

Chapter 2

Local Detection of Correlations in Composite Quantum Systems

Abstract We discuss a method for the detection of quantum discord between two subsystems, when access is limited to only one of them. The method is based on local operations on the accessible subsystem and makes use of the interaction to reveal the bipartite correlations via the measurable, local subsystem dynamics. We further present experimental realizations with a trapped ion and with single photons, as well as additional theoretical examples.

2.1 The Local Detection Protocol

In the present section we will review a method which was originally developed in the author's Diploma thesis (Gessner 2011), and published in Gessner and Breuer (2011). During the work on the present dissertation, this method was refined and extended (Gessner and Breuer 2013b; Gessner et al. 2014a, b). Here we present it from an updated perspective and restrict to providing only those details and derivations which have not been presented elsewhere.

In the previous chapter, we discussed the central role of correlated quantum states in many applications of quantum information theory, as well as for the dynamics of open quantum systems. The detection of correlations can pose a difficult task, especially when considering multi-particle systems, since standard methods usually require access to all subsystems (Mintert et al. 2005; Horodecki et al. 2009; Gühne and Tóth 2009; Auccaise et al. 2011; Silva et al. 2013; Walter et al. 2013). It was discussed in the previous chapter that a complete characterization of the correlation properties of large ensembles does not represent a scalable approach. Also in most quantum communication setups, each party typically has only local access to his share of the total quantum state.

Our strategy consists in detecting correlated quantum states in a possibly multipartite setting by accessing only a small subsystem. This has the advantage that operations are always limited to the small, low-dimensional, controllable Hilbert space of the accessible subsystem, regardless of the total size of the system. Since we will not even assume knowledge over the remaining subsystems, this scenario is applicable in an open quantum system context, where correlations may emerge between the a

well-controlled system and its inaccessible environment. As we discussed in the previous chapter, such system-environment correlations can have significant influence on the dynamics of the reduced subsystem. Factorizing initial conditions are indeed central to standard attempts to describe the dynamics of open quantum systems, for both the microscopic and the formal approach. Here, we harness the impact of correlations on the reduced dynamics to reveal the bipartite correlations to the subsystem.

2.1.1 Local Witness for Bipartite Quantum Discord

To develop the *local detection protocol*, we assume a high degree of control over the quantum system described by the Hilbert space $\mathcal{H}_S$, which is coupled to an inaccessible second system, described by $\mathcal{H}_E$. Here the subscripts S and E are inspired from the separation of the total system in system and environment, which can be an artificial decomposition of a many-body quantum system and does not have to describe an actual open system coupled to a heat bath. We further assume no knowledge about the second system, the interaction between the two systems and the properties of a given initial quantum state ρ of the composite system $\mathcal{H}_S \otimes \mathcal{H}_E$. The goal is to detect and possibly quantify the bipartite correlations contained in the quantum state ρ . Since the total state ρ is only partially accessible, the complete obtainable information at a single moment in time is contained in the corresponding reduced density matrix for the accessible subsystem, given, after diagonalization, by its spectral decomposition

$$\rho_S = \text{Tr}_E \rho = \sum_i p_i |i\rangle\langle i|, \quad (2.1)$$

with the *local eigenbasis* $|i\rangle$. Unless we encounter a pure state $\rho_S = |\varphi\rangle\langle\varphi|$, in which case we could conclude that the total state is a product state, this does not immediately allow us to draw conclusions about the correlations of the total state ρ . This is due to the fact that the statistical mixture of ρ_S can have two different causes. On the one hand, the observer's ignorance about the reduced state can be a result of the quantum correlations of the total state: A pure maximally entangled total state leads to maximally mixed reduced states in both subsystems. On the other hand, if the total state is already mixed, then the reduced state will be mixed, even in the absence of quantum correlations. To distinguish these two cases we need to take on further steps.

The general strategy will be to reveal the correlations via their effect on the local dynamics. Let $U(t)$ be the unitary time evolution operator describing the coherent evolution of the total system. Note that we assume unitary dynamics for convenience, and the local detection protocol does not rely on this assumption (Gessner and Breuer 2013b). The subsystem state after time t will be given by

$$\rho_S(t) = \text{Tr}_E\{U(t)\rho U^\dagger(t)\}. \quad (2.2)$$

This evolution contains the influence of the correlations which are possibly present in the initial state ρ . To detect these correlations, we will compare this evolution with the subsystem evolution after removing correlations from the total state with the help of a carefully designed local operation.

To be specific, we define a *local dephasing operation* to produce a classically correlated reference state where all discord-type correlations are removed. The action of the local dephasing operation on an arbitrary operator X is defined in terms of the eigenbasis $\{|i\rangle\}$ of ρ_S as (Gessner and Breuer 2011)

$$\Phi(X) = \sum_i |i\rangle\langle i| X |i\rangle\langle i|. \quad (2.3)$$

It is important to notice that this constitutes a *local* operation only on the Hilbert space $\mathcal{H}_S$, and does not affect the second subsystem at all. If we apply this operation locally to the total bipartite initial state ρ , we obtain

$$\rho' = (\Phi \otimes \mathbb{I})\rho = \sum_i (|i\rangle\langle i| \otimes \mathbb{I})\rho(|i\rangle\langle i| \otimes \mathbb{I}). \quad (2.4)$$

This operation corresponds to a local, non-selective measurement of ρ_S and induces complete dephasing in the local basis $\{|i\rangle\}$. Later in this chapter, we will demonstrate possibilities to realize this method with trapped ions and photonic polarization qubits. Recall from Sect. 1.2.1 that the correlations assessed by the quantum discord can indeed be defined on the basis of this dephasing operation. In fact, the state ρ is classical (zero discord) if and only if

$$\rho = \rho'. \quad (2.5)$$

Otherwise it contains quantum discord. This terminology is motivated by the fact that $\{|i\rangle\}$ can be interpreted as the eigenbasis of an observable. In the case of a classical state, one may perform non-selective measurements on this local observable without disturbing the total quantum state, and the quantum uncertainty for this observable vanishes (Girolami et al. 2013). Furthermore, uncorrelated product states are always classical, whereas all entangled states contain nonzero discord. For further details see Sect. 1.2.1.

Thus, we see that the local dephasing operation has no effect on the initial quantum state if there are no discord-type correlations present in ρ . Next, we study the effect of the local dephasing on the reduced density matrices of both subsystems. Since the dephasing, by construction, is a local operation on $\mathcal{H}_S$, the reduced state of the inaccessible subsystem will not be affected. Moreover, the dephasing is carried out in the eigenbasis of ρ_S , which means that ρ_S will also be invariant under such an operation. This implies that both reduced states of system and environment (or of the accessible and inaccessible subsystems) are not at all affected by the local dephasing.

Finally, we note that the state ρ' is always a state of zero discord, as it can be readily cast into the form of Eq. (1.3). Formal proofs of these statements can be found in Gessner (2011), Gessner and Breuer (2013b).

These properties allow for a straight-forward interpretation of the local dephasing operation: This locally implementable operation removes all discord-type correlations, while the local subsystem distributions are unaffected. We can therefore interpret the state ρ' as the incoherent counterpart to ρ (Gessner and Breuer 2013b). For example in the case of a pure state $\rho = |\Psi\rangle\langle\Psi|$, the local eigenbasis $\{|i\rangle\}$ corresponds to the local Schmidt basis of $|\Psi\rangle$, and the state ρ' is the incoherent mixture (with same weights) of the states which are coherently superposed to produce the entangled state $|\Psi\rangle$ (Gessner and Breuer 2013b). Yet, in order to directly check whether or not the total state changed under the influence of the local dephasing, we would still require access to both subsystems.

To reveal the possible change of the total state to the subsystem, we make use of the latter's *time evolution*. After preparing the reference state ρ' by locally dephasing the initial state ρ , we again observe the local time evolution in the accessible subsystem, given by

$$\rho'_S(t) = \text{Tr}_E\{U(t)\rho'U^\dagger(t)\}. \quad (2.6)$$

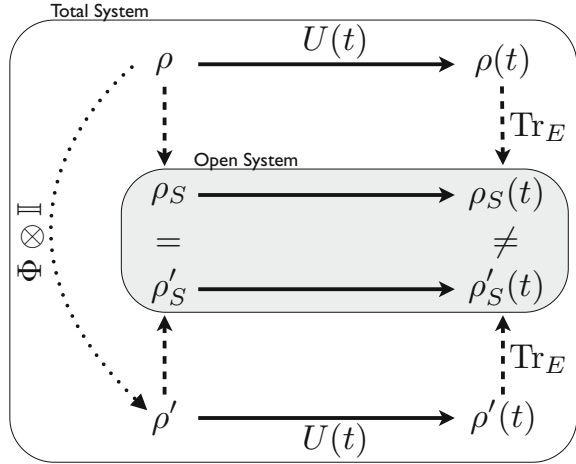
As discussed before, at $t = 0$ we will find that, in the local subsystem, we start out in the exact same reduced state as without the dephasing, $\rho_S(0) = \rho'_S(0)$. Furthermore, the unitary time evolution operator is the same, since we did not change the environment or the couplings between the two subsystems. One detail, however, did change: The discord-type correlations with the environment have been removed completely by the dephasing. Thus, if we find that the local time evolution changes due to the dephasing-induced removal of the quantum discord, we have found a witness for the initial quantum discord in the state ρ (Gessner and Breuer 2011, 2013b).

At the same time, this demonstrates the influence of initial system-environment correlations on the open-system dynamics. It becomes obvious that the reduced dynamics can no longer be defined in terms of a dynamical map which is independent of the environmental state, since the same initial state $\rho_S(0)$ is mapped onto two different final states. The fact that any difference between $\rho_S(t)$ and $\rho'_S(t)$ demonstrates the presence of quantum discord in the initial state ρ also implies that any approximate description of the resulting dynamics in terms of factorized initial conditions would be inappropriate (Laine et al. 2010; Gessner and Breuer 2011, 2013b). The steps of the local detection protocol are summarized in the diagram in Fig. 2.1.

2.1.2 Local Bound for the Minimum Entanglement Potential

The question now arises, can we go beyond mere detection and also quantify correlations using this protocol? In fact, the difference between two quantum states can

Fig. 2.1 The local detection protocol, as outlined in Sect. 2.1.1, is based on a local dephasing operation Φ which removes the discord-type correlations from the initial state ρ . If the subsystem dynamics is affected by this operation, the correlations of the initial state are revealed to observables of the subsystem. Adapted from Gessner et al. (2014a)



be quantified in terms of distance measures, such as the trace distance. Let us start by discussing the quantification of quantum discord in terms of trace-distance measures.

Considering the results of the previous section, an evident attempt to quantify the quantum discord of the state ρ is based on the disturbance (Luo 2008) which is induced by the local dephasing operation (2.4), for instance, via the trace distance (Gessner et al. 2014a)

$$D(\rho) = \|\rho - \rho'\|. \quad (2.7)$$

Due to the contractivity property of the trace distance under positive operations (Ruskai 1994), such as the partial trace and unitary time evolution, a lower bound for this quantity is given by the local trace distance, which can be measured in the accessible subsystem (Gessner and Breuer 2013b),

$$\begin{aligned} d(t) = \|\rho_S(t) - \rho'_S(t)\| &= \|\text{Tr}_E\{U(t)(\rho - \rho')U^\dagger(t)\}\| \\ &\leq \|U(t)(\rho - \rho')U^\dagger(t)\| \\ &= \|\rho - \rho'\|, \quad \forall t \geq 0. \end{aligned} \quad (2.8)$$

Since this holds for arbitrary $t \geq 0$, the best strategy to provide a tight bound to the global trace distance (2.7) consists in monitoring the local time evolution for long times and taking the maximum value (Gessner et al. 2014a)

$$d_{\max} = \max_t \|\rho_S(t) - \rho'_S(t)\| \leq \|\rho - \rho'\|. \quad (2.9)$$

From an information-theoretic perspective, geometric measures of discord-type correlations should determine the minimal distance to the set of all dephased or classical states (Horodecki et al. 2009; Modi et al. 2012). A priori, it is unclear

whether the dephasing-induced disturbance (2.7) minimizes this distance. Indeed, when compared to measures involving a minimization, one finds that, for certain mixed states, Eq. (2.7), overestimates the amount of correlations in ρ (Girolami et al. 2011; Campbell et al. 2011). Instead of the fixed dephasing in the eigenbasis, one could therefore consider to extend the dephasing process to arbitrary basis sets, defined by a set of orthogonal projection operators $\Pi = \{\Pi_1, \Pi_2, \dots\}$ as

$$\Phi_\Pi(\rho) = \sum_i (\Pi_i \otimes \mathbb{I}) \rho (\Pi_i \otimes \mathbb{I}). \quad (2.10)$$

Minimizing the distance over all projections,

$$D_{\min}(\rho) = \min_{\Pi} \|\rho - \Phi_\Pi(\rho)\|, \quad (2.11)$$

yields an information-theoretically sound measure for quantum discord, the *minimal dephasing disturbance*. When the accessible subsystem is a qubit, $\mathcal{H}_S = \mathbb{C}^2$, this measure coincides with the *minimum entanglement potential* (Nakano et al. 2013), which measures the maximal amount of entanglement which can be activated from discord in a measurement (Streltsov et al. 2011b; Piani et al. 2011; Adesso et al. 2014); see also Sect. 1.2.2. When ρ describes a pure state, this measure reduces to the *negativity* (Vidal and Werner 2002),

$$\mathcal{N}(\rho) = \frac{\|\rho^\Gamma\| - 1}{2}, \quad (2.12)$$

where ρ^Γ denotes the partial transpose of ρ in an arbitrary basis of one of the two subsystems (Piani and Adesso 2012). This result is expected since in the case of pure states, we know that the only type of correlation is entanglement. Furthermore, the trace distance and the negativity are both linear in the *Schmidt coefficients* λ_i . These are defined via a representation of the total state $\rho = |\Psi\rangle\langle\Psi|$ in terms of arbitrary bases of $\mathcal{H}_S$ and $\mathcal{H}_E$, followed by a singular value decomposition, which leads to (Schmidt 1907)

$$|\Psi\rangle = \sum_i \lambda_i |\varphi_i\rangle \otimes |\chi_i\rangle, \quad (2.13)$$

with the Schmidt basis $|\varphi_i\rangle \otimes |\chi_i\rangle$ of $\mathcal{H}_S \otimes \mathcal{H}_E$.

The question remains, which is the basis that minimizes the distance (2.11) in the above situation? The local eigenbasis, considered for dephasing in Eq. (2.4), here, is given by the local Schmidt basis $|\varphi_i\rangle$. In the case of $\mathcal{H}_S = \mathbb{C}^2$, the dephasing disturbance (2.7) in this particular basis $\Pi_0 = \{|\varphi_0\rangle\langle\varphi_0|, |\varphi_1\rangle\langle\varphi_1|\}$ is given by

$$\begin{aligned}
D(\rho) &= \|\rho - \Phi_{\Pi_0}(\rho)\| \\
&= \|\lambda_1 \lambda_0 |\varphi_1\rangle\langle\varphi_0| \otimes |\chi_1\rangle\langle\chi_0| + \lambda_1 \lambda_0 |\varphi_0\rangle\langle\varphi_1| \otimes |\chi_0\rangle\langle\chi_1|\| \\
&= \frac{1}{2} \text{Tr} \sqrt{C^\dagger C},
\end{aligned} \tag{2.14}$$

where the 2×2 matrix of operators on $\mathcal{H}_E$ can be represented in the local Schmidt basis as

$$C = \lambda_0 \lambda_1 \begin{pmatrix} 0 & |\chi_1\rangle\langle\chi_0| \\ |\chi_0\rangle\langle\chi_1| & 0 \end{pmatrix}. \tag{2.15}$$

Since $\lambda_{0,1} \geq 0$, the trace distance yields

$$D(\rho) = \lambda_0 \lambda_1 = \mathcal{N}(\rho). \tag{2.16}$$

Employing the same decomposition, the negativity yields (for arbitrary dimensions of $\mathcal{H}_S$)

$$\mathcal{N}(|\Psi\rangle\langle\Psi|) = \frac{1}{2} \sum_{\substack{i,j \\ i \neq j}} \lambda_i \lambda_j. \tag{2.17}$$

We find that the disturbance on a pure state induced by dephasing in its local qubit eigenbasis is quantified by the negativity. Since the *minimal* disturbance must also yield the negativity, we have shown that the local eigenbasis minimizes the expression (2.11) (Gessner et al. 2014b).

In general, however, we do not know whether the total state is pure or mixed. The goal is therefore to find a local lower bound for the minimal disturbance. This expression can indeed be estimated by dephasing in many different bases instead of only the local eigenbasis (Gessner et al. 2014b). Based on a particular selection of projection operators Π , we introduce the local distance

$$d_\Pi(t) = \|\rho_S(t) - \rho_S^\Pi(t)\|, \tag{2.18}$$

where

$$\rho_S^\Pi(t) = \text{Tr}_E\{U(t)\Phi_\Pi(\rho)U^\dagger(t)\} \tag{2.19}$$

describes the local time evolution after dephasing. Since the contractivity property which leads to the inequality (2.8) is independent of the dephasing basis, we can extend the statement to obtain a lower bound for the minimal dephasing disturbance (Gessner et al. 2014b)

$$d_{\min}(\rho) = \max_t \min_\Pi d_\Pi(t) \leq D_{\min}(\rho). \tag{2.20}$$

This quantity is accessible when control is restricted to the subsystem since both the dephasing operations and the reduced time evolution are processes which affect the local system. Note, however, that unlike to what we observed when dephasing was implemented in the local eigenbasis, here, we may find a nonzero difference between $\rho_S(0)$ and $\rho_S^\Pi(0)$.

The general procedure then consists in dephasing the initial state systematically in different bases. For each dephasing process, one observes the subsequent local time evolution. At each time t , the minimum value of all local distances (there exists one for each dephasing basis) is kept. Out of all the minimum values (one for each recorded time-step), one keeps the largest to obtain the tightest possible bound.

2.1.3 Efficacy of the Method

The local detection method is—in principle—always applicable, regardless of the environment to which the locally accessible system couples. Whether or not the presence of initial correlations is revealed, depends on the dynamics of system and environment. In the extreme case of no interaction between the two subsystems it is impossible to map the global correlations to the local subsystem dynamics (Gessner and Breuer 2013b).

The distance measures introduced in the previous section can be evaluated easily when, instead of the trace distance, the Hilbert–Schmidt distance, based on the Hilbert–Schmidt norm $\|X\|_2^2 = \text{Tr} X^\dagger X$, is used, even when the minimization procedure is included (Dakić et al. 2010). It further allows for analytic determination of the unitary expectation value and its variance (Gessner and Breuer 2013b) using Haar measure integration techniques (Gessner and Breuer 2013a). The resulting average local distance is given by (Gessner and Breuer 2011)

$$\int d\mu(U) \|\text{Tr}_E\{U(\rho - \rho')U^\dagger\}\|_2^2 = \frac{d_S^2 d_E - d_E}{d_S^2 d_E^2 - 1} \|\rho - \rho'\|_2^2, \quad (2.21)$$

where $d\mu$ denotes the Haar measure on the unitary group, and d_S and d_E are the dimensions of system and environment, respectively. Together with the corresponding variance, which can be found in Gessner and Breuer (2013b), this result can be used to assess the success probability of the local detection method. The unitary average value (2.21) shows that generic dynamical systems will reveal the quantum discord (right-hand side) to the open-system dynamics (left-hand side); see also Gessner (2011).

In many cases we can also establish a direct connection between the time average over the local Hilbert–Schmidt distance and the dephasing disturbance as quantified via the Hilbert–Schmidt distance (Gessner et al. 2014a). The Hilbert–Schmidt distance is, however, not contractive under positive maps, which can cause unphysical increases of these quantities under trivial operations (Ozawa 2000; Piani 2012). In

this thesis, we will mostly employ the trace distance, which does not suffer from these limitations (Paula et al. 2013).

The result (2.21) further shows that the local signal of these correlations shrinks as the dimension of the environment increases when the dynamical evolution is sufficiently generic to be described by the random-matrix average. For such complex system-environment interactions, this suggests the necessity of a finite-dimensional Hilbert space in order to be able to successfully reveal initial correlations using the present method. In the following sections, we present a series of theoretical and experimental applications of the local detection method to study its efficacy in different scenarios. These include examples of infinite-dimensional, memoryless environments where the correlations are revealed, and others where they cannot be detected.

2.2 Trapped-Ion Experiment

In this section we describe a trapped-ion experiment where the local detection protocol was implemented to detect discord-type correlations between the electronic qubit and the motional degree of freedom of the same trapped ion, while restricting to local operations on the electronic qubit (Gessner et al. 2014a).

2.2.1 Resonant Laser-Ion Interactions

Before describing the actual experiment regarding the detection of discord-type correlations, we derive a general theoretical description of the quantum dynamics induced by laser light on an electronic two-level transition as a function of the laser detuning, restricting the treatment to a single oscillator mode (Leibfried et al. 2003a; Meekhof et al. 1996; Gardiner et al. 1997). The derived expressions are then applied to the local detection scenario in Sect. 2.2.3. Our initial point in this section is the Hamiltonian (1.8), however, here we do not make use of the Lamb-Dicke limit (Roghani et al. 2011).

We start by transforming to a moving reference frame with the unitary operator $U(t) = e^{-i\omega_L\sigma_+\sigma_-t}$. Subsequently, we apply the *rotating wave approximation*, that is, we ignore rapidly oscillating terms corresponding to *sums* of optical frequencies over terms whose slow oscillation is determined by *differences* of optical frequencies; see, for example (Cohen-Tannoudji et al. 1992). This yields the time-independent Hamiltonian (Blockley et al. 1992; Vogel and Filho 1995)

$$H = -\hbar\Delta\sigma_+\sigma_- + \hbar\nu a^\dagger a + \frac{\hbar\Omega}{2}(\sigma_+e^{i\eta(a+a^\dagger)}e^{i\varphi} + \sigma_-e^{-i\eta(a+a^\dagger)}e^{-i\varphi}). \quad (2.22)$$

In the following, we consider different choices for the detuning Δ between the qubit transition and the laser frequency. To find the most relevant terms we transform to the interaction picture with respect to $H_0 = -\hbar\Delta\sigma_+\sigma_- + \hbar\nu a^\dagger a$. This transformation is effectively implemented via the replacements $a \rightarrow ae^{-i\nu t}$, $a^\dagger \rightarrow a^\dagger e^{i\nu t}$, $\sigma_\pm \rightarrow \sigma_\pm e^{\mp i\Delta t}$, which yield

$$H_I(t) = \frac{\hbar\Omega}{2} \left(\sigma_+ e^{i\eta(ae^{-i\nu t} + a^\dagger e^{i\nu t})} e^{-i\Delta t} e^{i\varphi} + \sigma_- e^{-i\eta(ae^{-i\nu t} + a^\dagger e^{i\nu t})} e^{i\Delta t} e^{-i\varphi} \right). \quad (2.23)$$

Using the Campbell-Baker-Hausdorff relation (Mandel and Wolf 1995), we obtain

$$e^{\pm i\eta(a^\dagger e^{i\nu t} + ae^{-i\nu t})} = e^{-\eta^2/2} \sum_{k,l=0}^{\infty} \frac{(\pm i\eta a^\dagger)^k (\pm i\eta a)^l}{k!l!} e^{i(k-l)\nu t}. \quad (2.24)$$

For additional resonance conditions $\Delta = m\nu$, $m \in \mathbb{Z}$, some of the terms do not oscillate at all. These will make the most relevant contribution to the dynamics as the oscillating terms can be ignored after a second application of the rotating wave approximation. This works best for a large ratio ν/Ω . Then, the rapid oscillations of frequencies ν and higher average to zero within the relevant time scale given by $1/\Omega$.

2.2.1.1 Carrier Transition

For $\Delta = 0$ we address the *carrier* transition since the laser is resonant with the atomic transition. The relevant terms are found by the condition $k = l$. Neglecting the counter-rotating terms we obtain

$$H_I^{(0)} = \frac{\hbar\Omega}{2} e^{-\eta^2/2} \sum_{k=0}^{\infty} \frac{(-\eta^2)^k (a^\dagger)^k a^k}{k!^2} (\sigma_+ e^{i\varphi} + \sigma_- e^{-i\varphi}). \quad (2.25)$$

This decomposes into a family of effective 2×2 -Hamiltonians $H_n^{(0)} = \hbar\Omega_n^{(0)}/2$ ($\sigma_+ e^{i\varphi} + \sigma_- e^{-i\varphi}$) with

$$\Omega_n^{(0)} = \Omega e^{-\eta^2/2} \sum_{k=0}^n \frac{(-\eta^2)^k n!}{(n-k)!k!^2}. \quad (2.26)$$

When $\eta \ll 1$, this frequency reduces to Ω , thereby reproducing the approximate expression given in Eq. (1.10) in this special case. Multiple application of the creation and annihilation operators on a Fock state $|n\rangle$ with n vibrational excitations yields

$$\begin{aligned}
(a^\dagger)^k |n\rangle &= \sqrt{\frac{(n+k)!}{n!}} |n+k\rangle, \\
a^k |n\rangle &= \sqrt{\frac{n!}{(n-k)!}} |n-k\rangle,
\end{aligned} \tag{2.27}$$

which leads to

$$(a^\dagger)^k a^k |n\rangle = \frac{n!}{(n-k)!} |n\rangle. \tag{2.28}$$

For an arbitrary state of the form $|\Psi(t)\rangle = \sum_{n=0}^{\infty} (g_n(t)|g, n\rangle + e_n(t)|e, n\rangle)$, where $|g\rangle$ and $|e\rangle$ denote ground and excited state of the two-level system, respectively, we solve Schrödinger's equation to obtain the following time evolution

$$\begin{aligned}
g_n(t) &= g_n(0) \cos\left(\frac{\Omega_n^{(0)} t}{2}\right) - i e^{-i\varphi} e_n(0) \sin\left(\frac{\Omega_n^{(0)} t}{2}\right), \\
e_n(t) &= e_n(0) \cos\left(\frac{\Omega_n^{(0)} t}{2}\right) - i e^{i\varphi} g_n(0) \sin\left(\frac{\Omega_n^{(0)} t}{2}\right).
\end{aligned} \tag{2.29}$$

Beyond coherent operations as discussed in Sect. 1.3.1.6, Rabi oscillations on the carrier transition are also used to estimate the ion's temperature. For a thermal distribution, $\rho(0) = |g\rangle\langle g| \otimes \sum_n p_n |n\rangle\langle n|$, we get

$$p_e(t) = \langle e | \rho_S(t) | e \rangle = \sum_n p_n \left| \sin\left(\frac{\Omega_n^{(0)} t}{2}\right) \right|^2 \tag{2.30}$$

where the Boltzmann-distributed populations of excited vibrational state can be expressed in terms of the average occupation number $\bar{n}$ as $p_n = \bar{n}^n / (\bar{n} + 1)^{n+1}$ (Leibfried et al. 2003a). This relation is often used to estimate the temperature of thermal excitations by fitting the experimentally measured Rabi-flop to the prediction (2.30) with the free parameter $\bar{n}$. This method has to be taken with some precaution since other dephasing effects (see Sect. 1.3.1.8) may also lead to a decay of the Rabi oscillations which is hard to distinguish from increased temperatures.

2.2.1.2 First Red Sideband Transition

If we choose a red detuning corresponding to the trap frequency, $\Delta = -\nu$, we drive the *first red sideband* transition. The time-independent terms in Eqs. (2.23) and (2.24) are those that satisfy the condition $l = k + 1$ in the sum corresponding to e^{ikx} and $k = l + 1$ for e^{-ikx} . The interaction Hamiltonian assumes a form close to the on-resonance Jaynes–Cummings Hamiltonian,

$$H_I^{(-1)} = i\eta \frac{\hbar\Omega}{2} e^{-\eta^2/2} \left(\sum_{k=0}^{\infty} \frac{(-\eta^2)^k (a^\dagger)^k a^k}{k!(k+1)!} \sigma_+ a e^{i\varphi} - \sigma_- a^\dagger e^{-i\varphi} \sum_{k=0}^{\infty} \frac{(-\eta^2)^k (a^\dagger)^k a^k}{k!(k+1)!} \right). \quad (2.31)$$

Here the state space splits up in families of pairs $\{|e, n\rangle, |g, n+1\rangle\}$ coupled via the Hamiltonians

$$H_n^{(-1)} = \frac{i\hbar\Omega_n^{(1)}}{2\sqrt{n+1}} (\sigma_+ a e^{i\varphi} - \sigma_- a^\dagger e^{-i\varphi}), \quad (2.32)$$

and effective Rabi frequencies

$$\Omega_n^{(1)} = \eta \sqrt{n+1} \Omega e^{-\eta^2/2} \sum_{k=0}^n \frac{(-\eta^2)^k n!}{k!(k+1)!(n-k)!}. \quad (2.33)$$

As already discussed in Sect. 1.3, the induced Jaynes–Cummings dynamics describes a variety of different physical systems, including cavity quantum electrodynamics, where light modes inside a cavity take on the role of the phonons.

2.2.1.3 First Blue Sideband Transition

The *first blue sideband* is addressed for $\Delta = \nu$. Dominant contributions to the e^{ikx} term are identified by $k = l + 1$, and by $l = k + 1$ for the e^{-ikx} term. We obtain the *anti Jaynes–Cummings* interaction,

$$H_I^{(+1)} = i\eta \frac{\hbar\Omega}{2} e^{-\eta^2/2} \left(\sigma_+ a^\dagger e^{i\varphi} \sum_{k=0}^{\infty} \frac{(-\eta^2)^k (a^\dagger)^k a^k}{k!(k+1)!} - \sum_{k=0}^{\infty} \frac{(-\eta^2)^k (a^\dagger)^k a^k}{k!(k+1)!} \sigma_- a e^{-i\varphi} \right). \quad (2.34)$$

This Hamiltonian couples the pairs $\{|e, n+1\rangle, |g, n\rangle\}$ with the 2×2 Hamiltonians

$$H_n^{(+1)} = \frac{i\hbar\Omega_n^{(1)}}{2\sqrt{n+1}} (\sigma_+ a^\dagger e^{i\varphi} - \sigma_- a e^{-i\varphi}). \quad (2.35)$$

This interaction cannot be found in cavity quantum electrodynamics experiments, since it would correspond to the creation of a photon, while at the same time the atom gets excited. Since in an ion trap this interaction is induced via an external laser, which can add or dissipate arbitrary amounts of energy into or out of the system, such processes can now occur. In fact, they form the basis of sideband cooling techniques, where vibrational excitations, phonons, are removed.

2.2.1.4 Arbitrary Order Sidebands

The previous considerations can be generalized to arbitrary order with $\Delta = \pm m\nu$, $m \in \mathbb{N}$ (Cahill and Glauber 1969; Wineland and Itano 1979; Leibfried et al. 2003a). We speak of the m th red (blue) sideband whenever the sign is negative (positive). Let us first consider the m th blue sideband. Equations (2.23) and (2.24) yield the conditions $k = l + m$, for e^{ikx} , and $l = k + m$, for e^{-ikx} , respectively. The Hamiltonian yields

$$H_I^{(+m)} = \frac{\hbar\Omega}{2} e^{-\eta^2/2} \left(\sigma_+ (i\eta a^\dagger)^m e^{i\varphi} \sum_{k=0}^{\infty} \frac{(-\eta^2)^k (a^\dagger)^k a^k}{k!(k+m)!} + \sum_{k=0}^{\infty} \frac{(-\eta^2)^k (a^\dagger)^k a^k}{k!(k+m)!} \sigma_- (-i\eta a)^m e^{-i\varphi} \right), \quad (2.36)$$

and splits up in 2×2 blocks of

$$H_n^{(+m)} = \sqrt{\frac{n!}{(n+m)!}} \frac{\hbar\Omega_n^{(m)}}{2} (i^m \sigma_+ (a^\dagger)^m e^{i\varphi} + (-i)^m \sigma_- a^m e^{-i\varphi}), \quad (2.37)$$

coupling $\{|g, n\rangle, |e, n+m\rangle\}$, with

$$\Omega_n^{(m)} = \eta^m \sqrt{\frac{(n+m)!}{n!}} \Omega e^{-\eta^2/2} \sum_{k=0}^n \frac{(-\eta^2)^k n!}{k!(k+m)!(n-k)!}. \quad (2.38)$$

It is possible to express this term as a function of the generalized Legendre polynomials (Morse and Feshbach 1953),

$$L_n^{(\alpha)}(x) = \sum_{k=0}^n (-1)^k \binom{n+\alpha}{n-k} \frac{x^k}{k!}, \quad (2.39)$$

as

$$\Omega_n^{(m)} = \eta^m \sqrt{\frac{n!}{(n+m)!}} \Omega e^{-\eta^2/2} L_n^{(m)}(\eta^2). \quad (2.40)$$

The evolution of a general state,

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} [g_n(t)|g, n\rangle + e_{n+m}(t)|e, n+m\rangle], \quad (2.41)$$

is given by

$$\begin{aligned} g_n(t) &= g_n(0) \cos\left(\frac{\Omega_n^{(m)} t}{2}\right) + (-i)^{m+1} e^{-i\varphi} e_{n+m}(0) \sin\left(\frac{\Omega_n^{(m)} t}{2}\right), \\ e_{n+m}(t) &= e_{n+m}(0) \cos\left(\frac{\Omega_n^{(m)} t}{2}\right) + i^{m-1} e^{i\varphi} g_n(0) \sin\left(\frac{\Omega_n^{(m)} t}{2}\right). \end{aligned} \quad (2.42)$$

By analogous methods the Hamiltonian for the m th red sideband yields

$$\begin{aligned} H_I^{(-m)} &= \frac{\hbar\Omega}{2} e^{-\eta^2/2} \left(\sum_{k=0}^{\infty} \frac{(-\eta^2)^k (a^\dagger)^k a^k}{k!(k+m)!} \sigma_+ (i\eta a)^m e^{i\varphi} \right. \\ &\quad \left. + \sigma_- (-i\eta a^\dagger)^m e^{-i\varphi} \sum_{k=0}^{\infty} \frac{(-\eta^2)^k (a^\dagger)^k a^k}{k!(k+m)!} \right), \end{aligned} \quad (2.43)$$

again splitting up in 2×2 blocks,

$$H_n^{(-m)} = \sqrt{\frac{n!}{(n+m)!}} \frac{\hbar\Omega_n^{(m)}}{2} (i^m \sigma_+ a^m e^{i\varphi} + (-i)^m \sigma_- (a^\dagger)^m e^{-i\varphi}), \quad (2.44)$$

coupling $\{|g, n+m\rangle, |e, n\rangle\}$. This evolves a general state,

$$|\Psi(t)\rangle = \sum_{n=0}^{\infty} [g_{n+m}(t) |g, n+m\rangle + e_n(t) |e, n\rangle], \quad (2.45)$$

according to

$$\begin{aligned} g_{n+m}(t) &= g_{n+m}(0) \cos\left(\frac{\Omega_n^{(m)} t}{2}\right) + (-i)^{m+1} e^{-i\varphi} e_n(0) \sin\left(\frac{\Omega_n^{(m)} t}{2}\right), \\ e_n(t) &= e_n(0) \cos\left(\frac{\Omega_n^{(m)} t}{2}\right) + i^{m-1} e^{i\varphi} g_{n+m}(0) \sin\left(\frac{\Omega_n^{(m)} t}{2}\right). \end{aligned} \quad (2.46)$$

While, so far, we assumed perfect resonance between laser and sideband frequency, one experimentally always has to account for detunings, with drifts of the trap frequency as dominant source. In the following section, we generalize the analytical description to include a small detuning from the sideband transition.

2.2.2 The Effect of Small Detunings

To be able to drop the idealized assumption that the sideband transition is addressed resonantly, we include the influence of an additional detuning $\delta \ll \Delta$. The rotating

wave approximation is still valid in this situation. However, a nonzero δ introduces additional time-dependent terms into the Hamiltonian, as can be seen by replacing Δ with $\Delta + \delta$ in Eq. (2.23). For red and blue sidebands, we obtain the time-dependent Hamiltonians

$$H_n^{(-m)}(t) = \sqrt{\frac{n!}{(n+m)!}} \frac{\hbar \Omega_n^{(m)}}{2} (i^m \sigma_+ a^m e^{i\varphi} e^{-i\delta t} + (-i)^m \sigma_- (a^\dagger)^m e^{-i\varphi} e^{i\delta t}), \quad (2.47)$$

and

$$H_n^{(+m)}(t) = \sqrt{\frac{n!}{(n+m)!}} \frac{\hbar \Omega_n^{(m)}}{2} (i^m \sigma_+ (a^\dagger)^m e^{i\varphi} e^{-i\delta t} + (-i)^m \sigma_- a^m e^{-i\varphi} e^{i\delta t}), \quad (2.48)$$

respectively. The Schrödinger equation leads for the red sidebands to the set of coupled differential equations

$$\begin{aligned} \dot{g}_{n+m}(t) &= (-i)^{m+1} e^{-i\varphi} e^{i\delta t} \frac{\Omega_n^{(m)}}{2} e_n(t), \\ \dot{e}_n(t) &= i^{m-1} e^{i\varphi} e^{-i\delta t} \frac{\Omega_n^{(m)}}{2} g_{n+m}(t). \end{aligned} \quad (2.49)$$

These can be solved by an ansatz of the form (Scully and Zubairy 1997)

$$\begin{aligned} g_{n+m}(t) &= \left(a_1 e^{i\tilde{\Omega}_n^{(m)} t/2} + a_2 e^{-i\tilde{\Omega}_n^{(m)} t/2} \right) e^{i\delta t/2}, \\ e_n(t) &= \left(b_1 e^{i\tilde{\Omega}_n^{(m)} t/2} + b_2 e^{-i\tilde{\Omega}_n^{(m)} t/2} \right) e^{-i\delta t/2}. \end{aligned} \quad (2.50)$$

The solution can be written in terms of the initial conditions as

$$\begin{aligned} g_{n+m}(t) &= \left(g_{n+m}(0) \left[\cos\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) - \frac{i\delta}{\tilde{\Omega}_n^{(m)}} \sin\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) \right] \right. \\ &\quad \left. + (-i)^{m+1} e^{-i\varphi} \frac{\Omega_n^{(m)}}{\tilde{\Omega}_n^{(m)}} e_n(0) \sin\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) \right) e^{i\delta t/2}, \\ e_n(t) &= \left(e_n(0) \left[\cos\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) + \frac{i\delta}{\tilde{\Omega}_n^{(m)}} \sin\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) \right] \right. \\ &\quad \left. - i^{m+1} e^{i\varphi} \frac{\Omega_n^{(m)}}{\tilde{\Omega}_n^{(m)}} g_{n+m}(0) \sin\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) \right) e^{-i\delta t/2}, \end{aligned} \quad (2.51)$$

where

$$\tilde{\Omega}_n^{(m)} = \sqrt{\left(\Omega_n^{(m)}\right)^2 + \delta^2}. \quad (2.52)$$

For the blue sidebands we obtain

$$\begin{aligned} \dot{g}_n(t) &= (-i)^{m+1} e^{-i\varphi} e^{i\delta t} \frac{\Omega_n^{(m)}}{2} e_{n+m}(t), \\ \dot{e}_{n+m}(t) &= i^{m-1} e^{i\varphi} e^{-i\delta t} \frac{\Omega_n^{(m)}}{2} g_n(t), \end{aligned} \quad (2.53)$$

with the solution

$$\begin{aligned} g_n(t) &= \left(g_n(0) \left[\cos\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) - \frac{i\delta}{\tilde{\Omega}_n^{(m)}} \sin\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) \right] \right. \\ &\quad \left. + (-i)^{m+1} e^{-i\varphi} \frac{\Omega_n^{(m)}}{\tilde{\Omega}_n^{(m)}} e_{n+m}(0) \sin\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) \right) e^{i\delta t/2}, \\ e_{n+m}(t) &= \left(e_{n+m}(0) \left[\cos\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) + \frac{i\delta}{\tilde{\Omega}_n^{(m)}} \sin\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) \right] \right. \\ &\quad \left. - i^{m+1} e^{i\varphi} \frac{\Omega_n^{(m)}}{\tilde{\Omega}_n^{(m)}} g_n(0) \sin\left(\frac{\tilde{\Omega}_n^{(m)} t}{2}\right) \right) e^{-i\delta t/2}. \end{aligned} \quad (2.54)$$

We can use this to explicitly write out the elements of the unitary time evolution operator,

$$U^{(\pm m)}(t_1, t_0) = \mathcal{T} \exp\left(-\frac{i}{\hbar} \int_{t_0}^{t_1} H^{(\pm m)}(\tau) d\tau\right), \quad (2.55)$$

where $H^{(\pm m)} = \bigoplus_{n=0}^{\infty} H_n^{(\pm m)}$, and $\mathcal{T}$ denotes the time-ordering operator. According to Eqs. (2.51) and (2.54), and using $U(t_1, t_0) = U(t_1, 0)U^\dagger(t_0, 0)$, we obtain

$$\begin{aligned} u_{gg}^{mn}(t_1, t_0) &:= \langle g, n | U^{(+m)}(t_1, t_0) | g, n \rangle = \langle g, n+m | U^{(-m)}(t_1, t_0) | g, n+m \rangle \\ &= \langle e, n+m | U^{(+m)}(t_1, t_0) | e, n+m \rangle^* = \langle e, n | U^{(-m)}(t_1, t_0) | e, n \rangle^* \\ &= \left[\cos\left(\frac{\tilde{\Omega}_n^{(m)}(t_1 - t_0)}{2}\right) - \frac{i\delta}{\tilde{\Omega}_n^{(m)}} \sin\left(\frac{\tilde{\Omega}_n^{(m)}(t_1 - t_0)}{2}\right) \right] e^{i\delta(t_1 - t_0)/2}, \end{aligned} \quad (2.56)$$

and

$$\begin{aligned}
 u_{eg}^{mn}(t_1, t_0) &:= \langle e, n+m | U^{(+m)}(t_1, t_0) | g, n \rangle = \langle e, n | U^{(-m)}(t_1, t_0) | g, n+m \rangle \\
 &= -\langle g, n | U^{(+m)}(t_1, t_0) | e, n+m \rangle^* = -\langle g, n+m | U^{(-m)}(t_1, t_0) | e, n \rangle^* \\
 &= -i^{m+1} e^{i\varphi} \frac{\Omega_n^{(m)}}{\tilde{\Omega}_n^{(m)}} \sin\left(\frac{\tilde{\Omega}_n^{(m)}(t_1 - t_0)}{2}\right) e^{-i\delta(t_1+t_0)/2}. \tag{2.57}
 \end{aligned}$$

2.2.3 The Local Detection Protocol for the First Blue Sideband

Having established these exact theoretical tools for the complete description of the dynamics of an atomic two-level system interacting with a harmonic oscillator mode, we can apply this to analyze the local detection protocol in the context of a trapped-ion experiment. We consider the electronic qubit of a single trapped ion as the accessible quantum system, which is coupled via laser-induced interactions to the a single mode of its own motion in the harmonic trap potential. This motional mode takes on the role of an environment. In this case it is to some extent possible to control the environment—nevertheless, coherent control over the ion’s motion is technically much more demanding and by far not as well-established as for the electronic qubit. For a complete measurement of the quantum state of qubit *and* motion, correlated operations on both degrees of freedom would be necessary. The present experiment makes use only of local operations on the qubit degree of freedom and is therefore—in principle—applicable independently of the environment to which the qubit couples. We shift the discussion about whether or not the scheme—in practice—would successfully reveal the presence of correlations for arbitrary environments to the end of this chapter.

The present experiment was performed in the quantum information group at Berkeley, and the experimental setup was already described in the introduction. Further details can be found in Ramm et al. (2013); Ramm (2014); Pruttivarasin (2014). The most important electronic levels of $^{40}\text{Ca}^+$ are displayed in Fig. 2.2.

After a combination of Doppler and sideband cooling, the ion is prepared in a low-temperature thermal state of motion, while its electronic degree of freedom is initialized in the ground state by optical pumping. The prepared state,

$$\rho_0 = \sum_{n=0}^{\infty} p_n |g\rangle\langle g| \otimes |n\rangle\langle n| \tag{2.58}$$

is an uncorrelated product state, characterized by the average thermal occupation number $\bar{n}$ via the probability distribution $p_n = \bar{n}^n / (\bar{n} + 1)^{n+1}$; see, for example, Leibfried et al. (2003a), section II.C.4. To create a correlated state of open system (qubit) and environment (motion), we drive the first blue sideband for a time t_0 .

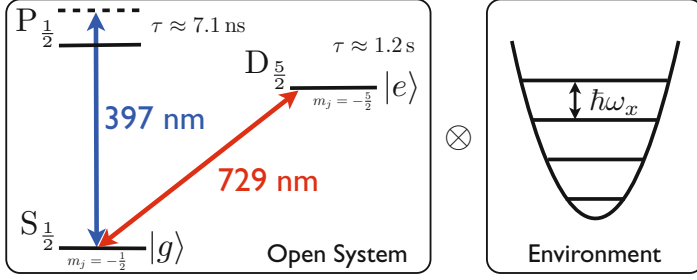


Fig. 2.2 The electronic qubit transition can be driven coherently via the quadrupole transition at 729 nm. Incoherent readout and dephasing operations are carried out via the 397 nm laser. When the detuning of the qubit laser corresponds to multiples of the trap frequency ω_x , the qubit is coupled via the laser to the harmonic ion motion. Compare also to Fig. 1.3. Adapted from Gessner et al. (2014a)

In Sect. 2.2.4, we present a general theoretical treatment of arbitrary red or blue sidebands. The sideband resonance is chosen to match the trap frequency along the x direction, that is, we select the motional frequency $\nu = \omega_x$. The experimental trap frequencies of the linear Paul trap are $(\omega_x, \omega_y, \omega_z) = 2\pi \times (2.8, 2.6, 0.2) \text{ MHz}$.

2.2.3.1 Preparation

The initial state ρ_0 evolves under the influence of the first blue sideband,

$$\begin{aligned} \rho(t_0) &= U^{(+1)}(t_0, 0)\rho_0 U^{(+1)\dagger}(t_0, 0) \\ &= \sum_{n=0}^{\infty} p_n \left[|u_{gg}^{1n}(t_0, 0)|^2 |g, n\rangle\langle g, n| + u_{eg}^{1n}(t_0, 0)^* u_{gg}^{1n}(t_0, 0) |g, n\rangle\langle e, n+1| \right. \\ &\quad \left. + u_{eg}^{1n}(t_0, 0) u_{gg}^{1n}(t_0, 0)^* |e, n+1\rangle\langle g, n| + |u_{eg}^{1n}(t_0, 0)|^2 |e, n+1\rangle\langle e, n+1| \right], \end{aligned} \quad (2.59)$$

which describes a correlated quantum state. This first time evolution therefore represents the *preparation* of the correlated initial state. The next step is the detection of these correlations by local means. Of the total correlated state, we can only access the reduced density matrix of the qubit, which now is described by

$$\rho_S(t_0) = \text{Tr}_E \rho(t_0) = \sum_{n=0}^{\infty} p_n \left[|u_{gg}^{1n}(t_0, 0)|^2 |g\rangle\langle g| + |u_{eg}^{1n}(t_0, 0)|^2 |e\rangle\langle e| \right]. \quad (2.60)$$

Figure 2.3 shows the key steps of the local detection protocol, tomography data, and the time evolution of $\langle e | \rho_S(t_0) | e \rangle$ for different initial temperatures of the motional state. For the time evolution, an average with slightly fluctuating parameters was performed to account for magnetic field noise and instabilities of the trap frequency.

Typically, the detuning was assumed to fluctuate according to a Gaussian distribution with both center and variance of the order of $2\pi \times 1$ kHz (Gessner et al. 2014a).

To reveal the correlations to the local dynamics, we implement the local dephasing operation (2.4) in the eigenbasis of the qubit. This local eigenbasis is always given by $|g\rangle$ and $|e\rangle$, as can be seen from Eq. (2.60) and which is confirmed experimentally by state tomography shown in Fig. 2.3b.

2.2.3.2 Implementing the Local Dephasing Operation

The key step of the local detection protocol consists in the removal of coherences from the *total* quantum state by a *local* dephasing operation. In the present setting, coherences in the local eigenbasis can be removed in a controlled way by applying a far-detuned laser on the 397 nm transition, see Fig. 2.2. Such a laser will induce an AC-Stark shift (Cohen-Tannoudji et al. 1992) on the ground state $|g\rangle$, while the excited state $|e\rangle$ is not affected. If the detuning is sufficiently large, the population of the qubit ground state will not be affected, which is important, since we only need to remove certain coherences. In this limit, one can describe the dynamics of the relevant electronic states with the laser mode via an effective Hamiltonian (Schneider et al. 1997; Gerry and Knight 2005; Foot 2007)

$$H_{\text{eff}} = \hbar \frac{\Omega_f^2}{4\Delta_f} (|f\rangle\langle f| + (|f\rangle\langle f| - |g\rangle\langle g|)b^\dagger b), \quad (2.61)$$

where b is the annihilation operator of the laser light mode and $|f\rangle = |P_{1/2}\rangle$ describes the short-lived excited $P_{1/2}$ state, which is addressed by the 397 nm laser light. We see that this Hamiltonian does not induce population transfer, but instead induces an energy shift. Thus, during the application of this off-resonant laser, the ground state will accumulate a phase shift with respect to the unperturbed excited state. This phase shift oscillates with frequency $\Omega_f^2/4\Delta_f$, and can be precisely manipulated via the pulse duration. This oscillation of the relative phase was also experimentally confirmed via Ramsey experiments (Ramm 2014).

By averaging the subsequent time evolution over a series of phase shifts whose average value is zero, we effectively remove all coherences and with it the quantum discord of the *total* state, while only manipulating the local state of the qubit. To ensure that indeed the ground-state population is not affected by accidental photon scattering events from the 397 nm laser (Ozeri et al. 2005; Uys et al. 2010), we estimate the number of scattered photons (Gessner et al. 2014a). The scattering rate is given by Γp_f , where the $P_{1/2}$ -state population is obtained from the steady-state solution of the optical Bloch equations (Cohen-Tannoudji et al. 1992),

$$p_f = \frac{s/2}{1 + s + (2\Delta_f/\Gamma)^2}, \quad (2.62)$$

with the saturation parameter $s = 2\Omega_f^2/\Gamma^2$ and the decay rate $\Gamma = 1/\tau$ of the $P_{1/2}$ state, which has a lifetime of $\tau \approx 7.1$ ns. All achievable phase shifts are reached within the oscillation period

$$T = \frac{8\pi\Delta_f}{\Omega_f^2}, \quad (2.63)$$

which therefore determines the longest duration of all required dephasing pulses. We further write $s = I/I_{\text{sat}}$ with (Foot 2007)

$$I_{\text{sat}} = \frac{\pi}{3} \frac{hc}{\lambda^3 \tau}. \quad (2.64)$$

Inserting $\lambda = 397$ nm leads to $I_{\text{sat}} = 46.8$ mW/cm², and we find $s \approx 255$, which is consistent with independent measurements of the laser beam size in the focus and the intensities of the lasers entering the trap. For the experimental parameters $T = 25$ μ s, $\Delta = 2\pi \times 400$ GHz, we obtain an estimation of 3.5×10^{-4} scattering events during the longest dephasing pulse.

Hence, this dephasing method indeed does not affect the populations and describes the local dephasing operation,

$$\begin{aligned} \rho'(t_0) &= (\Phi \otimes \mathbb{I})\rho(t_0) \\ &= \sum_{i \in \{e, g\}} (|i\rangle\langle i| \otimes \mathbb{I})\rho(t_0)(|i\rangle\langle i| \otimes \mathbb{I}) \\ &= \sum_{n=0}^{\infty} p_n \left[|u_{gg}^{1n}(t_0, 0)|^2 |g, n\rangle\langle g, n| + |u_{eg}^{1n}(t_0, 0)|^2 |e, n+1\rangle\langle e, n+1| \right]. \end{aligned} \quad (2.65)$$

This produces the reference state $\rho'(t_0)$ which differs from the original state $\rho(t_0)$ only by lacking quantum discord. The trace-distance dephasing disturbance (2.7) is easily evaluated and yields

$$D(\rho(t_0)) = \sum_{n=0}^{\infty} p_n |u_{eg}^{1n}(t_0, 0)u_{gg}^{1n}(t_0, 0)|. \quad (2.66)$$

2.2.3.3 Detection

Finally, the correlations (2.66) are detected by monitoring the subsequent time evolution in the local subsystem after dephasing. At t_0 there is no difference between the local states with and without dephasing, as confirmed by the state tomography in Fig. 2.3c. The local time evolution of the excited state population

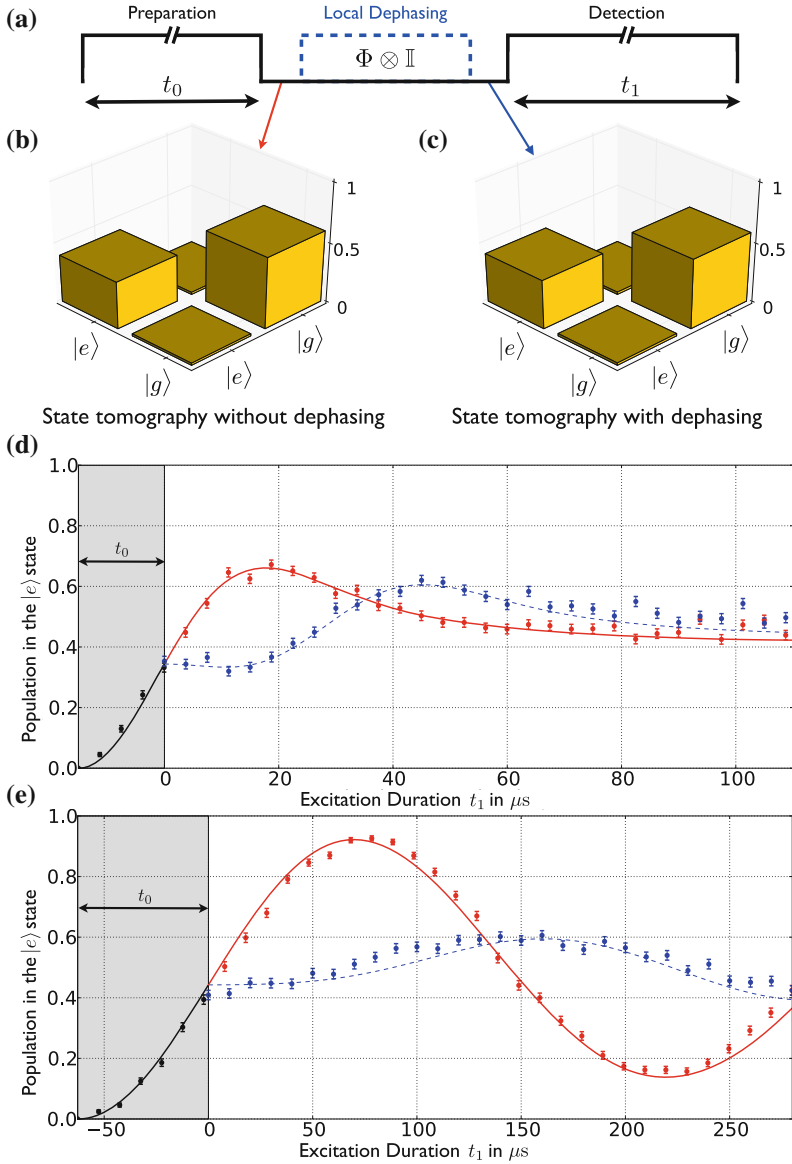


Fig. 2.3 a The experimental sequence consists in the comparison of the time evolution with and without dephasing. b, c Full state tomography of the qubit system confirms that dephasing in the local eigenbasis does not affect the local initial state. d, e The subsequent time evolution is dramatically influenced by the removal of initial discord-type correlations by local dephasing, which is shown for different environmental thermal distributions: parameters are $\bar{n} = 5.9$ in d and $\bar{n} = 0.2$ in e. The error bars describe the statistical error $\sigma_p = \sqrt{p(1-p)/N}$ for $N = 1000$ measurements per timestep. Adapted from Gessner et al. (2014a)

$$\begin{aligned}
& \langle e | \rho'_S(t_1 + t_0) | e \rangle \\
&= \langle e | \text{Tr}_E \{ U^{(+1)}(t_1 + t_0, t_0) \rho'(t_0) U^{(+1)}(t_1 + t_0, t_0)^\dagger \} | e \rangle \\
&= \sum_{n=0}^{\infty} p_n [|u_{gg}^{1n}(t_0, 0)|^2 |u_{eg}^{1n}(t_1 + t_0, t_0)|^2 + |u_{eg}^{1n}(t_0, 0)|^2 |u_{gg}^{1n}(t_1 + t_0, t_0)|^2], \quad (2.67)
\end{aligned}$$

can differ significantly from the evolution of the unperturbed initial state. This difference

$$\begin{aligned}
d_e(t_0, t_1) &= \langle e | \rho_S(t_1 + t_0) - \rho'_S(t_1 + t_0) | e \rangle \\
&= \sum_{n=0}^{\infty} p_n [u_{eg}^{1n}(t_0, 0)^* u_{gg}^{1n}(t_0, 0) u_{eg}^{1n}(t_1 + t_0, t_0) u_{gg}^{1n}(t_1 + t_0, t_0) + \text{c.c.}] \\
&\quad (2.68)
\end{aligned}$$

directly determines the local trace distance

$$d(t_0, t_1) = \| \rho_S(t_1 + t_0) - \rho'_S(t_1 + t_0) \| = |d_e(t_0, t_1)|, \quad (2.69)$$

since the coherences in the qubit system are always zero. Any nonzero value of $d(t_0, t_1)$ establishes a witness of quantum discord in the initial state $\rho(t_0)$, and provides a lower bound to Eq. (2.66). Figure 2.4 compares the measured local trace distance evolution with the theoretical prediction obtained from the parameters of the measured, unperturbed Rabi oscillation. The parameters are $\Omega \approx 2\pi \times 100$ kHz and $\eta = 0.04$. We find strong signatures of the correlations in the local dynamics, even for the higher-temperature environment, which was prepared only by Doppler cooling. When additionally sideband cooling is implemented, the deviations almost reach the maximum value of $1/2$.

To find the closest lower bound on the dephasing disturbance of the total states for each initial state, characterized by t_0 , we maximize the local trace distance over all measured times t_1 . The result can be seen in Fig. 2.5. The local witness almost reaches the actual value of the global correlations for the low-temperature environment. In the case of higher temperature, the contribution of many different frequencies to the time evolution does not lead to a full revival. The dashed line shows the theoretically computed upper limit for the maximum local distance within an experimentally motivated, observable time window, which shows that our measurements provide the experimentally closest possible lower bound.

2.2.3.4 Temperature Dependence of the Local Signal

In the experiment, the local signature of the correlations was studied for two different environmental temperatures. The case of a sideband-cooled environment is special in a sense that we almost couple to a pure Fock state, which allows to adjust the

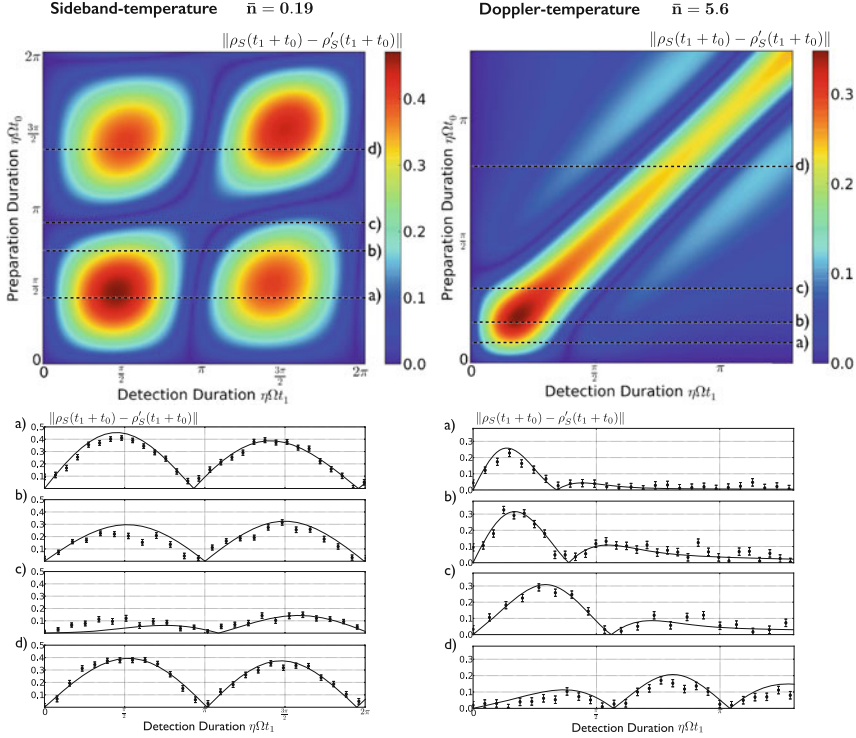


Fig. 2.4 The local trace distance is theoretically predicted [Eq. (2.69)] for the parameters obtained from independent measurements of the unperturbed Rabi oscillations. After different preparation times, we employ the local detection protocol and monitor the local trace distance experimentally, for two different initial environmental temperature states. The *error bars* label statistical errors over 1000 experimental repetitions per timestep. Adapted from Gessner et al. (2014a)

detection duration t_1 such that the local bound is tight (Gessner et al. 2014a). In general, this is not possible, since each environmental state couples the system with a different Rabi frequency, which can be seen from the dependence of $\Omega_n^{(m)}$ on n in Eqs. (2.33) and (2.40). For the first blue sideband, this dependence is approximately given by $\Omega_n^{(1)} \approx \eta\sqrt{n+1}\Omega$, when $\eta \ll 1$. Does this mean that the local signature of the initial correlations vanishes in the high-temperature limit? In fact, numerical analysis of the local signal (2.69) suggests that the locally accessible lower bound converges to a finite value in the present scenario when $\bar{n} \rightarrow \infty$, as we will show in the following.

To estimate the temperature-dependence of the local signal we consider $\delta = 0$, and, for convenience we fix the preparation and detection times to $t_0 = t_1 = \pi/(2\eta\Omega)$. After inserting these parameters, together with Eqs. (2.56) and (2.57) with $m = 1$ into the expression (2.69) for the local signal, we obtain

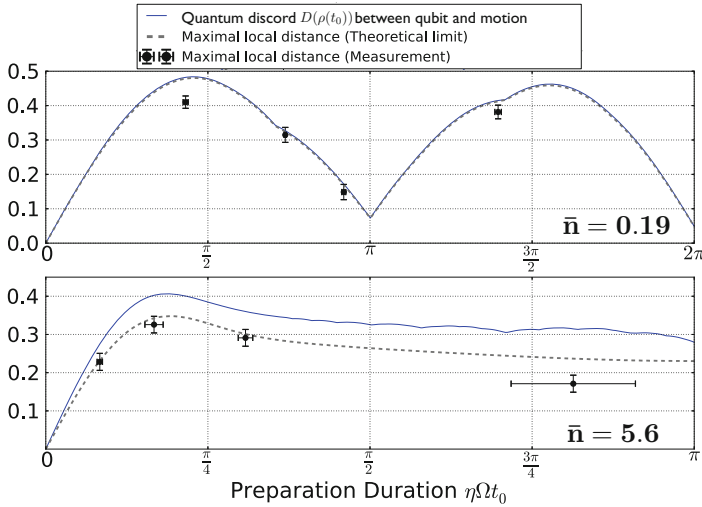


Fig. 2.5 The maximum local trace distance provides a lower bound for the dephasing disturbance (2.7). While this bound is almost tight for the low-temperature initial state, the contributions of many different Rabi frequencies limit the reachable maximum value at finite temperatures; see also Sect. 2.2.3.4 for further discussions. Adapted from Gessner et al. (2014a)

$$d(t_0, t_1)|_{t_0=t_1=\frac{\pi}{2\eta\Omega}} = \frac{1}{2} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(\bar{n}+1)^{n+1}} \sin^2\left(\frac{\pi\sqrt{n+1}}{2}\right). \quad (2.70)$$

One can indeed readily verify that the above choice of t_0 and t_1 , which corresponds to the $\pi/2$ -time of the ground-state Rabi oscillation on the first sideband, leads to the maximum local signal of $d = 1/2$ when $\bar{n} = 0$; see also the supplementary material of Gessner et al. (2014a) for further details. To observe the dependence on temperature, we plot Eq. (2.70) as a function of $\bar{n}$ in Fig. 2.6. We find that after an initial reduction of the local signal, the recordable local signal approaches the finite value $1/4$ in the limit $\bar{n} \rightarrow \infty$, and does not decay to zero as one might have expected. Note that it might even be possible to achieve a stronger signal by changing the parameters t_0 and t_1 . In the limit $\bar{n} \rightarrow \infty$, an increasing number of environmental states become relevant for the dynamics of the electronic two-level system. While this result confirms the efficacy of the method also in the limit of large effective environmental dimensions, extrapolation of these results to even higher values of $\bar{n}$ would require a careful analysis of the validity of previously made assumptions, such as the time-averaged, effectively static potential, and the controlled resonant interaction of the ion with a laser.

Note that, in the light of the discussion of Sect. 2.1.3, this result also implies that the present Jaynes–Cummings-type interaction between system and environment is not generic in the sense that it is not well described by the Haar-measure average over all unitary evolutions, since in that case we expect the signal to approach zero in the high-dimensional limit; see also Gessner (2011).

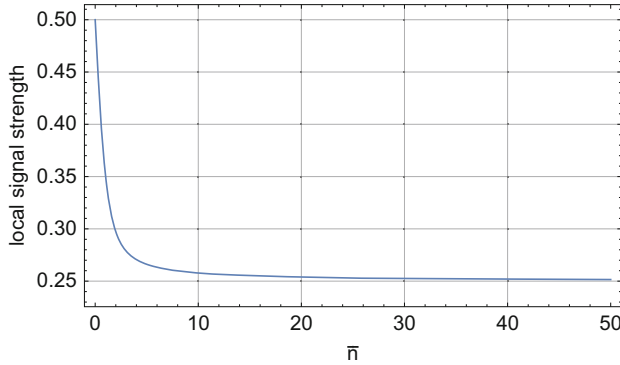


Fig. 2.6 The local signal strength, as quantified by the trace distance (2.70), is plotted for the special case where preparation time and detection time are both chosen as $t_0 = t_1 = \pi/(2\eta\Omega)$. While for $\bar{n} = 0$ this leads to the maximum value of $1/2$, the signal decays as $\bar{n}$ increases, which was also experimentally observed in Figs. 2.3 and 2.5 for $\bar{n} \approx 5.6$. For larger values of $\bar{n}$, the signal does not decay to zero and instead reaches an asymptotic value of $1/4$. The sum in Eq. (2.70) was evaluated numerically over 10000 environmental states

2.2.3.5 Time-Averaged Local Hilbert–Schmidt Distance

In the previous sections, we have discussed the local detection of trace-distance quantum discord based on the contraction property. When instead the Hilbert–Schmidt distance is employed, an interesting relation between the total distance and the time-averaged local distance can be derived. Nevertheless, we must recall the caveat mentioned already in Sect. 2.1.3, which implies that the absolute information content of the Hilbert–Schmidt distance is limited when employed as a formal information-theoretic measure of discord-type correlations (Piani 2012).

The dephasing disturbance, expressed by the Hilbert–Schmidt distance reads

$$\begin{aligned}
 D_{HS}(\rho(t_0)) &= \|\rho(t_0) - \rho'(t_0)\|_2^2 \\
 &= 2 \sum_{n=0}^{\infty} p_n^2 |u_{eg}^{1n}(t_0, 0)|^2 |u_{gg}^{1n}(t_0, 0)|^2.
 \end{aligned} \tag{2.71}$$

As mentioned in Sect. 2.1.3, employing methods from random matrix theory, one can show that the unitary Haar-measure average of the local Hilbert–Schmidt distance reproduces the total Hilbert–Schmidt distance except for a dimension-dependent prefactor (Gessner and Breuer 2011, 2013a, b), see Eq. (2.21). Assuming an ergodicity-type connection between the average over all unitary time evolution operators and the time average for a given Hamiltonian, one can expect to draw similar conclusions from the time average (Gessner and Breuer 2013b).

The local Hilbert–Schmidt distance is given by

$$\|\rho_S(t_1 + t_0) - \rho'_S(t_1 + t_0)\|_2^2 = 2|d_e(t_0, t_1)|^2. \quad (2.72)$$

For on-resonance sideband interactions ($\delta = 0$), this yields

$$\begin{aligned} & \|\rho_S(t_0 + t_1) - \rho'_S(t_0 + t_1)\|_2^2 \\ &= \frac{1}{2} \sum_{n,m} p_n p_m \sin(\Omega_n^{(1)} t_0) \sin(\Omega_n^{(1)} t_1) \sin(\Omega_m^{(1)} t_0) \sin(\Omega_m^{(1)} t_1). \end{aligned} \quad (2.73)$$

The time average over t_1 can be evaluated analytically, using

$$\begin{aligned} & \langle \sin(\Omega_n^{(1)} t_1) \sin(\Omega_m^{(1)} t_1) \rangle_{t_1} \\ &= \frac{1}{2} \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T dt_1 [\cos((\Omega_n^{(1)} - \Omega_m^{(1)}) t_1) - \cos((\Omega_n^{(1)} + \Omega_m^{(1)}) t_1)] \\ &= \frac{1}{2} \delta_{\Omega_n^{(1)} \Omega_m^{(1)}}. \end{aligned} \quad (2.74)$$

For relevant frequencies of low-temperature thermal states, $\Omega_n^{(1)}$ and $\Omega_m^{(1)}$ coincide only if $n = m$. We finally obtain

$$\begin{aligned} \langle \|\rho_S(t_0 + t_1) - \rho'_S(t_0 + t_1)\|_2^2 \rangle_{t_1} &= \sum_n p_n^2 \cos^2\left(\frac{\Omega_n^{(1)} t_0}{2}\right) \sin^2\left(\frac{\Omega_n^{(1)} t_0}{2}\right) \\ &= \frac{1}{2} \|\rho(t_0) - \rho'(t_0)\|_2^2 \\ &= \frac{1}{2} D_{HS}(\rho(t_0)). \end{aligned} \quad (2.75)$$

As anticipated, the time-average of the local Hilbert–Schmidt distance over the detection time t_1 is directly proportional to $D_{HS}(\rho(t_0))$. The corresponding experimental data is shown in Fig. 2.7.

2.2.4 Generalization to Arbitrary Sidebands

The theoretical treatment from the previous sections can be extended to the dynamics on arbitrary red and blue sidebands. Starting from the uncorrelated, thermal state (2.58), a pulse on the m th red sideband produces the correlated state

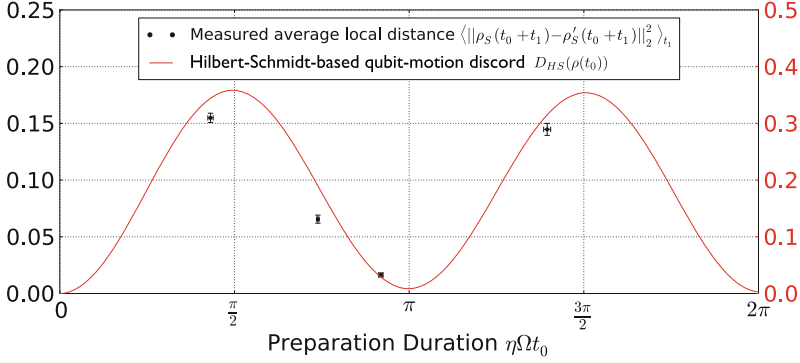


Fig. 2.7 The measurement data is evaluated in terms of the Hilbert–Schmidt distance instead of the trace distance, which shows that the local time average (2.75) is proportional to the Hilbert–Schmidt distance (2.71) of the total states. Adapted from Gessner et al. (2014a)

$$\begin{aligned}
 \rho(t_0) &= U^{(-m)}(t_0, 0)\rho_0 U^{(-m)}(t_0, 0)^\dagger \\
 &= \sum_{n=0}^{\infty} p_{n+m} \left[|u_{gg}^{mn}(t_0, 0)|^2 |g, n+m\rangle\langle g, n+m| \right. \\
 &\quad + u_{eg}^{mn}(t_0, 0)^* u_{gg}^{mn}(t_0, 0) |g, n+m\rangle\langle e, n| \\
 &\quad + u_{eg}^{mn}(t_0, 0) u_{gg}^{mn}(t_0, 0)^* |e, n\rangle\langle g, n+m| \\
 &\quad \left. + |u_{eg}^{mn}(t_0, 0)|^2 |e, n\rangle\langle e, n| \right]. \tag{2.76}
 \end{aligned}$$

Note that, if the initial state is prepared with low temperature, there will be almost no signal from the red sideband pulse, since the first m populations are skipped in the sum. This expresses the simple fact that there are no resonant red sideband states to couple to, when the ion is already prepared in $|g, 0\rangle$. For blue sideband transitions, we can easily generalize (2.59) to arbitrary m beyond 1. Both cases of red and blue sidebands can be combined to a single expression as

$$\begin{aligned}
 \rho(t_0) &= U^{(s)}(t_0, 0)\rho(0)U^{(s)}(t_0, 0)^\dagger \\
 &= \sum_{n=0}^{\infty} p_{n+x} \left[|u_{gg}^{mn}(t_0, 0)|^2 |g, n+x\rangle\langle g, n+x| \right. \\
 &\quad + u_{eg}^{mn}(t_0, 0)^* u_{gg}^{mn}(t_0, 0) |g, n+x\rangle\langle e, n+x+s| \\
 &\quad + u_{eg}^{mn}(t_0, 0) u_{gg}^{mn}(t_0, 0)^* |e, n+x+s\rangle\langle g, n+x| \\
 &\quad \left. + |u_{eg}^{mn}(t_0, 0)|^2 |e, n+x+s\rangle\langle e, n+x+s| \right], \tag{2.77}
 \end{aligned}$$

with $m = |s|$ and $x = \max\{0, -s\}$. The corresponding reduced state of the two-level system is for $s \neq 0$ given by

$$\rho_S(t_0) = \text{Tr}_E \rho(t_0) = \sum_{n=0}^{\infty} p_{n+x} [|u_{gg}^{mn}(t_0, 0)|^2 |g\rangle\langle g| + |u_{eg}^{mn}(t_0, 0)|^2 |e\rangle\langle e|]. \quad (2.78)$$

This state is always diagonal in the computational basis $|g\rangle$ and $|e\rangle$. The reference state after dephasing yields

$$\begin{aligned} \rho'(t_0) &= (\Phi \otimes \mathbb{I})\rho(t_0) \\ &= \sum_{n=0}^{\infty} p_{n+x} [|u_{gg}^{mn}(t_0, 0)|^2 |g, n+x\rangle\langle g, n+x| \\ &\quad + |u_{eg}^{mn}(t_0, 0)|^2 |e, n+x+s\rangle\langle e, n+x+s|]. \end{aligned} \quad (2.79)$$

This leads to a trace distance of

$$D(\rho(t_0)) = \|\rho(t_0) - \rho'(t_0)\| = \sum_{n=0}^{\infty} p_{n+x} |u_{eg}^{mn}(t_0, 0) u_{gg}^{mn}(t_0, 0)|, \quad (2.80)$$

and a squared Hilbert–Schmidt distance of

$$D_{HS}(\rho(t_0)) = \|\rho(t_0) - \rho'(t_0)\|_2^2 = 2 \sum_{n=0}^{\infty} p_{n+x}^2 |u_{eg}^{mn}(t_0, 0)|^2 |u_{gg}^{mn}(t_0, 0)|^2. \quad (2.81)$$

Analogous to the considerations in the previous section, we find for the generalized expression for the excited state population after a subsequent time evolution

$$\begin{aligned} &\langle e | \rho'_S(t_1 + t_0) | e \rangle \\ &= \langle e | \text{Tr}_E \{ U^{(s)}(t_1 + t_0, t_0) \rho'(t_0) U^{(s)}(t_1 + t_0, t_0)^\dagger \} | e \rangle \\ &= \sum_{n=0}^{\infty} p_{n+x} [|u_{gg}^{mn}(t_0, 0)|^2 |u_{eg}^{mn}(t_1 + t_0, t_0)|^2 + |u_{eg}^{mn}(t_0, 0)|^2 |u_{gg}^{mn}(t_1 + t_0, t_0)|^2]. \end{aligned} \quad (2.82)$$

The local trace- and Hilbert–Schmidt distances can then be evaluated using Eqs. (2.69) and (2.72), based on the resulting difference in the excited state population, which yields

$$\begin{aligned} d_e(t_0, t_1) &= \langle e | \rho_S(t_1 + t_0) - \rho'_S(t_1 + t_0) | e \rangle \\ &= \sum_{n=0}^{\infty} p_{n+x} [u_{eg}^{mn}(t_0, 0)^* u_{gg}^{mn}(t_0, 0) u_{eg}^{mn}(t_1 + t_0, t_0) u_{gg}^{mn}(t_1 + t_0, t_0) + \text{c.c.}]. \end{aligned} \quad (2.83)$$

2.2.5 Extension of the Experimental Technique

The experimental techniques, developed in the course of the described trapped-ion experiment, enable to dephase in the computational basis of the qubit. In Sect. 2.1.2, we have seen that, in order to avoid overestimation of the discord-type correlations for all classes of mixed states, one should consider dephasing over a large set of basis sets. Dephasing in different bases is also important to increase the flexibility of the method to ensure applicability in situations where the local eigenbasis is not the computational basis. In this section we briefly discuss how the AC-Stark shift method can be easily extended to achieve this.

To induce dephasing in a different basis, a combination of the AC-Stark shift dephasing procedure with single-qubit unitary rotations is sufficient. Let us assume that dephasing Φ_{Π_A} can be implemented in the basis $\Pi_A = \{|1\rangle\langle 1|, |2\rangle\langle 2|, \dots\}$ (recall Eq. (2.10) for the definition of the notation). To achieve dephasing in another basis $\Pi_B = \{|\varphi_1\rangle\langle \varphi_1|, |\varphi_2\rangle\langle \varphi_2|, \dots\}$, we consider the unitary operation,

$$U = \sum_i |\varphi_i\rangle\langle i|, \quad (2.84)$$

which rotates one basis into the other. As discussed in Sect. 1.3.1, on trapped-ion qubits such operations are typically implemented with fidelities of 0.995 (Schindler et al. 2013). The desired dephasing operation Φ_{Π_B} is then achieved as a combination of unitary rotations and the implementable dephasing Φ_{Π_A} as (Gessner et al. 2014a)

$$\begin{aligned} U \Phi_{\Pi_A} (U^\dagger X U) U^\dagger &= \sum_{ijklm} |\varphi_i\rangle\langle i|k\rangle\langle k|l\rangle\langle \varphi_l|X|\varphi_m\rangle\langle m|k\rangle\langle k|j\rangle\langle \varphi_j| \\ &= \sum_i |\varphi_i\rangle\langle \varphi_i|X|\varphi_i\rangle\langle \varphi_i| = \Phi_{\Pi_B}(X). \end{aligned} \quad (2.85)$$

2.3 Photonic Experiment

After having discussed the experimental realization of the local detection protocol with a single trapped ion, we now turn to an implementation with photons, which are sent through linear and nonlinear optical elements. This section describes an experiment which was carried out in the Key Laboratory of Quantum Information in Hefei, China (Tang et al. 2015). Since photons are often used as traveling carriers of quantum information (Duan et al. 2001), which can be sent over long distances, it is important to characterize the correlations encoded into their quantum states under experimentally realistic constraints. When photons are sent through noisy environments, decoherence effects can be detrimental to the capacity of the quantum channel. A turbulent atmosphere can cause decay of orbital angular momentum entanglement (Paterson 2005), but also the typically more robust polarization qubits are subject to decoherence in optical fibers.

Decoherence is a manifestation of interaction with another degree of freedom, which produces correlated states that, on the level of the reduced system, appear as incoherent mixtures. As we have seen in the previous sections, these correlations can influence the time evolution of the reduced system and thereby be revealed. In the present experiment, we consider a photon in a birefringent fiber, which induces interactions between the polarization qubit and the photon's frequency degree of freedom. Formally, this resembles the coupling of a two-level system to a harmonic oscillator which we discussed in the previous section—however, here, the environment is composed of a *continuum* of modes, instead of a single mode as the harmonic motion of the trapped ion.

For the experiment we apply the local detection method in a typical two-party quantum communication scenario. We suppose that Alice prepares a photon in a possibly correlated state of qubit and motion and sends it to Bob. Being only able to access the qubit degree of freedom, Bob then tries to identify and estimate these correlations making use of interactions between both degrees of freedom, which, in the spirit of open-system theory, we consider system (polarization qubit) and environment (continuum of frequency modes).

2.3.1 The Pre-initial State

On Alice's side, the single photons are created by a quantum dot, where spectral diffusion¹ (Sallen et al. 2010) generates a mixed frequency state, given by a sum over closely spaced frequencies (Tang et al. 2015),

$$\rho_E = \sum_{\omega} \Delta\omega G(\omega) |\omega\rangle \langle \omega|, \quad (2.86)$$

with a Lorentzian line shape

$$G(\omega) = \frac{1}{\pi} \frac{\delta\omega}{\delta\omega^2 + (\omega - \omega_0)^2}, \quad (2.87)$$

and measured parameters of $\omega_0 = 2\pi c/(914 \text{ nm})$, and $1/\delta\omega = 9.703 \pm 0.124 \text{ ps}$, with c denoting the speed of light.

To ensure proper normalization of ρ_E , we have introduced a description by a discrete sum of frequency states $|\omega\rangle$, which form a complete orthonormal set, that is, $\langle \omega | \omega' \rangle = \delta_{\omega, \omega'}$ and $\sum_{\omega} |\omega\rangle \langle \omega| = \mathbb{I}$. Positivity and normalization of ρ_E are warranted since $\Delta\omega G(\omega) \geq 0$ and $\sum_{\omega} \Delta\omega G(\omega) = 1$. After determining the relevant quantities, such as the trace distance, we will perform the continuum limit $\Delta\omega \rightarrow 0$.

The total pre-initial state describes an uncorrelated product state, $\rho_{pi} = \rho_0 \otimes \rho_E$, where the state of the polarization qubit is represented in the basis $\{|H\rangle, |V\rangle\}$ as

¹Random jumps of a narrow resonance caused by environmental fluctuations.

$$\rho_0 = \begin{pmatrix} \alpha & \beta e^{i\varphi} \\ \beta e^{-i\varphi} & 1 - \alpha \end{pmatrix}. \quad (2.88)$$

The states $|H\rangle$ and $|V\rangle$ represent horizontal and vertical polarization eigenstates, respectively. The parameters β and α describe the qubit quantum state. While the following theoretical treatment is applicable for arbitrary values of $\alpha, \beta \in \mathbb{R}$ (as long as ρ_0 describes a valid quantum state), for the ease of notation, we will restrict to the case of $\alpha = 1/2$, which describes the experiment as well. This equal superposition between $|H\rangle$ and $|V\rangle$ is realized by sending the qubit through a 45° polarization plate.

2.3.2 Preparation of the Initial State

In order to generate polarization-frequency correlations, Alice sends the pre-initial state,

$$\begin{aligned} \rho_{pi} = \sum_{\omega} \Delta\omega G(\omega) & \left(\frac{1}{2} |H, \omega\rangle \langle H, \omega| + \beta e^{i\varphi} |H, \omega\rangle \langle V, \omega| \right. \\ & \left. + \beta e^{-i\varphi} |V, \omega\rangle \langle H, \omega| + \frac{1}{2} |V, \omega\rangle \langle V, \omega| \right), \end{aligned} \quad (2.89)$$

through a birefringent calcite crystal of length L . The induced interaction between polarization and frequency degrees of freedom is described by

$$U_{\text{cal}}(t) : \begin{cases} |H, \omega\rangle \rightarrow |H, \omega\rangle \\ |V, \omega\rangle \rightarrow e^{-i\omega t} |V, \omega\rangle \end{cases}, \quad (2.90)$$

where the time spent inside the crystal is given by $t = \Delta n_{\text{cal}} L / c$. The birefringence Δn_{cal} describes the difference between the refraction indices for the two polarization states and can be measured from the shift of the single-photon spectra with and without the calcite crystal in a Michelson interferometer (Tang et al. 2015). The experimental parameters are $\Delta n_{\text{cal}} = 0.179 \pm 0.001$ and $L = 35.92$ mm. This evolution results in the generation of the correlated initial state

$$\begin{aligned} \rho_{SE} &= U_{\text{cal}}(t) \rho_{pi} U_{\text{cal}}^\dagger(t) \\ &= \sum_{\omega} \Delta\omega G(\omega) \left(\frac{1}{2} |H, \omega\rangle \langle H, \omega| + \beta e^{i(\omega t + \varphi)} |H, \omega\rangle \langle V, \omega| \right. \\ & \quad \left. + \beta e^{-i(\omega t + \varphi)} |V, \omega\rangle \langle H, \omega| + \frac{1}{2} |V, \omega\rangle \langle V, \omega| \right). \end{aligned} \quad (2.91)$$

The reduced state for the qubit is obtained by tracing over the frequency degrees of freedom,

$$\rho_S = \begin{pmatrix} 1/2 & \beta C(t) e^{i\Psi(t)} \\ \beta C(t) e^{-i\Psi(t)} & 1/2 \end{pmatrix}, \quad (2.92)$$

with

$$C(t) = \sum_{\omega} \Delta\omega G(\omega) e^{i(\omega - \omega_0)t}. \quad (2.93)$$

and $\Psi(t) = \omega_0 t + \varphi$. The initial phase φ can be adjusted, such that it cancels the time-dependent phase for any fixed time t , that is, $\Psi(t) \equiv 0$, which implies the choice $\varphi = -\omega_0 t$. The function $C(t)$ is real valued since $G(\omega)$ is symmetric about $\omega = \omega_0$.

Before this state is sent to Bob, Alice hides the choice of her local eigenbasis by performing an additional random local unitary basis rotation of the qubit (HWP0 in Fig. 2.8). This rotation does not affect the correlation properties of the quantum state. Formally, this rotation can be taken into account by replacing the polarization eigenbasis $|H\rangle$ and $|V\rangle$ with the rotated eigenvectors. We can therefore neglect this additional local rotation in the theoretical treatment.

2.3.3 Local Dephasing Operation

The first step on Bob's side consists in obtaining the local eigenbasis of ρ_S . To this end the removable mirror RM1 is placed to guide the photon into the tomography section T2, see Fig. 2.8. Diagonalization of (2.92) yields the eigenvalues,

$$\begin{aligned} \lambda_0 &= 1/2 + \beta C(t), \\ \lambda_1 &= 1/2 - \beta C(t), \end{aligned} \quad (2.94)$$

and eigenvectors,

$$\begin{aligned} |0\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|H\rangle + |V\rangle), \\ |1\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}} (|H\rangle - |V\rangle). \end{aligned} \quad (2.95)$$

Dephasing in the local eigenbasis is described by

$$\Phi(X) = |0\rangle\langle 0|X|0\rangle\langle 0| + |1\rangle\langle 1|X|1\rangle\langle 1|. \quad (2.96)$$

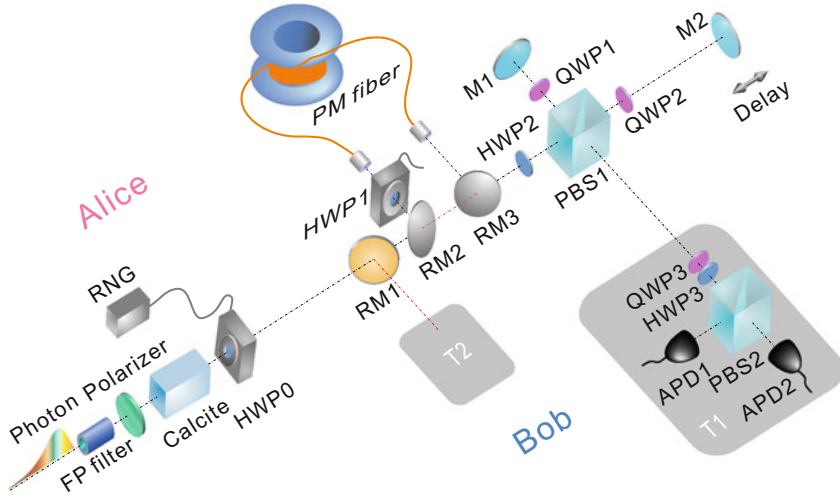


Fig. 2.8 Alice prepares a polarization-frequency correlated single photon state using a combination of polarizer and calcite crystal. A half-wave plate (HWP0) whose orientation is driven by a random number generator hides Alice's choice for the local eigenbasis before the state is sent to Bob whose task it is to detect and estimate the correlations with local operations on the qubit degree of freedom. Removable mirrors (RM1, RM2 and RM3) are employed to realize the different steps of the local detection protocol. First, qubit tomography (T2) is performed to obtain the local eigenbasis. The dephasing is performed using a half-wave plate (HWP1) followed by a long polarization-maintaining fiber. Original and dephased state are submitted to a unitary time evolution generated by a Michelson interferometer, which consists in another half-wave plate (HWP2), a polarizing beam splitter (PBS1), and two interferometer arms comprised of mirrors (M1 and M2) and quarter-wave plates (QWP1 and QWP2). The second mirror M2 is movable to vary the phase shift. Finally, another qubit tomography reveals whether the removal of discord-type correlations by local dephasing had an observable effect on the subsequent qubit dynamics, which then serves as a witness and lower bound for the correlations in the initial state. The tomography sections (T1 and T2) consist of quarter-wave plate (QWP3) and half-wave plate (HWP3), a polarizing beam splitter (PBS2) and two avalanche single photon detectors (APD1 and APD2). Adapted from Tang et al. (2015)

This produces the classically correlated reference state (with $\varphi = -\omega_0 t$, see above, in Eq. (2.91)),

$$\begin{aligned}
 \rho'_{SE} &= (\Phi \otimes \mathbb{I})\rho_{SE} \\
 &= \frac{1}{2} \sum_{\omega} \Delta\omega G(\omega) \left[|H, \omega\rangle\langle H, \omega| + |V, \omega\rangle\langle V, \omega| \right. \\
 &\quad + \beta(e^{i(\omega-\omega_0)t} + e^{-i(\omega-\omega_0)t})|H, \omega\rangle\langle V, \omega| \\
 &\quad \left. + \beta(e^{-i(\omega-\omega_0)t} + e^{i(\omega-\omega_0)t})|V, \omega\rangle\langle H, \omega| \right].
 \end{aligned} \tag{2.97}$$

To realize the local dephasing operation on the polarization qubit, the photon is sent through a very long polarization-maintaining fiber. Considering the setup

described in Fig. 2.8, this is achieved by removing the mirror RM1, and placing the mirrors RM2 and RM3. Before entering the fiber, the photon passes a computer-controlled half-wave plate (HWP1) which rotates the eigenstates $|0\rangle$ and $|1\rangle$ onto the principal axes of the fiber. The fiber also shows a small birefringence of $\Delta n_{\text{pmf}} = 3 \times 10^{-4}$, which we exploit to generate a dephasing effect after long interaction times. The interaction in the fiber is described by

$$U_{\text{pmt}} : \begin{cases} |0, \omega\rangle & \longrightarrow |0, \omega\rangle \\ |1, \omega\rangle & \longrightarrow e^{-i\omega s} |1, \omega\rangle \end{cases}, \quad (2.98)$$

where $s = \Delta n_{\text{pmf}} l / c \gg 1 / \delta\omega$ describes the time the photon spends inside the fiber. We first rewrite the effect of this dynamics in terms of $|H\rangle$ and $|V\rangle$,

$$U_{\text{pmt}} : \begin{cases} |H, \omega\rangle & \longrightarrow a|H, \omega\rangle + b|V, \omega\rangle \\ |V, \omega\rangle & \longrightarrow b|H, \omega\rangle + a|V, \omega\rangle \end{cases}, \quad (2.99)$$

where we introduced

$$a = \frac{1}{2} (1 + e^{-i\omega s}), \quad b = \frac{1}{2} (1 - e^{-i\omega s}). \quad (2.100)$$

This dynamics evolves the initial state (2.91), again with the choice $\varphi = -\omega_0 t$, into the state

$$\begin{aligned} \rho_{SE}'' &= U_{\text{pmt}} \rho_{SE} U_{\text{pmt}}^\dagger \\ &= \sum_{\omega} \Delta\omega G(\omega) \left[\frac{1}{2} |H, \omega\rangle \langle H, \omega| + \frac{1}{2} |V, \omega\rangle \langle V, \omega| \right. \\ &\quad + \beta (e^{i(\omega-\omega_0)t} a b^* + e^{-i(\omega-\omega_0)t} a^* b) |H, \omega\rangle \langle H, \omega| \\ &\quad + \beta (e^{i(\omega-\omega_0)t} a^* b + e^{-i(\omega-\omega_0)t} a b^*) |V, \omega\rangle \langle V, \omega| \\ &\quad + \beta (e^{i(\omega-\omega_0)t} |a|^2 + e^{-i(\omega-\omega_0)t} |b|^2) |H, \omega\rangle \langle V, \omega| \\ &\quad \left. + \beta (e^{i(\omega-\omega_0)t} |b|^2 + e^{-i(\omega-\omega_0)t} |a|^2) |V, \omega\rangle \langle H, \omega| \right]. \end{aligned} \quad (2.101)$$

To realize that this state indeed describes the desired dephased state ρ_{SE}' , Eq. (2.97), to a good approximation, it is important to recall that the time spent inside the fiber is much larger than the inverse spectral width of the frequency state, $\delta\omega \cdot s \gg 1$. Hence, the rapid oscillations of the function

$$|a|^2 = \frac{1}{2} (1 + \cos \omega s). \quad (2.102)$$

cannot be resolved since the uncertainty for the time s in the fiber exceeds the period of this oscillation. We must therefore replace $|a|^2$ by its mean value, obtained by averaging over one period

$$|a|^2 \rightarrow \langle |a|^2 \rangle = \frac{1}{2}. \quad (2.103)$$

Similarly,

$$|b|^2 \rightarrow \langle |b|^2 \rangle = \frac{1}{2}, \quad ab^* \rightarrow \langle ab^* \rangle = 0. \quad (2.104)$$

Inserting this into Eq. (2.101), we see that indeed ρ''_{SE} reduces to ρ'_{SE} , as given by Eq. (2.97).

2.3.4 Reduced Distributions

Before we proceed, we confirm that the reduced states of ρ_{SE} and ρ'_{SE} coincide, which should be the case by construction of the local dephasing operation. To this end, we take the difference between the two total states

$$\Delta\rho_{SE} = \rho_{SE} - \rho'_{SE} = \sum_{\omega} \Delta\omega G(\omega) f(\omega) [|H, \omega\rangle\langle V, \omega| - |V, \omega\rangle\langle H, \omega|], \quad (2.105)$$

where we introduced the imaginary-valued function

$$f(\omega) = \frac{\beta}{2} (e^{i(\omega-\omega_0)t} - e^{-i(\omega-\omega_0)t}) = i\beta \sin[(\omega - \omega_0)t]. \quad (2.106)$$

The equivalence of the environmental states is readily confirmed due to the orthogonality of $|H\rangle$ and $|V\rangle$,

$$\rho_E - \rho'_E = \text{Tr}_S\{\rho_{SE} - \rho'_{SE}\} = 0. \quad (2.107)$$

To confirm this also for the qubit states, we trace over the frequency degrees of freedom, obtaining

$$\rho_S - \rho'_S = \text{Tr}_E\{\rho_{SE} - \rho'_{SE}\} = \sum_{\omega} \Delta\omega G(\omega) f(\omega) [|H\rangle\langle V| - |V\rangle\langle H|]. \quad (2.108)$$

We now perform the continuum limit $\Delta\omega \rightarrow 0$, leading to

$$\sum_{\omega} \Delta\omega G(\omega) f(\omega) \longrightarrow \int d\omega G(\omega) f(\omega) = 0. \quad (2.109)$$

The integral vanishes since $G(\omega)$ is symmetric about $\omega = \omega_0$, while $f(\omega)$ is anti-symmetric. To summarize, we have confirmed that the reduced distributions of original and dephased states coincide for both the environmental degree of freedom and the qubit, $\rho_S - \rho'_S = 0$.

2.3.5 Total Trace Distance

To obtain the trace distance between the states ρ_{SE} and ρ'_{SE} , we diagonalize the operator $\Delta\rho_{SE}$, Eq. (2.105), which yields the eigenvalues

$$\mu_{\pm}(\omega) = \pm i \Delta\omega G(\omega) f(\omega) \quad (2.110)$$

and eigenstates

$$|\mu_{\pm}(\omega)\rangle = \frac{1}{\sqrt{2}}(|H\rangle \pm i|V\rangle) \otimes |\omega\rangle. \quad (2.111)$$

The trace distance of ρ_{SE} and ρ'_{SE} corresponds to the trace norm of $\Delta\rho_{SE}$

$$\begin{aligned} D(\rho_{SE}) &= \|\rho_{SE} - \rho'_{SE}\| \\ &= \frac{1}{2} \text{Tr} \sqrt{\Delta\rho_{SE}^{\dagger} \Delta\rho_{SE}} \\ &= \frac{1}{2} \sum_{\omega} (|\mu_{-}(\omega)| + |\mu_{+}(\omega)|) \\ &= \sum_{\omega} |\Delta\omega G(\omega) f(\omega)| \\ &= \frac{\beta}{2} \sum_{\omega} \Delta\omega G(\omega) |e^{i(\omega-\omega_0)t} - e^{-i(\omega-\omega_0)t}|. \end{aligned} \quad (2.112)$$

We finally perform the continuum limit to obtain the trace-distance dephasing disturbance (2.7),

$$D(\rho_{SE}) = \frac{\beta}{2} \int d\omega G(\omega) |e^{i(\omega-\omega_0)t} - e^{-i(\omega-\omega_0)t}|, \quad (2.113)$$

which quantifies the quantum discord in ρ_{SE} .

2.3.6 Open-System Evolution Depending on Initial Correlations

By now we have established a reference state, which was produced by local dephasing of the original, unknown state ρ_{SE} . We know that the two total states differ if the initial state contained discord-type correlations, but since we cannot measure the full state, we try to map the influence of the correlations onto the local, observable dynamics of the qubit. To this end, both, the original state ρ_{SE} and the dephased state ρ'_{SE} are subjected to the same time evolution, which is observed locally on the qubit degree of freedom.

This dynamics is generated by a Michelson delay setup, which consists of a half-wave plate (HWP2) with an adjustable angle $\eta/2$, followed by a polarizing beam splitter (PBS1), see Fig. 2.8. The beam splitter then sends the two polarization components $|\eta\rangle$ and $|\eta_\perp\rangle$, into different arms of an interferometer. The two polarization states accumulate a relative phase shift which depends on the difference of the two interferometer path lengths. This is described by

$$U(\eta, \tau) : \begin{cases} |\eta, \omega\rangle & \longrightarrow |\eta, \omega\rangle \\ |\eta_\perp, \omega\rangle & \longrightarrow e^{-i\omega\tau} |\eta_\perp, \omega\rangle \end{cases}, \quad (2.114)$$

where $\tau = 2x/c$ corresponds to the coordinate of the delayable mirror (M2). The half-wave plate HWP2 generates the basis vectors

$$|\eta\rangle = \begin{pmatrix} \cos \eta \\ \sin \eta \end{pmatrix}, \quad |\eta_\perp\rangle = \begin{pmatrix} -\sin \eta \\ \cos \eta \end{pmatrix}. \quad (2.115)$$

Using

$$|H\rangle = \cos \eta |\eta\rangle - \sin \eta |\eta_\perp\rangle, \quad |V\rangle = \sin \eta |\eta\rangle + \cos \eta |\eta_\perp\rangle, \quad (2.116)$$

we express the interferometer dynamics in terms of the basis $|H\rangle$ and $|V\rangle$,

$$U(\eta, \tau) : \begin{cases} |H, \omega\rangle & \longrightarrow \cos \eta |\eta, \omega\rangle - \sin \eta e^{-i\omega\tau} |\eta_\perp, \omega\rangle \\ |V, \omega\rangle & \longrightarrow \sin \eta |\eta, \omega\rangle + \cos \eta e^{-i\omega\tau} |\eta_\perp, \omega\rangle \end{cases}. \quad (2.117)$$

2.3.7 Local Trace Distance

To determine the evolution of the local trace distance as a function of the correlations we study the evolution of the difference operator $\Delta\rho_{SE} = \rho_{SE} - \rho'_{SE}$; using Eqs. (2.105) and (2.117):

$$\begin{aligned} & U(\eta, \tau)(\rho_{SE} - \rho'_{SE})U^\dagger(\eta, \tau) \\ &= \sum_{\omega} \Delta\omega G(\omega) f(\omega) \left[\sin \eta \cos \eta |\eta, \omega\rangle \langle \eta, \omega| + \cos^2 \eta e^{i\omega\tau} |\eta, \omega\rangle \langle \eta_\perp, \omega| \right. \\ &\quad \left. - \sin^2 \eta e^{-i\omega\tau} |\eta_\perp, \omega\rangle \langle \eta, \omega| - \sin \eta \cos \eta |\eta_\perp, \omega\rangle \langle \eta_\perp, \omega| - \text{H.c.} \right] \\ &= \sum_{\omega} \Delta\omega G(\omega) f(\omega) \left[e^{i\omega\tau} |\eta, \omega\rangle \langle \eta_\perp, \omega| - e^{-i\omega\tau} |\eta_\perp, \omega\rangle \langle \eta, \omega| \right]. \end{aligned} \quad (2.118)$$

The qubit dynamics is given after the partial trace over the frequency space

$$\begin{aligned}
\rho_S(\eta, \tau) - \rho'_S(\eta, \tau) &= \text{Tr}_E\{U(\eta, \tau)(\rho_{SE} - \rho'_{SE})U^\dagger(\eta, \tau)\} \\
&= \sum_{\omega} \Delta\omega G(\omega) f(\omega) [e^{i\omega\tau} |\eta\rangle\langle\eta_\perp| - e^{-i\omega\tau} |\eta_\perp\rangle\langle\eta|]. \quad (2.119)
\end{aligned}$$

We represent this operator in the basis $\{|\eta\rangle, |\eta_\perp\rangle\}$ as

$$\rho_S(\eta, \tau) - \rho'_S(\eta, \tau) = \begin{pmatrix} 0 & z \\ z^* & 0 \end{pmatrix}, \quad (2.120)$$

where

$$z = \sum_{\omega} \Delta\omega G(\omega) f(\omega) e^{i\omega\tau}. \quad (2.121)$$

Since this result does not depend on η , we omit the argument η in the following. The trace distance yields

$$\begin{aligned}
d(\tau) &= \|\rho_S(\tau) - \rho'_S(\tau)\| \\
&= |z| \\
&= \left| \sum_{\omega} \Delta\omega G(\omega) f(\omega) e^{i\omega\tau} \right| \\
&= \frac{\beta}{2} \left| \sum_{\omega} \Delta\omega G(\omega) (e^{i(\omega-\omega_0)t} - e^{-i(\omega-\omega_0)t}) e^{i\omega\tau} \right|. \quad (2.122)
\end{aligned}$$

In the continuum limit we obtain,

$$d(\tau) = \frac{\beta}{2} \left| \int d\omega G(\omega) (e^{i(\omega-\omega_0)t} - e^{-i(\omega-\omega_0)t}) e^{i\omega\tau} \right|. \quad (2.123)$$

Comparison with Eq. (2.113) confirms that the general contractivity property, as expressed via the inequality

$$\|\text{Tr}_E\{U(\eta, \tau)(\rho_{SE} - \rho'_{SE})U^\dagger(\eta, \tau)\}\| \leq \|\rho_{SE} - \rho'_{SE}\|, \quad (2.124)$$

is satisfied here, as a consequence of the triangle inequality.

We can further analytically simplify the local trace distance (2.123), since due to a sufficiently sharp, optical frequency range, $\omega_0 \gg \delta\omega > 0$, the support of the function $G(\omega)$ restricts to a good approximation to the positive domain,

$$\int_{-\infty}^{\infty} G(\omega) \simeq \int_0^{\infty} G(\omega) = 1. \quad (2.125)$$

The Fourier transform is given by

$$\int_0^\infty G(\omega) e^{\pm i\omega t} d\omega = e^{\pm i\omega_0 t} C(t), \quad (2.126)$$

with

$$C(t) = e^{-\delta\omega|t|}. \quad (2.127)$$

Using this in the expression for the local trace distance (2.123), we obtain

$$d(\tau) = \frac{\beta}{2} \left| e^{-\delta\omega|t+\tau|} - e^{-\delta\omega|t-\tau|} \right|. \quad (2.128)$$

Figure 2.9 shows the local trace distance, for $\beta = 1/2$ in (2.88), as a function of $x = c\tau/2$ for different angles of the half-wave plate HWP2. As becomes evident from Eq. (2.117), this angle determines the parts of the polarization state that are

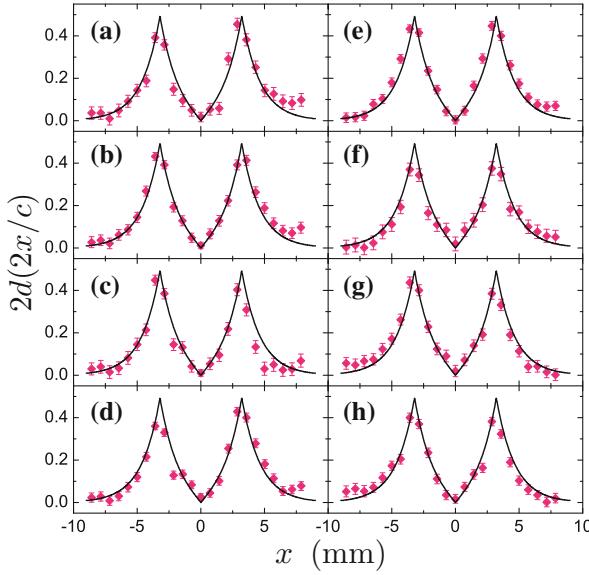


Fig. 2.9 The trace distance evolution of the qubit reveals the initial correlations to the local subsystem. The time variable $\tau = 2x/c$ is determined by the difference x between the path lengths of the interferometer's arms, which is adjusted via the position of the mirror M2, see Fig. 2.8. The plots *a–h* show the evolution for different orientations of the half-wave plate HWP2, specified by the angles $\eta = n\pi/16$ with $n = 0, \dots, 7$. The data was obtained for an initial state (2.91) whose correlation properties (2.113) are determined via the length of the calcite crystal leading to interaction time $t = \Delta n_{\text{cal}} L/c$ with $L = 35.92$ mm. The *black lines* represent the theoretical prediction (2.128), which is independent of η , and therefore makes identical predictions for all subfigures *a–h*, as confirmed by the experimental data. Adapted from Tang et al. (2015)

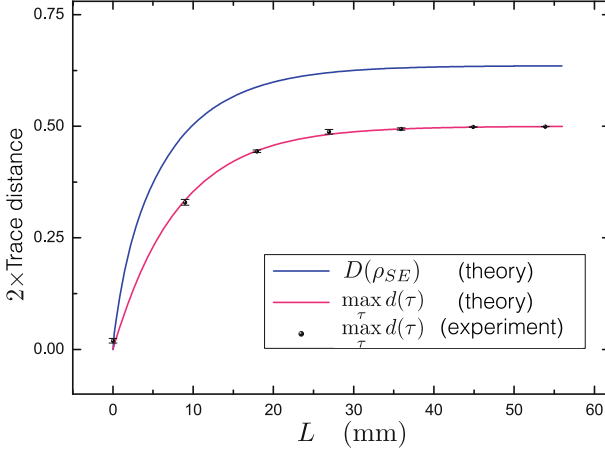


Fig. 2.10 For each initial state (2.91), the maximum value of the local trace distance (2.129) bounds the total trace distance (2.113) from below. The dynamics does not reveal the full correlations to the local system, but the theoretical upper limit is experimentally reached. These functions hardly depend on $\delta\omega$ in Eq. (2.87) when the propagation time is rescaled accordingly, and as long as (2.125) is valid. Adapted from Tang et al. (2015)

sent into the short and long interferometer arms after passing the beam splitter PBS1. The maximum value represents the closest lower bound to the total trace distance. Using Eq. (2.128), the maximum local distance is found as

$$\max_{\tau} d(\tau) = \frac{\beta}{2}(1 - e^{-2\delta\omega\tau}). \quad (2.129)$$

This procedure is repeated for different lengths of the calcite crystal, which varies the correlations in the initial state. The nature of the polarization-frequency interaction, induced by the birefringence of the calcite crystal, converts the initial coherence of the polarization state gradually into entanglement with the frequency modes, by delocalizing the initial populations over both degrees of freedom. The longer the photon spends inside the crystal, the stronger are the correlations at the end of the evolution, as illustrated by the blue line in Fig. 2.10. The maximum values of (2.129), obtained via local monitoring of the polarization state, are compared to the theoretically determined total trace distance and the predicted theoretical bound of the local distance in Fig. 2.10. Even though the local bound does not reach the actual value of the total trace distance, it provides a clear signature of the influence of the initial polarization-frequency correlations on the local dynamics of the qubit, even when the environmental state represents a continuum of infinite-dimensional modes.

The induced, effective open-system dynamics of the polarization degree of freedom is described by a monotonous decay of coherences. This is an irreversible decoherence process whose time scale is determined by $\delta\omega$ in the spectral distribution

of the environmental state (2.87), and such an environment is therefore considered Markovian² (Liu et al. 2011). Numerical studies of the total trace distance (2.113) and of the observable local distance (2.129) (as plotted in Fig. 2.10) confirm that the observed results do not depend on $\delta\omega$, as long as the environmental frequency range can be assumed positive, as in Eq. (2.125). Since changing the width $\delta\omega$ of the spectral distribution also changes the time scale of the exponential decay of coherences, the propagation time in the calcite crystal must be rescaled accordingly.

In conclusion, the present all-optical experiment demonstrates the applicability of the local detection method in the presence of a memoryless environment, generated by a continuum of modes with a Lorentzian spectral density. Upon appropriate rescaling of the time scale, the result is independent of the spectral width. In the next section, we will theoretically explore the limitations of the method, by studying the example of spontaneous emission.

2.4 Theoretical Studies of Further Examples

So far, in this chapter, we have introduced a method to detect discord-type correlations in a bipartite scenario with local operations, and presented experimental realizations with photons and ions. Recently, another experimental realization with polarization qubits was reported, where correlations are created with two spatial propagation modes (Cialdi et al. 2014). In this section we discuss the application of the method in different scenarios theoretically. Some applications were already discussed in Gessner (2011), Gessner and Breuer (2011, 2013a,b), mostly in the context of random, generic systems, which we do not review here.

2.4.1 Atom-Photon Correlations During Spontaneous Decay

In the course of the preceding sections, we have often tried to test the limits of the efficacy of the local detection method, for example by studying the high-temperature limit in the trapped-ion environment in Sect. 2.2, and a broad frequency distribution of the photonic environment in Sect. 2.3. So far, numerical studies always confirmed that the method successfully reveals the system-environment correlations to the local dynamics.

In the trapped-ion experiment, the electronic qubit exchanges excitations with a rather simple model environment, described by only a single frequency mode. The frequency degree of freedom of a single photon contains a continuum of modes. Here, the resulting irreversible dynamics is described by a pure decoherence process, where no excitations are transferred between the system and the environment

²A non-Markovian environment can be generated in this scenario by inducing additional peaks and structure in the environmental spectral distribution, as was realized in the experiment reported in Liu et al. (2011).

(Liu et al. 2011). In this section, we combine the infinite-dimensional Hilbert-space, represented by a continuum of modes, with the excitation-exchanging coupling. To this end, we apply the local detection protocol to the extreme case of the spontaneous, irreversible emission process into an infinite environment. Our goal is to study the influence of the atom-field entanglement which builds up in the transient state, when an initial atomic excitation decays spontaneously into the free-space modes of the electromagnetic field. In this case we do not expect any back-action of the correlations on the system dynamics. Indeed, we will find that in this case, employing the local dephasing at any time during the decay process of an excited atom will not produce any effect on the atomic dynamics.

Given the difficulty to detect a spontaneously emitted photon in a random spatial direction (Volz et al. 2006), the local detection method, in principle, represents a convenient alternative to other entanglement detection methods³ since it only demands access to the controllable open system, which here is described by the atom. The fact that no effect on the local dynamics can be observed when the entanglement is destroyed by local dephasing indicates that the effect of the correlations on this particular irreversible dynamics are negligible.

2.4.1.1 Atomic Evolution with and Without Local Dephasing: Resolvent Method

Following standard treatments of spontaneous emission (Weisskopf and Wigner 1930; Cohen-Tannoudji et al. 1992), we assume a unitary evolution $U(t) = e^{-iHt/\hbar}$ which transfers excitations between the atomic states $|e\rangle$ and $|g\rangle$ and the modes $|\mathbf{k}\rangle$ of the electromagnetic field. The Hamiltonian is given by $H = H_0 + V$ with

$$H_0 = E_e|e\rangle\langle e| + E_g|g\rangle\langle g| + \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} \quad (2.130)$$

and

$$V = \sum_{\mathbf{k}} (g_{\mathbf{k}} a_{\mathbf{k}}^\dagger |g\rangle\langle e| + g_{\mathbf{k}}^* a_{\mathbf{k}} |e\rangle\langle g|). \quad (2.131)$$

The combined states $\{|g, \mathbf{k}\rangle, |e, \mathbf{k}\rangle\}$ form a basis of the free Hamiltonian, $H_0|i, \mathbf{k}\rangle = (E_i + \hbar\omega_{\mathbf{k}})|i, \mathbf{k}\rangle$, $i \in \{e, g\}$ and $\omega_{eg} = E_e - E_g$. We start with the dynamics

$$\begin{aligned} U(t)|e, 0\rangle &= u_{00}(t)|e, 0\rangle + \sum_{\mathbf{k}} u_{\mathbf{k}0}(t)|g, \mathbf{k}\rangle, \\ U(t)|g, \mathbf{k}\rangle &= u_{0\mathbf{k}}(t)|e, 0\rangle + \sum_{\mathbf{k}'} u_{\mathbf{k}'\mathbf{k}}(t)|g, \mathbf{k}'\rangle. \end{aligned} \quad (2.132)$$

³Note that in Volz et al. (2006), the entanglement between the atomic state and the emitted photon's polarization degree of freedom is investigated, instead of the atom-field entanglement which we discuss here.

Irreversibility is later introduced when the transition amplitudes are determined and by performing the continuum limit of the sum over all modes. We use the resolvent method (Cohen-Tannoudji et al. 1992) to determine the matrix elements non-perturbatively

$$\begin{aligned} u_{00}(t) &= \langle e, 0 | U(t) | e, 0 \rangle, \\ u_{\mathbf{k}0}(t) &= \langle g, \mathbf{k} | U(t) | e, 0 \rangle, \\ u_{0\mathbf{k}}(t) &= \langle e, 0 | U(t) | g, \mathbf{k} \rangle, \\ u_{\mathbf{k}\mathbf{k}'}(t) &= \langle g, \mathbf{k} | U(t) | g, \mathbf{k}' \rangle. \end{aligned} \quad (2.133)$$

Unitarity requires $u_{0\mathbf{k}}(t) = u_{\mathbf{k}0}^*(-t)$ for time-independent Hamiltonians.

The initial state $|e, 0\rangle$ evolves after time t_0 into

$$|\Psi(t_0)\rangle = u_{00}(t_0)|e, 0\rangle + \sum_{\mathbf{k}} u_{\mathbf{k}0}(t_0)|g, \mathbf{k}\rangle. \quad (2.134)$$

Local dephasing in the local eigenbasis $\{|e\rangle, |g\rangle\}$ erases all except the off-diagonal matrix elements and generates the classically correlated reference state,

$$\rho'(t_0) = (\Phi \otimes \mathbb{I})|\Psi(t_0)\rangle\langle\Psi(t_0)|, \quad (2.135)$$

from the initially entangled state $\rho(t_0) = |\Psi(t_0)\rangle\langle\Psi(t_0)|$. When dephasing is implemented after time t_0 , the difference between the two states is given by

$$\rho(t_0) - \rho'(t_0) = \sum_{\mathbf{k}} (u_{00}(t_0)u_{\mathbf{k}0}^*(t_0)|e, 0\rangle\langle g, \mathbf{k}| + u_{00}^*(t_0)u_{\mathbf{k}0}(t_0)|g, \mathbf{k}\rangle\langle e, 0|). \quad (2.136)$$

Further evolution from t_0 to t yields, after tracing out the electromagnetic field modes,

$$\begin{aligned} \rho_S(t, t_0) - \rho'_S(t, t_0) &= \text{Tr}_E\{U(t - t_0)(\rho(t_0) - \rho'(t_0))U^\dagger(t - t_0)\} \\ &= 2\Re \left[\sum_{\mathbf{k}} u_{00}(t_0)u_{\mathbf{k}0}^*(t_0)u_{00}(t - t_0)u_{0\mathbf{k}}^*(t - t_0) \right] \sigma_z, \end{aligned} \quad (2.137)$$

where $\Re$ denotes the real part of a complex number.

The unitary time evolution operator is expressed in terms of the resolvent $G(z) = 1/(z - H)$ as (Cohen-Tannoudji et al. 1992),

$$U(t) = \lim_{\eta \rightarrow 0^+} \frac{1}{2\pi i} \int_{-\infty}^{\infty} dE e^{-iEt/\hbar} [G(E - i\eta) - G(E + i\eta)]. \quad (2.138)$$

Projecting this matrix onto the subspace spanned by $|e, 0\rangle$ and assuming that the radiative level shift Δ_e and the decay constant Γ are energy-independent, we obtain (see, for instance, Cohen-Tannoudji et al. (1992), III.C.1)

$$u_{00}(t) = e^{-i\tilde{E}_e t/\hbar - \Gamma|t|/2}, \quad (2.139)$$

where $\tilde{E}_e = E_e + \hbar\Delta_e$. If the above assumptions are satisfied, this expression is exact. In order to obtain $u_{\mathbf{k}0}(t)$, we use Eq. (C.29), III.C.2 in Cohen-Tannoudji et al. (1992):

$$\langle g, \mathbf{k} | G(E \pm i\eta) | e, 0 \rangle \simeq \frac{g_{\mathbf{k}}}{(E \pm i\eta - \tilde{E}_g - \hbar\omega_{\mathbf{k}})(E - \tilde{E}_e \pm i\hbar(\Gamma/2))}, \quad (2.140)$$

where, again, the radiative level shift Δ_g yields $\tilde{E}_g = E_g + \hbar\Delta_g$. Equation (2.138) can be integrated using a contour integral if we choose the integration path appropriately. This requires the numerator $e^{-izt/\hbar}$ to tend towards zero for $|z| \rightarrow \infty$. Inserting $z = |z|(\cos\phi + i\sin\phi)$ yields

$$|e^{-izt/\hbar}| = |e^{-i|z|(\cos\phi + i\sin\phi)t/\hbar}| = |e^{-it|z|\cos\phi/\hbar} e^{|z|t\sin\phi/\hbar}| \leq e^{|z|t\sin\phi/\hbar}. \quad (2.141)$$

Hence, for $t > 0$ we must close the contour below the x-axis, with $\sin\phi < 0$ for $\phi \in (\pi, 2\pi)$. Since the only poles of $G(z)$ are on the x-axis, it follows from Cauchy's integral theorem that the contribution of $G(E - i\eta)$ is zero. Conversely, for $t < 0$ the contour must be closed above the x-axis, $\sin\phi > 0$ for $\phi \in (0, \pi)$ and in this case the term $G(E + i\eta)$ does not contribute. We start with $t > 0$:

$$\begin{aligned} u_{\mathbf{k}0}(t) &= \lim_{\eta \rightarrow 0^+} \frac{1}{2\pi i} \int_{-\infty}^{\infty} dE e^{-iEt/\hbar} \langle g, \mathbf{k} | G(E + i\eta) | e, 0 \rangle \\ &\simeq \lim_{\eta \rightarrow 0^+} \frac{1}{2\pi i} \int_{-\infty}^{\infty} dE e^{-iEt/\hbar} \frac{g_{\mathbf{k}}}{(E + i\eta - \tilde{E}_g - \hbar\omega_{\mathbf{k}})(E - \tilde{E}_e + i\hbar(\Gamma/2))}. \end{aligned} \quad (2.142)$$

Substituting E by a complex number z and integrating in the complex plane over a contour from $R \in \mathbb{R}$ to $-R$, which is closed below the real axis as described before we obtain by the residue theorem

$$\oint_C f(z)dz = \int_{-R}^{-R} f(z)dz + \int_{\text{arc}} f(z)dz = 2\pi i (\text{Res}(f, z_0) + \text{Res}(f, z_1)), \quad (2.143)$$

where

$$f(z) = e^{-izt/\hbar} \frac{g_{\mathbf{k}}}{(z - z_0)(z - z_1)}, \quad (2.144)$$

has two complex poles of first order at $z_0 = \tilde{E}_g + \hbar\omega_{\mathbf{k}} - i\eta$ and $z_1 = \tilde{E}_e - i\hbar(\Gamma/2)$. The residues are given by

$$\begin{aligned} \text{Res}(f, z_0) &= \frac{e^{-iz_0t/\hbar}}{z_0 - z_1}, \\ \text{Res}(f, z_1) &= \frac{e^{-iz_1t/\hbar}}{z_1 - z_0}. \end{aligned} \quad (2.145)$$

After the limits $R \rightarrow \infty$ and $\eta \rightarrow 0^+$ we finally obtain

$$u_{\mathbf{k}0}(t) = g_{\mathbf{k}} \frac{e^{-i(\tilde{E}_g/\hbar + \omega_{\mathbf{k}})t} - e^{-i\tilde{E}_e t/\hbar - \Gamma t/2}}{\hbar\omega_{\mathbf{k}} - (\tilde{E}_e - \tilde{E}_g) + i\hbar\Gamma/2}. \quad (2.146)$$

Analogously, for $t < 0$ we get

$$\begin{aligned} u_{\mathbf{k}0}(t) &\simeq \lim_{\eta \rightarrow 0^+} \frac{1}{2\pi i} \int_{-\infty}^{\infty} dE e^{-iEt/\hbar} \frac{g_{\mathbf{k}}}{(E - i\eta - \tilde{E}_g - \hbar\omega_{\mathbf{k}})(E - \tilde{E}_e - i\hbar(\Gamma/2))} \\ &= g_{\mathbf{k}} \frac{e^{-i(\tilde{E}_g/\hbar + \omega_{\mathbf{k}})t} - e^{-i\tilde{E}_e t/\hbar + \Gamma t/2}}{\hbar\omega_{\mathbf{k}} - (\tilde{E}_e - \tilde{E}_g) - i\hbar\Gamma/2}. \end{aligned} \quad (2.147)$$

Using the unitarity condition for time-independent Hamiltonians we get ($t > 0$)

$$u_{0\mathbf{k}}(t) = u_{\mathbf{k}0}^*(-t) \simeq g_{\mathbf{k}}^* \frac{e^{-i(\tilde{E}_g/\hbar + \omega_{\mathbf{k}})t} - e^{-i\tilde{E}_e t/\hbar - \Gamma t/2}}{\hbar\omega_{\mathbf{k}} - (\tilde{E}_e - \tilde{E}_g) + i\hbar\Gamma/2}. \quad (2.148)$$

We now use this to determine the difference in the open-system evolution of the atomic electronic levels, after the entanglement with the electromagnetic field has been removed and replaced by classical correlations. To this end, we insert the derived expressions into Eq. (2.137), which yields

$$\begin{aligned} \rho_S(t, t_0) - \rho'_S(t, t_0) &\simeq \frac{2}{\hbar} \Re \sum_{\mathbf{k}} |g_{\mathbf{k}}|^2 e^{-\Gamma t} \left(\frac{e^{-i(\tilde{\omega}_{eg} - \omega_{\mathbf{k}})t + \Gamma t/2} - e^{-i(\tilde{\omega}_{eg} - \omega_{\mathbf{k}})(t-t_0) + \Gamma(t-t_0)/2}}{(\omega_{\mathbf{k}} - \tilde{\omega}_{eg} - i\Gamma/2)^2} \right. \\ &\quad \left. + \frac{-e^{-i(\tilde{\omega}_{eg} - \omega_{\mathbf{k}})t_0 + \Gamma t_0/2} + 1}{(\omega_{\mathbf{k}} - \tilde{\omega}_{eg} - i\Gamma/2)^2} \right) \sigma_z, \end{aligned} \quad (2.149)$$

with the radiation-shifted atomic resonance $\hbar\tilde{\omega}_{eg} = \tilde{E}_e - \tilde{E}_g$. We now evaluate the sum in the continuum limit (Weisskopf and Wigner 1930),

$$\begin{aligned}
 & \rho_S(t, t_0) - \rho'_S(t, t_0) \\
 & \simeq \frac{2|\mu_{eg}|^2}{3(2\pi)^2\hbar\epsilon_0 c^3} \frac{2}{\hbar} \Re \int_0^\infty d\omega_{\mathbf{k}} \omega_{\mathbf{k}}^3 e^{-\Gamma t} \left(\frac{e^{-i(\tilde{\omega}_{eg}-\omega_{\mathbf{k}})t+\Gamma t/2} - e^{-i(\tilde{\omega}_{eg}-\omega_{\mathbf{k}})(t-t_0)+\Gamma(t-t_0)/2}}{(\omega_{\mathbf{k}} - \tilde{\omega}_{eg} - i\Gamma/2)^2} \right. \\
 & \quad \left. + \frac{-e^{-i(\tilde{\omega}_{eg}-\omega_{\mathbf{k}})t_0+\Gamma t_0/2} + 1}{(\omega_{\mathbf{k}} - \tilde{\omega}_{eg} - i\Gamma/2)^2} \right) \sigma_z \\
 & \approx \frac{|\mu_{eg}|^2 \tilde{\omega}_{eg}^3}{3\pi^2 \hbar^2 \epsilon_0 c^3} e^{-\Gamma t} \Re \int_{-\infty}^\infty d\omega_{\mathbf{k}} \left(\frac{e^{-i(\tilde{\omega}_{eg}-\omega_{\mathbf{k}})t+\Gamma t/2} - e^{-i(\tilde{\omega}_{eg}-\omega_{\mathbf{k}})(t-t_0)+\Gamma(t-t_0)/2}}{(\omega_{\mathbf{k}} - \tilde{\omega}_{eg} - i\Gamma/2)^2} \right. \\
 & \quad \left. + \frac{-e^{-i(\tilde{\omega}_{eg}-\omega_{\mathbf{k}})t_0+\Gamma t_0/2} + 1}{(\omega_{\mathbf{k}} - \tilde{\omega}_{eg} - i\Gamma/2)^2} \right) \sigma_z, \tag{2.150}
 \end{aligned}$$

where μ_{eg} denotes the dipole moment of the atomic transition, and ϵ_0 is the dielectric constant of the vacuum. Due to a narrow resonance of the transition amplitudes at $\omega_{\mathbf{k}} = \tilde{\omega}_{eg}$, we approximate the $\omega_{\mathbf{k}}^3$ -term in the integral with the term $\tilde{\omega}_{eg}^3$. We evaluate the remaining integral with the same contour integral methods as before. Closing the contour above the x-axis, we find a second order pole at $z_0 = \tilde{\omega}_{eg} + i\Gamma/2$ whereas the residue for functions of the type $f(z) = e^{izt}/(z - z_0)^2$ is given by

$$\text{Res}(f, z_0) = ite^{iz_0 t}. \tag{2.151}$$

Finally, this leads to a vanishing difference in the evolution of the atom:

$$\rho_S(t, t_0) - \rho'_S(t, t_0) \simeq \frac{2|\mu_{eg}|^2}{3(2\pi)^2\hbar\epsilon_0 c^3} \frac{2}{\hbar} e^{-\Gamma t} \Re \{it - i(t - t_0) - it_0\} \sigma_z = 0, \tag{2.152}$$

independently of the observation time t and the dephasing time t_0 .

2.4.1.2 Evolution of Atom-Field Negativity

Before concluding that the atom-field entanglement cannot be detected in the atomic evolution based on the local detection method, we have to demonstrate that, within the same approximations, the atom-field entanglement is finite. Based on (2.16) we can use the trace-distance dephasing disturbance (2.7) to determine the atom-field negativity of the transient state $|\Psi(t_0)\rangle$, as given in Eq. (2.134). To this end, we determine the trace norm of the difference operator $M(t_0) = \rho(t_0) - \rho'(t_0)$, which is given in Eq. (2.136). We obtain

$$M^\dagger(t_0)M(t_0) = |u_{00}(t_0)|^2 (|e\rangle\langle e| + |g\rangle\langle g|) \otimes N(t_0) (|0\rangle\langle 0| + |\Psi_E(t_0)\rangle\langle\Psi_E(t_0)|), \quad (2.153)$$

with the pure state

$$|\Psi_E(t_0)\rangle = \frac{1}{\sqrt{N(t_0)}} \sum_{\mathbf{k}} u_{\mathbf{k}0}(t_0) |\mathbf{k}\rangle, \quad (2.154)$$

and the normalization constant $N(t_0) = \sum_{\mathbf{k}} |u_{\mathbf{k}0}(t_0)|^2$. We express $N(t_0)$ using Eqs. (2.146) and employ the same approximations as before:

$$\begin{aligned} N(t_0) &= \frac{1}{\hbar^2} \sum_{\mathbf{k}} |g_{\mathbf{k}}|^2 \frac{1 - 2 \cos[(\omega_{\mathbf{k}} - \tilde{\omega}_{eg})t_0] e^{-\Gamma t_0/2} + e^{-\Gamma t_0}}{(\omega_{\mathbf{k}} - \tilde{\omega}_{eg})^2 + \Gamma^2/4} \\ &\simeq \frac{1}{\hbar^2} \frac{|\mu_{eg}|^2}{6\pi^2 \epsilon_0 \hbar c^3} \int_0^\infty d\omega_{\mathbf{k}} \omega_{\mathbf{k}}^3 \frac{1 - 2 \cos[(\omega_{\mathbf{k}} - \tilde{\omega}_{eg})t_0] e^{-\Gamma t_0/2} + e^{-\Gamma t_0}}{(\omega_{\mathbf{k}} - \tilde{\omega}_{eg})^2 + \Gamma^2/4} \\ &\approx \frac{1}{\hbar^2} \frac{|\mu_{eg}|^2 \tilde{\omega}_{eg}^3}{6\pi^2 \epsilon_0 \hbar c^3} \int_0^\infty d\omega_{\mathbf{k}} \frac{1 - 2 \cos[(\omega_{\mathbf{k}} - \tilde{\omega}_{eg})t_0] e^{-\Gamma t_0/2} + e^{-\Gamma t_0}}{(\omega_{\mathbf{k}} - \tilde{\omega}_{eg})^2 + \Gamma^2/4} \\ &= \frac{1}{\hbar^2} \frac{|\mu_{eg}|^2 \tilde{\omega}_{eg}^3}{6\pi^2 \epsilon_0 \hbar c^3} \frac{2\pi}{\Gamma} (1 - e^{-\Gamma t_0}). \end{aligned} \quad (2.155)$$

Using (2.139), we obtain the atom-field negativity via the trace distance from (2.153),

$$\begin{aligned} \mathcal{N}(|\Psi(t_0)\rangle\langle\Psi(t_0)|) &= \frac{1}{2} \text{Tr} \sqrt{M^\dagger(t_0)M(t_0)} \\ &= 2 |u_{00}(t_0)| \sqrt{N(t_0)} \\ &= 2 \sqrt{\frac{|\mu_{eg}|^2 \tilde{\omega}_{eg}^3}{3\pi \epsilon_0 \hbar^3 c^3 \Gamma} e^{-\Gamma t_0} (1 - e^{-\Gamma t_0})}. \end{aligned} \quad (2.156)$$

The temporal behavior described by (2.156) matches the physical intuition: At initial times $t_0 \sim 0$, the atom is described by the excited state $|e\rangle$, whereas after long waiting times $t_0 \rightarrow \infty$, the atom has decayed with high probability into the ground state $|g\rangle$. In both cases, the system is completely decoupled from the environment, and, hence, no atom-field entanglement is present in those limits. However, at transient times t_0 , we find finite amount of entanglement, as quantified by the trace distance of the total atom-field states with and without quantum correlations, which coincides with the negativity. Since the local state difference (2.152) vanishes also for transient values of t_0 , we find that the local detection method is not able to reveal these dynamically in the present context.

2.4.1.3 Discussion

As indicated in the beginning of this section, the atomic evolution is not influenced by removing the entanglement between the radiation field and the atom during the spontaneous decay process. This can be attributed to the complete lack of structure in the environmental frequency modes, and a system-environment coupling which induces exchange of excitations. The resulting irreversible exponential decay of the atomic population has the unique property that it can be interrupted at arbitrary moments, and the subsequent dynamics always has the exact same shape. Slight modulations thereof, for example, by higher-order corrections (Cohen-Tannoudji et al. 1992) or coupling the atom to a lossy cavity (Puri and Agarwal 1986), are expected to change this behavior (Breuer and Petruccione 2002).

2.4.2 *Many-Mode Extension of the Trapped-Ion Experiment: A Proposal*

The example of an infinite-size continuum of free-space modes shows that there exists a limit in which the bipartite correlations no longer significantly influence the local subsystem dynamics. In contrast, the trapped-ion experiment showed strong signatures of the correlations on the qubit dynamics when the ion couples to a single mode where only few discrete energy levels are occupied. The question is, what happens in between, when the open system couples to a finite-size mesoscopic environment? At which size can we neglect the influence of correlations on the reduced dynamics and how long is it possible to locally detect these correlations dynamically?

In this section we discuss possible extensions of the trapped-ion experiment, where the number of harmonic modes can be tuned to probe the transition from small model environments with low effective dimensions to an almost irreversible and memoryless decay into a continuum of modes. To this end, instead of a single ion, we consider a full chain of N ions. We now find $3N$ motional modes, whereas depending on the specific geometric arrangement of the laser system one can restrict to the N modes in just one spatial dimension. How can we entangle a single qubit with several motional modes at the same time? Driving one of the sidebands resonantly would still only couple the ion to the addressed single eigenmode, and the theoretical treatment would be equivalent to the one presented in Sect. 2.2.

A possibility to generate interactions with many modes at the same time are fast pulses on *local* sidebands. This corresponds to a sideband pulse which is fast enough such that its Fourier bandwidth contains all eigenmodes (Brown et al. 2011; Harlander et al. 2011), which is possible if sufficient laser power is available. These local sideband pulses are essentially local, spin-dependent kicks, which displace the ion in the laser focus on time scales which are too fast for the rest of the chain to follow the movement. The created local vibrational excitations are non-equilibrium

wave-packets of the phonon system (Haze et al. 2012; Toyoda et al. 2013) and can be used to study energy transport within the ion chain (Ramm et al. 2014; Ramm 2014); see also Sect. 5.4.

An interaction of a single electronic qubit with an ensemble of modes can be generated based on the following scheme:

- Apply a spin-dependent kick on the selected ion.
- Let the system evolve freely for time t .
- Apply another spin-dependent kick, 180° out of phase to the first one.

To illustrate the mechanism of this sequence, let us assume that the ion was prepared in a product state of $|\varphi\rangle = |0\rangle$ and $\rho_m = |0\rangle\langle 0|$, which denote the respective ground states of qubit and motion. The first spin-dependent kick promotes the ion in a superposition state (Monroe et al. 1996)

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|0, 0_1\rangle + |1, \alpha_1\rangle), \quad (2.157)$$

where $|\alpha_1\rangle$ denotes a coherent state on the Fock space describing the motion of the addressed ion. Such an excitation is not an eigenstate of the full coupled ion chain, and will therefore evolve during t , which constitutes the next step. During the free evolution, the populations of the electronic qubit do not evolve. Finally, the phonon evolution is mapped back onto the electronic degree of freedom with a second displacement. The 180° out-of-phase shift of the second kick ensures that at $t = 0$, the excitation is mapped right back to the initial state. This combination of $\pi/2$ -pulses (creating equal superpositions of ground and excited states) and free evolution basically constitutes a Ramsey scheme (Ramsey 1990) on the local phonons.

The advantage of this scheme is that it can be implemented with arbitrary ion numbers N . Furthermore the ratio of the trap frequencies in axial and radial directions controls how densely the motional frequencies along the observable direction are packed, which allows to tune the spectral density of the phonon bath. When the inter-ion distance reduces, it becomes experimentally more challenging to focus the laser only onto a single ion. This technical problem may be circumvented using the fact that the outer ions usually show significantly larger spacings than those in the center when the chain becomes large, which facilitates the local addressing, see also the inset in Fig. 1.2. A study of a related scheme (De Chiara et al. 2008) demonstrated that the trace-distance information backflow from the phonons to the qubit—a quantifier of non-Markovian effects—peaks at the structural phase transition (Borrelli et al. 2013), which occurs when the axial trap frequency exceeds a threshold value and the ions change their arrangement from a linear to a zig-zag structure (Birkel et al. 1992; Waki et al. 1992).

Using the above Ramsey scheme, the qubit can be coupled to a tunable bath of phonons. To explore the influence of correlations on the qubit dynamics, several possibilities emerge. The most natural choice would be to submit correlated states to the Ramsey dynamics, and to check for a different resulting qubit evolution when additionally the local dephasing operation is implemented before the first spin-

dependent kick. Correlated states of spin and motion may be generated by the same spin-dependent displacement operations or other sideband pulses. Alternatively, one may interrupt the Ramsey scheme to implement a dephasing operation after the first spin-dependent kick.

2.4.3 *Quantum Phase Transition in a Transverse-Field Ising Chain*

Finally, we study another scenario of a many-body interacting system. We now move from an interacting multi-phonon environment to a system of interacting spins which describes a quantum magnet and shows interesting phenomena, such as a quantum phase transition (Sachdev 1999). A detailed study of this model will be provided in Sect. 2.4.

Quantum phase transitions are essentially crossings or avoided crossings of energy levels, as a function of an external control parameter, which involve the ground state. After such a crossing, a new state has lower energy than the ground state in the previous quantum phase, which renders it the new ground state. Quite generically, this new ground state exhibits different properties than the original ground state, including, for example its entanglement properties. This makes the ground-state quantum correlations perceptive to the quantum phase transition, as confirmed in a series of case studies (Osterloh et al. 2002; Osborne and Nielsen 2002; Vidal et al. 2003; Wu et al. 2004).

Detecting quantum correlations in a large crystal can, however, not be achieved on the basis of conventional detection schemes—which require access to each individual spin. For this reason, a direct experimental verification of diverging entanglement quantifiers at the critical point of a phase transition has not yet been achieved. Such an experiment would directly demonstrate the existence of a striking link between concepts from quantum information and solid state physics. In controlled quantum systems, Hamiltonians that mimic the dynamics of solid-state systems can be designed, and local access is given to individual constituents; recall Sect. 1.3 for a brief overview. Thus, in this last study, we apply the local detection protocol to detect the quantum correlations in the ground state of large spin chains, by considering only one of the spins as the accessible subsystem and treating the remaining spins as environment (Gessner et al. 2014b).

To be precise, we study a family of spin chain models with variable-range interactions, given by

$$H = - \sum_{\substack{i,j=1 \\ (i < j)}}^N \frac{J_0}{|i-j|^\alpha} \sigma_x^{(i)} \sigma_x^{(j)} - B \sum_{i=1}^N \sigma_y^{(i)}. \quad (2.158)$$

As discussed in Sects. 1.3.1.7 and 1.3.2, this Hamiltonian can be simulated efficiently with a chain of trapped ions, and cold atoms on tilted optical lattices. The interaction range is determined by the continuous parameter α , and in trapped-ion experiments this parameter can be tuned within the values $0 < \alpha < 3$ (Kim et al. 2010), whereas typically, these experiments work around $\alpha \approx 1$. A detailed study of this model and the associated quantum phase transition will be provided in Chap. 4. In this section, we restrict to the application and interpretation of the local detection protocol, for both zero- and nonzero temperature states.

The Hamiltonian (2.158) shows a quantum phase transition from (anti-)ferromagnet to paramagnet when the external magnetic field B exceeds the strength of the spin-spin interactions, which are determined by J_0 . The sign of J_0 determines whether the system behaves as a ferromagnet ($J_0 > 0$) or anti-ferromagnet ($J_0 < 0$) when $B = 0$. While both cases are scrutinized in Chap. 4, in this section we restrict to the ferromagnetic case $J_0 > 0$. The theoretical treatment, however, is easily transferred to the anti-ferromagnet. In the ferromagnetic phase, for example at $B = 0$, the ground state⁴ is described by superposition states of Greenberger-Horne-Zeilinger (GHZ) type (Greenberger et al. 1989; Dür et al. 2000; Štelmachovič and Bužek 2004),

$$|\text{GHZ}_\varphi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\ldots\rangle + e^{i\varphi}|\downarrow\downarrow\ldots\rangle). \quad (2.159)$$

In such a configuration, each spin is entangled with the rest of the chain.⁵ The ground-state entanglement reduces when B increases since the spins start to align along the y direction of the external magnetic field. In the extreme case of $B \gg J_0$, the ground state forms a completely uncorrelated product state $|\uparrow_y\rangle^{\otimes N}$. Hence, we expect the bipartite ground-state entanglement between a randomly selected spin and the rest of the chain to decay monotonously as the external magnetic field is scanned from zero to large fields, where the transition from entangled to separable becomes sharper with increasing system size.

2.4.3.1 Ground-State Negativity from the Evolution of the Single-Spin Magnetization

Let us first consider the case of the pure ground state $|\Psi_0\rangle$ of H , as given in (2.158). We apply the local detection protocol to the state $|\Psi_0\rangle$, by considering the left-most spin the accessible subsystem, whereas the remaining spins are treated as part of an inaccessible reservoir. In this case, the initial state $|\Psi_0\rangle$ does not evolve in time, since it is the ground state, hence, an eigenstate of H . Dephasing in the local eigenbasis,

⁴See also Heisenberg (1928), Bethe (1931) for pioneering work on quantum magnetism.

⁵Multipartite entanglement will only be introduced in the next chapter. We already remark here that the GHZ-states contain global multipartite entanglement, but when one or more particles are traced out, no entanglement remains in the subsystem (Dür et al. 2000).

however, populates a combination of excited states, which can lead to time evolution. The signatures of this time evolution on the single spin are then employed to detect and estimate the ground-state entanglement.

The Z_2 symmetry of the Hamiltonian H (invariance under a 180° rotation around the y -axis) requires that the reduced, single-spin density matrices are diagonal in the eigenbasis of σ_y , for all eigenstates. For further details we refer to Chap. 4. Therefore, the local dephasing is always implemented by projecting non-selectively in the eigenbasis of σ_y of the left-most spin. The resulting dephasing operation yields the reference state $(\Phi \otimes \mathbb{I})|\Psi_0\rangle\langle\Psi_0|$, which, in general, is no longer an eigenstate of the system. Recall from Sect. 2.1.2 that the dephasing in the eigenbasis minimizes the trace distance over all possible dephasing bases in the case of a pure state. Since the local spin is described by a two-level system, we obtain the negativity (2.12) as a figure of merit for the quantum correlations as [see Eqs. (2.7) and (2.16)]

$$D(|\Psi_0\rangle\langle\Psi_0|) = \| |\Psi_0\rangle\langle\Psi_0| - (\Phi \otimes \mathbb{I})|\Psi_0\rangle\langle\Psi_0| \| = \mathcal{N}(|\Psi_0\rangle\langle\Psi_0|). \quad (2.160)$$

To obtain a locally observable lower bound on this quantity, we monitor the time evolution of the dephased state on the single-spin level

$$\rho_S(t) = \text{Tr}_E\{U(t)(\Phi \otimes \mathbb{I})|\Psi_0\rangle\langle\Psi_0|U^\dagger(t)\}, \quad (2.161)$$

with $U(t) = e^{-iHt/\hbar}$. Any deviation from $\rho_S(0)$ indicates the presence of entanglement in $|\Psi_0\rangle$ and the trace distance

$$d(t) = \|\rho_S(t) - \rho_S(0)\| \quad (2.162)$$

provides a lower bound on the negativity $\mathcal{N}(|\Psi_0\rangle\langle\Psi_0|)$ at all times t . Since the local state $\rho_S(t)$ must remain diagonal in the eigenbasis of σ_y , the trace distance (2.162) is fully determined by the magnetization $m_y(t) = \text{Tr}\{\rho_S(t)\sigma_y\}$ of the observable spin along the y direction as

$$d(t) = \frac{1}{2}|m_y(t) - m_y(0)|. \quad (2.163)$$

The maximum local distance

$$d_{\max} = \max_t d(t) \leq \mathcal{N}(|\Psi_0\rangle\langle\Psi_0|) \quad (2.164)$$

is plotted in Fig. 2.11. We find that for very small values of B , despite large amounts of entanglement in the ground state (as quantified by $D(|\Psi_0\rangle\langle\Psi_0|)$ and plotted by the green dashed line in Fig. 2.11b), the dynamics does not reveal those correlations to the local spin state. When we approach the quantum critical point around $B \simeq J_0$, a strong, visible signal emerges and almost reaches the upper bound which is given by the negativity of the total state. As the external field is further increased, the total correlations decrease, and consequently the local signal disappears. The distinct

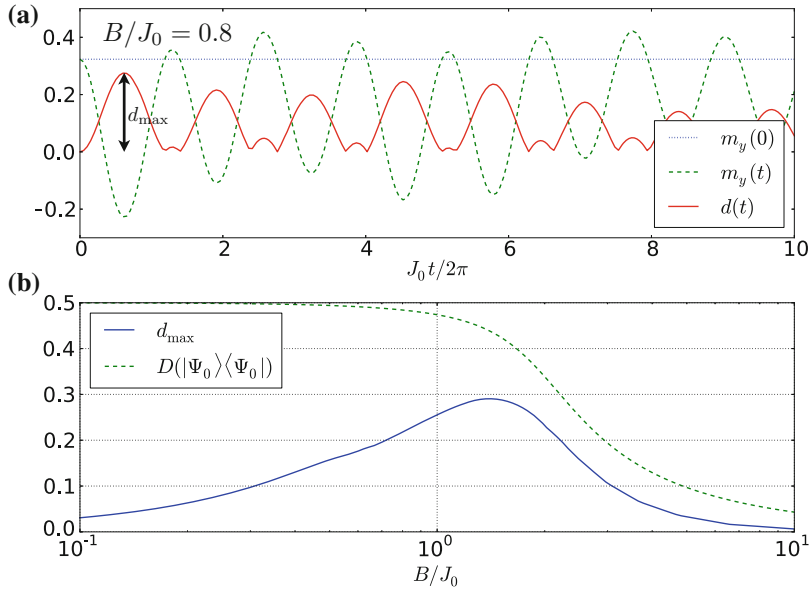


Fig. 2.11 Monitoring the dephasing-induced time evolution (2.163) of the local single-spin magnetization along the y -axis is sufficient to provide a lower bound for the total ground state negativity (2.160). The maximal local trace distance [maximized over t , see Eq. (2.164)] shows a peak close to the critical point of the quantum phase transition, of which predecessors can be observed in this small system of $N = 7$ spins. Observation of the time evolution over a longer interval can further improve the lower bound. The total trace distance $D(|\Psi_0\rangle\langle\Psi_0|)$ is equal to the ground-state negativity, see Eq. (2.160). Adapted from Gessner et al. (2014b)

peak identifies the critical point of the quantum phase transition. An explanation for this behavior can be given after realizing that the dephasing operation connects the ground-state properties to the excitation spectrum, since these are revealed via the dephasing-induced dynamics. Indeed, the excitation spectrum undergoes a significant structural change around the critical field of the quantum phase transition, which induces complex dynamics which is more likely to reveal correlation properties to the local system, as we will discuss in further detail in Chap. 4.

Typical parameters are $J_0 \approx 2\pi \times 500$ Hz and coherence times are usually around 3 ms. In the simulations, we therefore restrict the observable time window to a range of 5 ms. The simulations are implemented with a `python` code employing the `qutip` package (Johansson et al. 2013).

2.4.3.2 Local Bound for the Minimum Entanglement Potential of Thermal States

For finite-sized ferromagnetic systems, there exists a small energy gap Δ between the ground state and the first excited state of H . This gap vanishes in the thermodynamic limit, $N \rightarrow \infty$, where the ground state becomes two-fold degenerate. Despite very low temperatures, it is therefore more realistic to consider the thermal state

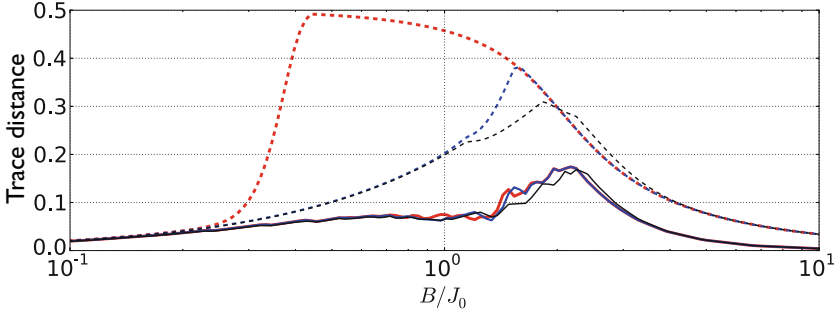


Fig. 2.12 For low, finite temperatures, the minimum dephasing disturbance $D_{\min}$ (dashed lines), defined in Eq. (2.11), vanishes when the excitation gap between the almost-degenerate ground states is smaller than the thermal energy kT (see Fig. 2.13). The parameters are $N = 7$ spins with $kT = 10^{-5}$ (thick, red), $kT = 0.1$ (medium, blue), and $kT = 1$ (black, thin line). The local signature $d_{\min}$ (corresponding continuous lines), defined in Eq. (2.20), is, however, only weakly affected by finite temperatures. The simulations consider a set of 20 uniformly distributed pairs of qubit basis states for the different dephasing bases that occur in the definition (2.11) of the minimum dephasing disturbance. Adapted from Gessner et al. (2014b) (color figure online)

$$\rho_{\beta} = \frac{e^{-\beta H}}{\text{Tr} e^{-\beta H}}, \quad (2.165)$$

instead of the pure ground state, with $\beta = 1/kT$, where k denotes the Boltzmann constant and T the temperature. Even in the zero-temperature limit, the degeneracy produces an incoherent mixture of the two ground states instead of a pure state. This has the effect that the entanglement, and even the discord-type correlations vanish: The mixture of the two ground states

$$\rho_{\infty|B=0} = \frac{1}{2} (|\text{GHZ}_{\varphi}\rangle\langle\text{GHZ}_{\varphi}| + |\text{GHZ}_{\varphi+\pi}\rangle\langle\text{GHZ}_{\varphi+\pi}|), \quad (2.166)$$

describes a state of zero discord since the opposing phase factors lead to complete cancellation of the coherences when both states are mixed. The minimum dephasing disturbance $D_{\min}$, as introduced in Eq. (2.11), therefore yields zero for the low-temperature thermal ground state (2.166) when $B = 0$, and increases to a nonzero value when the energy gap between the ground states exceeds the thermal excitation energy kT . It finally recovers the negativity of the energetically lower-lying state, as can be seen from the dashed lines in Fig. 2.12 and the inset in Fig. 2.13. From these figures we can also observe how the sharpness of this transition increases with decreasing T . A lower bound for $D_{\min}$ can be obtained by dephasing in a large ensemble of basis states, and monitoring the subsequent time evolutions, which yields the optimal local witness $d_{\min}$, as defined in Eq. (2.20). While the total correlations are strongly affected by the mixing process, the local dynamical signature remains largely unaffected, and still shows a maximum in the vicinity of the quantum phase transition.

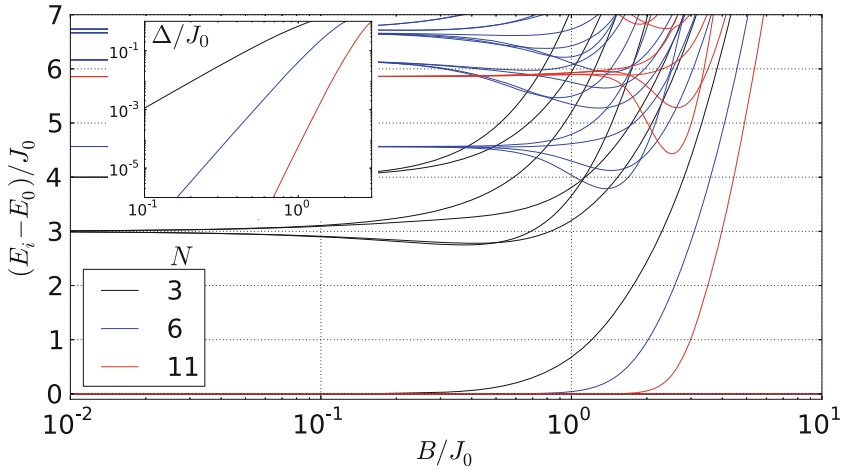


Fig. 2.13 Lower part of the excitation spectrum of the ferromagnetic spin chain (2.158) at $\alpha = 1$ for different system sizes. The gap Δ between the ground- and first excited state (*inset*) determines at which temperature the low-temperature thermal state (2.165) turns from a pure ground state to an incoherent mixture of both ground states. Unlike the pure ground state, which contains GHZ-type entanglement (2.159), the incoherent mixture (2.166) contains neither entanglement nor discord-type correlations. In the thermodynamic limit the two ground states are degenerate for values of B below the critical field. A detailed discussion of the spectrum is provided in Chap. 4

In the present study, we have established a dynamical witness for the critical field of the quantum phase transition based on entanglement. It is based on measurements of dephasing-induced dynamics of the the single-spin magnetization. This dynamics is entirely determined by the excitation spectrum which is populated by the local dephasing operation. The fact that we observe a strong peak at the phase transition point indicates a connection of the quantum phase transition to the excitation spectrum, which we will study in further detail in Chap. 4.

2.5 Discussion

We have seen in this chapter that the coupling of a small controllable subsystem to additional, inaccessible degrees of freedom can be harnessed to reveal global properties, such as bipartite correlations, via the local subsystem dynamics. The basic mechanism which enables the local detection protocol presented in this chapter is the strong influence of correlations on the subsequent time evolution of an open quantum system. The method can always be applied when full coherent control is available over a part of the system. Whether or not the method successfully reveals the presence of correlations through the local dynamics depends on the time evolution of the full system.

The examples studied in this chapter have explored the limitations of the method for different couplings, environments, and physical settings. Scenarios where the

detection was successful included the dissipative coupling to a single-mode harmonic oscillator, which was studied in the trapped-ion experiment. Numerical simulations also suggest an observable local signature of correlations in a high-temperature limit. Moreover, the photonic experiment demonstrated a clearly observable signature of initial correlations in the presence of coupling to a continuum of modes that lead to irreversible, memoryless dephasing of a qubit. The important difference to the trapped-ion case is that, here, the coupling does not induce exchange of excitations between system and environment. A completely memoryless, dissipative environment was studied theoretically using the example of spontaneous emission of an atomic excitation into the free-field modes. In this case, there is no influence of the atom-field entanglement on the atomic subdynamics. Taken together, these findings suggest that the efficacy of the method does not rely on non-Markovian memory effects or structured environments, as long as the reduced dynamics is described by pure dephasing (no excitation exchange). For dissipative environments (including excitation exchange), a structured environment, leading to back-action on the system, seems to be a necessary requirement for a local signature of initial correlations based on the present method.

The border where the influence of initial correlations on the reduced dynamics vanishes due to dispersion into many modes of a high-dimensional system may be explored thoroughly with a proposed generalization of the trapped-ion experiment. Based on the intuition developed in this chapter, we expect the local signal to decrease when the environment becomes sufficiently large and spectrally broad, to be effectively memoryless. In this case, when we find irreversible dissipation, free of revivals, we expect little influence of the initial correlations on the open-system dynamics. Similarly, theoretical studies in the context of a lossy cavity, described by coupling to a continuum of modes of variable spectral width, may also directly test the dependence of the method on non-Markovian effects in the presence of dissipative couplings.

The separation into system and environment can formally be carried over to few-body or many-body systems, where local control of some of the constituents is possible, while complete control over the state of the full system is not available. To verify the presence of discord-type correlations, which are necessary for carrying out specific quantum information protocols, one may employ the local detection method, given that interactions between the constituents are present. To this end, one treats the accessible subsystem as the “open system” and the remaining constituents of the interacting system as “environment”. The method is expected to be useful when the total system becomes too large to employ tomographic reconstruction methods or to measure complicated entanglement witnesses, which require measurements on all parties. In this case, one may restrict to controlling a low-dimensional subsystem, which is more feasible. In the case of excitation-exchanging interactions, however, we must consider the limitation of the method in the extreme case when the environmental part becomes effectively memoryless, as discussed above. The discussion of the example of the spin-chain model furthermore raises the intriguing question about the connection between ground-state quantum correlations, the excitation spectrum and the quantum phase transition, which will be analyzed in Chap. 4.

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