Chapter 1
Hyperinflation Theories: An Abridged Survey

1 Introduction

According to Cagan's definition hyperinflation starts in a month when the price level increases at least by 50 % and it ends when the price level drops below 50 % and stays there by at least one year. This was Cagan's empirical criteria to select episodes of hyperinflation. Using this definition Hanke and Krus (2013) were able to identify 56 episodes of hyperinflation worldwide since the first one in France (1796) up to the last one that occurred in Zimbabwe (2007). However, this number should be taken as a lower bound estimate if one takes into account a proper definition of hyperinflation. I will define hyperinflation as a pathology that arises when the price of money goes to zero in finite time.

This chapter presents an abridged survey of hyperinflation models that were built to explain such episodes. These models explain this phenomenon either through fundamentals (exogenous variables) or bubbles. From the outset I would like to stress a basic principle: the model has to be hit by an exogenous variable change for the endogenous variable to change. Otherwise, the model would yield a bubble, i.e., endogenous variables changing, holding the exogenous variables constant. Several models that I will present here would be classified in this category. However, I would claim that in all hyperinflations episodes this phenomenon has been produced by fundamentals, namely financing the fiscal deficit by issuing money. Thus, I will paraphrase Milton Friedman and state that hyperinflation is always and everywhere a fiscal problem.

The stylized facts of hyperinflation experiences are the following: (1) the real quantity of money decreases and it approaches zero while hyperinflation lasts; (2) the rate of inflation skyrockets; (3) the public deficit is financed by issuing money; (5) the length of time that hyperinflation lasts is variable and it depends on the experience of each country; (5) a foreign currency is substituted for the local currency, first as a unit of account and reserve of value and later on as a means of

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payments; and (6) hyperinflation stops overnight with a small social cost or no cost at all, and a change in the monetary-fiscal policy regime. A theoretical framework to explain hyperinflation has to account for these facts.

This chapter is organized as follows. First, in Sect. 2, I present Cagan's (1956) classical model of hyperinflation that assumes a semi-log demand for money, an adaptive expectation mechanism and an exogenous money supply process. For the sake of completeness I also present Cagan's model using the rational expectations hypothesis. The second model, in Sect. 3, is Kalecki's (1962) model of hyperinflation that assumes that the rate of monetary growth is endogenous due to a fiscal deficit to be financed by issuing money. To the best of my knowledge this is the first paper that used such assumption. The third model, in Sect. 4 has the same ingredients of Cagan's model but replaces the exogenous money supply by the hypothesis that money supply is endogenous and seigniorage is constant. This model will be named the constant seigniorage adaptive expectations model (CSAE). The fourth model, in Sect. 5, is Kiguel's (1989) extension of the seigniorage model that explains hyperinflation through a-once-for-all change in the public deficit that generates a hyperinflation disequilibrium path. This model will be named the increasing seigniorage model (IS). The fifth model, in Sect. 6, is the rational expectations version of the constant seigniorage model (CSRE) due to Sargent and Wallace (1987) and Bruno and Fischer (1990), which yields a high inflation path produced by multiple equilibria and an ad hoc shock that displaces the economy from an unstable to a stable equilibrium. However, such high inflation path is not a hyperinflation. The sixth model, in Sect. 7, is Sargent et al.'s (2009) extension of the hyperinflation models of Sargent and Wallace (1987), Marcet and Sargent (1989) and Marcet and Nicolini (2003), that assumes shifts in the deficit and destabilizing expectations (VSDE). Section 8 concludes.

2 Cagan's Model

Cagan's model has three equations. The first is a demand for money with semilog specification:

$$\log m = -\alpha \pi^e \tag{1.1}$$

where m is real cash balance, α the semi-elasticity of demand for real quantity of money with respect to the expected rate of inflation and π^e the expected rate of inflation. The second equation is the adaptive expectation mechanism, according to which the expected rate of inflation is revised based on the difference between the current rate of inflation and the expected rate:

$$\dot{\pi}^e = \beta \left(\pi - \pi^e\right) \tag{1.2}$$

The parameter β determines the speed of adjustment of expected rates to current rates of inflation. The adjustment is instantaneous if $\beta \to \infty$. It can be shown that the expected rate of inflation is an exponentially weighted average of past rates of inflation.[1] The third equation of Cagan's model assumes that the rate of growth (μ) of the quantity of money is exogenous:

$$\mu = \frac{d \log M}{dt} = \mu(t) \tag{1.3}$$

To solve this model, I begin by taking time derivatives of both sides of the demand for money and we use the adaptive expectation mechanism to get:

$$\mu - \pi = -\alpha \dot{\pi}^e = -\alpha\beta \left(\pi - \pi^e\right)$$

This equation can be rewritten as:

$$\mu - \alpha\beta\pi^e = \left(1 - \alpha\beta\right)\pi$$

By using the demand for money (1.1) to substitute the expected rate of inflation into this expression produces the following result:

$$\mu + \beta \log m = \left(1 - \alpha\beta\right)\pi$$

By taking time derivatives of both sides of this equation I obtain:

$$\dot{\mu} + \beta\left(\mu - \pi\right) = \left(1 - \alpha\beta\right)\dot{\pi}$$

By rearranging this expression we reduce Cagan's model to a relation between two observable variables, the rate of inflation and the rate of monetary growth:

$$\dot{\pi} = \frac{\beta}{1 - \alpha\beta}\left(\mu - \pi\right) + \frac{1}{1 - \alpha\beta}\dot{\mu}, \alpha\beta \neq 1 \tag{1.4}$$

This is a linear first order differential equation for the rate of inflation. The rate of change in inflation depends on two terms: (1) the gap between the rate of monetary growth and the rate of inflation and (2) the acceleration of the rate of monetary growth. Figure 1.1 gives the phase diagram of this equation when the rate of monetary growth is constant. In steady state, the rate of inflation is equal to the rate of growth of the quantity of money. This figure assumes that $1 - \alpha\beta > 0$, otherwise the equation is not stable. Thus, Cagan's model would yield a bubble solution only when $1 - \alpha\beta < 0$.

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[1]The solution of the adaptive expectation mechanism differential equation is given by: $\pi^e(t) = \int_{-\infty}^{t} \beta\pi e^{-\beta(t-x)} dx, \int_{-\infty}^{t} \beta e^{-\beta(t-x)} dx = 1$.

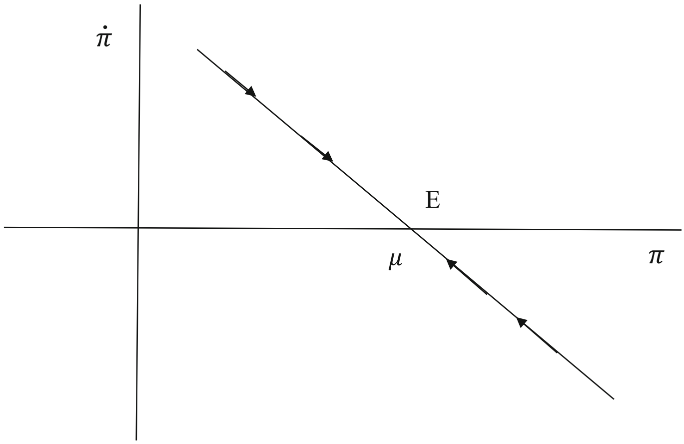

Fig. 1.1 Cagan's AE model: phase diagram

The solution of the differential equation of Cagan's model is given by:

$$\pi(t) = \tilde{\mu}(t) + \frac{1}{1-\alpha\beta}\left[\mu(t) - \tilde{\mu}t\right] \tag{1.5}$$

where $\tilde{\mu}$, an exponentially weighted average of past rates of monetary growth, could be taken as a particular trend rate, is given by:

$$\tilde{\mu}(t) = \int_{-\infty}^{t} \frac{\beta}{1-\alpha\beta}\mu e^{-\frac{\beta}{1-\alpha\beta}(t-x)}dx$$

The rate of inflation has two components. The first, the inertia of the process, is given by the trend rate of monetary growth. The second component depends on the gap between the current rate of monetary growth and the trend rate.

Cagan's model has a bifurcation point when $\alpha\beta = 1$, since the model is stable when $1 - \alpha\beta > 0$ and unstable when $1 - \alpha\beta < 0$. If $\alpha\beta = 1$, the inflation rate is given by:

$$\pi = \mu + \frac{1}{\beta}\dot{\mu}, \alpha\beta = 1 \tag{1.6}$$

The rate of inflation is equal to the rate of growth of the quantity of money plus a term that takes into account the change in the rate of growth of money.

2.1 Rational Expectations

The adaptive expectation mechanism is an error-learning device. It assumes that people learn from their forecast errors, but even so this mechanism implies that they persist being wrong for a long period of time. In the 70s it was abandoned when the rational expectations alternative became available. The rational expectations hypothesis supposes that people make forecast consistent with the relevant economic theory using all the information available at the time the forecast is made. People do make forecast errors but the errors are unpredictable given the information at hand. When the variables are not stochastic rational expectations are equivalent to assume perfect foresight:

$$\pi^e = \pi \tag{1.7}$$

Cagan's model becomes forward looking under the rational expectations hypothesis. In order to solve the model, I take time derivatives of the demand for money function (1.1) and I use the perfect foresight hypothesis to write:

$$\mu - \pi = -\alpha \dot{\pi}^e = -\alpha \dot{\pi}$$

From this expression I obtain the differential equation for the rate of inflation:

$$\dot{\pi} = \frac{1}{\alpha} (\pi - \mu) \tag{1.8}$$

The rate of change in inflation is proportional to the difference between the inflation rate and the rate of monetary growth. Figure 1.2 shows the phase diagram

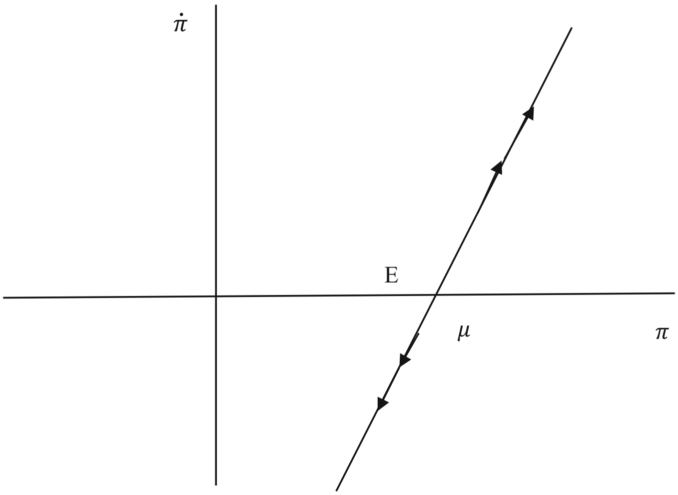

Fig. 1.2 Cagan's RE model: phase diagram

of this equation. The model is unstable and a bubble, either hyperinflation or hyperdeflation, can exist as shown by the arrows.

The solution of the linear differential equation has two components: the fundamental and the bubble solution:

$$\pi(t) = \int_t^\infty \frac{1}{\alpha}\mu e^{-\frac{1}{\alpha}(x-t)}dx + Ce^{\frac{1}{\alpha}t} \tag{1.9}$$

The fundamental solution states that the rate of inflation is an exponentially weighted average of the future rates of growth of the quantity of money. When the constant C is equal to zero there is no bubble.

3 Kalecki's Hyperinflation Model

Cagan's model did not pay attention to the fact that the rate of monetary growth is endogenous in hyperinflation environments. The basic hypothesis of all hyperinflation models that I will present in the following sections assumes that money finances the fiscal deficit. This hypothesis was first used by Kalecki's (1962) overlooked paper, which has been neglected by the profession.

Kalecki's model has three equations: (1) $V = V(\pi^e)$; (2) $\pi^e = \pi$; and (3) $\dot{M} = Pf$. The first equation states that velocity of money depends on the expected rate of inflation ($V'(\pi^e) > 0$); the second is the perfect foresight hypothesis and the third equation assumes that money finances the public deficit.[2] It should be pointed out that Kalecki's solution of his model is not correct because when taking time derivatives of the quantity equation ($MV = PY$) he assumed $\dot{V} = 0$. Below, I will present the solution of his model.

By using the quantity theory of money $MV = Py$ I can write:

$$\frac{\dot{M}}{M} + \frac{\dot{V}}{V} = \frac{\dot{P}}{P}$$

where I assume that the rate of growth of real income is equal to zero. Since

$$\dot{M} = Pf, \dot{V} = \frac{\partial V}{\partial \pi}\dot{\pi}, \frac{\dot{P}}{P} = \pi$$

I substitute these results into the quantity theory equation to obtain the following differential equation:

[2]Kalecki's (1962, p. 277) states that "$\dot{M}$ is the increase in the money in circulation per unit of time which occurs trough the channels of budget deficit and the expansion of banking credit to business."

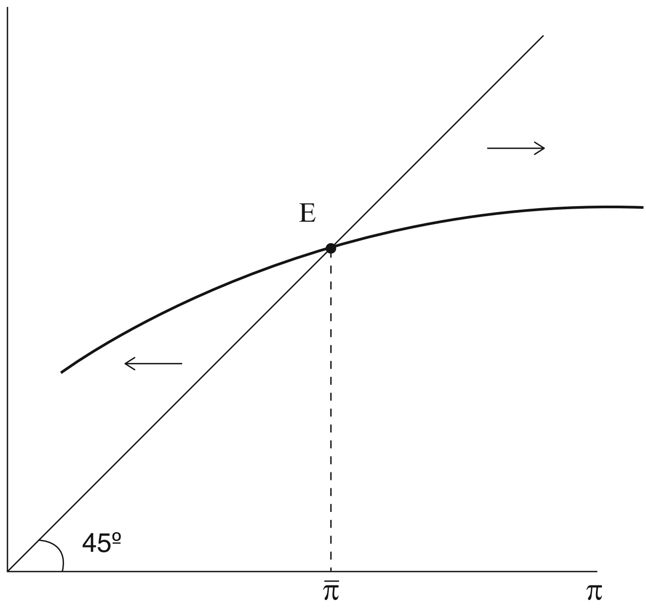

Fig. 1.3 Kalecki's model: phase diagram

$$\gamma(\pi)\,\dot{\pi} = \pi - \frac{f}{y}V(\pi) \tag{1.10}$$

The coefficient $\gamma(\pi)$ is the semi-elasticity of income velocity with respect to the rate of inflation: $\gamma(\pi) = (1/V)\,\partial V/\partial\pi > 0$.

According to Kalecki this semi-elasticity decreases when the rate of inflation increases because he assumed the velocity function to be concave. Figure 1.3 presents the phase diagram of this differential equation. The equilibrium point E is unstable as shown by the arrows. Thus, this model can only generate a bubble hyperinflation.

4 Constant Seigniorage Adaptive Expectations Model (CSAE)

The constant seigniorage adaptive expectations model assumes that money finances the fiscal deficit. Thus, the government flow budget constraint is given by:

$$\frac{\dot{M}}{P} = f \tag{1.11}$$

where seigniorage ($\dot{M}/P$) is equal to the constant public deficit (f). Therefore, the rate of monetary growth depends on the fiscal deficit and the real cash balances, according to:

$$\frac{\dot{M}}{M} = \frac{f}{m}$$

Taking the time derivative of real cash balance ($m = M/P$) the flow budget constraint can be written as:

$$\dot{m} = f - m\pi = f - \tau(m) \tag{1.12}$$

where the inflation tax revenue, $\tau(m) = \pi m$, is a function of the real cash balance. From now on I present all the models using the same technique, namely this differential equation for the real quantity of money and its corresponding phase diagram. This unifying approach allows highlighting both the differences and similarities among the models.

The constant seigniorage and adaptive expectations model consist of three equations: (1) a demand for money, $\log m = -\alpha\pi^e$; (2) an adaptive expectation mechanism $\dot{\pi}^e = \beta(\pi - \pi^e)$ and (3) a constant deficit (f) to be financed by money, where $m = M/P$ is real cash balance, π^e the expected rate of inflation, M the stock of money, P the price level, π the rate of inflation, $\dot{X}$ the time derivative of X and α and β positive parameters. After some algebra, the inflation tax revenue $\tau = m\pi$ is given by:

$$\tau = \frac{1}{1 - \alpha\beta}(f + \beta m \log m) \tag{1.13}$$

From this expression it is straightforward to obtain the differential equation for the real quantity of money:

$$\dot{m} = -\frac{\beta}{1 - \alpha\beta}(\alpha f + m \log m) \tag{1.14}$$

I have two cases: (1) $1 - \alpha\beta > 0$ and (2) $1 - \alpha\beta < 0$. This second case will not be analyzed since it yields no hyperinflation.[3]

Figure 1.4 shows the phase diagram for the first case, where $\alpha\beta < 1$. There are two equilibrium points. The low inflation equilibrium, point B, is stable, as shown by the arrows. The high inflation equilibrium, point A, is unstable. The path AC is a bubble because hyperinflation happens in spite of the fiscal deficit being constant. When there is a bubble ($m \to 0$) the inflation tax increases. Thus, the demand for money becomes inelastic with respect to the inflation rate.

[3]When $1 - \alpha\beta < 0$ there is a possibility of a hyperdeflation bubble.

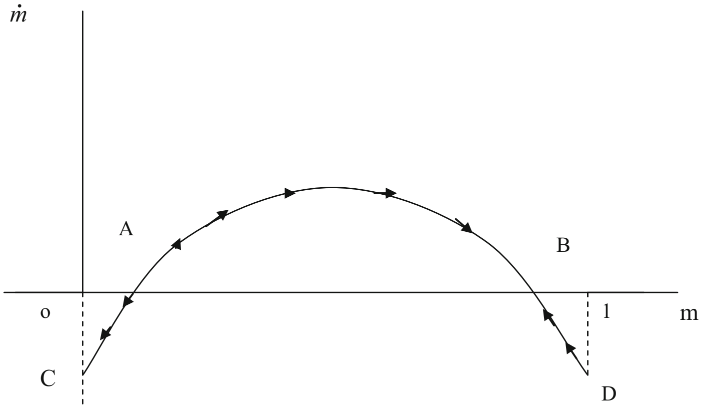

Fig. 1.4 CSAE model: phase diagram

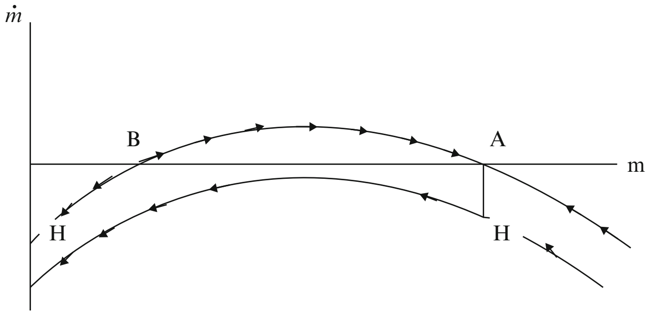

Fig. 1.5 Fiscal crisis model: phase diagram

5 Seigniorage Model Under a Fiscal Crisis (FC)

Kiguel's model is formed by the same three equations of the previous model, but the public deficit to be financed by issuing money is not constant. The public deficit to be financed by money is a step function, with the following specification:

$$f(t) = \begin{cases} f_0 < f^*, & \text{if } t < 0 \\ \bar{f} > f^* = \frac{e^{-1}}{\alpha}, & \text{if } t \geq 0 \end{cases} \tag{1.15}$$

where f^* is the maximum deficit that can be financed through the inflation tax in a permanent basis. Thus, at the moment t there is a fiscal crisis, namely the fiscal deficit increases to an unsustainable level ($\bar{f} > f^*$). Figure 1.5 shows the phase diagram for this case, under the hypothesis that the parameters are such that $\alpha\beta < 1$. The economy before time t was at the low inflation equilibrium (point A). Then, at time t it shifts from A to H, entering the hyperinflation path HH, as the real quantity of money goes to zero as indicated by the arrows.

6 Constant Seigniorage Rational Expectations Model (CSRE)

The Sargent and Wallace (1987) and Bruno and Fischer (1990) model replaces the adaptive expectations mechanism by the rational expectations hypothesis. The model is formed by the same equations of the constant seigniorage model except that now there is perfect foresight ($\pi^e = \pi$). It is straightforward to show that the differential equation for m is given by:

$$m = f - m\pi = f + \frac{m \log m}{\alpha} \tag{1.16}$$

Figure 1.6 is the phase diagram of this model. The low inflation point (B) is unstable and the high inflation point (A) is stable, as shown by the arrows. There is no hyperinflation bubble. However, there is a possibility of a hyperdeflation bubble (BD).

It is assumed that the unstable point B is the equilibrium of the economy. The hyperinflation path, according to Sargent and Wallace (1987) is yielded in this model if the economy moves from point B to point A. But such a path is not a hyperinflation and there is no rationale for such a movement. This shift is completely *ad hoc*.

7 Changes in Fiscal Deficit and Destabilizing Expectations Model (VSDE)

According to Sargent et al. (2009) the hyperinflation models of Sargent and Wallace (1987), Marcet and Sargent (1989) and Marcet and Nicolini (2003) could not discriminate among alternative hypotheses that explain the origins and the stabilizations of big inflations because they assumed a constant government deficit

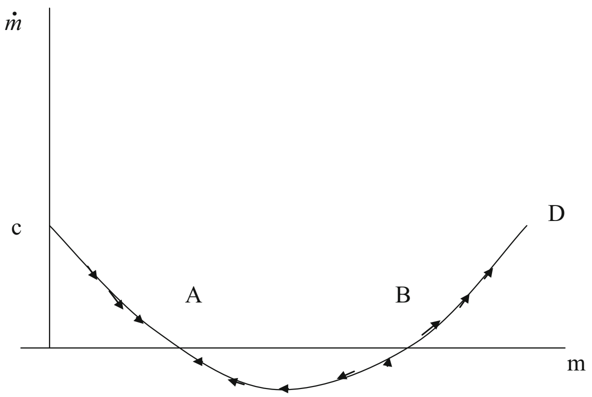

Fig. 1.6 CSRE model: phase diagram

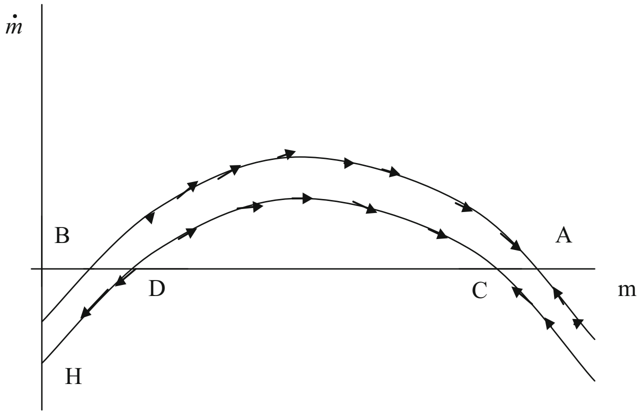

Fig. 1.7 VSDE model: phase diagram

financed by issuing money. Thus, Sargent et al. (2009) assume a variable deficit that follows a finite state Markov chain. The agents' expected rate of inflation is based on a constant gain learning algorithm that is similar to adaptive expectations, with the adaptive expectations coefficient changing due to a learning process.

Sargent, Williams and Zha's model has the same three equations of the constant deficit model, a linear demand for money, an expectation mechanism and a public deficit financed by money. Different from Kiguel's model they do not assume that the fiscal deficit increases to an unsustainable level. Figure 1.7 shows the phase diagram of the model. There are two equilibrium points. The low inflation equilibrium is stable (points A and C). The high inflation equilibrium is unstable (points D and B). When the deficit increases, the rate of inflation also increases, since the economy moves from point A to point C. If the economy is not moved by fundamentals but by destabilizing expectations it can jump to a point at the left of point D and enter into a hyperinflation path. By the same token, a cosmetic reform, defined as a reform that does not alter fiscal fundamentals, can temporarily lower the rate of inflation bringing the economy to a point at the right side of point C.

8 Conclusion

Cagan's model can explain hyperinflation, either by fundamentals or by bubble. However, his assumption of an exogenous money supply is at odds with the stylized facts of hyperinflation economies, since the central bank issues money to finance the public deficit. Kalecki's hyperinflation model was the first to use the hypothesis that money issue is used to finance the fiscal deficit. However, his model can yield a hyperinflation bubble but not a hyperinflation based on fundamentals.

Financing a constant level of real government expenditures through money issue, using Cagan's demand for money function and introducing some *ad hoc* rigidity hypothesis, such as adaptive expectations (or sluggish money market adjustment), yields multiple equilibrium. This framework can only produce a hyperinflation bubble when the demand for money is inelastic with respect to the inflation rate. If the fiscal deficit to be financed by issuing money jumps to an unsustainable level, a dynamic path takes the economy from the stable low inflation equilibrium to a disequilibrium hyperinflation path.

Financing a constant level of real government expenditures through money issue, using Cagan's demand for money function, and introducing rational expectations yields multiple equilibria. This framework can produce a dynamic path that takes the economy from the unstable low inflation equilibrium to the stable high inflation equilibrium. This is not a hyperinflation path and there is no rationale for such a shift. Furthermore, in this framework there is no hyperinflation bubble.

I may conclude that to explain hyperinflation it is not suitable to assume a constant real fiscal deficit to be financed by issuing money. This hypothesis can only generate, under certain conditions, a bubble hyperinflation. Thus, the mechanics of hyperinflation has to be explained by an increasing fiscal deficit that requires increasing the rate of growth of money being printed.

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