received during the channel estimation phase.

2.1 Introduction

Supporting the ever increasing wireless throughput demand is the primary factor driving the industry and academia alike toward the fifth generation (5G) wireless systems. To attain a satisfied quality of service (QoS), 5G networks need not only to provide a large aggregate capacity, but also guarantee high worst-case rates for all UEs, including those located at the cell edge, i.e., close to interfering BSs [1–3]. New technologies are being introduced to improve the performance of cell-edge UEs from current levels. Equipping BSs with a large number of antennas, widely known as massive multiple-input multiple-output (MIMO), has emerged as one of the most promising solutions [4–6]. Besides the large diversity gain it brings along, spatial

dimensions available at massive MIMO BSs can also be used to suppress interference at cell-edge UEs, thus shed light to new design aspects of inter-cell interference mitigation. To this end, we design and analyze a linear transmission scheme, termed *cell-edge-aware* (CEA) zero forcing (ZF) precoder, that significantly improves the data rate of UEs at the cell edge, as well as the overall network throughput.

2.1.1 Motivation and Related Work

A considerable amount of research has investigated the use of multi-cell joint signal processing for cell-edge performance improvement [7–9]. The common idea behind joint processing techniques is to organize BSs in clusters, where BSs lying in the same cluster share information on the data to be transmitted to all UEs in the cluster. Although this information allows BSs to coordinate their transmission and jointly serve all UEs with an improved system throughput, it comes at the cost of heavy signaling overhead and backhaul latency, which defy the purpose of its implementation [10].

As the benefits of joint processing are often outweighed by the increased latency and overhead, a more practical alternative to increase the cell-edge throughput can be found in coordinated beamforming, or *precoding*, schemes [11–13]. Under coordinated precoding, each BS acquires additional channel state information (CSI) of UEs in neighboring cells, but no data information is shared between the various BSs. The additional CSI can then be exploited to control the crosstalk generated at UEs in other cells, e.g., by using multiple BS antennas to steer the crosstalk toward the nullspace of the neighboring UEs. This approach is especially attractive for massive MIMO BSs, due to the abundance of spatial dimensions provided by the large antenna arrays [14].

Remarkable attempts to design and analyze a coordinated precoder for massive MIMO cellular networks are made in [15, 16]. The current paper differs from and generalizes the latter works in two key aspects

1. *Design:* Unlike [15, 16], where each BS suppresses the interference at all edge UEs in all neighboring cells, we specifically target those neighboring UEs close to the BS coverage area. Therefore, our precoder employs fewer spatial dimensions to mitigate inter-cell interference, leaving more degrees of freedom to each BS to better multiplex its own associated UEs [17].
2. *Analysis:* While [15, 16] assume a symmetric hexagonal cellular network, we consider a generalized model with random topology. Hexagonal models can lead to substantial performance overestimation, as demonstrated in [18, 19], whereas our analysis accounts for the randomness of practical cellular deployments.

2.1.2 Approach and Summary of Results

In this work, we propose a CEA-ZF precoder for massive MIMO cellular networks. Exploiting the excess spatial degrees of freedom made available by each massive MIMO BS, our CEA-ZF precoder suppress inter-cell interference at the cell-edge UEs in a distributed manner, thus boosting the received signal-to-interference ratio (SIR) of these most vulnerable of these most vulnerable UEs. In order to evaluate the performance of our scheme, we combine random matrix theory with stochastic geometry and analyze the coverage and rate performance of the proposed CEA-ZF precoder, as well as those of a conventional *cell-edge-unaware* (CEU) ZF precoder, in a general setting that accounts for random BS deployment and interference affecting both the channel estimation and data transmission phases. Our key results and contributions can be summarized as follows:

- We propose a new linear precoder for the downlink of massive MIMO cellular networks, which we denote as the CEA-ZF precoder, where some spatial dimensions are used to suppress inter-cell interference at the cell-edge neighboring UEs, and the remaining degrees of freedom are used to multiplex UEs within the cell. Our precoder works in a distributed manner, and it boosts network coverage and rate performance compared to CEU-ZF precoding.
- We develop a general framework to analyze the SIR distribution and coverage of massive MIMO cellular networks for both the proposed CEA-ZF and the CEU-ZF precoder. Our analysis is tractable and captures the effects of multi-antenna transmission, spatial multiplexing, path loss and small-scale fading, network load and BS deployment density, imperfect channel estimation, and random network topology.
- Through our analysis, which is validated via simulation results, we demonstrate that the proposed CEA-ZF precoder outperforms the CEU-ZF precoder in terms of aggregate per-cell data rate and coverage probability. Moreover, the CEA-ZF precoding guarantees a significantly larger 95 %-likely rate, i.e., it improves the worst data rate that any UE can expect to achieve.
- We quantify the effect of imperfect CSI, and reveal the importance of controlling the amount of pilot contamination received during the channel estimation phase, e.g., through smart pilot allocation schemes. We also study the system performance as a function of the network load, showing that it is beneficial to schedule the optimal number of UEs per BS.

The remainder of this chapter is organized as follows. We introduce the system model in Sect. 2.2. In Sect. 2.3, we analyze the SIR and network coverage under CEA-ZF and CEU-ZF precoding, also studying a special case that provides intuitions behind the gain attained by using different precoding schemes. We show the simulations that confirm the accuracy of our analysis as well as the numerical results to quantify the benefits of CEA-ZF precoding and obtain design insights in Sect. 2.4. This chapter is concluded in Sect. 2.5 (Main notations are summarized in Table 2.1).

Table 2.1 Notation summary

Notation	Definition
$\Phi_b; \lambda$	PPP modeling the location of BSs; BS deployment density
$N; K$	Number of transmit antennas per BSs; number of scheduled UEs per BS
$P_t; \alpha$	BS transmit power; path loss exponent
$C_i; C_i^N$	First-order Voronoi cell for BS i ; cell neighborhood for BS i
$C_i^E = C_i \cup C_i^N$	Extended cell for BS i
$r_{iik}; r_{\bar{i}ik}$	Distance between UE k in cell i and its serving BS i and second closest BS $\bar{i}$, respectively
$\mathbf{x}_{ijk} \sim CN(0, \mathbf{I}_N)$	Small-scale fading between BS i and UE k in cell j
$\mathbf{w}_{u,ik}; \mathbf{w}_{a,ik}$	CEU-ZF and CEA-ZF precoding vector, respectively, at BS i for its UE k
$M; F$	Available number of pilots; pilot reuse factor
I_p	Interference during training phase
$I_u; I_a$	Interference during data transmission phase for CEU-ZF and CEA-ZF, respectively
τ_{ijk}	Standard deviation of CSI error between BS i and UE k in cell j
θ	SIR decoding threshold
$P_c(\theta); \rho_{95}$	Coverage probability; 95 %-likely rate

2.2 System Model

We consider the downlink of a cellular network that consists of randomly deployed BSs, whose location follows a homogeneous Poisson point process (PPP) Φ_b of spatial density λ in the Euclidean plane. In this network, each BS transmits with power P_t and is equipped with a large number of antennas, N . Single-antenna UEs are distributed as a homogeneous PPP with sufficient high density on the plane, such that each BS has at least $K \leq N$ candidate UEs in its cell to serve. In light of its higher spectral efficiency, we consider spatial multiplexing at the BSs, where in each time–frequency resource block (RB) each BS simultaneously serves the K UEs in its cell [4].

We assume that UEs associate to the BS that provides the largest average received power. Due to the homogeneous nature of the network, this results in a distance-based association rule.¹ In this sense, the set of UE locations that are associated to BS i located at $\mathbf{z}_i \in \mathbb{R}^2$ are defined by a classical Voronoi tessellation on the plane, denoted by V_i^1 and given by [22, 23]

$$V_i^1 = \{\mathbf{z} \in \mathbb{R}^2 | \|\mathbf{z} - \mathbf{z}_i\| \leq \|\mathbf{z} - \mathbf{z}_k\|, \forall \mathbf{z}_k \in \Phi_b \setminus \mathbf{z}_i\}. \quad (2.1)$$

¹Different association rules apply when transmit power or large-scale fading vary among BSs, resulting in a weighted Voronoi diagram [20, 21].

We note that the set V_i^1 contains all locations for which BS i is the closest. Such definition is identical to that of a traditional cell, thus we equivalently denote V_i^1 as C_i .

In order to identify the neighboring UEs for each BS, we find it useful to generalize the above definition of Voronoi cell to the second order. More precisely, the second-order Voronoi tessellation $V_{i,j}^2$ denotes the set of UE locations for which the BSs in $\mathbf{z}_i$ and $\mathbf{z}_j$ are the two closest, and it is given by [22, 23]

$$V_{i,j}^2 = \{\mathbf{z} \in \mathbb{R}^2 \mid \cap_{l \in \{i,j\}} \{\|\mathbf{z} - \mathbf{z}_l\| \leq \|\mathbf{z} - \mathbf{z}_k\|\}, \quad \forall \mathbf{z}_k \in \Phi_b \setminus \{\mathbf{z}_i, \mathbf{z}_j\}\}. \quad (2.2)$$

Using the second-order Voronoi tessellation, we can now define the notion of extended cell C_i^E for BS i , given by (i) all UEs for which BS i is the closest, and (ii) all UEs for which BS i is the second closest. The extended cell C_i^E is given by

$$C_i^E = \cup_j V_{i,j}^2, \quad \forall \mathbf{z}_j \in \Phi_b \setminus \mathbf{z}_i. \quad (2.3)$$

According to the above definition, each UE that lies in C_i^E sees BS i as either its closest or second closest BS. The extended cell C_i^E includes the UEs located in C_i that are served by BS i , as well as the neighboring UEs which are most vulnerable to interference generated by BS i . These UEs constitute the cell neighborhood for BS i , which we denote by C_i^N and define as follows:

$$C_i^N = C_i^E \setminus C_i. \quad (2.4)$$

Figure 2.1 illustrates the concepts of first-order and second-order Voronoi tessellation, cell neighborhood, and extended cell. Figure 2.1a shows a realization of first-order Voronoi tessellation, where each BS i covers a cell C_i . Figure 2.1b depicts the corresponding second-order Voronoi tessellation, where each pair of BSs (i, j)

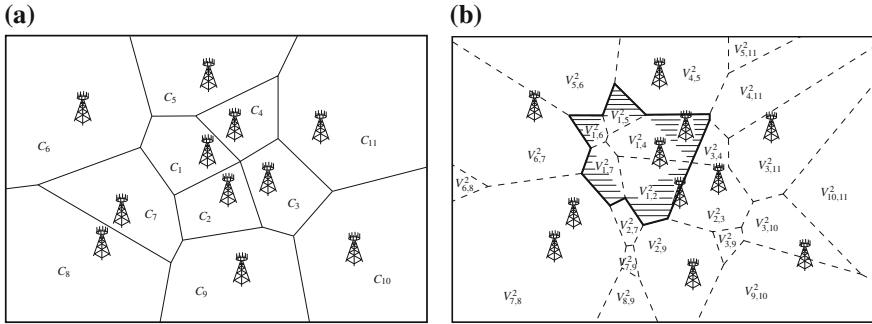


Fig. 2.1 Examples of (a) first-order and (b) second-order Voronoi tessellation. In (a), solid lines delimit first-order Voronoi cells C_i . In (b), dashed lines delimit second-order Voronoi cells $V_{i,j}^2$, solid lines delimit the extended cell C_i^E , and a shadowed region indicates the cell neighborhood C_i^N .

identifies a region $V_{i,j}^2$ (delimited by dashed lines), such that UEs located in $V_{i,j}^2$ have BS i and BS j as their closest and second closest, or vice versa. Figure 2.1b also shows the extended cell C_1^E for BS 1 (delimited by solid lines), which is composed by the first-order cell C_1 and by the neighborhood C_1^N (shadowed region).

In the following, by using the concepts of extended cell and cell neighborhood, we propose a CEA-ZF precoding scheme where each BS not only spatially multiplexes the associated UEs in C_i , but also suppresses the interference caused at the most vulnerable neighboring UEs in C_i^N . We note that each BS i can easily obtain a list of UEs in C_i^N by means of reference signal received power (RSRP) estimation. In fact, downlink RSRP measurements for a list of neighboring BSs are periodically sent by each UE for handover purposes [24].

2.2.1 Channel Model and Estimation

In this network, we model the channels between any pair of antennas as independent and identically distributed (i.i.d.) and quasi-static, i.e., the channel is constant during a sufficiently long coherence block, and varies independently from block to block.² Moreover, we assume that each channel is narrowband and affected by two attenuation components, namely, small-scale Rayleigh fading, and large-scale path loss.³ As such, the channel matrix from BS i to its K associated UEs can be written as

$$\mathbf{H}_i = \mathbf{R}_i^{\frac{1}{2}} \mathbf{X}_i, \quad (2.5)$$

where $\mathbf{R}_i = \text{diag}\{r_{i11}^{-\alpha}, \dots, r_{iKK}^{-\alpha}\}$ is the path loss matrix, with r_{ijk} denoting the distance from the BS i to UE k in cell j , i.e., associated with BS j . The constant α represents the path loss exponent, whereas $\mathbf{X}_i = [\mathbf{x}_{i11}, \dots, \mathbf{x}_{iKK}]^H$ is the $K \times N$ fading matrix, where $\mathbf{x}_{ijk} \sim CN(0, \mathbf{I}_N)$ is the channel fading vector between BS i and UE k in cell j . Due to the interference-limited nature of massive MIMO cellular networks, we neglect the effect of thermal noise [4].

In order to simultaneously amplify the desired signal at the intended UEs and suppress interference at other UEs, each BS requires CSI from all the UEs it serves. This CSI is obtained during the training phase, where some RBs are used for the transmission of pilot signals. Since the number of pilots, i.e., the number of RBs allocated to the training phase, is limited, these pilots must be reused across cells. Pilot reuse implies that the estimate for the channel between a BS and one of its UEs

²Note that the results obtained through the machinery of random matrix theory can be modified to model transmit antenna correlation [25].

³For the sake of tractability, the analysis presented here does not consider shadowing. Note that the results involving large-system approximations can be adjusted to account for the presence of shadowing as in [20]. Moreover, a generalized gamma approximation can still be used under shadowing, since the channel attenuation at a given UE follows a Rayleigh distribution [21].

is contaminated by the channels between the BS and UEs in other cells which share the same pilot [4–6, 26].

Pilot contamination can be a limiting factor for the performance of massive MIMO. In order to mitigate such phenomenon, nonuniversal pilot reuse has been proposed, where neighboring cells use different sets of mutually orthogonal pilots [15, 19]. Under nonuniversal pilot reuse, the total set of available pilot sequences is divided into subgroups, and different subgroups are assigned to adjacent cells. For a pilot reuse factor F , the same subgroup of orthogonal pilot sequences is reused in every F cells.

We denote by $M = \kappa L$ the number of available orthogonal pilots, with L being the number of symbols that can be transmitted within a time–frequency coherence block, and κ being the fraction of symbols that are allocated for channel estimation. For a time-division duplexing (TDD) system with $L = 2 \times 10^4$ and $\kappa = 5\%$, there would be $M = 1000$ orthogonal pilots, and therefore a pilot reuse factor $F = 7$ would allow to estimate the channels of 142 UEs per cell [10].⁴ As a general rule, a pilot reuse factor $F > 3$ is recommended in order to mitigate pilot contamination [28]. In this regard, we assume that there are sufficient pilot sequences to support a large enough pilot reuse factor, such that each BS can estimate the CSI of UEs in its own cell and in adjacent cells.

By using the MMSE criterion for pilot-based channel estimation, we can express the estimated small-scale fading $\hat{\mathbf{x}}_{ijk}$ between BS i and UE k in cell j as [17]

$$\mathbf{x}_{ijk} = \sqrt{1 - \tau_{ijk}^2} \hat{\mathbf{x}}_{ijk} + \tau_{ijk} \mathbf{q}_{ijk}, \quad (2.6)$$

where $\mathbf{q}_{ijk} \sim CN(\mathbf{0}, \mathbf{I}_N)$ is the normalized estimation error and τ_{ijk}^2 is the error variance, given by [9]

$$\tau_{ijk}^2 = \frac{1}{1 + \mathbb{E}[\gamma_{ijk}^{\text{CSI}}] \cdot \frac{M}{FK}}. \quad (2.7)$$

In (2.7), $\gamma_{ijk}^{\text{CSI}}$ is the SIR of the received pilot signal at the BS, given by

$$\gamma_{ijk}^{\text{CSI}} = \frac{r_{ijk}^{-\alpha} \|\mathbf{x}_{ijk}\|^2}{\sum_{l \in \Phi_p} r_{ilk}^{-\alpha} \|\mathbf{x}_{ilk}\|^2} \quad (2.8)$$

where Φ_p indicates the set of BSs that have their UEs reusing the same pilot as UE k in cell j and are thus generating pilot contamination. The estimated channel matrix at BS i can therefore be written as

⁴In a TDD system, downlink channels can be estimated through uplink pilots thanks to channel reciprocity. This makes the training time proportional to the number of UEs. A frequency-division duplexing (FDD) system requires a considerably longer training time, proportional to the number of BS antennas, and is therefore less suitable for massive MIMO [4, 27].

$$\hat{\mathbf{H}}_i = \mathbf{R}_i^{\frac{1}{2}} \hat{\mathbf{X}}_i. \quad (2.9)$$

2.2.2 Downlink Transmission

For the downlink transmission, we introduce two precoding schemes: (i) the conventional CEU-ZF precoder and (ii) the proposed CEA-ZF precoder.

2.2.2.1 Conventional CEU-ZF Precoding

With conventional zero forcing transmission, each BS i calculates the precoding vector to its UE k as [25]

$$\mathbf{w}_{u,ik} = \frac{1}{\sqrt{\zeta_{u,i}}} \left(\hat{\mathbf{H}}_i^H \hat{\mathbf{H}}_i \right)^{-1} \hat{\mathbf{h}}_{iik}, \quad (2.10)$$

where $\hat{\mathbf{H}}_i = [\hat{\mathbf{h}}_{ii1}, \dots, \hat{\mathbf{h}}_{iiK}]$ is the estimated channel matrix, and $\zeta_{u,i} = \text{tr}[(\hat{\mathbf{H}}_i^H \hat{\mathbf{H}}_i)]$ is a power normalization factor.

Note that ZF precoding aims at mitigating intra-cell crosstalk caused by spatial multiplexing to attain better SIR, where each BS projects signal for each intended UE onto the null space of other UEs receiving service in the same cell. In a multiuser MIMO system, it has been shown that ZF outperform maximum ratio transmission (MRT) in terms of per-cell sum-rate [15]. When the system dimensions make the ZF matrix inversion in (2.10) computationally expensive, a simpler truncated polynomial expansion can be employed with similar performance [29].

2.2.2.2 Proposed CEA-ZF Precoding

Unlike CEU-ZF precoding, where all spatial dimensions available at each BS i are used to multiplex UEs within cell C_i , the proposed CEA-ZF precoder exploits some spatial dimensions to suppress interference at the most vulnerable UEs, i.e., those lying in the BS's cell neighborhood C_i^N . In this fashion, BS i sacrifices some of its intra-cell power gain while trying to be a good neighbor who reduces the inter-cell interference to each of its adjacent cells. If such interference suppression is performed by all BSs in a distributed manner, the cell-edge performance as well as the overall data rate of the network can be improved. An illustration of the basic features of CEA-ZF is given in Fig. 2.2, where a multi-antenna BS spatially multiplexes its in-cell UEs while simultaneously suppressing interference at its neighboring UEs.

For BS i , we denote by K' the number of UEs lying in the cell neighborhood, where we omit the subscript i for notational convenience. We note that K' indicates the number of UEs for which BS i is the second closest, i.e., the number of neighboring

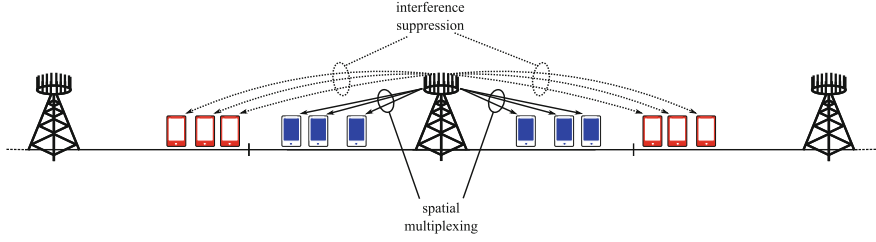


Fig. 2.2 Illustration of CEA-ZF, where a BS performs spatial multiplexing for in-cell UEs (*blue*) and interference suppression for neighboring UEs (*red*)

UEs for BS i . The proposed CEA-ZF precoder between BS i and UE k in cell i is then given by

$$\mathbf{w}_{a,ik} = \frac{1}{\sqrt{\zeta_{a,i}}} \left(\sum_{l=1}^K \hat{\mathbf{h}}_{iil} \hat{\mathbf{h}}_{iil}^H + \sum_{l=1}^{K'} \hat{\mathbf{h}}_{i\bar{i}l} \hat{\mathbf{h}}_{i\bar{i}l}^H \right)^{-1} \hat{\mathbf{h}}_{iik}, \quad (2.11)$$

with $\hat{\mathbf{h}}_{i\bar{i}l}$ denoting the estimated channel between BS i and the l -th neighboring UE, where the notation $\bar{i}$ indicates that BS i is the second closest BS for that particular UE. The constant $\zeta_{a,i}$ is chosen as an average power normalization factor, given by $\zeta_{a,i} = \sum_{k=1}^K \|\mathbf{w}_{a,ik}\|^2$.

It should be noted that the CEA-ZF precoder in (2.11) can be seen as a generalization of the two-cell precoder proposed in [11] to a nonsymmetric and non-pairwise scenario.

2.3 Coverage Analysis

In this section, we analyze the downlink SIR coverage of a massive MIMO cellular network with conventional CEU-ZF precoding and the proposed CEA-ZF precoding, and we provide a special case that helps us to grasp intuitions about the performance gain attained from CEA-ZF.

2.3.1 Preliminaries

2.3.1.1 Coverage Probability

In our analysis, the performance metric of interest is the coverage probability, defined as the probability that the received SIR γ at a generic UE is above a given threshold θ , i.e.,

$$P_c(\theta) = \mathbb{P}(\gamma \geq \theta), \quad \theta > 0. \quad (2.12)$$

We note that the coverage probability $P_c(\theta)$ provides information on the SIR distribution across the network, and it allows to evaluate the data rate performance at the cell edge.

2.3.1.2 SIR at a Typical UE

By applying Slivnyak's theorem to the stationary PPP of BSs, it is sufficient to evaluate the SIR of a typical UE at the origin [30]. In the following, we denote as *typical* the UE k associated with BS i , with a received signal given by

$$y_{ik} = P_i \mathbf{h}_{ik} \mathbf{w}_{ik} s_{ik} + \sum_{l=1, l \neq k}^K P_i \mathbf{h}_{il} \mathbf{w}_{il} s_{il} + \sum_{j \neq i}^{\infty} \sum_{l=1}^K P_j \mathbf{h}_{jl} \mathbf{w}_{jl} s_{jl}, \quad (2.13)$$

where $\mathbf{w}_{ik}$ is the normalized precoding vector from the serving BS i to the typical UE, and s_{ik} is the corresponding unit-power signal, i.e., $\mathbb{E}[|s_{ik}|^2] = 1$. The vector $\mathbf{w}_{ik}$ can take different forms, depending on the precoding scheme employed. The SIR at the typical UE can be written as

$$\gamma_{ik} = \frac{|\mathbf{h}_{ik} \mathbf{w}_{ik}|^2}{\sum_{l=1, l \neq k}^K |\mathbf{h}_{il} \mathbf{w}_{il}|^2 + I}, \quad (2.14)$$

where the first summation in the denominator represents the intra-cell interference, while I denotes the aggregate out-of-cell interference. The latter is given by

$$I = \sum_{j \neq i} \frac{g_{jik}}{r_{jik}^\alpha}, \quad (2.15)$$

where $g_{jik} = \sum_{l=1}^K |\mathbf{x}_{jik} \mathbf{w}_{jl}|^2$ is the effective small-scale fading from interfering BS j to UE k in cell i . From results in [31], we note that if the precoding vectors $\{\mathbf{w}_{jl}\}_{l=1}^K$ at BS j are mutually independent and satisfy $\sum_{l=1}^K \|\mathbf{w}_{jl}\|^2 = 1$, the effective channel fading is distributed as $g_{jik} \sim \Gamma(K, 1/K)$.

2.3.1.3 CSI Error

Under sufficient nonuniversal pilot reuse, a generic BS i can estimate the channels of all in-cell UEs as well as the channels of neighboring UEs. From (2.7) and (2.8), the CSI error variance for an in-cell UE and for a neighboring UE can be, respectively, written as

$$\tau^2 = \frac{1}{1 + \frac{t^{-\alpha}}{I_p} \cdot \frac{M}{FK}}, \quad (2.16)$$

$$\bar{\tau}^2 = \frac{1}{1 + \frac{s^{-\alpha}}{I_p} \cdot \frac{M}{FK}}, \quad (2.17)$$

where t and s denote the distance between a typical UE and its closest and second closest BS, respectively, and I_p is the pilot interference received during the training phase.

Under reuse factor F , clusters of F adjacent cells choose different subgroups of pilot sequences and do not cause interference, i.e., pilot contamination to each other. Therefore, each BS receives pilot contamination only from UEs lying outside the cluster of F cells, whose mean area can be calculated as F/λ [30]. This area can be approximated with a circle $B(0, R_e)$ of radius $R_e = \sqrt{F/(\lambda\pi)}$ [32], yielding the following mean interference

$$\begin{aligned} \mathbb{E}[I_p] &= \mathbb{E} \left[\sum_{x \in \Phi_p \cap B^c(0, R_e)} h_{x,o} \|x\|^{-\alpha} \right] \\ &= \frac{2(\lambda\pi/F)^{\frac{\alpha}{2}}}{\alpha - 2}, \end{aligned} \quad (2.18)$$

where $B^c(0, R_e)$ denotes the complement set of $B(0, R_e)$. By approximating the interference I_p with its mean [9] and by substituting (2.18) into (2.16) and (2.17), the CSI error can be expressed as a function of t and s as follows:

$$\tau^2 \approx \frac{1}{1 + \frac{M(\alpha-2)F^{\frac{\alpha}{2}-1}}{2K(\lambda\pi)^{\frac{\alpha}{2}}t^\alpha}}, \quad (2.19)$$

$$\bar{\tau}^2 \approx \frac{1}{1 + \frac{M(\alpha-2)F^{\frac{\alpha}{2}-1}}{2K(\lambda\pi)^{\frac{\alpha}{2}}s^\alpha}}. \quad (2.20)$$

2.3.2 Coverage Probability Under CEU-ZF

We now derive the coverage probability under CEU-ZF precoding. An approximation of the conditional SIR under CEU-ZF can be obtained in the large-system regime as follows.

Lemma 2.1 *Conditioned on the out-of-cell interference I_u and the intra-cell distance r_{iil} , $l \in \{1, \dots, K\}$, when $K, N \rightarrow \infty$ with $\beta = K/N < 1$, the SIR achieved by CEU-ZF precoding converges almost surely to the following quantity*

$$\gamma_{u,ik} \rightarrow \frac{(1 - \tau_{iik}^2)(1 - \beta)N}{(\tau_{iik}^2 r_{iik}^{-\alpha} + I_u)(r_{iik}^{\alpha} + R_k)}, \quad (2.21)$$

where R_k is given by

$$R_k = \sum_{l=1, l \neq k}^K r_{iil}^{\alpha}. \quad (2.22)$$

Proof See Appendix section “Proof of Lemma 2.1”.

Deriving the coverage probability requires knowledge of the distribution of R_k , which is the sum of $(K - 1)$ i.i.d. random variables (r.v.s) r_{iil}^{α} . The distance r_{iil} is a r.v. that follows a Rayleigh distribution $f_c(r)$, given by [33]

$$f_c(r) = 2\pi\lambda r e^{-\lambda\pi r^2}. \quad (2.23)$$

It can then be shown that r_{iil}^{α} follows a Weibull distribution with shape and scale parameters $2/\alpha$ and $(\lambda\pi)^{-\frac{\alpha}{2}}$, respectively [34]. As such, the distribution of R_k can be approximated by a generalized Gamma distribution as follows [35].

Assumption 1 The probability density function (pdf) $f_{R_k}(r)$ and cumulative density function (CDF) $F_{R_k}(r)$ of the r.v. R_k can be approximated as follows:

$$f_{R_k}(r) \approx \frac{\eta \mu^{\mu} r^{\eta\mu-1}}{\Omega^{\mu} \Gamma(\mu)} \exp\left(-\frac{\mu r^{\eta}}{\Omega}\right), \quad (2.24)$$

$$F_{R_k}(r) \approx \frac{1}{\Gamma(\mu)} \Gamma\left(\mu, \frac{\mu r^{\eta}}{\Omega}\right), \quad (2.25)$$

where $\Gamma(s, x) = \int_0^x t^{s-1} e^{-t} dt$ is the lower incomplete gamma function, and $\Omega = \mathbb{E}[R_k^{\eta}]$ is a scale parameter, given by

$$\Omega = \left[\frac{\mu^{\frac{1}{\eta}} \Gamma(\mu) \mathbb{E}[R_k]}{\Gamma\left(\mu + \frac{1}{\eta}\right)} \right]^{\eta} \quad (2.26)$$

while μ and η are solutions of the following equations:

$$\frac{\Gamma^2\left(\mu + \frac{1}{\eta}\right)}{\Gamma(\mu) \Gamma\left(\mu + \frac{2}{\eta}\right) - \Gamma^2\left(\mu + \frac{1}{\eta}\right)} = \frac{\mathbb{E}[R_k]}{\mathbb{E}[R_k^2] - \mathbb{E}[R_k]}, \quad (2.27)$$

$$\frac{\Gamma^2\left(\mu + \frac{2}{\eta}\right)}{\Gamma(\mu) \Gamma\left(\mu + \frac{4}{\eta}\right) - \Gamma^2\left(\mu + \frac{2}{\eta}\right)} = \frac{\mathbb{E}[R_k^2]}{\mathbb{E}[R_k^4] - \mathbb{E}[R_k^2]}. \quad (2.28)$$

The quantities $\mathbb{E}[R_k]$, $\mathbb{E}[R_k^2]$, and $\mathbb{E}[R_k^4]$ are the first, second, and fourth moment of the r.v. R_k , respectively, and can be calculated as

$$\mathbb{E}[R_k] = \frac{K-1}{(\lambda\pi)^{\frac{\alpha}{2}}} \Gamma\left(1 + \frac{\alpha}{2}\right), \quad (2.29)$$

$$\mathbb{E}[R_k^2] = \frac{K-1}{(\lambda\pi)^\alpha} \left[\Gamma(1 + \alpha) + (K-2) \Gamma^2\left(1 + \frac{\alpha}{2}\right) \right], \quad (2.30)$$

$$\begin{aligned} \mathbb{E}[R_k^4] = & \frac{K-1}{(\lambda\pi)^{2\alpha}} \left[(K-2)(K-3)(K-4) \Gamma^4\left(1 + \frac{\alpha}{2}\right) \right. \\ & + 3(K-2) \Gamma^2(1 + \alpha) + 4(K-2) \Gamma\left(1 + \frac{\alpha}{2}\right) \Gamma\left(1 + \frac{3\alpha}{2}\right) \\ & \left. + \Gamma(1 + 2\alpha) + 6(K-2)(K-3) \Gamma(1 + \alpha) \Gamma^2\left(1 + \frac{\alpha}{2}\right) \right]. \end{aligned} \quad (2.31)$$

By using the approximated distribution of R_k , we can now obtain the coverage probability of a massive MIMO cellular network under CEU-ZF.

Theorem 2.1 *The coverage probability of a massive MIMO cellular network under CEU-ZF precoding can be approximated as*

$$\mathbb{P}(\gamma_{u,ik} \geq \theta) \approx \frac{1}{\Gamma(\mu)} \int_0^\infty \Gamma\left(\mu, \frac{\mu}{\Omega} \left[\frac{(1-\tau^2)(1-\beta)Nr^\alpha}{\theta\left(\tau^2 + \frac{2\pi\lambda r^2}{\alpha-2}\right)} - r^\alpha \right]^\eta\right) f_c(r) dr, \quad (2.32)$$

where τ^2 is given in (2.19), and $f_c(r)$ is given by (2.23).

Proof See Appendix section “Proof of Theorem 2.1”.

Here, it should be noted that although coverage probability of a multiuser MIMO cellular network with conventional CEU-ZF precoding has also been derived in [31, 36, 37], the result in (2.32) provides an approximation that involves only one integration and is therefore easier to be evaluated. The accuracy of this approximation will be verified in Fig. 2.4.

2.3.3 Coverage Probability Under CEA-ZF

We now derive the coverage probability under the proposed CEA-ZF precoder. Similar to the CEU-ZF, an approximation of the conditional SIR under CEA-ZF can be obtained in the large-system regime as follows.

Lemma 2.2 *Conditioned on the out-of-cell interference I_a , the intra-cell distance r_{iil} with $l \in \{1, \dots, K\}$, the distance r_{iik} between the typical UE and its second*

closest BS, and the standard deviation τ_{ik} of the corresponding CSI error, when $K, N \rightarrow \infty$ with $\beta = K/N < 1$ and $\beta' = K'/N < 1$, the SIR of CEA-ZF converges almost surely to a quantity given by

$$\gamma_{a,ki} \rightarrow \frac{(1 - \tau_{ik}^2)(1 - \beta - \beta')N}{\left(\tau_{ik}^2 r_{ik}^{-\alpha} + \tau_{ik}^2 r_{ik}^{-\alpha} + I_a\right)(r_{ik}^{\alpha} + R_k)} \quad (2.33)$$

Proof See Appendix section “Proof of Lemma 2.2”.

Using the above results, we are now able to derive the coverage probability under CEA-ZF precoding.

Theorem 2.2 *The coverage probability of a massive MIMO cellular network under CEA-ZF can be obtained as*

$$\mathbb{P}(\gamma_{a,ik} \geq \theta) \approx \int_0^\infty \int_t^\infty \Gamma\left(\mu, \frac{\mu}{\Omega} \left[\frac{(1 - \tau^2)(1 - 2\beta)Ns^\alpha}{\theta \left(\bar{\tau}^2 + \tau^2 \frac{s^\alpha}{i^\alpha} + \frac{2\pi\lambda s^2}{\alpha - 2} \right)} - t^\alpha \right]^\eta\right) \frac{f_{s|c}(s, t)f_c(t)}{\Gamma(\mu)} ds dt, \quad (2.34)$$

where τ^2 and $\bar{\tau}^2$ are given in (2.19) and (2.20), respectively, and

$$f_{s|c}(s, t) = 2\pi\lambda s e^{-\lambda\pi(s^2 - t^2)}. \quad (2.35)$$

Proof See Appendix section “Proof of Theorem 2.2”.

Equations (2.32) and (2.34) quantify how some of the key features of a cellular network, i.e., deployment strategy, interference, and impairments in the channel estimation phase, affect the coverage probability provided by massive MIMO BSs. These equations will be used in our numerical study in the following section, which investigates the impact of different system parameters on the network performance. Before going to that directly, we provide a special case that helps us grasp the first order understanding from network parameter impacts, stated in the following corollary.

Corollary 2.1 *In the case that $\tau^2 = \bar{\tau}^2 = 0$, when $\alpha = 4$ and $N \rightarrow \infty$, the coverage probability under CEU-ZF and CEA-ZF converges respectively as follows:*

$$\mathbb{P}(\gamma_{u,ik} \geq \theta) \rightarrow 1 - \frac{2(K - 1)\theta}{(1 - \beta)N} \approx 1 - \frac{2\beta\theta}{1 - \beta}, \quad (2.36)$$

$$\mathbb{P}(\gamma_{a,ik} \geq \theta) \rightarrow 1 - \frac{4(K - 1)(K + 4)\theta^2}{(1 - 2\beta)^2 N^2} \approx 1 - \left(\frac{2\beta\theta}{1 - 2\beta} \right)^2. \quad (2.37)$$

Proof See Appendix section “Proof of Corollary 2.1”.

Several conclusions can be drawn from the above corollary, we state them as follows:

- CEA-ZF outperforms CEU-ZF in terms of coverage probability only when $\beta < \frac{1}{2} - \frac{\sqrt{\theta^2 + 2\theta}}{4 + 2\theta}$, or equivalently, $\frac{2(4+\theta)K}{2+\theta-\sqrt{\theta^2+2\theta}} < N$. This observation implies that CEA-ZF requires sufficient spatial dimensions to attain better performance.
- When BS antenna number N grows to be large, the coverage probability under CEU-ZF and CEA-ZF increases as $1/N$ and $1/N^2$, respectively. This observation demonstrates that CEA-ZF is more effective precoding scheme in massive MIMO cellular networks.

2.4 Numerical Results and Discussion

In this section, we first show simulation results that confirm the accuracy of our analytical framework. After that, we provide numerical results to show the performance gain attained by the proposed CEA-ZF precoder, and we discuss how data rate and coverage are affected by the number of scheduled UEs and the channel estimation accuracy.

2.4.1 Simulation Validations

Unless differently specified, we use the following parameters for path loss exponent, BS density, and number of UE, respectively: $\alpha = 3.8$, $\lambda = 10^{-6}$, and $K = 10$.

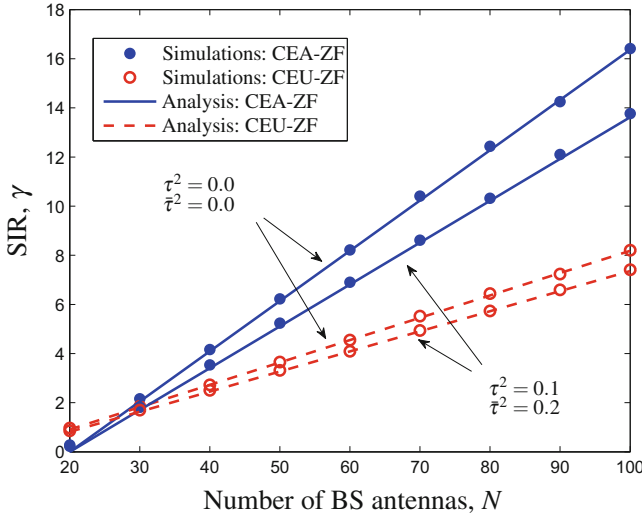


Fig. 2.3 Downlink SIR for CEU-ZF and CEA-ZF versus number of BS antennas, for scenarios with and without CSI error

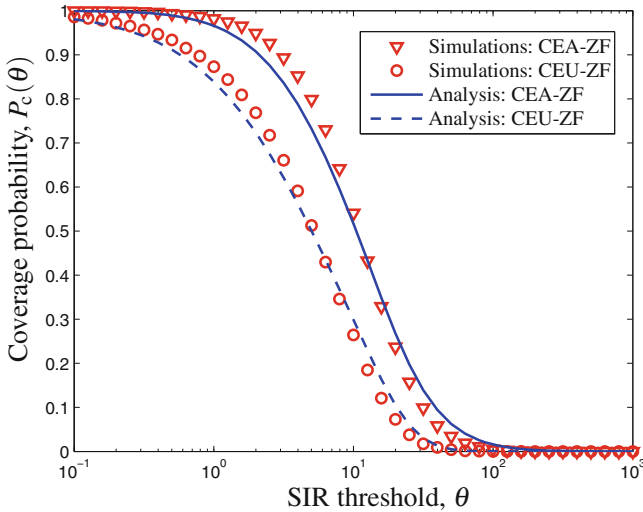


Fig. 2.4 Coverage probability under CEU-ZF and CEA-ZF precoding

In Fig. 2.3a, we depict the downlink SIR achieved by a typical UE of a massive MIMO cellular network as a function of the number of BS antennas N , under different precoding schemes and transmit CSI errors. The figure shows a negligible difference between simulations and analytical results, which confirms the accuracy of Lemmas 2.1 and 2.2. We also note that the SIR values obtained for ZF precoding are consistent with the ones obtained in [17].

Figure 2.4 compares the simulated coverage probability to the analytical results derived in Theorems 2.1 and 2.2. The coverage probability is plotted versus the SIR threshold at the typical UE. The figure shows that analytical results and simulations fairly well match and follow the same trend, thus confirming the accuracy of the theorems.

2.4.2 Numerical Results

Unless otherwise stated, the following system parameters will be used: BS deployment density $\lambda = 10^{-6}$, number of scheduled UEs per cell $K = 20$, path loss exponent $\alpha = 3.8$, and pilot reuse factor $F = 7$. We note that neglecting the training time does not affect the fairness of our performance comparison between CEU-ZF and CEA-ZF precoding.

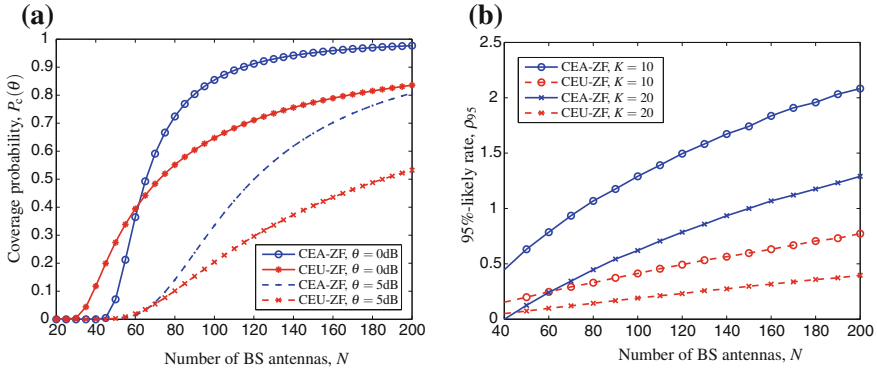


Fig. 2.5 Impact of antenna number on network performance: **a** coverage probability versus number of BS antennas for $K = 20$ scheduled UEs per cell, under CEU-ZF and CEA-ZF precoding. **b** 95 %-likely rate versus number of BS antennas, with CEU-ZF and CEA-ZF precoding

2.4.2.1 Impact of Large Antenna Array

In Fig. 2.5, we show the effect of BS antenna number N on the network performance. Figure 2.5a depicts the coverage probability under CEU-ZF and CEA-ZF precoding as a function of the number of BS antennas N , for two different SIR thresholds θ . The following can be observed: (i) with a sufficient number of BS antennas, i.e., $N > 3K$, CEA-ZF outperforms CEU-ZF precoding, and (ii) CEA-ZF requires significantly fewer antennas to achieve high coverage probabilities compared to CEU-ZF precoding.

In Fig. 2.5b, we compare the 95 %-likely rate under the two precoding schemes. The 95 %-likely rate (denoted by ρ_{95}) is defined as the rate achievable by at least 95 % of the UEs in the network, and it can be regarded as the worst rate any scheduled UE may expect to receive when located at the cell edge [3, 4]. While the 95 %-likely rates of both CEU-ZF and CEA-ZF precoding benefit from a larger number of BS antennas N , the proposed CEA-ZF precoder achieves a significantly larger 95 %-likely rate compared to conventional CEU-ZF, and the gain increases as N grows.

In summary, the proposed CEA-ZF precoder outperforms conventional CEU-ZF precoding from several perspectives. CEA-ZF provides better coverage than CEU-ZF, especially in the massive MIMO regime, i.e., when BSs are equipped with a large number of antennas, N . While CEA-ZF can attain high coverage probability with reasonable values of N , a significantly larger number of antennas is required to achieve the same coverage under CEU-ZF precoding. The proposed CEA-ZF precoder also achieves a larger sum-rate per cell, and a significantly larger 95 %-likely rate. The latter is especially important, being the worst data rate that any scheduled UE can expect to receive.

2.4.2.2 Impact of Imperfect CSI

We now study the impact of the channel estimation error on coverage and edge rates. To this end, we vary the CSI error variance τ^2 and $\bar{\tau}^2$ at in-cell UEs and neighboring UEs, respectively, while keeping their ratio constant as $\mathbb{E}[\bar{\tau}^2]/\mathbb{E}[\tau^2] = 1.8$. In the following, we set the SIR threshold as $\theta = 0$ dB and number of BS antennas as $N = 100$.

Figure 2.6a shows the coverage probability as a function of the CSI error for various values of scheduled UEs per cell. Although the presence of a CSI error degrades the coverage probability of both CEU-ZF and CEA-ZF precoding, it can be seen that CEA-ZF significantly outperforms conventional CEU-ZF for low-to-moderate values of the CSI error. Under a large CSI error, CEA-ZF still performs as well as CEU-ZF as long as the number of scheduled UEs per cell is controlled, e.g., $K = 10$ or $K = 20$ in the figure.

Figure 2.6b depicts the 95 %-likely rate as a function of the CSI error variance for CEU-ZF and CEA-ZF precoding. Once again, CEA-ZF significantly outperforms conventional CEU-ZF for low-to-moderate values of the CSI error, while the 95 %-likely rates of both precoders degrade and achieve similar values under very poor CSI quality, i.e., large values of τ^2 and $\bar{\tau}^2$.

As expected, pilot contamination can negatively affect the achievable data rates by degrading the quality of the CSI available at the BSs. In the presence of very large channel estimation errors, the performance of CEA-ZF precoding degrades and converges to the one of conventional CEU-ZF precoding. In fact, the cell-edge suppression mechanism employed by CEA-ZF relies on the accuracy of the measured channels, and the promised gains in terms of coverage and 95 %-likely rate cannot be achieved. It is therefore desirable to control the amount of pilot contamination received during the channel estimation phase, for example by designing appropriate pilot allocation schemes.

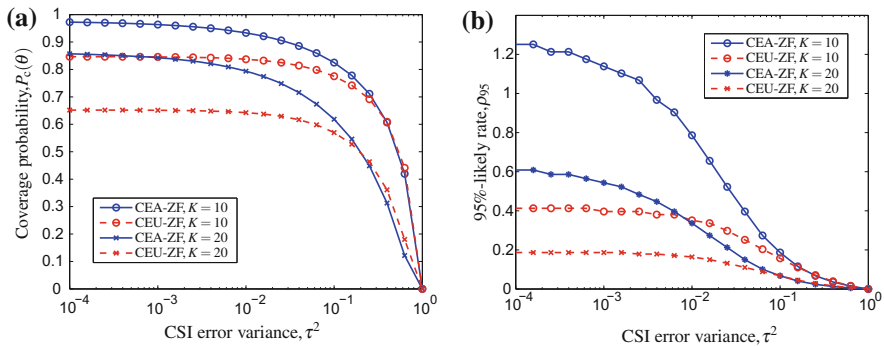


Fig. 2.6 Comparison of coverage probability and 95 %-likely rate with CSI error variance. In (a), Coverage probability plotted as a function of CSI error variance. In (b), 95 %-likely rate plotted as a function of CSI error variance

2.5 Conclusion

In this chapter, we proposed the CEA-ZF precoder that exploits the excess spatial degrees of freedom available at massive MIMO BSs to suppress inter-cell interference at the most vulnerable UEs in the network. The CEA-ZF specifically targets those neighboring UEs close to the BS coverage area, thus requiring fewer spatial dimensions to mitigate inter-cell interference, and leaving more dimensions for intra-cell spatial multiplexing. Moreover, it can be implemented in a distributed fashion.

In order to model practical deployments, we analyzed the performance of CEA-ZF and conventional CEU-ZF precoding in a random asymmetric cellular network. By using CEA-ZF precoder, we showed that a better network coverage is attainable. More importantly, the 95 %-likely rate, namely, the minimum data rate that any UE can expect to achieve, is significantly improved. The latter is of particular interest, given the ambitious edge rate requirements set for 5G, which aims at uninterrupted user experience. Our study also quantified the impact of imperfect CSI, confirming the importance of controlling the amount of pilot contamination during the channel estimation phase.

Appendix

Proof of Lemma 2.1

We treat the out-of-cell interference as noise, then as $K, N \rightarrow \infty$ with $\beta = K/N < 1$, the SIR converges to [25]

$$\gamma_{u,ik} \rightarrow \frac{1 - \tau_{ik}^2}{\Upsilon \cdot r_{ik}^{-\alpha} \tau_{ik}^2 + \Psi \cdot I_u}, \quad a.s. \quad (2.38)$$

where Υ and Ψ are given by

$$\Upsilon = \frac{1}{\phi} \frac{1}{N} \text{tr} \mathbf{R}_i^{-1}, \quad (2.39)$$

$$\Psi = \frac{\psi}{\frac{N}{K} \phi^2 - \psi} \frac{1}{K} \frac{1}{N} \text{tr} \mathbf{R}_i^{-1} \quad (2.40)$$

and ψ and ϕ are given, respectively, by

$$\psi = \frac{1}{N} \text{tr} \left(\mathbf{I}_N + \frac{K}{N} \frac{1}{\phi} \mathbf{I}_N \right)^{-2} = \frac{1}{\left(1 + \frac{\beta}{\phi} \right)^2}, \quad (2.41)$$

$$\phi = \frac{1}{N} \text{tr} \left[\left(1 + \frac{\beta}{\phi} \right) \mathbf{I}_N \right]^{-1} = \frac{1}{1 + \frac{\beta}{\phi}}. \quad (2.42)$$

By solving (2.41) and (2.42), we obtain $\phi = 1 - \beta$ and $\psi = \phi^2$. By substituting ψ and ϕ in (2.39) and (2.40), respectively, the following holds

$$\gamma = \psi = \frac{1}{N} \frac{1}{1 - \beta} \sum_{l=1}^K r_{iil}^\alpha. \quad (2.43)$$

Lemma 2.1 then follows by substituting (2.43) into (2.38).

Proof of Theorem 2.1

Consider a typical UE k in cell i and located at the origin. We denote the distance between the typical UE and its serving BS i as $r_{iik} = t$. As such, we can approximate the out-of-cell interference I_u by its mean, which can be computed as

$$\begin{aligned} \mathbb{E}[I_u] &= \mathbb{E} \left[\sum_{x \in \Phi_b \setminus i} \frac{g_{xik}}{\|x\|^\alpha} \right] \\ &\stackrel{(a)}{=} \int_t^\infty r^{-\alpha} r dr = \frac{2\pi\lambda t^{-(\alpha-2)}}{\alpha-2}, \end{aligned} \quad (2.44)$$

where (a) is obtained by applying the Campbell's theorem [30]. By substituting (2.44) into (2.21), the conditional SIR received at the typical UE can be approximated as

$$\gamma_{u,ik} \approx \frac{(1 - \tau_{iik}^2)(1 - \beta)N}{\left(\tau_{iik}^2 t^{-\alpha} + \frac{2\pi\lambda t^{-(\alpha-2)}}{\alpha-2} \right) (t^\alpha + R_k)}. \quad (2.45)$$

The coverage probability can then be calculated as

$$\begin{aligned} \mathbb{P}(\gamma_{u,ik} \geq \theta) &\approx \mathbb{E} \left[\mathbb{P} \left(R_k \leq \frac{(1 - \tau_{iik}^2)(1 - \beta)N t^\alpha}{\left(\tau_{iik}^2 + \frac{2\pi\lambda t^2}{\alpha-2} \right) \theta} - t^\alpha \middle| r_{iik} = t \right) \right] \\ &= \frac{1}{\Gamma(\mu)} \mathbb{E} \left[\Gamma \left(\mu, \frac{\mu}{\Omega} \left[\frac{(1 - \tau^2)(1 - \beta)N t^\alpha}{\theta \left(\tau^2 + \frac{2\pi\lambda t^2}{\alpha-2} \right)} - t^\alpha \right]^\eta \right) \right]. \end{aligned} \quad (2.46)$$

Since the pdf of t has been given in (2.23), Theorem 2.1 then follows from (2.46) by deconditioning t .

Proof of Lemma 2.2

We start with the minimum-mean-square-error (MMSE) version of the CEA-ZF precoder, which includes a regularization term $\rho \mathbf{I}_N$, given by

$$\tilde{\mathbf{w}}_{a,ik} = \frac{1}{\sqrt{\tilde{\zeta}_{a,i}}} \left(\rho N \mathbf{I}_N + \sum_{l=1}^K \hat{\mathbf{h}}_{iil} \hat{\mathbf{h}}_{iil}^H + \sum_{l=1}^{K'} \hat{\mathbf{h}}_{i\bar{l}l} \hat{\mathbf{h}}_{i\bar{l}l}^H \right)^{-1} \hat{\mathbf{h}}_{iik}, \quad (2.47)$$

where $\tilde{\zeta}_{a,i} = \sum_{k=1}^K \|\tilde{\mathbf{w}}_{a,ik}\|^2$. The SIR at the typical UE can be written as

$$\gamma_{a,ik} = \frac{|\mathbf{h}_{iik}^H \tilde{\mathbf{w}}_{a,ik}|^2}{\sum_{l \neq k} |\mathbf{h}_{iik}^H \tilde{\mathbf{w}}_{a,il}|^2 + \sum_{l=1}^K |\mathbf{h}_{iik}^H \tilde{\mathbf{w}}_{a,\bar{l}l}|^2 + I_a}. \quad (2.48)$$

Substituting (2.9) into the numerator of (2.48), and using the matrix inversion lemma and the rank-1 permutation lemma [25], the received signal power converges to the following limit in the large-system regime:

$$|\hat{\mathbf{h}}_{iik}^H \tilde{\mathbf{w}}_{a,ik}|^2 \rightarrow \frac{1}{\zeta_{a,i}} \frac{(r_{iik}^{-\alpha} \Lambda_i)^2}{(1 + r_{iik}^{-\alpha} \Lambda_i)^2}, \quad \text{as } N \rightarrow \infty \quad (2.49)$$

where Λ_i is the solution of the following fix point equation [38]

$$\Lambda_i = \frac{1}{\rho + \frac{1}{N} \sum_{l=1}^K \frac{r_{iil}^{-\alpha}}{1 + \Lambda_l r_{iil}^{-\alpha}} + \frac{1}{N} \sum_{l=1}^{K'} \frac{r_{i\bar{l}l}^{-\alpha}}{1 + \Lambda_l r_{i\bar{l}l}^{-\alpha}}}. \quad (2.50)$$

We next deal with the first two summations in the denominator of (2.48), which are the intra-cell interference from the serving BS and the inter-cell interference from the second closest interfering BS. Similarly, by using the rank-1 permutation again, the large-system limit for these interference read as

$$\sum_{l \neq k} |\mathbf{h}_{iik}^H \tilde{\mathbf{w}}_{a,il}|^2 = \sum_l \frac{\frac{1}{\zeta_{a,i}} \frac{r_{iil}^{-\alpha}}{N} \left(-\frac{\partial \Lambda_i}{\partial \rho} \right)}{(1 + r_{iil}^{-\alpha} \Lambda_i)^2} \left[\frac{(1 - \tau_{iik}^2) r_{iik}^{-\alpha}}{(1 + r_{iik}^{-\alpha} \Lambda_i)^2} + \tau_{iik}^2 r_{iik}^{-\alpha} \right],$$

and

$$\sum_l |\mathbf{h}_{iik}^H \tilde{\mathbf{w}}_{a,\bar{l}l}|^2 = \sum_l \frac{\frac{1}{\zeta_{a,\bar{i}}} \frac{r_{i\bar{l}l}^{-\alpha}}{N} \left(-\frac{\partial \Lambda_{\bar{i}}}{\partial \rho} \right)}{(1 + r_{i\bar{l}l}^{-\alpha} \Lambda_{\bar{i}})^2} \left[\frac{(1 - \tau_{iik}^2) r_{iik}^{-\alpha}}{(1 + r_{iik}^{-\alpha} \Lambda_{\bar{i}})^2} + \tau_{iik}^2 r_{iik}^{-\alpha} \right],$$

respectively. For the power normalization factor $\tilde{\zeta}_{a,i}$ and $\tilde{\zeta}_{a,\bar{i}}$, the deterministic equivalence under large-system regime can be derived in a similar way as

$$\tilde{\zeta}_{a,i} \rightarrow \frac{1}{N} \sum_{l=1}^K \frac{r_{iil}^{-\alpha} \left(-\frac{\partial \Lambda_i}{\partial \rho} \right)}{\left(1 + r_{iil}^{-\alpha} \Lambda_i \right)^2}, \quad \tilde{\zeta}_{a,\bar{i}} \rightarrow \frac{1}{N} \sum_{l=1}^K \frac{r_{iil}^{-\alpha} \left(-\frac{\partial \Lambda_{\bar{i}}}{\partial \rho} \right)}{\left(1 + r_{iil}^{-\alpha} \Lambda_{\bar{i}} \right)^2}.$$

As such, we have the deterministic equivalence of SIR being as

$$\mathcal{V}_{a,ik} = \frac{\left(1 - \tau_{iik}^2 \right) \frac{(r_{iik}^{-\alpha} \Lambda_i)^2}{(1 + r_{iik}^{-\alpha} \Lambda_i)^2} \left| \frac{1}{N} \sum_{l=1}^K \frac{r_{iil}^{-\alpha} \left(-\frac{\partial \Lambda_i}{\partial \rho} \right)}{\left(1 + r_{iil}^{-\alpha} \Lambda_i \right)^2} \right|^{-1}}{\frac{(1 - \tau_{iik}^2) r_{iik}^{-\alpha}}{(1 + r_{iik}^{-\alpha} \Lambda_i)^2} + \tau_{iik}^2 r_{iik}^{-\alpha} + \frac{(1 - \tau_{iik}^2) r_{iik}^{-\alpha}}{(1 + r_{iik}^{-\alpha} \Lambda_{\bar{i}})^2} + \tau_{iik}^2 r_{iik}^{-\alpha} + I_a}. \quad (2.51)$$

Finally, by letting $\rho \rightarrow 0$, each term in (2.51) that contains Λ_i , respectively, converges to

$$\frac{(r_{iik}^{-\alpha} \Lambda_i)^2}{(1 + r_{iik}^{-\alpha} \Lambda_i)^2} = \frac{(r_{iik}^{-\alpha} \rho \Lambda_i)^2}{(\rho + r_{iik}^{-\alpha} \rho \Lambda_i)^2} \rightarrow 1, \quad (2.52)$$

$$\frac{(1 - \tau_{iik}^2) r_{iik}^{-\alpha}}{(1 + r_{iik}^{-\alpha} \Lambda_i)^2} = \frac{(1 - \tau_{iik}^2) \rho^2 r_{iik}^{-\alpha}}{(\rho + \rho r_{iik}^{-\alpha} \Lambda_i)^2} \rightarrow 0, \quad (2.53)$$

$$\left| \frac{1}{N} \sum_{l=1}^K \frac{r_{iil}^{-\alpha} \left(-\frac{\partial \Lambda_i}{\partial \rho} \right)}{(1 + r_{iil}^{-\alpha} \Lambda_i)^2} \right|^{-1} \rightarrow \frac{(1 - \beta - \beta') N}{\sum_{l=1}^K r_{iil}^{\alpha}}. \quad (2.54)$$

Lemma 2.2 then follows by substituting (2.52), (2.53), and (2.54) into (2.51).

Proof of Theorem 2.2

We consider a typical UE k of BS i that locates at the origin, and denote the distance between the UE and its associated BS as $r_{iik} = t$ and the distance from the UE to its second closest BS as $r_{iik} = s$. As such, we can approximate the out-of-cell interference I_a by its mean based on Campbell's theorem [30], as follows

$$\begin{aligned} \mathbb{E}[I_a] &= \mathbb{E} \left[\sum_{x \in \Phi_b \setminus \{i, \bar{i}\}} \frac{g_{xik}}{\|x\|^\alpha} \right] \\ &= \frac{2\pi \lambda s^{-(\alpha-2)}}{\alpha - 2}. \end{aligned} \quad (2.55)$$

By substituting (2.55) into (2.33), the conditional SIR received at the typical UE can be approximated as

$$\gamma_{a,ik} \approx \frac{(1 - \tau_{ik}^2)(1 - \beta - \beta')N}{\left(\tau_{ik}^2 t^{-\alpha} + \tau_{ik}^2 s^{-\alpha} + \frac{2\pi\lambda s^{-(\alpha-2)}}{\alpha-2}\right)(t^\alpha + R_k)}. \quad (2.56)$$

The coverage probability can then be approximated as

$$\begin{aligned} \mathbb{P}(\gamma_{a,ik} \geq \theta) &\approx \mathbb{P}\left(R_k \leq \frac{(1 - \tau_{ik}^2)(1 - \beta - \beta')N\theta^{-1}}{\tau_{ik}^2 t^{-\alpha} + \tau_{ik}^2 s^{-\alpha} + \frac{2\pi\lambda s^{-(\alpha-2)}}{\alpha-2}} - t^\alpha\right) \\ &= \frac{1}{\Gamma(\mu)} \mathbb{E}\left[\gamma\left(\mu, \frac{\mu}{\Omega} \left[\frac{(1 - \tau^2)(1 - \beta - \beta')Ns^\alpha}{\theta(\bar{\tau}^2 + \tau^2 \frac{s^\alpha}{t^\alpha} + \frac{2\pi\lambda s^2}{\alpha-2})} - t^\alpha\right]^\eta\right)\right]. \end{aligned} \quad (2.57)$$

Furthermore, from the mass transport theorem [30], we have the following relationship:

$$\mathbb{E}[|C_i^E|] = 2\mathbb{E}[|C_i|]. \quad (2.58)$$

where $|\cdot|$ denotes the Lebesgue measure. As a result, the expectation of K' can be calculated as

$$\begin{aligned} \mathbb{E}[K'] &\stackrel{(a)}{=} \lambda_u \mathbb{E}[|C_i^E|] - \lambda_u \mathbb{E}[|C_i|] \\ &= K, \end{aligned} \quad (2.59)$$

where (a) follows from the fact that mean number of UEs in area A is given by $\lambda_u |A|$. Using this fact, we approximate the random variable K' by its mean K . Theorem 2.2 then follows from (2.57) by deconditioning on t and s , with their pdf given in (2.23) and (2.35), respectively.

Proof of Corollary 2.1

Let $\tau^2 = 0$, $\bar{\tau}^2 = 0$, and $\alpha = 4$. We start with the asymptotic result for the network coverage probability under CEU-ZF. Using the Fubini's theorem [34], coverage probability in (2.32) can be written as

$$\mathbb{P}(\gamma_{u,ik} \geq \theta) \approx \frac{1}{\Gamma(\mu)} \int_0^\infty \exp\left(-\left(\frac{\Omega}{\mu}t\right)^{\frac{1}{\eta}} \frac{2\pi(\lambda\pi)^2\theta}{(1-\beta)(\alpha-2)N}\right) t^{\mu-1} e^{-t} dt. \quad (2.60)$$

As $N \rightarrow \infty$, we have

$$\exp\left(-\left(\frac{\Omega}{\mu}t\right)^{\frac{1}{\eta}} \frac{2\pi(\lambda\pi)^2\theta}{(1-\beta)(\alpha-2)N}\right) \sim 1 - \left(\frac{\Omega}{\mu}t\right)^{\frac{1}{\eta}} \frac{2\pi(\lambda\pi)^2\theta}{(1-\beta)(\alpha-2)N}. \quad (2.61)$$

By substituting (2.61) into (2.60), the coverage probability under CEU-ZF read as

$$\mathbb{P}(\gamma_{u,ik} \geq \theta) \sim 1 - \frac{\Gamma\left(\frac{1}{\eta} + \mu\right)}{\Gamma(\mu)} \left(\frac{\Omega}{\mu}t\right)^{\frac{1}{\eta}} \frac{(\lambda\pi)^2\theta}{(1-\beta)N}, \quad \text{as } N \rightarrow \infty \quad (2.62)$$

and (2.36) follows from using (2.26) and (2.29) into (2.62).

Next, we deal with the asymptotic result for the network coverage probability under CEA-ZF. When $\tau^2, \bar{\tau}^2 \rightarrow 0$, we have the following expression for coverage probability in (2.34) by using the Fubini's theorem [34],

$$\begin{aligned} & \mathbb{P}(\gamma_{a,ik} \geq \theta) \\ & \approx \frac{1}{\Gamma(\mu)} \int_0^\infty \left(1 + \frac{(\Omega t/\mu)^{\frac{1}{\eta}} (\lambda\pi)^2\theta}{(1-2\beta)N}\right) \exp\left(-\frac{(\Omega t/\mu)^{\frac{1}{\eta}} (\lambda\pi)^2\theta}{(1-2\beta)N}\right) t^{\mu-1} e^{-t} dt \end{aligned} \quad (2.63)$$

As $N \rightarrow \infty$, we have

$$\exp\left(-\frac{(\Omega t/\mu)^{\frac{1}{\eta}} (\lambda\pi)^2\theta}{(1-2\beta)N}\right) \sim 1 - \frac{(\Omega t/\mu)^{\frac{1}{\eta}} (\lambda\pi)^2\theta}{(1-2\beta)N}. \quad (2.64)$$

By substituting (2.64) into (2.63), the coverage probability under CEA-ZF read as

$$\mathbb{P}(\gamma_{a,ik} \geq \theta) \sim 1 - \frac{\Gamma\left(\frac{2}{\eta} + \mu\right)}{\Gamma(\mu)} \left(\frac{\Omega}{\mu}t\right)^{\frac{2}{\eta}} \frac{(\lambda\pi)^4\theta^2}{(1-2\beta)^2N^2}, \quad \text{as } N \rightarrow \infty \quad (2.65)$$

and (2.37) follows from using (2.26) and (2.30) into (2.65).

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