

Chapter 2

Sampled-Data Control with Actuators Saturation

2.1 Introduction

With the rapid development of intelligent instrument and digital measurement, modern control systems tend to be controlled by digital signal processing approaches, i.e. the control input signals are kept constant via a ZOH during the sampling instants and are only allowed to change at the discrete time instants. Thus, sampled-data control problem has been a hot research topic and numerical essential approaches have been derived in the literature, which include three main models: discrete-time model [1], impulsive model [2] and input delay model [3]. It is worthwhile to mention that, in [3], a time-dependent Lyapunov functional approach is proposed to model the sampled-data system as a continue system with time-varying delay in the control input. Since the works of [3], sampled-data control schemes for sampled-data systems have been thoroughly investigated, for instance, neural networks [4], vehicle active suspension systems [5], semilinear parabolic systems [6], fuzzy systems [7], etc. Besides, it is more difficult to analyze the synchronization of LSNSs due mainly to the couplings, nonlinearities and complex dynamical behaviours. To this regard, there is a vital need to fully address the sampled-data control strategies for the synchronization of LSNSs, and various control criteria have been established over the past decades [8, 9].

Due to the safety or technological constraints, the practical physical actuators can only generate bounded amplitude signals. So the actuators saturation are inherent and ubiquitous limitation in many control systems. From this perspective, plenty of control criteria for handling the actuators saturation nonlinearity have been adequately studied, which include two important approaches: the one with a local sector bound nonlinearity description [10–12] and the one with a polytopic representation [13, 14]. Among the existed literatures on the sampled-data controller design of LSNSs, most of them assume that the actuators are fully accessible and always working under the linear condition [15]. However, such an assumption is restrictive for designing

controller of LSNSs. Accordingly, the synchronization of LSNSs subjected to actuators saturation via sampled-data controller is a logical next step with both theoretical significance and practical importance.

From motivation mentioned above, the main objective of this chapter is to design a sampled-data controller that can ensure the LSNSs are asymptotically synchronous subject to actuators saturation. By constructing a novel time-dependent Lyapunov functional, we make full use of the available information about the sampling pattern. Additionally, the actuators saturation are taken into account in the form of the generalized sector bound condition. Furthermore, by utilizing the property of the network topology matrix, we derive the sufficient conditions in the framework of the stability analysis for decoupled systems. The obtained synchronization criteria can be recast as two optimization cases aiming at maximizing the upper bound or enlarging estimates of the filed of attraction. Finally, numerical examples are exploited to illustrate the effectiveness and usefulness of the proposed sampled-data control strategy.

2.2 Preliminaries

Consider the following LSNSs that consist of N identical coupled nodes with each node being an n -dimensional dynamical system:

$$\dot{x}_i(t) = Ax_i(t) + Bf(x_i(t)) + c \sum_{j=1}^N G_{ij} \Gamma x_j(t) + \sigma(u_i(t)), \quad i = 1, \dots, N, \quad (2.1)$$

where $x_i(t)$ and $u_i(t)$ are, respectively, the state variable and the control input of the node i , $G = (G_{ij})_{N \times N}$ is an outer-coupling configuration matrix representing the topological structure of the network, where G_{ij} is defined as follows: if there is a connection between node i and node j ($j \neq i$), then $G_{ij} = G_{ji} > 0$; otherwise, $G_{ij} = 0$, and the diagonal elements are defined by $G_{ii} = -\sum_{j=1, j \neq i}^N G_{ij}$. Γ is a constant inner-coupling matrix between two connected nodes, and c is a constant denoting the coupling strength. It is assumed that $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a continuous vector-valued function and satisfies the following sector-bounded condition:

$$[f(x) - f(y)]^T [f(x) - f(y) - F(x - y)] \leq 0, \quad \forall x, y \in \mathbb{R}^n, \quad (2.2)$$

where F is constant matrix of appropriate dimension. The function $\sigma(u_i(t)): \mathbb{R}^n \rightarrow \mathbb{R}^n$ is the saturation function defined as follows

$$\sigma(u_i(t)) = [\sigma_1(u_{i1}(t)) \ \sigma_2(u_{i2}(t)) \ \cdots \ \sigma_n(u_{in}(t))]^T, \quad (2.3)$$

where $\sigma_l(u_{il}(t)) = \text{sign}(u_{il}(t)) \min\{u_{il}, |u_{il}(t)|\}$, and u_{il} is the known saturation level for the l th element of the vector u_i .

Let $s(t)$ be a solution of the following isolated target node:

$$\dot{s}(t) = As(t) + Bf(s(t)), \quad (2.4)$$

with initial condition $s(0) \in \mathbb{R}^n$. Let $\varsigma_i(t) = x_i(t) - s(t)$ be the error state of the node i , one can obtain the following error dynamics of LSNSs (2.1):

$$\dot{\varsigma}_i(t) = A\varsigma_i(t) + Bg(\varsigma_i(t)) + c \sum_{j=1}^N G_{ij} \Gamma \varsigma_j(t) + \sigma(u_i(t)), \quad (2.5)$$

where

$$g(\varsigma_i(t)) = f(x_i(t)) - f(s(t)).$$

It can be found from (2.2) that

$$g(\varsigma_i(t))^T [g(\varsigma_i(t)) - F\varsigma_i(t)] \leq 0, \quad (2.6)$$

which is equivalent to

$$\begin{bmatrix} \varsigma_i(t) \\ g(\varsigma_i(t)) \end{bmatrix}^T \begin{bmatrix} \mathfrak{U} & \mathfrak{V} \\ * & I \end{bmatrix} \begin{bmatrix} \varsigma_i(t) \\ g(\varsigma_i(t)) \end{bmatrix} \leq 0, \quad (2.7)$$

where

$$\mathfrak{U} = \frac{F_1^T F_2}{2} + \frac{F_2^T F_1}{2}, \quad \mathfrak{V} = -\frac{F_1^T + F_2^T}{2}.$$

Throughout this chapter, we assume that only discrete measurements of $x_i(t)$ and $s(t)$ can be used for synchronization of LSNSs (2.1), that is, we only have the measurements $x_i(t)$ and $s(t)$ at the sampling instant t_k . Furthermore, the control signal is assumed to be generated by using a ZOH function with a sequence of hold times

$$0 = t_0 < t_1 < \cdots < t_k < \cdots < \lim_{k \rightarrow +\infty} t_k = +\infty.$$

In this chapter, the sampling is not required to be periodic, and there are two constants $\theta_1 > 0$ and $\theta_2 \geq \theta_1$ such that

$$\theta_k := t_{k+1} - t_k \in [\theta_1, \theta_2], \quad \forall k \in \mathbb{N}, \quad (2.8)$$

where $0 < \theta_1 \leq \theta_2 < +\infty$.

Remark 2.1 The existed strategies about variable sampling for synchronization of LSNSs are only dependent on the upper bound θ_2 but disregard the information of the lower bound θ_1 , which are restrictive and only reflect a few idea situations. Thus, the

strategy proposed in the chapter can provide more flexibility and less conservative results compared to the most of relevant literatures.

Then, for error system (2.5), our objective is to design the following set of sampled-data state feedback controllers:

$$u_i(t) = K \varsigma_i(t_k), \quad t_k \leq t < t_{k+1}, \quad i = 1, \dots, N, \quad (2.9)$$

where K is the state feedback controller gain to be designed.

Substituting (2.9) into (2.5) leads to

$$\begin{aligned} \dot{\varsigma}_i(t) = & A \varsigma_i(t) + Bg(\varsigma_i(t)) + c \sum_{j=1}^N G_{ij} \Gamma \varsigma_j(t) \\ & + \sigma(K \varsigma_i(t_k)), \quad i = 1, \dots, N, \quad t_k \leq t < t_{k+1}. \end{aligned} \quad (2.10)$$

Define the following nonlinear function:

$$\phi(K \varsigma_i(t_k)) = \begin{cases} u_{il} - K_l \varsigma_i(t_k), & \text{if } K_l \varsigma_i(t_k) > u_{il}, \\ 0, & \text{if } -u_{il} \leq K_l \varsigma_i(t_k) \leq u_{il}, \\ -u_{il} - K_l \varsigma_i(t_k), & \text{if } K_l \varsigma_i(t_k) < -u_{il} \end{cases} \quad (2.11)$$

From the above definition, system (2.10) can be rewritten as

$$\begin{aligned} \dot{\varsigma}_i(t) = & A \varsigma_i(t) + Bg(\varsigma_i(t)) + c \sum_{j=1}^N G_{ij} \Gamma \varsigma_j(t) + K \varsigma_i(t_k) \\ & + \phi(K \varsigma_i(t_k)), \quad i = 1, \dots, N, \quad t_k \leq t < t_{k+1}, \end{aligned} \quad (2.12)$$

which is equivalent to

$$\dot{\varsigma}(t) = (\hat{A} + \hat{C})\varsigma(t) + \hat{B}\mu(\varsigma(t)) + \hat{K}\varsigma(t_k) + \psi(\hat{K}\varsigma(t_k)), \quad t_k \leq t < t_{k+1}, \quad (2.13)$$

where $\hat{A}_{Nn \times Nn} = I_N \otimes A$, $\hat{B} = I \otimes B$, $\hat{C} = c(G \otimes \Gamma)$, $\hat{K} = I \otimes K$ and

$$\varsigma(t) = \begin{bmatrix} \varsigma_1(t) \\ \varsigma_2(t) \\ \vdots \\ \varsigma_N(t) \end{bmatrix}, \quad \mu(\varsigma(t)) = \begin{bmatrix} g(\varsigma_1(t)) \\ g(\varsigma_2(t)) \\ \vdots \\ g(\varsigma_N(t)) \end{bmatrix}, \quad \psi(\hat{K}\varsigma(t_k)) = \begin{bmatrix} \phi(K \varsigma_1(t_k)) \\ \phi(K \varsigma_2(t_k)) \\ \vdots \\ \phi(K \varsigma_N(t_k)) \end{bmatrix}.$$

Consider a matrix $S \in \mathbb{R}^{n \times n}$ and define the following polyhedral sets:

$$\mathcal{S}_i = \{ \varsigma_i \in \mathbb{R}^n : |(K_l - S_l) \varsigma_i| \leq u_{il}, \quad l = 1, 2, \dots, n \}, \quad i = 1, \dots, N. \quad (2.14)$$

If $\varsigma_i(t_k) \in \mathcal{S}_i$, then the following sector condition concerning the nonlinearity $\phi(K\varsigma_i(t_k))$ [16],

$$\phi(K\varsigma_i(t_k))^T V (\phi(K\varsigma_i(t_k)) + S\varsigma_i(t_k)) \leq 0, \quad (2.15)$$

can be satisfied for any diagonal matrix $V \in \mathbb{R}^n > 0$.

Define an ellipsoidal set $\mathcal{E}(P)$:

$$\mathcal{E}(P) = \{\varsigma \in \mathbb{R}^n : \varsigma^T P \varsigma \leq \rho\},$$

with a positive-definite matrix $P \in \mathbb{R}^{n \times n}$ and a scalar $\rho > 0$.

Lemma 2.2 ([7]) *Considering system (2.13), the following inequality is true*

$$\|\varsigma(t)\|^2 \leq \theta \|\varsigma(t_k)\|^2, \quad t_k \leq t < t_{k+1}. \quad (2.16)$$

Definition 2.3 LSNSs (2.1) is said to be local exponentially synchronized if the error dynamic (2.13) is exponentially stable, i.e. there exist two constants $\alpha > 0$ and $\beta > 0$ such that

$$\|\varsigma(t)\| \leq \beta e^{-\alpha t}, \quad \forall t \geq 0. \quad (2.17)$$

We are now in a position to formulate the sampled-data exponential synchronization problem to be addressed in this chapter as follows:

Design sampled-data controllers in the form of (2.9) such that the error system (2.13) is exponentially stable, that is, LSNSs (2.1) is exponentially synchronized.

2.3 Main Results

In this section, the exponential stability of error system (2.13) is investigated based on the time-dependent Lyapunov functional approach, and sufficient condition is derived to guarantee the system stability and synthesize the sampled-data controllers in the form of (2.9).

Theorem 2.4 *Given scalars $\alpha > 0$, $\theta_2 > \theta_1 > 0$ and matrix F , if there exist matrices $P > 0$, $X > 0$, $V > 0$, $\begin{bmatrix} U_1 & U_2 & U_3 \\ * & U_4 & U_5 \\ * & * & U_6 \end{bmatrix} > 0$, $H = [H_1 \ H_2 \ H_3 \ H_4 \ H_5]$, F_1 , F_2 , S and a scalar $\varrho > 0$, for $i = 1, \dots, N$, such that*

$$\begin{bmatrix} P & (K_l - S_l)^T \\ * & u_{il}^2 \end{bmatrix} \geq 0, \quad l = 1, \dots, n, \quad (2.18)$$

$$\Xi_1^i(\bar{\theta}) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \bar{\theta}H_1^T \\ * & \Xi_{22} & \Xi_{23} & F_2B & F_2 & \bar{\theta}H_2^T \\ * & * & \Xi_{33} - \bar{\theta}e^{-2\alpha\theta_2}U_4 & -H_4 & \Xi_{35} + \Pi_{35} & \bar{\theta}H_3^T \\ * & * & * & -2\rho & 0 & \bar{\theta}H_4^T \\ * & * & * & * & -2V + \Pi_{55} & \bar{\theta}H_5^T \\ * & * & * & * & * & -e^{-2\alpha\theta_2}\bar{\theta}U_1 \end{bmatrix} < 0, \quad (2.19)$$

$$\Xi_2^i(\bar{\theta}) = \begin{bmatrix} \Xi_{11} + 2\alpha\bar{\theta}X & \Xi_{12} + \bar{\theta}X & \Xi_{13} - 2\alpha\bar{\theta}X & \Xi_{14} & \Xi_{15} \\ * & \Xi_{22} + \bar{\theta}U_1 & \Xi_{23} + \Theta_{23} & F_2B & F_2 + \bar{\theta}U_3 \\ * & * & \Xi_{33} + \Theta_{33} & -H_4 & \Xi_{35} + \bar{\theta}U_5 \\ * & * & * & -2\rho & 0 \\ * & * & * & * & -2V + \bar{\theta}U_6 \end{bmatrix} < 0, \quad (2.20)$$

where

$$\Xi_{11} = 2\alpha P - X + F_1A + c\lambda_i F_1\Gamma + A^T F_1^T + c\lambda_i \Gamma^T F_1^T + H_1 + H_1^T, \quad (2.21)$$

$$\Xi_{12} = P - F_1 + A^T F_2^T + c\lambda_i \Gamma^T F_2^T + H_2,$$

$$\Xi_{13} = -e^{-2\alpha\theta_2}U_2 + X + F_1K + H_3 - H_1^T,$$

$$\Xi_{14} = F_1B + \rho F^T + H_4, \quad \Xi_{15} = -e^{-2\alpha\theta_2}U_3 + F_1 + H_5,$$

$$\Xi_{22} = -F_2 - F_2^T, \quad \Xi_{23} = F_2K - H_2^T,$$

$$\Xi_{33} = e^{-2\alpha\theta_2}U_2 + e^{-2\alpha\theta_2}U_2^T - X - H_3^T - H_3,$$

$$\Xi_{35} = -S^T V^T - H_5 + e^{-2\alpha\theta_2}U_3, \quad \Pi_{35} = -\bar{\theta}e^{-2\alpha\theta_2}U_5 - 2\alpha\bar{\theta}S^T V^T,$$

$$\Pi_{55} = -\bar{\theta}e^{-2\alpha\theta_2}U_6 - 4\alpha\bar{\theta}V, \quad \Theta_{23} = \bar{\theta}U_2 - \bar{\theta}X^T, \quad \Theta_{33} = \bar{\theta}U_4 + 2\alpha\bar{\theta}X,$$

where $\bar{\theta} \in \{\theta_1, \theta_2\}$, $\lambda_i, i = 1, \dots, N$ are the eigenvalues of the matrix G and satisfying $\lambda_N \leq \dots \leq \lambda_2 < \lambda_1 = 0$. Then, for the initial condition $\mathcal{E}(P)$, the LSNSs (2.1) are locally exponentially synchronized with decay rate α .

Proof Choose a Lyapunov functional as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad t \in [t_k, t_{k+1}), \quad (2.22)$$

where

$$V_1(t) = e^{2\alpha t} \varsigma(t)^T \hat{P} \varsigma(t),$$

$$V_2(t) = (t_{k+1} - t) \int_{t_k}^t e^{2\alpha s} \pi(s, t_k)^T \hat{U} \pi(s, t_k) ds,$$

$$V_3(t) = (t_{k+1} - t) e^{2\alpha t} (\varsigma(t) - \varsigma(t_k))^T \hat{X} (\varsigma(t) - \varsigma(t_k)),$$

$$V_4(t) = -2e^{2\alpha t} (t - t_k) \psi(\hat{K} \varsigma(t_k))^T \hat{V} (\psi(\hat{K} \varsigma(t_k))) + \hat{S} \varsigma(t_k),$$

with

$$\pi(t, t_k) = \begin{bmatrix} \dot{\varsigma}(t) \\ \varsigma(t_k) \\ \psi(\hat{K}\varsigma(t_k)) \end{bmatrix}, \quad \hat{U} = \begin{bmatrix} \hat{U}_1 & \hat{U}_2 & \hat{U}_3 \\ * & \hat{U}_4 & \hat{U}_5 \\ * & * & \hat{U}_6 \end{bmatrix}.$$

Along the trajectory of the error system (2.13), taking the derivative of $V(t)$, we have

$$\dot{V}_1(t) = 2e^{2\alpha t} \varsigma(t)^T \hat{P} \dot{\varsigma}(t) + 2\alpha e^{2\alpha t} \varsigma(t)^T \hat{P} \varsigma(t), \quad (2.23)$$

$$\begin{aligned} \dot{V}_2(t) &= (t_{k+1} - t) e^{2\alpha t} \pi(t, t_k)^T \hat{U} \pi(t, t_k) - \int_{t_k}^t e^{2\alpha s} \pi(s, t_k)^T \hat{U} \pi(s, t_k) ds \\ &\leq (t_{k+1} - t) e^{2\alpha t} \pi(t, t_k)^T \hat{U} \pi(t, t_k) - e^{2\alpha t} e^{-2\alpha\theta_2} \int_{t_k}^t \pi(s, t_k)^T \hat{U} \pi(s, t_k) ds \\ &= (t_{k+1} - t) e^{2\alpha t} \pi(t, t_k)^T \hat{U} \pi(t, t_k) - e^{2\alpha t} e^{-2\alpha\theta_2} \int_{t_k}^t \dot{\varsigma}(s)^T \hat{U}_1 \dot{\varsigma}(s) ds \\ &\quad - (t - t_k) e^{2\alpha t} e^{-2\alpha\theta_2} \begin{bmatrix} \varsigma(t_k) \\ \psi(\hat{K}\varsigma(t_k)) \end{bmatrix}^T \begin{bmatrix} \hat{U}_4 & \hat{U}_5 \\ * & \hat{U}_6 \end{bmatrix} \begin{bmatrix} \varsigma(t_k) \\ \psi(\hat{K}\varsigma(t_k)) \end{bmatrix} \\ &\quad - 2e^{2\alpha t} e^{-2\alpha\theta_2} \times (\varsigma(t) - \varsigma(t_k))^T [\hat{U}_2 \ \hat{U}_3] \begin{bmatrix} \varsigma(t_k) \\ \psi(\hat{K}\varsigma(t_k)) \end{bmatrix}, \end{aligned} \quad (2.24)$$

$$\begin{aligned} \dot{V}_3(t) &= -e^{2\alpha t} (\varsigma(t) - \varsigma(t_k))^T \hat{X} (\varsigma(t) - \varsigma(t_k)) \\ &\quad + 2(t_{k+1} - t) e^{2\alpha t} (\varsigma(t) - \varsigma(t_k))^T \hat{X} \dot{\varsigma}(t) \\ &\quad + 2\alpha(t_{k+1} - t) e^{2\alpha t} (\varsigma(t) - \varsigma(t_k))^T \hat{X} (\varsigma(t) - \varsigma(t_k)), \end{aligned} \quad (2.25)$$

$$\begin{aligned} \dot{V}_4(t) &= -2e^{2\alpha t} \psi(\hat{K}\varsigma(t_k))^T \hat{V} (\psi(\hat{K}\varsigma(t_k))) + \hat{S}\varsigma(t_k) \\ &\quad - 4\alpha e^{2\alpha t} (t - t_k) \psi(\hat{K}\varsigma(t_k))^T \hat{V} (\psi(\hat{K}\varsigma(t_k))) + \hat{S}\varsigma(t_k). \end{aligned} \quad (2.26)$$

On the other hand, let

$$\hat{\pi}(t, t_k) = [\varsigma(t)^T \ \dot{\varsigma}(t)^T \ \varsigma(t_k)^T \ \mu(\varsigma(t))^T \ \psi(\hat{K}\varsigma(t_k))^T]^T.$$

For any matrix $\hat{U}_1 > 0$, the following inequality holds

$$(e^{-\alpha\theta_2} \dot{\varsigma}(s) + e^{\alpha\theta_2} \hat{U}_1^{-1} H \hat{\pi}(t, t_k))^T \hat{U}_1 (e^{-\alpha\theta_2} \dot{\varsigma}(s) + e^{\alpha\theta_2} \hat{U}_1^{-1} H \hat{\pi}(t, t_k)) \geq 0, \quad (2.27)$$

where $H = [H_1 \ H_2 \ H_3 \ H_4 \ H_5]$.

Integrating the above inequality from t_k to t leads to the following inequality,

$$\begin{aligned} 0 \leq & e^{-2\alpha\theta_2} \int_{t_k}^t \dot{\varsigma}(s)^T \hat{U}_1 \dot{\varsigma}(s) ds + (t - t_k) e^{2\alpha\theta_2} \hat{\pi}(t, t_k)^T H^T \hat{U}_1^{-1} H \hat{\pi}(t, t_k) \\ & + 2\hat{\pi}(t, t_k)^T H^T (\varsigma(t) - \varsigma(t_k)). \end{aligned} \quad (2.28)$$

Based on the descriptor systems approach, the following equation is ensured

$$\begin{aligned} 0 = & 2e^{2\alpha t} \left[\varsigma(t)^T \hat{F}_1 + \dot{\varsigma}(t)^T \hat{F}_2 \right] [-\dot{\varsigma}(t) + (\hat{A} + \hat{C})\varsigma(t) \\ & + \hat{B}\mu(\varsigma(t)) + \hat{K}\varsigma(t_k) + \psi(\hat{K}\varsigma(t_k))], \end{aligned} \quad (2.29)$$

for any appropriately dimensioned matrices \hat{F}_1 and \hat{F}_2 . Moreover, the satisfaction of condition (2.2) implies that for any scalar $\varrho > 0$, the following inequality holds:

$$0 \leq -2\varrho\mu(\varsigma(t))^T [\mu(\varsigma(t)) - \hat{F}\varsigma(t)]. \quad (2.30)$$

Using (2.23)–(2.26) and adding the right-hand sides of (2.28)–(2.30) to $\dot{V}(t)$. Let

$$\hat{\Xi}_1(\bar{\theta}) = \begin{bmatrix} \hat{\Xi}(\bar{\theta}) & \bar{\theta} \hat{H}^T \\ * & -e^{-2\alpha\theta_2} \bar{\theta} \hat{U}_1 \end{bmatrix} + \Psi_1(\bar{\theta}), \quad (2.31)$$

$$\hat{\Xi}_2(\bar{\theta}) = \hat{\Xi}(\bar{\theta}) + \hat{\Lambda}(\bar{\theta}), \quad (2.32)$$

where

$$\hat{\Xi}(\bar{\theta}) = \begin{bmatrix} \hat{\Xi}_{11} & \hat{\Xi}_{12} & \hat{\Xi}_{13} & \hat{\Xi}_{14} & \hat{\Xi}_{15} \\ * & \hat{\Xi}_{22} & \hat{\Xi}_{23} & \hat{F}_2 \hat{B} & \hat{F}_2 \\ * & * & \hat{\Xi}_{33} & -\hat{H}_4 & \hat{\Xi}_{35} \\ * & * & * & -2\varrho & 0 \\ * & * & * & * & -2\hat{V} \end{bmatrix}, \quad \Psi_1(\bar{\theta}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ * & * & \bar{\theta} \gamma_{33} & 0 & \bar{\theta} \gamma_{35} & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & \bar{\theta} \gamma_{55} & 0 \\ * & * & * & * & * & 0 \end{bmatrix},$$

$$\hat{\Lambda}(\theta_k) = \theta_k \begin{bmatrix} 2\alpha \hat{X} & \hat{X} & -2\alpha \hat{X} & 0 & 0 \\ * & \hat{U}_1 & \hat{U}_2 - \hat{X}^T & 0 & \hat{U}_3 \\ * & * & \hat{U}_4 + 2\alpha \hat{X} & 0 & \hat{U}_5 \\ * & * & * & 0 & 0 \\ * & * & * & * & \hat{U}_6 \end{bmatrix},$$

$$\begin{aligned}
\Upsilon_{33} &= -e^{-2\alpha\theta_2}\hat{U}_4, \quad \Upsilon_{35} = -e^{-2\alpha\theta_2}\hat{U}_5 - 2\alpha\hat{S}^T\hat{V}^T, \\
\Upsilon_{55} &= -e^{-2\alpha\theta_2}\hat{U}_6 - 4\alpha\hat{V}, \\
\hat{\Xi}_{11} &= 2\alpha\hat{P} - \hat{X} + \hat{F}_1(\hat{A} + \hat{C}) + (\hat{A} + \hat{C})^T\hat{F}_1^T + \hat{H}_1 + \hat{H}_1^T, \\
\hat{\Xi}_{12} &= \hat{P} - \hat{F}_1 + (\hat{A} + \hat{C})^T\hat{F}_2^T + \hat{H}_2, \\
\hat{\Xi}_{13} &= -e^{-2\alpha\theta_2}\hat{U}_2 + \hat{X} + \hat{F}_1\hat{K} + \hat{H}_3 - \hat{H}_1^T, \\
\hat{\Xi}_{14} &= \hat{F}_1\hat{B} + \varrho\hat{F}^T + \hat{H}_4, \quad \hat{\Xi}_{15} = -e^{-2\alpha\theta_2}\hat{U}_3 + \hat{F}_1 + \hat{H}_5, \\
\hat{\Xi}_{22} &= -\hat{F}_2 - \hat{F}_2^T, \quad \hat{\Xi}_{23} = \hat{F}_2\hat{K} - \hat{H}_2^T, \\
\hat{\Xi}_{33} &= e^{-2\alpha\theta_2}\hat{U}_2 + e^{-2\alpha\theta_2}\hat{U}_2^T - \hat{X} - \hat{H}_3^T - \hat{H}_3, \\
\hat{\Xi}_{35} &= -\hat{S}^T\hat{V}^T - \hat{H}_5 + e^{-2\alpha\theta_2}\hat{U}_3,
\end{aligned} \tag{2.33}$$

we get

$$\begin{aligned}
\dot{V}(t) &< e^{2\alpha t}\hat{\pi}(t, t_k)^T \left\{ \hat{\Xi}_2(0) + (t_{k+1} - t)\hat{\Lambda}(I) + (t - t_k)\hat{\Upsilon}(I) \right\} \hat{\pi}(t, t_k) \\
&< e^{2\alpha t}\hat{\pi}(t, t_k)^T \left\{ \frac{t_{k+1} - t}{\theta_k}(\hat{\Xi}_2(0) + \hat{\Lambda}(\theta_k)) + \frac{t - t_k}{\theta_k}(\hat{\Xi}_2(0) + \hat{\Upsilon}(\theta_k)) \right\} \hat{\pi}(t, t_k) \\
&< e^{2\alpha t}\hat{\pi}(t, t_k)^T \left\{ \frac{t_{k+1} - t}{\theta_k}\hat{\Xi}_2(\theta_k) + \frac{t - t_k}{\theta_k}(\hat{\Xi}_2(0) + \hat{\Upsilon}(\theta_k)) \right\} \hat{\pi}(t, t_k),
\end{aligned}$$

where

$$\hat{\Upsilon}(\theta_k) = \theta_k \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & \Upsilon_{33} & 0 & \Upsilon_{35} \\ * & * & * & 0 & 0 \\ * & * & * & * & \Upsilon_{55} \end{bmatrix} + \theta_k \hat{H}^T e^{2\alpha\theta_2} \hat{U}_1^{-1} \hat{H}.$$

Since the eigenvalues of G satisfy $0 = \lambda_1 > \lambda_2 \geq \dots \geq \lambda_N$ and G is irreducible, there exists an orthogonal matrix W , which satisfies $W^T G W = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_N\}$. Pre- and post-multiplying both sides of (2.31) and (2.32) with $\text{diag}\{W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n\}$ and $\text{diag}\{W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n, W^T \otimes I_n\}$, respectively. We get

$$\tilde{\Xi}_1(\bar{\theta}) = \begin{bmatrix} \tilde{\Xi}(\bar{\theta}) & \bar{\theta}\hat{H}^T \\ * & -e^{-2\alpha\theta_2}\bar{\theta}\hat{U}_1 \end{bmatrix} + \Psi_1(\bar{\theta}), \tag{2.34}$$

$$\tilde{\Xi}_2(\bar{\theta}) = \tilde{\Xi}(\bar{\theta}) + \Psi_2(\bar{\theta}), \tag{2.35}$$

where

$$\tilde{\mathcal{E}}(\bar{\theta}) = \begin{bmatrix} \tilde{\mathcal{E}}_{11} & \tilde{\mathcal{E}}_{12} & \hat{\mathcal{E}}_{13} & \hat{\mathcal{E}}_{14} & \hat{\mathcal{E}}_{15} \\ * & \hat{\mathcal{E}}_{22} & \hat{\mathcal{E}}_{23} & \hat{F}_2 \hat{B} & \hat{F}_2 \\ * & * & \hat{\mathcal{E}}_{33} & -\hat{H}_4 & \hat{\mathcal{E}}_{35} \\ * & * & * & -2\varrho & 0 \\ * & * & * & * & -2\hat{V} \end{bmatrix},$$

$$\begin{aligned} \tilde{\mathcal{E}}_{11} &= 2\alpha\hat{P} - \hat{X} + \hat{F}_1\hat{A} + c(W^T G W \otimes F_1\Gamma) + \hat{A}^T \hat{F}_1^T \\ &\quad + c(W^T G W \otimes \Gamma^T F_1^T) + \hat{H}_1 + \hat{H}_1^T, \\ \tilde{\mathcal{E}}_{12} &= \hat{P} - \hat{F}_1 + \hat{A}^T \hat{F}_2^T + c(W^T G W \otimes \Gamma^T F_2^T) + \hat{H}_2. \end{aligned}$$

As (2.19) implies (2.31) < 0 and (2.34) < 0 . According to Schur complement, (2.31) < 0 implies

$$\hat{\mathcal{E}}_2(0) + \hat{\Upsilon}(\theta_1) < 0, \quad \hat{\mathcal{E}}_2(0) + \hat{\Upsilon}(\theta_2) < 0,$$

and

$$\hat{\mathcal{E}}_2(0) + \hat{\Upsilon}(\theta_k) = \frac{\theta_k - \theta_1}{\theta_2 - \theta_1}(\hat{\mathcal{E}}_2(0) + \hat{\Upsilon}(\theta_2)) + \frac{\theta_2 - \theta_k}{\theta_2 - \theta_1}(\hat{\mathcal{E}}_2(0) + \hat{\Upsilon}(\theta_1)) < 0. \quad (2.36)$$

Moreover, As (2.20) implies (2.32) < 0 and (2.35) < 0 , from (2.32) < 0 , the following inequality is true

$$\hat{\mathcal{E}}_2(\theta_k) = \frac{\theta_k - \theta_1}{\theta_2 - \theta_1}\hat{\mathcal{E}}_2(\theta_2) + \frac{\theta_2 - \theta_k}{\theta_2 - \theta_1}\hat{\mathcal{E}}_2(\theta_1) < 0. \quad (2.37)$$

By considering (2.36) and (2.37), we get

$$\dot{V}(t) < 0, \quad t \in [t_k, t_{k+1}). \quad (2.38)$$

It is noted that $V_l(t_k) = \lim_{t \rightarrow t_{k+1}^-} V_l(t) = 0$, ($l = 2, 3$). Thus, integrating (2.38) from t_k to t_{k+1}^- leads to

$$\begin{aligned} &e^{2\alpha t_{k+1}} \varsigma(t_{k+1})^T \hat{P} \varsigma(t_{k+1}) - 2\theta_k e^{2\alpha t_{k+1}} \psi(\hat{K} \varsigma(t_k))^T \hat{V}(\psi(\hat{K} \varsigma(t_k)) \\ &\quad + \hat{S} \varsigma(t_k)) - e^{2\alpha t_k} \varsigma(t_k)^T \hat{P} \varsigma(t_k) < 0. \end{aligned} \quad (2.39)$$

Using Schur complement to (2.18), it is easy to get that

$$(K_l - S_l)^T (K_l - S_l) \leq u_{il}^2 P. \quad (2.40)$$

On the other hand, it is noted that for any initial condition $\varsigma(t_0) \in \mathcal{E}(I \otimes P)$, we have $\varsigma_i(t_0) \in \mathcal{E}(P)$. Based on (2.40), belongs to \mathcal{S} . Thus,

$$\begin{aligned} \psi(K\varsigma_i(t_0))^T V(\psi(K\varsigma_i(t_0)) + S\varsigma_i(t_0)) &\leq 0, \\ \psi(\hat{K}\varsigma(t_0))^T \hat{V}(\psi(\hat{K}\varsigma(t_0)) + \hat{S}\varsigma(t_0)) &\leq 0, \end{aligned} \quad (2.41)$$

which combining (2.39) means

$$e^{2\alpha t_1} \varsigma(t_1)^T \hat{P}\varsigma(t_1) - \varsigma(t_0)^T \hat{P}\varsigma(t_0) < 0. \quad (2.42)$$

For any initial condition $\varsigma(t_1) \in \mathcal{E}(I \otimes P)$, we have $\varsigma_i(t_1) \in \mathcal{E}(P)$. Based on (2.40), belongs to \mathcal{S} . Thus,

$$\begin{aligned} \psi(K\varsigma_i(t_1))^T V(\psi(K\varsigma_i(t_1)) + S\varsigma_i(t_1)) &\leq 0, \\ \psi(\hat{K}\varsigma(t_1))^T \hat{V}(\psi(\hat{K}\varsigma(t_1)) + \hat{S}\varsigma(t_1)) &\leq 0, \end{aligned} \quad (2.43)$$

which combining (2.39) means

$$e^{2\alpha t_2} \varsigma(t_2)^T \hat{P}\varsigma(t_2) - e^{2\alpha t_1} \varsigma(t_1)^T \hat{P}\varsigma(t_1) < 0. \quad (2.44)$$

Repeating the above process, we can find that

$$\begin{aligned} \phi(\hat{K}\varsigma(t_k))^T \hat{V}(\phi(\hat{K}\varsigma(t_k)) + \hat{S}\varsigma(t_k)) &\leq 0, \quad k \geq 0, \\ e^{2\alpha t_{k+1}} \varsigma(t_{k+1})^T \hat{P}\varsigma(t_{k+1}) - e^{2\alpha t_k} \varsigma(t_k)^T \hat{P}\varsigma(t_k) &< 0. \end{aligned} \quad (2.45)$$

Based on Lemma 2.2 and (2.45), we can conclude that for $t_k \leq t < t_{k+1}$

$$\begin{aligned} \|\varsigma(t)\|^2 &\leq \theta \|\varsigma(t_k)\|^2 \\ &= \frac{\theta}{\lambda_{\min}(P) e^{2\alpha t_k}} e^{2\alpha t_k} \lambda_{\min}(P) \|\varsigma(t_k)\|^2 \\ &\leq \frac{\theta}{\lambda_{\min}(P) e^{2\alpha t_k}} e^{2\alpha t_k} \varsigma(t_k)^T \hat{P}\varsigma(t_k) \\ &\leq \frac{\theta}{\lambda_{\min}(P) e^{2\alpha t_k}} \varsigma(t_0)^T \hat{P}\varsigma(t_0) \\ &\leq \frac{\theta}{\lambda_{\min}(P)} e^{-2\alpha t} e^{2\alpha t_0} \\ &\leq e^{2\alpha \theta_2} \frac{\theta}{\lambda_{\min}(P)} e^{-2\alpha t}. \end{aligned} \quad (2.46)$$

Thus, by Definition 2.3, for any initial condition $\varsigma(t_0) \in \mathcal{E}(I \otimes P)$, the corresponding trajectory of system (2.13) converges exponentially to the origin, i.e. the network system (2.1) achieves locally exponentially synchronization with the leader (2.4). This completes the proof.

Remark 2.5 The orthogonal matrix W is introduced to let the outer-coupling configuration matrix G transform to a diagonal matrix $\text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_N\}$, which means that the coupling between each nodes of the coupled networks is eliminated. By utilizing the property of the network topology matrix, we can recast the synchronization problem into the stability of decoupled error systems. The decoupled method proposed in this chapter is supposed to provide less conservative results.

Remark 2.6 By constructing the $(t_k - t_{k+1})$ -dependent terms $V_2(t)$ and $V_3(t)$ in (2.22), which are motivated from the time-dependent Lyapunov functional method, our method can capture the characteristics of the sampled-data systems and make good use of the available information of the sampling pattern. So the proposed method is less conservative.

Remark 2.7 The two LMIs (2.19) and (2.20) are convex in θ_1 and θ_2 , for the reason that $\mathcal{E}_m^i(\hat{\theta}) = \frac{\hat{\theta} - \theta_1}{\theta_2 - \theta_1} \mathcal{E}_m^i(\theta_2) + \frac{\theta_2 - \hat{\theta}}{\theta_2 - \theta_1} \mathcal{E}_m^i(\theta_1) < 0$, ($m = 1, 2; i = 1, 2, 3$). So (2.19) and (2.20) are feasible for all $\hat{\theta} \in [\theta_1, \theta_2]$, where θ_1 and θ_2 are the lower and the upper bounds of the sampling intervals.

Depending on Theorem 2.4, we can obtain the following corollary.

Corollary 2.8 *The error system (2.13) is exponentially stable with a small enough decay rate, if (2.18)–(2.20) are true when $\alpha = 0$.*

Next, we will design the sampled-data controllers in the form of (2.9) to make LSNSs (2.1) exponentially synchronized. The following theorem presents a sufficient condition of the existence of the desired sampled-data controllers based on Theorem 2.4.

Theorem 2.9 *Given scalars $\alpha > 0$, $\theta_2 > \theta_1 > 0$, κ , if there exist matrices $\bar{P} > 0$, $\bar{X} > 0$, $\bar{V} > 0$, $\begin{bmatrix} \bar{U}_1 & \bar{U}_2 & \bar{U}_3 \\ * & \bar{U}_4 & \bar{U}_5 \\ * & * & \bar{U}_6 \end{bmatrix} > 0$, $\bar{H} = [\bar{H}_1 \ \bar{H}_2 \ \bar{H}_3 \ \bar{H}_4 \ \bar{H}_5]$, \bar{K} , $\bar{\mathcal{F}}$, \bar{S} and a scalar $\bar{\varrho} > 0$, such that the following LMIs hold,*

$$\begin{bmatrix} \bar{P} & (\bar{K}_l - \bar{S}_l)^T \\ * & u_{il}^2 \end{bmatrix} \geq 0, \quad l = 1, \dots, n; \quad i = 1, \dots, N, \quad (2.47)$$

$$\Omega_1^i(\bar{\theta}) = \begin{bmatrix} \bar{\mathcal{E}}_{11} & \bar{\mathcal{E}}_{12} & \bar{\mathcal{E}}_{13} & \bar{\mathcal{E}}_{14} & \bar{\mathcal{E}}_{15} & \bar{\theta} \bar{H}_1^T \\ * & \bar{\mathcal{E}}_{22} & \bar{\mathcal{E}}_{23} & \kappa \bar{B} \bar{\varrho} & \kappa \bar{V}^T & \bar{\theta} \bar{H}_2^T \\ * & * & \bar{\mathcal{E}}_{33} - \bar{\theta} e^{-2\alpha\theta_2} \bar{U}_4 - \bar{H}_4 & \bar{\mathcal{E}}_{35} + \bar{\Pi}_{35} & & \bar{\theta} \bar{H}_3^T \\ * & * & * & -2\bar{\varrho} & 0 & \bar{\theta} \bar{H}_4^T \\ * & * & * & * & -2\bar{V}^T + \bar{\Pi}_{55} & \bar{\theta} \bar{H}_5^T \\ * & * & * & * & * & -e^{-2\alpha\theta_2} \bar{\theta} \bar{U}_1 \end{bmatrix} < 0, \quad (2.48)$$

$$\Omega_2^i(\bar{\theta}) = \begin{bmatrix} \bar{\Xi}_{11} + 2\alpha\bar{\theta}\bar{X} & \bar{\Xi}_{12} + \bar{\theta}\bar{X} & \bar{\Xi}_{13} - 2\alpha\bar{\theta}\bar{X} & \bar{\Xi}_{14} & \bar{\Xi}_{15} \\ * & \bar{\Xi}_{22} + \bar{\theta}\bar{U}_1 & \bar{\Xi}_{23} + \bar{\Theta}_{23} & \kappa B\bar{\varrho} & \kappa\bar{V}^T + \bar{\theta}\bar{U}_3 \\ * & * & \bar{\Xi}_{33} + \bar{\Theta}_{33} & -\bar{H}_4 & \bar{\Xi}_{35} + \bar{\theta}\bar{U}_5 \\ * & * & * & -2\bar{\varrho} & 0 \\ * & * & * & * & -2\bar{V}^T + \bar{\theta}\bar{U}_6 \end{bmatrix} < 0, \quad (2.49)$$

where

$$\begin{aligned} \bar{\Xi}_{11} &= 2\alpha\bar{P} - \bar{X} + A\bar{\mathcal{F}}^T + c\lambda_i\Gamma\bar{\mathcal{F}}^T + \bar{\mathcal{F}}A^T + c\lambda_i\bar{\mathcal{F}}\Gamma^T + \bar{H}_1 + \bar{H}_1^T, \\ \bar{\Xi}_{12} &= \bar{P} - \bar{\mathcal{F}}^T + \kappa\bar{\mathcal{F}}A^T + c\lambda_i\kappa\bar{\mathcal{F}}\Gamma^T + \bar{H}_2, \\ \bar{\Xi}_{13} &= -e^{-2\alpha\theta_2}\bar{U}_2 + \bar{X} + \bar{K} + \bar{H}_3 - \bar{H}_1^T, \quad \bar{\Xi}_{14} = B\bar{\varrho} + \bar{\mathcal{F}}F^T + \bar{H}_4, \\ \bar{\Xi}_{15} &= -e^{-2\alpha\theta_2}\bar{U}_3 + \bar{V}^T + \bar{H}_5, \quad \bar{\Xi}_{22} = -\kappa\bar{\mathcal{F}}^T - \kappa\bar{\mathcal{F}}, \quad \bar{\Xi}_{23} = \kappa\bar{K} - \bar{H}_2^T, \\ \bar{\Xi}_{33} &= e^{-2\alpha\theta_2}\bar{U}_2 + e^{-2\alpha\theta_2}\bar{U}_2^T - \bar{X} - \bar{H}_3^T - \bar{H}_3, \quad \bar{\Xi}_{35} = -\bar{S}^T - \bar{H}_5 + e^{-2\alpha\theta_2}\bar{U}_3, \\ \bar{\Pi}_{35} &= -\bar{\theta}e^{-2\alpha\theta_2}\bar{U}_5 - 2\alpha\bar{\theta}\bar{S}^T, \quad \bar{\Pi}_{55} = -\bar{\theta}e^{-2\alpha\theta_2}\bar{U}_6 - 4\alpha\bar{\theta}\bar{V}^T, \\ \bar{\Theta}_{23} &= \bar{\theta}\bar{U}_2 - \bar{\theta}\bar{X}^T, \quad \bar{\Theta}_{33} = \bar{\theta}\bar{U}_4 + 2\alpha\bar{\theta}\bar{X}, \end{aligned}$$

where $\bar{\theta} \in \{\theta_1, \theta_2\}$, λ_i are the eigenvalues of the matrix G and satisfying $\lambda_N \leq \dots \leq \lambda_2 < \lambda_1 = 0$. Then, for the initial condition $\mathcal{E}(\bar{\mathcal{F}}^{-1}\bar{P}\bar{\mathcal{F}}^{-T})$, the LSNSs (2.1) are locally exponentially synchronized with decay rate α . Furthermore, the desired state feedback controller gains are given as

$$K_i = \bar{K}_i\bar{\mathcal{F}}_i^{-T}, \quad i = 1, \dots, N. \quad (2.50)$$

Proof Letting

$$\begin{aligned} F_1 &= \bar{\mathcal{F}}, \quad F_2 = \kappa F_1, \quad \bar{\mathcal{F}} = \bar{\mathcal{F}}^{-1}, \quad \bar{V} = V^{-1}, \quad \bar{\varrho} = \varrho^{-1}, \quad \bar{K} = K\bar{\mathcal{F}}^T, \quad \bar{P} = \bar{\mathcal{F}}P\bar{\mathcal{F}}^T, \\ \bar{X} &= \bar{\mathcal{F}}X\bar{\mathcal{F}}^T, \quad \bar{H}_m = \bar{\mathcal{F}}H_m\bar{\mathcal{F}}^T, \quad (m = 1, 2, 3), \quad \bar{H}_4 = \bar{\mathcal{F}}H_4\bar{\varrho}^T, \quad \bar{H}_5 = \bar{\mathcal{F}}H_5\bar{V}^T, \\ U_n &= \bar{\mathcal{F}}U_n\bar{\mathcal{F}}^T, \quad (n = 1, 2, 4), \quad U_l = \bar{\mathcal{F}}U_l\bar{V}^T, \quad (l = 3, 5), \\ U_6 &= \bar{V}U_6\bar{V}^T, \quad \bar{S} = S\bar{\mathcal{F}}^T, \end{aligned}$$

and block diagonal matrices

$$\begin{aligned} \bar{\mathcal{F}}_1 &= \text{diag}\{\bar{\mathcal{F}}, I\}, \quad \bar{\mathcal{F}}_2 = \text{diag}\{\bar{\mathcal{F}}, \bar{\mathcal{F}}, \bar{\mathcal{F}}, \bar{\varrho}, \bar{V}, \bar{\mathcal{F}}\}, \\ \bar{\mathcal{F}}_3 &= \text{diag}\{\bar{\mathcal{F}}, \bar{\mathcal{F}}, \bar{\mathcal{F}}, \bar{\varrho}, \bar{V}\}. \end{aligned}$$

By pre- and post-multiplying (2.18) with $\bar{\mathcal{F}}_1$ and $\bar{\mathcal{F}}_1^T$, respectively, we can get (2.47). By pre- and post-multiplying (2.19) with $\bar{\mathcal{F}}_2$ and $\bar{\mathcal{F}}_2^T$, respectively, we can get (2.48). By pre- and post-multiplying (2.20) with $\bar{\mathcal{F}}_3$ and $\bar{\mathcal{F}}_3^T$, respectively, we can get (2.49). This completes the proof.

Remark 2.10 If there just one node is LSNSs, the synchronization problem is formulated as Master-Slaver synchronization with master system (2.4), slave system (2.1) and controller (2.9). Then we can get the master-slave synchronization scheme by changing the parameters to $i = 1, \lambda_1 = 0$ in Theorem 2.9.

On the other way, according on Theorem 2.9, we can get the following corollary.

Corollary 2.11 *The LSNSs (2.1) are exponentially synchronized with a small enough decay rate, if (2.47)–(2.49) are true when $\alpha = 0$. Furthermore, the desired state feedback sampled-data controller matrices are presented in (2.50).*

According to different performance requirements, we can transform the proposed theoretical conditions into LMI-based optimization problems and obtain the corresponding controllers.

Case 1: If preset the lower bound θ_1 and the upper bound θ_2 of the sampling interval, our main objective is to maximize the initial condition $\mathcal{E}(\tilde{\mathcal{F}}^{-1} \bar{P} \tilde{\mathcal{F}}^{-T})$, in which the LSNSs are locally exponentially synchronize with the leader. Given a set $\mathcal{E}(R)$ which satisfies $\beta \mathcal{E}(R) \subset \mathcal{E}(\tilde{\mathcal{F}}^{-1} \bar{P} \tilde{\mathcal{F}}^{-T})$, it is easy to obtain $\varsigma^T \beta^2 \tilde{\mathcal{F}}^{-1} \bar{P} \tilde{\mathcal{F}}^{-T} \varsigma \leq \varsigma^T R \varsigma \leq 1$. By using Schur complement, this inequality can imply

$$\begin{bmatrix} -R & -\beta I \\ * & \bar{P} - \tilde{\mathcal{F}} - \tilde{\mathcal{F}}^T \end{bmatrix} \leq 0. \quad (2.51)$$

When θ_2 is fixed, we can maximize the initial ellipsoidal set by settling the following optimization problem

$$\begin{aligned} \min \quad & \beta^{-1} \\ \text{s.t.} \quad & (2.47)(2.48)(2.49)(2.51). \end{aligned} \quad (2.52)$$

Case 2: If fix the lower bound θ_1 of the sampling interval and the initial ellipsoidal set $\mathcal{E}(\tilde{\mathcal{F}}^{-1} \bar{P} \tilde{\mathcal{F}}^{-T})$, we focus on how large the upper bound θ_2 can reach, in which the LSNSs are locally exponentially synchronize with the leader. When β is given, we can maximize the upper bound θ_2 by settling the following optimization problem

$$\begin{aligned} \min \quad & \theta_2^{-1} \\ \text{s.t.} \quad & (2.47)(2.48)(2.49)(2.51)|_{\beta \text{ is given}}. \end{aligned} \quad (2.53)$$

Since there exist some items include θ_2 multiplying by variable matrices in (2.48) and (2.49), so we cannot use the function “mincx” in LMITools directly to maximize θ_2 . The alternative method is to iteratively increase θ_2 and straightforward to experiment the feasibility of (2.47)–(2.49) and (2.51) by using the function “feasp” in LMITools.

2.4 Numerical Example

In this section, a numerical example will be presented for the purpose of illustrating the effectiveness of the proposed techniques in the previous sections.

Consider LSNSs (2.1) that consist of three linearly coupled identical Chua's chaotic circuit, which is a class of typical benchmark three dimensional chaotic system, with parameters described as follows

$$A = \begin{bmatrix} -\frac{18}{7} & 9 & 0 \\ 1 & -1 & 1 \\ 0 & 14.28 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{27}{7} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$f_l(x_{il}(t)) = \frac{1}{2}(|x_{il}(t) + 1| - |x_{il}(t) - 1|).$$

It is demonstrable that (2.2) is true with $F = \text{diag}\{1, 1, 1\}$.

The LSNSs include 3 nodes. The outer-coupling configuration matrix and the inner-coupling matrix are given as

$$G = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2.54)$$

The eigenvalues of G are: 0, -1 , -3 . The saturation levels are assumed to be $u_{il} = 1$, the lower bound $\theta_1 = 0.05$, $\kappa = 0.5$, $\alpha = 0.05$ and the coupling strength $c = 0.5$.

Case 1: Set $R = I_{3 \times 3}$, then $\varsigma^T R \varsigma \leq 1$ is a sphere in three-dimensions. By solving optimization problem (2.52), we get different β corresponding to different sampling upper bound θ_2 , the results are shown in Table 2.1.

Moreover, Fig. 2.1 shows the surface for the admissible initial ellipsoidal set, which are changed by θ_2 . The three ellipsoidal sets (outer, middle and inner) are corresponding to different θ_2 .

From Table 2.1 and Fig. 2.1, it is easy to obtain that the smaller sampling upper bound θ_2 can led to larger ellipsoidal set.

When $\theta_2 = 0.3$, by calculating the optimization problem (2.52), one gets

Table 2.1 β for different sampling upper bound θ_2

θ_2	0.10	0.20	0.30
β	0.3027	0.2786	0.1653

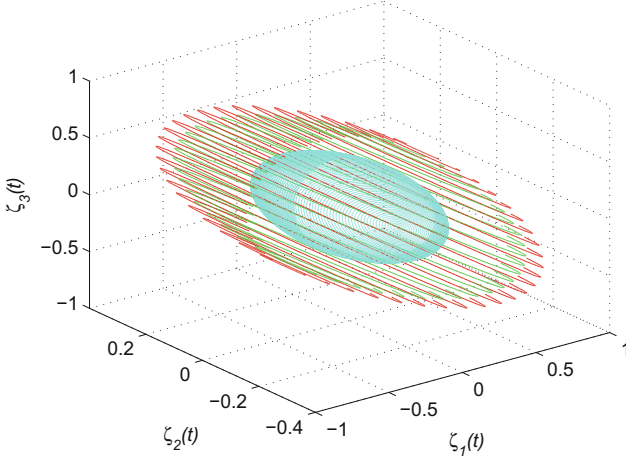


Fig. 2.1 The surface for the admissible initial ellipsoidal set corresponding to three different θ_2

$$\bar{\mathcal{F}} = \begin{bmatrix} 0.3164 & -0.0752 & -0.1756 \\ -0.0053 & 0.0514 & 0.0250 \\ -0.1313 & 0.0712 & 0.2931 \end{bmatrix}, \quad \bar{P} = \begin{bmatrix} 0.5038 & -0.0767 & -0.3310 \\ -0.0767 & 0.0740 & 0.0890 \\ -0.3310 & 0.0890 & 0.4987 \end{bmatrix},$$

$$\bar{K} = \begin{bmatrix} -0.3957 & -0.1971 & 0.0689 \\ -0.0029 & -0.1198 & -0.2303 \\ -0.0546 & 0.3224 & -0.2421 \end{bmatrix}.$$

The gain matrix K of the desired sampled-data controller is obtained as

$$K = \begin{bmatrix} -2.0668 & -4.2120 & 0.3320 \\ -0.8790 & -2.0960 & -0.6704 \\ 0.1944 & 7.5531 & -2.5734 \end{bmatrix}, \quad (2.55)$$

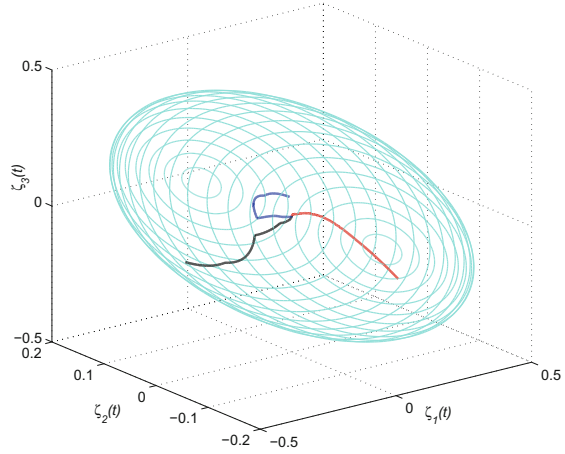
and the initial condition satisfies

$$\mathcal{E} = \left\{ \varsigma^T \begin{bmatrix} 4.9290 & 1.5613 & 1.8439 \\ 1.5613 & 29.9533 & -3.0857 \\ 1.8439 & -3.0857 & 6.5390 \end{bmatrix} \varsigma \leq 1 \right\}. \quad (2.56)$$

Remark 2.12 The smallest semi axis of the ellipsoidal set is 0.1813, which is larger than the sphere radius 0.1653. That verifies $\beta \mathcal{E}(R) \subset \mathcal{E}(\bar{\mathcal{F}}^{-1} \bar{P} \bar{\mathcal{F}}^{-T})$, i.e. the obtain maximized sphere set $\beta \mathcal{E}(R)$ is inside the initial ellipsoidal set $\mathcal{E}(\bar{\mathcal{F}}^{-1} \bar{P} \bar{\mathcal{F}}^{-T})$. By maximizing the β , we get the maximized initial ellipsoidal set.

The initial values of nodes in LSNSs and the unforced leader node are chosen as $x_1(0) = [0.7 \ -0.3 \ 0.1]^T$, $x_2(0) = [0.3 \ -0.3 \ 0.5]^T$, $x_3(0) = [0.3 \ -0.1 \ 0.1]^T$, and

Fig. 2.2 The state trajectories of three nodes are restricted in the ellipsoidal set



$s(0) = [0.5 \ -0.2 \ 0.3]^T$. Then we obtain that $\varsigma_1(0) = [0.2 \ -0.1 \ -0.2]^T$, $\varsigma_2(0) = [-0.2 \ -0.1 \ 0.2]^T$, $\varsigma_3(0) = [-0.2 \ 0.1 \ -0.2]^T$. It is easy to check $\varsigma_i(0) \in \mathcal{E}$, ($i = 1, 2, 3$), which implies that all the initial values of the error systems are inside the initial ellipsoidal set.

By implementing the control gain K shown in (2.55), the state trajectories of three nodes in error systems (2.10) are shown in Fig. 2.2. As seen in Fig. 2.2, the initial conditions of three nodes are inside the ellipsoidal set, and the states orbits are also restricted in the ellipsoidal set and finally tend to original point. I.e. all the error trajectories starting from inside of the ellipsoidal set will remain in it and converge to the origin eventually. That means by using the proposed method, all states trajectories of each nodes in LSNSs can converge to the one dominated by the isolated target node.

Furthermore, in order to demonstrate more clearly the effectiveness of the obtained controller, we define $err(t) = \sum_{j=1}^3 \sqrt{\sum_{i=1}^3 [x_{ij}(t) - s_i(t)]^2}$. The total synchronization error $err(t)$ is presented in Fig. 2.3, which shows that the synchronization between each nodes and the leader node can be achieved in a short time.

In addition, Fig. 2.4 depicts the simulation result of leader and the first node of LSNSs. For the isolated leader node (2.4) exhibits chaotic behavior, the three nodes of LSNSs are also exhibits chaotic behavior (for space limitation, we just show the chaotic behavior of the first node).

From Figs. 2.2, 2.3 and 2.4, we can get that the states of each nodes in the LSNSs (2.1) are indeed well locally exponentially synchronized to the states of leader (2.4) node by the designed sampled-data controller.

Case 2: We fix the admissible initial condition in (2.56) and the lower bound θ_1 , then the admissible upper bound $\theta_2 = 0.294$ can be obtained by solving the optimization problem (2.53) with the corresponding controller.

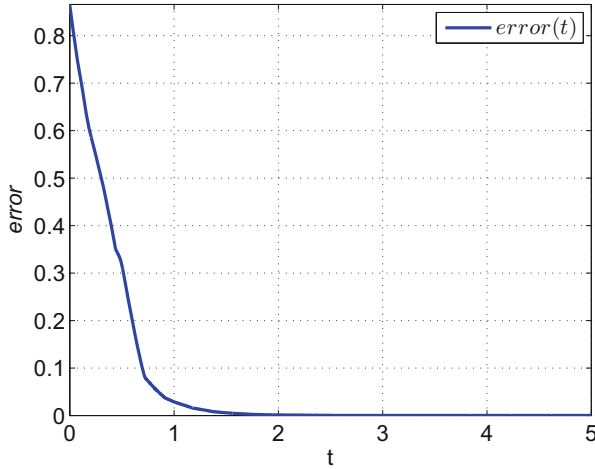


Fig. 2.3 The total synchronization error $err(t)$ of the LSNSs

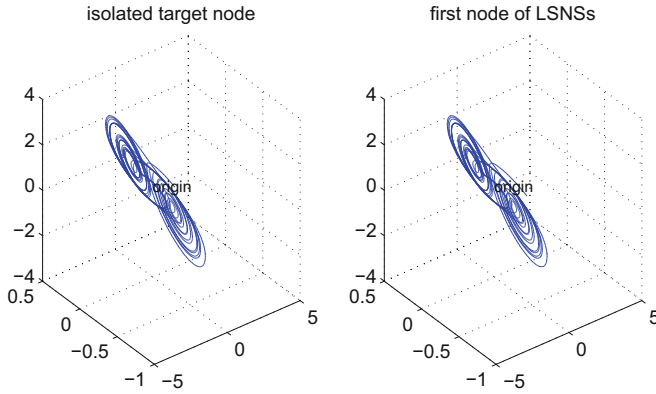


Fig. 2.4 The double scroll attractor of the Leader system (2.4) and one node of LSNSs (2.1)

$$K = \begin{bmatrix} -1.7231 & -4.6867 & 0.4265 \\ -1.0254 & -1.8875 & -0.7619 \\ -0.0689 & 8.4550 & -2.5103 \end{bmatrix}. \quad (2.57)$$

Initial conditions of all nodes are given as

$$x_i(0) = [0.6 - 0.1i \ 0.4 - 0.1i \ 0.1 + 0.1i]^T, \quad (i = 1, 2, 3),$$

and $s(0) = [0.2 \ 0.2 \ 0.2]^T$. These imply that $\varsigma_i(0) = x_i(0) - s(0)$ are in the admissible ellipsoidal set. Under the gain matrix K with the parameter given by (2.57), the evolutions for the norm of error systems $\|\varsigma_i(t)\|$, ($i = 1, 2, 3$) are presented in

Fig. 2.5, which mean that the error states of each nodes can decrease to zero in a short time.

The control inputs $u_i(t)$ are invariable between two sampling instants, which are exhibited in Fig. 2.6.

Subsequently, according to Remark 2.10, if there is just one node in LSNSs. By settling the optimization problem (2.52), different β corresponding to different sampling upper bound θ_2 are shown in Table 2.2.

Compared to the results in Table 2.1, the results in Table 2.2 show that, the admissible upper bound $\theta_2 = 0.37$ is larger than that of when $i = 3$; and for $\theta_2 = 0.3$, the maximized initial ellipsoidal set is also larger than the obtained result

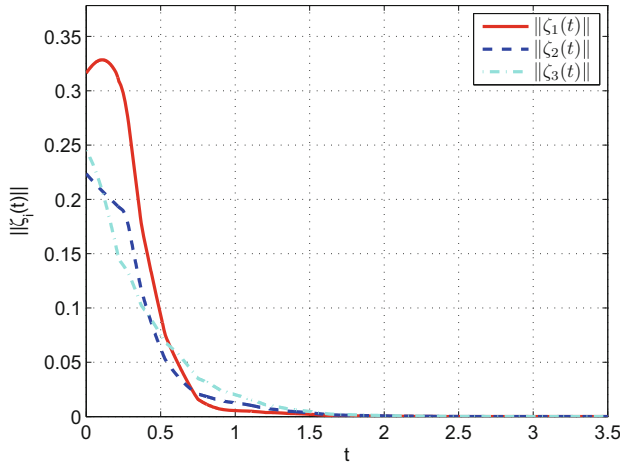


Fig. 2.5 The norm of error systems $\|\zeta_i(t)\|$, ($i = 1, 2, 3$)

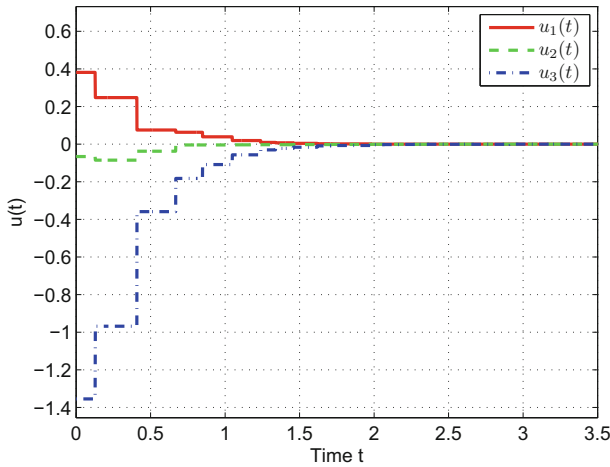


Fig. 2.6 The control input $u_i(t)$

Table 2.2 β for different sampling upper bound θ_2

θ_2	0.30	0.34	0.37
β	0.2286	0.1652	0.0985

when $i = 3$. Furthermore, when $\theta_2 = 0.37$, the corresponding control matrix is obtained as

$$K = \begin{bmatrix} -2.4959 & -5.9537 & 0.5773 \\ -0.6175 & -1.2174 & -0.7788 \\ 0.9808 & 10.2114 & -2.6196 \end{bmatrix}, \quad (2.58)$$

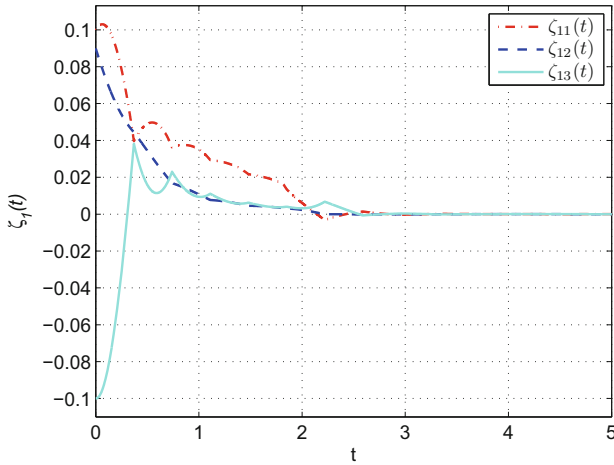
and the initial condition satisfies

$$\mathcal{E} = \left\{ \varsigma^T \begin{bmatrix} 15.0730 & -5.3629 & 3.4825 \\ -5.3629 & 99.3088 & -11.5748 \\ 3.4825 & -11.5748 & 16.7348 \end{bmatrix} \varsigma \leq 1 \right\}. \quad (2.59)$$

The initial values of slave and master systems are given as $x_1(0) = [0.42 \ -0.34 \ 0.05]^T$, $s(0) = [0.32 \ -0.43 \ 0.15]^T$. It is easy to check $\varsigma_1(0) \in \mathcal{E}$. Under the gain matrix K given in (2.58), we set each sampling intervals as constants 0.37, the state trajectories $\varsigma_1(t)$ are shown in Fig. 2.7.

From simulation results, the obtain sampled-data controller (2.58) can also guarantee the synchronization between the controlled slave system and the master system for all the invariable sampling intervals which satisfy $\theta_k = \theta_2$.

The above results can well imply that the LSNSs (2.1) are locally exponentially synchronized with the leader (2.4) in a short time under the sampled-data controller.

**Fig. 2.7** The error system states $\varsigma_1(t)$ for constant sampling $\theta_k = 0.37$. ($k = 0, 1, 2, \dots$)

2.5 Conclusion

This chapter investigates the design of sampled-data controller for synchronization of LSNSs with actuators saturation. A novel time-dependent Lyapunov functional and sector bound condition are utilized to make full use of the sampling pattern and describe the special properties of the saturation, respectively. Sufficient conditions are established for existence of the desired controller, and can be transformed to two optimization problems, in which the region of stability can be enlarged or the sampling upper bound can be maximized. Moreover, the effectiveness of the proposed approach is demonstrated by numerical examples, i.e. it is validated by the examples that the obtained sampled-data controller can guarantee the synchronization of the LSNSs subjected to actuators saturation.

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