

Chapter 2

State-of-the-Art for the BIEM

Abstract In this chapter, general remarks about the BIEM, fundamental solutions, and modern computational techniques for inhomogeneous 2D elastic domains are discussed, together with applications to seismic wave propagation problems. More specifically, a closer look with scales of hundreds of km reveals the Earth is both strongly inhomogeneous with a sharp gradient of variation of its material properties and also heterogeneous due to the existence of free and subsurface relief, non-parallel layers, cavities, inclusions, cracks and faults, and finally underground engineering constructions. The Earth's varying surface geology, the existence of a geomaterial depth-dependent gradient, of topography, and of nonlinear stress–strain states in the general geological region of interest, is the cause of significant spatial variations of seismic ground motion that can lead to large amplifications during earthquakes. A quantitative prediction of strong ground motion manifestation at a given site involves dealing with the source of seismic waves, their path to that site, and the effects of local conditions. A possible way of shedding some light on the understanding of the site-response phenomena and their sensitivity to the type and properties of the seismic source, of inhomogeneity and heterogeneity in the wave path, is in developing high-performance numerical methods for the simulation of seismic wave propagation phenomena.

2.1 The BIEM as an Effective Computational Tool

Mechanical models describing seismic wave propagation must include the following key parameters:

- characteristics of the seismic source, e.g., seismic moment, kinematical and geometrical properties of the fault system, etc.;
- wave path structure and material inhomogeneity;
- local geological conditions such as layering surface and subsurface relief and the existence of discontinuities at different length scales, such as cracks, inclusions, and faults and;

- mechanical characteristics and fabric of the geological material, including anisotropy, relative porosity, degree of saturation, physical and geometrical non-linearity, etc.

In addition, it may be necessary to consider the three-dimensional nature of the seismic motion and its coupling with thermal, mass transport and chemical processes. To this date, it has proven impossible to include all these effects in seismic design codes. This is due to several reasons, the primary one being the sheer complexity of the mechanical models describing the seismic scenarios that may develop at a given geographical location. The state of the art proposed here follows the review in Manolis and Dineva (2015) and will discuss two main aspects:

- A. The potential of the BIEM to treat elastodynamic problems for inhomogeneous and heterogeneous geological media. The BIEM advantages and disadvantages compared to other methods available in the literature, such as analytical, numerical, and hybrid techniques.
- B. BIEM results for seismic wave propagation in elastic/poroelastic, homogeneous/inhomogeneous, isotropic/anisotropic geological media with heterogeneities such as free and subsurface relief peculiarities, the presence of alluvial basins and of different types of inclusions, cavities, cracks, tunnels, pipelines, non-parallel layers, etc.

2.1.1 Available Computational Techniques for Elastodynamics of Inhomogeneous Media

Elastic wave motion in complex media is nowadays modeled using a variety of analytical/semi-analytical, numerical, and hybrid methods. In reference to the first category, commonly used methods are wave function expansion techniques (Pao and Mow 1971), ray theory and its modifications (Babich 1956; Pao and Gajewski 1977), method of bicharacteristics Boyadzhiev (2015), various matrix equation methods, reflectivity methods, wave number integration methods (Luco and Apsel 1983; Apsel and Luco 1983; Wuttke 2005), and finally mode-matching techniques (Fäh 1992; Panza et al. 2009). In general, all these approaches are restricted to special types of inhomogeneous media with simple geometries, plus a heterogeneity length scale that is considerably larger than the predominant wavelengths. Generally speaking, numerical techniques are now thought of as being the preferred way for studying elastic wave motion in inhomogeneous and heterogeneous media. Although they are suitable for analyzing media with complex structure, they require much computational effort in terms of both run-times and memory space. Among currently used numerical approaches such as FEM and FDM, BIEM has become quite popular over the last decades because of their efficient handling of infinite and semi-infinite domains connected with the modeling of the Earth (Beskos 1987a,b, Manolis and Beskos 1988; Manolis and Davies 1993; Dominguez 1993; Beskos 1997; Bonnet

1995; Aliabadi 2001). Next, hybrid techniques based on the BIEM in combination with other (mostly domain-type) methods are discussed in the following references:

- (a) Zienkiewicz et al. (1977); Beer (1986), where the FEM-BIEM technique is proposed;
- (b) Mogilevskaya and Crouch (2001, 2002), where the BIEM is combined with a series expansions approach;
- (c) Dineva et al. (2003); Panza et al. (2009), where a hybrid modal summation–BIE method is proposed for site effect estimation of a seismic region in a laterally varying media;
- (d) Wuttke et al. (2011); Dineva et al. (2012a), where a hybrid wave number integration method is combined with the BIEM for synthesis of synthetic seismic signals in a complex geological media;
- (e) Gil-Zepeda et al. (2003), who proposed a hybrid direct BIEM–discrete wave number method to simulate the seismic response of stratified alluvial valleys and;
- (f) Manolis et al. (2013, 2015), where a hybrid FD-BIEM was used to evaluate the seismic wave field development within a key geological cross section of an underground subway construction project.

2.1.2 BIEM for Inhomogeneous and Heterogeneous Media

Among currently used numerical approaches, the BIEM has become quite popular over the last decade because of efficient handling of heterogeneous materials. This is achieved by reformulating the corresponding boundary-value problem with integral equations only along the boundaries via the combination of reciprocity theorems and fundamental solutions of the governing equation (see Rizzo et al. 1985a,b; Beskos 1987a,b; Kobayashi 1987; Manolis and Beskos 1988). This technique is developed based on a transformation of the governing partial differential equations, which involve unknown fields inside and along the boundary of a domain, into an integral equation involving fields on the boundary only.

The work stages when applying the BIEM for the solution of a problem in mathematical physics are (see Fig. 2.1) as follows:

- Formulation of the original initial BVP by PDE, BC, and IC;
- Reformulation of the IBVP via BIE by the use of a reciprocity theorem and a fundamental solution of the PDE;
- Discretization and collocation procedure, evaluation of the asymptotic behavior of the fundamental solution, and the type of integrals obtained after behavior (regular, singular, the type of singularity);
- Formation of local and global matrices of influence, followed by formation and solution of the algebraic system of equation with respect to the unknown field variables;
- Verification study and validation study and;

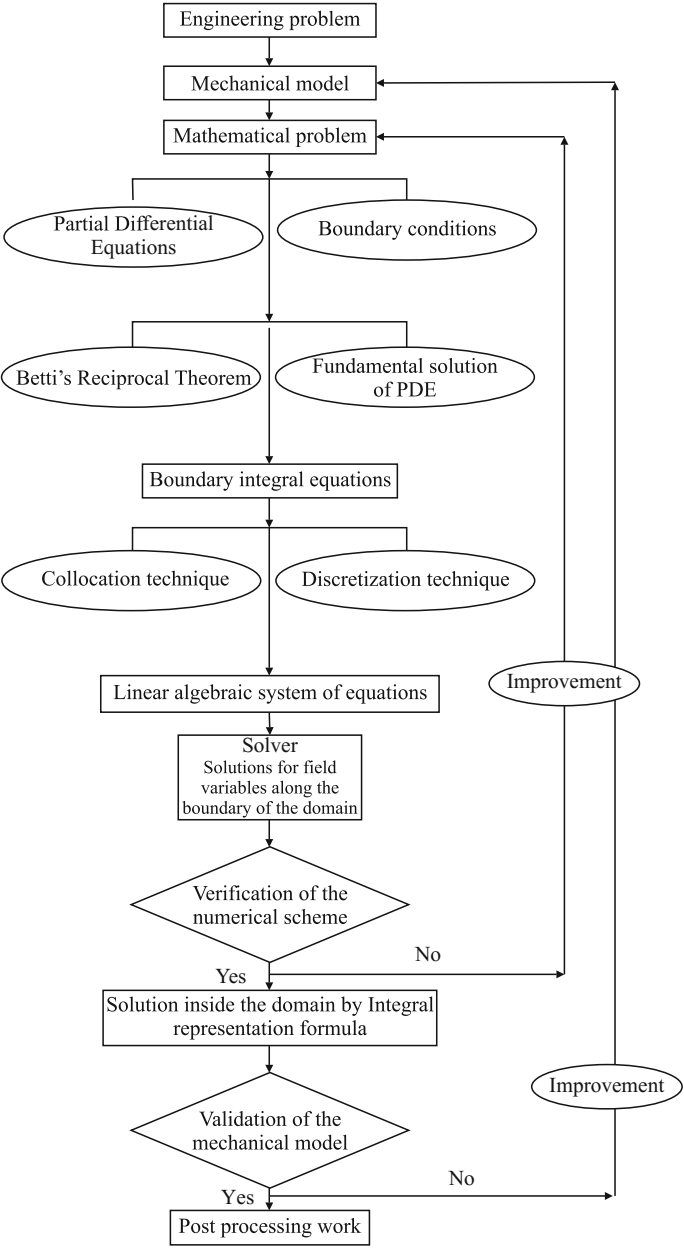


Fig. 2.1 The general BIEM flowchart

- Numerical simulations to evaluate the effects of the key parameters of the proposed mechanical model on its response.

The following BIEM formulations are available for the solution of transient problems:

- (a) BIE methods based on the use of time-dependent fundamental solution (see Mansur and Lima-Silva (1992));
- (b) BIE methods based on fundamental solutions defined in the Laplace or Fourier transform domain, with concurrent use of the corresponding inverse integral transform (see Manolis and Beskos (1988));
- (c) Dual reciprocity-BEM based on the elastostatic fundamental solution with the inertial terms treated as external body forces (see Albuquerque et al. (2003a, 2004));
- (d) Convolution quadrature time-domain BIEM augmented with the convolution quadrature formula of Lubich (1988) for approximating of the Riemann time convolution integrals that arise in time-dependent BIEs. Note that the last formulation leads to a time-stepping numerical scheme (see Schanz and Antes (1997); Furukawa et al. (2014)).

The following comments on the mesh-sensitivity of the available dynamic BEMs are collected in Garcia-Sanchez and Zhang (2007):

- (i) In the conventional time-domain BEM, the spatial discretization and the time discretization are not independent of each other, which is important for the quality and the stability of the BIEM. The situation for anisotropic elastic solids becomes even more complicated due to the directional dependence of the wave velocities;
- (ii) In the frequency domain, the mesh-sensitivity is governed by the ratio of nodal spacing to wave length, and for high frequencies, this may cause some serious difficulties due to the substantially large memory, storage, and computing time, which may exceed the available computer resources and;
- (iii) the non-conventional time-domain BIEM based on the Lubich quadrature is less sensitive to both the spatial and the time mesh size.

The advantages of the BIEM for its use in seismic mechanics are well known and have been discussed in detail in the literature. They are as follows:

- (a) Reduction in the size of the problem dimensionality and in the size of the resulting algebraic system, in contrast to other numerical domain methods;
- (b) Possibility to model lateral inhomogeneity in contrast to analytical methods;
- (c) Solution at each internal point in the domain is expressed in terms of boundary values without recourse to domain discretization, and this facility is very important when wave propagation problems are being solved in multilayered solids;
- (d) Flexibility to model relief peculiarities, in contrast to analytical methods and finite difference methods which encounter problems with implementing conditions on boundaries of complex geometric shapes;

- (e) The possibility to obtain directly, with no other intermediate source of error, displacements, and tractions;
- (f) The semi-analytical character of the method is important for accuracy reasons insofar as it is based on the fundamental solution or Green's function of the considered problem;
- (g) High level of accuracy is achieved, since numerical quadrature techniques are directly applied to the boundary integral equations, which are an exact solution of the problem at hand;
- (h) Boundary integral equation/boundary element methods have proven to be powerful tools for formulating and numerically treating exterior boundary-value problems. The fundamental solution used in the construction of the boundary integral equations obeys the radiation condition, and thus, infinitely extended boundaries are automatically incorporated, in contrast to other numerical methods where special transmitting and/or non-reflecting viscous boundaries have to be used. When a BIEM is used for half-space problems, such as those that occur frequently in elastodynamics, and the full-space fundamental solution is used, a truncated discretized model of the half-space surface is required. Therefore, a truncation problem can exist in modeling a half-space problem, even with BIE methods. However, the truncation issue is different compared to finite elements. With finite elements, the domain itself and not just a bounding surface must be truncated.
- (i) High accuracy in the computation of SCF near cavities and SIF near cracks, since solution at internal points is expressed in terms of boundary values without recourse to domain discretization.

The difficulties when work with the BIEM are as follows:

- (a) Fundamental solutions of the governing equations must be derived in a closed form, and in this respect, the method has limitations in solving nonlinear problems;
- (b) Overall accuracy in the BIEM results is sensitive to the degree of precision with which both singular and regular boundary integrals are evaluated;
- (c) Poor implementation of distributed body forces, because domain discretization appears and the method loses its main advantage of only surface discretization;
- (d) A crack in 2D solids is a line with two coincident faces, which leads to a degeneration of the classical displacement BEM formulation. Alternative BIE formulations have been developed and are discussed below in Sect. 2.2.2 and;
- (e) BIE implementation may be inefficient for large-scale problems because it results in densely populated matrices requiring large amounts of computation time.

The majority of results in the open literature are obtained by DBIEM. The 2D scattering and diffraction of SH waves of arbitrary angle of incident from irregular canyon-shaped topography has been formulated since the 1970s years by Wong and Jenings (1975). Most recently, Zhang and Chopra (1991) studied scattering of elastic waves by topographic irregularities. Direct BIEM has been applied to solve two-dimensional scattering of harmonic elastic waves by canyons (Sanchez-Sesma et al.

1985; Sanchez-Sesma and Campillo 1993), alluvial deposits (Dravinski 1982a, b), and ridges (Sanchez-Sesma et al. 1982), for different types of waves and shapes of the scatterers. The direct approach has been extended to 3D problems by Sanchez-Sesma (1983); Kim and Papageorgiou (1993); Papageorgiou and Pei (1998); Kawano et al. (1994); Reinoso (1994). Taking into account all the results produced by different authors, it is concluded that the DBIEM is a reliable and powerful tool for simulating ground motion in irregular multilayered sites.

Displacements and tractions in the IBIEM are formulated in terms of unknown fictitious ‘source densities,’ which do not have a direct physical interpretation. The source densities are distributed over the physical or an auxiliary boundary. The unknown displacements and/or tractions are obtained by integrating the indirect BIE solution for the fictitious sources. Thus, the solution to a problem via the IBIEM is always a two-step procedure. This shortcoming has contributed to the popularity of the DBIEM over the IBIEM. The IBIEM has been applied by Sanchez-Sesma and Campillo (1991) to study the diffraction of P, SV, and Rayleigh waves by topographic irregularities on the surface of a half-space. The seismic response of 2D alluvial valleys was dealt with by Sanchez-Sesma et al. (1993), whereas 3D site effect cases were studied by Ortiz-Aleman et al. (1998); Gil-Zepeda et al. (2002). The 3D response of 2D topography, the so-called 2.5D problem, has also been considered by Luco et al. (1990); Pedersen et al. (1994).

In what follows, there were listed the methods used to obtain fundamental solutions or Green’s functions for inhomogeneous continua. Note that the mathematical definition of a ‘fundamental solution’ is the solution of a differential equation with a singular Dirac’s function at the right-hand side, while the term ‘Green’s function’ is understood as a fundamental solution, which in addition satisfies boundary conditions. A Green’s function, if it exists, is unique, while a fundamental solution is not, and it is defined with a precision to a solution of the homogeneous differential equation.

2.1.3 Fundamental Solutions for Continuously Inhomogeneous Media and BIEM Formulations

Fundamental solutions are particular solutions of the differential operator in question for point forces in space and time, applied in unbounded domain. In turn, they serve as kernel functions in integral equation formulations that are redefined for both source (where the load is applied) and receiver (where the response is measured) points. The key role played by the fundamental solution is to reduce a given BVP formulated by governing partial differential equations together with boundary and initial conditions, into a system of BIE through the use of reciprocal theorem. As previously mentioned, the mechanical interpretation of the fundamental solution is the response of a solid to a unit body force concentrated at a given point and at a given moment of time. It is for this reason that the recovery of the fundamental solutions

in analytical form, or at least easy to calculate numerical form, is so important. A detailed review of the fundamental solutions and Green's functions in elastodynamics can be found in Kausel (2006). Most of the fundamental solutions are derived by Fourier, Laplace, or Radon transforms. The notion of a fundamental solution gradually became clear during the late nineteenth century. At the beginning, it was thought that only fundamental solutions could be applied via the convolution of distributions to the solution of linear partial differential equations with constant coefficients. The transient point force solution of the elastodynamic equation for homogeneous isotropic media was derived by Stokes (1849). This well-known result, known as the elastodynamic fundamental solutions for homogeneous isotropic and unbound media, plays a fundamental role in elastodynamics. What follows next is to consider the available fundamental solutions for homogeneous and inhomogeneous elastic media and to discuss possible BIEM formulations and their solutions.

The analytically derived elastodynamic fundamental solutions for homogeneous continua are as follows:

(a) In the time domain:

- i. For 2D anti-plane wave motion: Morse and Feshbach (1953), Eringen and Suhubi (1975), Cole et al. (1978), Mansur and Brebbia (1982a, b), Manolis (1983);
- ii. For 2D in-plane wave motion: Volterra (1894), Eason et al. (1956), Achenbach (1973);
- iii. For 3D: Stokes (1849), Love (1944), Morse and Feshbach (1953), Maruyama (1963), Achenbach (1973), Niwa et al. (1975b), Karabalis and Beskos (1984).

(b) In the frequency domain:

- i. For 2D anti-plane wave motion: Morse and Feshbach (1953), Eringen and Suhubi (1975);
- ii. For 2D in-plane wave motion: Doyle (1966), Cruse and Rizzo (1968), Kitahara et al. (1989), Dominguez (1993), Barra and Telles (1996);
- iii. For 3D: Doyle (1966), Cruse and Rizzo (1968), Kitahara et al. (1989), Dominguez (1993).

Early BIEM formulations for homogeneous elastodynamics based on the above-cited fundamental solutions are as follows:

- (a) For 2D anti-plane wave motion: in the time domain, see Friedman and Shaw (1962), Manolis (1983), Manolis and Beskos (1988), Beskos (1987a, b, 1997), and in the frequency domain, see Banaugh and Goldsmith (1963a, b), Manolis and Beskos (1988), Beskos (1987a, b, 1997).
- (b) For 2D in-plane wave motion: in the time domain, see Niwa et al. (1986), Manolis (1983), Antes (1985), Manolis and Beskos (1988), Beskos (1987a, b, 1997), and in the frequency domain, see Banaugh and Goldsmith (1963b), Cruse and Rizzo (1968), Niwa et al. (1975a, 1976), Dominguez (1993), Manolis (1983).

- (c) For 3D: in time domain, see Karabalis and Beskos (1984), Banerjee et al. (1986), Beskos (1987a, b, 1997), and in the frequency domain, see Rizzo et al. (1985a, b), Nakagawa and Kitahara (1986); Kitahara et al. (1989), Beskos (1987a, b, 1997).

BVPs for inhomogeneous materials are described by PDE with variable coefficients, for which extension of the many available analytical and numerical methods of solution for homogeneous materials cannot be easily done. It is known that differential operators with variable coefficients do not necessarily have fundamental solutions, but at the same time, BIE-based formulations presuppose their existence. Thus, to the authors' best knowledge, an inhomogeneous material with a general material profile does not possess available in analytical form fundamental solution, even for elastostatics. The general differential operators with variable coefficients do not necessarily have fundamental solutions (see Itô (2000)).

In what follows, we list the methods used to obtain elastodynamic fundamental solutions for limited cases of inhomogeneous continua:

- (a) Solution of the original partial differential equation:
Unfortunately, such solutions are generally not available. Some exceptions to this are as follows: (1) Hook (1962), who succeeded in obtaining a fundamental solution in closed form for the displacements in a vertically inhomogeneous medium with linear velocity profile of the P and SV waves along the direction of inhomogeneity due to transient in time point source, and (2) Watanabe and Takeuchi (2002), who derived a fundamental solution for the case of linear profiles of P and SV wave velocities in a radially inhomogeneous solid due to transient and time-harmonic sources.
- (b) Approximate fundamental solutions:
These solutions correctly describe the main part of the fundamental solution, but are not required to satisfy the original differential equations, apart from the singularity point (see Beskos 1997; Mikhailov 2002; Bai et al. 2002).
- (c) Use of available fundamental solutions for homogeneous materials:
The best example is the DR-BEM based on the fundamental solution for homogeneous case, whereby the resulting integral formulation includes both surface and domain integrals (see Clements 1980; Ang et al. 1996).
- (d) Use of available fundamental solutions for homogeneous materials by the reduction of the partial differential equation with variable coefficients to one with constant coefficients:
The path followed herein for recovering such types of solutions is (1) to use a simple algebraic transformation for the displacement vector (see Ben-Menahem 1987), so as to bring about a governing PDE of motion with constant coefficients, albeit at the cost of introducing a series of constraints on the types of material profiles; (2) to carefully examine these constraints, which reveal a rather rich range of possible variations of the elastic module in both vertical and lateral directions; (3) to use Fourier, Laplace, or Radon integral transform in order to obtain the fundamental solution for the differential operator with constant coefficients; and finally (4) to apply inverse algebraic transformation for the displacement in order to obtain the solution defined in the original domain.

Following the above procedures, results are obtained for two subgroups: The first one concerns graded materials with constant velocity profile, while the second one presents solutions for inhomogeneous materials with position-dependent velocity profiles. We will evaluate these two groups separately.

Fundamental solutions and subsequent BIEM realizations for elastic inhomogeneous continua with a constant velocity profiles can be found for: (1) anti-plane wave motion with time- and frequency-dependent fundamental solutions for graded material of exponential and trigonometric type in Daros (2008) and with frequency-dependent fundamental solution for exponential type of material gradient in Manolis et al. (2012), see Chap. 6 and (2) in-plane wave motion with frequency-dependent fundamental solutions for graded material of quadratic type in Manolis et al. (2004) and Dineva et al. (2006), see Chap. 8, of exponential and trigonometric type in Rangelov et al. (2005), see Chap. 4, and Dineva et al. (2007), see Chap. 6.

The following BIEM realizations for elastic inhomogeneous continua with variable velocity profile are available:

(a) In the time domain:

- (i) anti-plane wave motion, see Watanabe (1982), Watanabe and Payton (2004), Daros (2008, 2013), Sanchez-Sesma et al. (2001), Luzon et al. (2003, 2004) for a linear velocity profile. Additionally, power function velocity profile is considered in Watanabe and Payton (2004) and in Daros (2013) and
- (ii) in-plane wave motion, see Hook (1962), Watanabe and Takeuchi (2002), Sanchez-Sesma et al. (2001), Luzon et al. (2009) for linear velocity profiles.

(b) In the frequency domain:

- (i) anti-plane wave motion, see Watanabe and Payton (2004) for linear and power type of velocity profiles and Daros (2008), Sanchez-Sesma et al. (2001), and Kuvashinov and Mulder (2006) for linear velocity profiles;
- (ii) in-plane wave motion, see Watanabe and Takeuchi (2002), Sanchez-Sesma et al. (2001), Luzon et al. (2009).

Finally, all aforementioned fundamental solutions, besides being useful in their own right, serve as kernels in BIE-based formulations for solving complex problems numerically. Some such formulations are given in Ang et al. (1996) and Clements (1998) for the general second-order elliptic partial differential operator with non-constant coefficients that are functions of two spatial variables, in Xu and Kamiya (1998) for the inhomogeneous Poisson's equation whose linear part is governed by the Laplace operator, and in Itagaki (2000), who used the DR-BEM to handle Helmholtz's equation with a spatially dependent source term.

2.1.4 Green's Functions and BIEM Formulations for a Homogeneous Half-Space

Green's functions play an important role in the solution of linear ordinary and partial differential equations in science and engineering, including the field of seismic wave propagation in geological regions with complex mechanical and geometrical properties. Combined with BIE, the Green's function provides an elegant and yet powerful tool in investigating the complex responses of geomaterials and of buried civil engineering structures. If a BIEM based on Green's functions is available, then a smaller set of boundary data appears in conjunction with the surface representations, which in turn requires fewer boundary elements for the discretization process, increases accuracy, and reduces overall CPU execution time and memory requirements. Unfortunately, Green's functions for a given differential operator are known for relatively few simple cases of geometry and boundary conditions (Kausel 2006). This is the reason for using BIE formulations based on the fundamental solutions for point forces in the full space and then producing variants that are applicable to more specific cases.

The half-space Green's function (see Lamb 1904; Hook 1962; Ewing et al. 1957; Johnson 1974; Apsel 1979) could be used in place of a fundamental solution, and as a result, the presence of the half-space would be analytically incorporated into the formulation. However, for the most cases, the Lamb function is not available in closed algebraic form, but rather in the form of infinite frequency integrals which, for the present purpose, would need to be evaluated numerically. This is a matter of numerical sensitivity and, in practice, expensive. In Guan et al. (1998), a transient Green's function due to a suddenly applied line load in an isotropic and homogeneous half-space is derived. Utilizing the solution in the frequency and wave number domains given by Kobayashi (1983), the time-domain solution is obtained after first inverting the Laplace transform following the procedure of Eason (1964). The inversion of the Fourier transform is then carried out explicitly with respect to wave number. A half-space fundamental solution based on the full-space one is derived in Kobayashi and Nishimura (1980), while a two-dimensional frequency-dependent fundamental solution for a homogeneous half-space is presented in Kobayashi (1983). In Banerjee and Mamoon (1990), the half-space solution for the periodic point force is derived using a technique developed by Mindlin (1936) who solved the corresponding 3D static problem. A BIE formulation using Lamb's solution for elastic half-space problems was first proposed in Rizzo et al. (1985b). In a follow-up study (Gonsalves et al. 1990), half-space problems were considered by BIEM formulations using both full and half-space Green's function.

In Pan (1997), there is a discussion of advantages and disadvantages for using the half-space Green's function, when wave propagation problems in half-space with heterogeneity are being solved. It is shown that the truncation problem is avoided because the Somigliana integral formula has only an integral over the finite surface of the heterogeneity embedded in the half-space. However, the computation of BIE based on the Lamb's half-space Green's function is very time-consuming. In Pan

(1997), this is further discussed by performing numerical experiment for a time-harmonic wave radiation from a spherical void in a half-space with discretization of 8 elements. It took 2.8 s to solve the BIE based on the full-space Green's function and 820 s for the BIEM with half-space Green's function. In Apsel and Luco (1987) and Chapel (1987) Lamb's fundamental tensors are used to formulate the axisymmetric BIE for a layered viscoelastic half-space, thereby obviating discretization of the half-space surface.

2.1.5 BIEM Based on the Green's Function for a Layered Half-Plane

Although a homogeneous half-space represents the simplest soil model of practical importance, it is rather limited since most subgrades present stratification into horizontal layers due to the geological process of sedimentation. To account for the variability along the vertical direction, a great effort has been made in the evaluation of the Green's function for the case of horizontally layered media. Availability of layered half-space fundamental solutions eliminates the use of traction-free surface and interfaces between layers and greatly simplifies the BIEM solution procedure. Analytical solutions for the Green's function of a layered half-space do not exist except in the simple case of a homogeneous half-space, see Banerjee and Mamoon (1990). Numerical techniques to compute these functions have been first introduced by Thomson (1950) and Haskell (1953) by the propagator method in the wave number domain and then using an inverse Fourier Transform, see Bouchon and Aki (1977). In the early 1980s, Kennett (1983) proposed a new algorithm based on the reflection–transmission coefficients which has been shown to be more stable. More recently, different formulations based on the function for a layered half-space have been proposed by several authors, following Kausel and Peek (1982), Apsel (1979), and Kausel (1981). In Pais (1988), a 2D BIEM based on the Green's function originally proposed by Kausel and Peek (1982) is developed for dynamic analysis of soil-foundation interaction.

In Clouteau (1990), it is shown that the Green's functions of the layered half-space based on the Kennett's algorithm coupled with an inverse fast Hankel transform (Chapel and Tsakaladis 1985) may be efficiently used in a BIEM formulation, provided that singularities are efficiently accounted for using the adapted regularization procedures in Aubry and Clouteau (1991).

The Green's function for layered half-space used in Pais (1988) is closed form and does not need numerical integration. In Guan and Novak (1994a, b), a transient Green's function for surface loads over rectangular and strip regions, respectively, is developed. In Hisada et al. (1993a, b), a direct BIEM based on the combination of the exact Green's function of the layered half-space and of the normal mode solution derived and developed in Harkider (1964) is proposed. The method is applied for solution of seismic wave propagation in multilayered 2D and 3D basins. The

advantages of this method are illustrated in Hisada (1994, 1995), where seismograms for 2D anti-plane and in-plane sedimentary basin models are computed. A 3D BEM has been developed in Pan et al. (2001) for the analysis of composite laminates with holes, where 3D layered Green's functions have been implemented into the BEM formulation. Since layered Green's functions satisfy exactly the interlaminar continuity conditions along the interfaces and top and bottom free surfaces a priori, discretization along these interfaces and surfaces is unnecessary.

2.1.6 BIEM Based on the Green's Function for a Continuously Inhomogeneous in Depth Half-Plane

In here we look for Green's functions for the half-plane, which need to satisfy an extra boundary condition in the form of a traction-free horizontal surface. The usual approach is to consider this constraint directly, which adds extra correction terms to the fundamental solution. These terms tend to decay rapidly for points far below the free surface, causing the solution to converge to that for the full space. An analytical-numerical solution for the wave field produced by time-harmonic Rayleigh waves in an elastic half-space of constant density and whose shear modulus increases linearly with depth can be found in Vardoulakis and Vrettos (1988). The dynamic response of a half-space with constant density and an exponential variation of the stiffness under a time-harmonic, tangential line force on the surface is examined in Vrettos (1991a, b) by applying the Fourier transform method with pole residue integration. The treatment of in-plane problems is more complicated, since the system of the governing partial differential equations can be decoupled only in restrictive cases of the material parameter variation. For instance, Vrettos (1990) studied wave propagation in a half-space with constant density and Poisson's ratio, but with a shear modulus varying with depth according to a bounded exponential function. The propagation of Love waves in a non-homogeneous layer of finite thickness overlying an isotropic semi-infinite medium, with trigonometric variation in the material parameters, was solved by Kakar and Kakar (2012) by the method of separation of variables. Along the same lines, we have the work by Rangelov and Manolis (2014), see Chap. 5, Sect. 5.2.1, on the derivation of Green's functions for point sources and dipoles in the half-space with quadratic and exponential types of material inhomogeneities, assumed to hold proportionally for both shear modulus and density. This work can be viewed as a continuation of an earlier derivation in Rangelov and Manolis (2010), see Chap. 5, Sect. 5.1, of Green's functions for point sources in the half-space with quadratic types of material inhomogeneities varying proportionally for both Lamé modulus and density. Analytical solutions for the steady-state response of layered acoustic as well as elastic formations that are valid at high frequencies and can serve as Green's functions for a BIE-based formulation to analyze 3D fluid-structure interaction problems appear in Tadeu and Antonio (2001).

In closing, among other recent work on the computation of specialized Green's functions, we would like to mention also the result of Guzina and Pak (1996) for smoothly inhomogeneous half-space, while in Rajapakse and Wang (1991), a Green's function for the orthotropic half-planesubjected to a set of time-harmonic buried load is derived. In Apsel and Luco (1983), Luco and Apsel (1983), Hisada (1994, 1995), Pak and Guzina (2002), and Kausel (1981), a Green's function for the stratified half-plane is derived, but the computational burden can make the corresponding BIE approach impractical or expensive.

The following conclusions can be done from the above state of the art:

- (a) There is still a paucity of studies developing BIEM codes for computation of synthetic seismograms accounting for the broader system made up of seismic source, wave path, and the local site with its specific geometrical and mechanical properties;
- (b) The conventional BIEM based on the fundamental solution consumes much CPU time and memory for solution of large geological multilayered regions with complex geometry and thus becomes non-effective;
- (c) The BIEM based on more sophisticated Green's function increases the solution accuracy and reduces the CPU time and memory, but their availability is limited and;
- (d) The existence of very complex Green's function very often leads to impossible situations for solution of BIEs, so the best case is when the Green's function (if it exists) decreases both the computational efforts and the CPU time and memory.

2.2 BIEM Application in Seismic Wave Propagation Problems

2.2.1 *Elastic Waves in Crack-Free Media*

2.2.1.1 Homogeneous Isotropic and Anisotropic Media

The dynamic behavior of a homogeneous solid containing multiple inclusions and/or cavities is an important engineering problem with applications in many modern technological fields such as material science, nondestructive testing evaluation, computational geophysics, and fracture mechanics, see Dineva and Manolis (2001a, b), Meguid and Wang (1995), Datta and Shah (2008). Elastostatic problems for multiple inclusions were solved by BIE-based methods in Graffi (1998), Yao et al. (2003), Leite and Venturini (2006), Kong et al. (2002), Tan et al. (1992), and Dong et al. (2003), where in most cases, the substructuring technique was used. In Mogilevskaya and Crouch (2001), the problem of an infinite plane with any number of randomly distributed, perfectly bonded circular elastic inclusions with arbitrary elastic properties is solved by combining the series expansion technique with a direct BIE technique.

Regarding in-plane wave motions, we have the essential papers by Cruse (1968) and Cruse and Rizzo (1968) on a Laplace-transformed, direct-type BIEM elastodynamic formulation. Using this type of formalism, a series of results appeared for the diffraction of plane elastic waves by cavities (e.g., Kobayashi 1983). Furthermore, Ohutsu and Uesugi (1985) and Luco and Barros (1994) developed an indirect-type BIEM to analyze planar wave scattering in a half-plane with surface relief plus a buried cavity. The presence of the free surface can be treated either by discretization followed by imposition of the boundary conditions as in the previous papers, or through the derivation of a specialized Green's function (Kontoni et al. 1987) so that evaluation of surface integrals along the free surface is unnecessary.

In more detail, elastodynamic problems for multiple inclusions solved by BEM are presented in Providakis et al. (1993), where stress reduction around a circular cavity in a dynamically loaded plate is achieved by introducing auxiliary cavities around the reference cavity. Solution of this plane stress problem was carried out in the Laplace transform domain, with the time-domain response recovered numerically by using the inverse transformation. In Gao et al. (2001), the scattering of flexural waves is studied with calculation of the dynamic SCF in a thin plate with a cutout by using the DR-BEM. Furthermore, the dynamic stress field around cavities of arbitrary shape in infinitely extended bodies under plane stress/strain conditions was numerically determined in Manolis and Beskos (1981). In Niwa et al. (1986), the Fourier transform method is used to solve elastic wave scattering by heterogeneity in the half-space. The heterogeneity considered may either be a cavity, an elastic inclusion, or a fluid inclusion, while the surrounding half-space is assumed to be a homogeneous. Scattering of plane harmonic scalar waves by multiple inclusions in a half-plane is studied in Dravinski and Yu (2011). The basic BVP for all cases of P, SV, and SH waves was reformulated in terms of the displacement BIEM using frequency-dependent fundamental solutions for in-plane/anti-plane wave motion, augmented with substructuring (or subregions, see Venturini 1992). More recently, BIEM simulations of dynamic stress fields that develop in finite-sized solids containing multiple inclusions of arbitrary shape are presented for in-plane strain in Parvanova et al. (2013, 2014b) and for anti-plane strain in Parvanova et al. (2014a, c). By using an indirect BIEM, Benites et al. (1992) investigated SH wave scattering by a system of multiple cavities in both an infinite plane and a half-plane.

Various BIE-based formulations for solution of scattering problems can be found in Manolis and Beskos (1981); Niwa et al. (1986); Nolet et al. (1989); Rus and Gallego (2005); Antes et al. (1991); Schanz and Antes (1997); Zhang and Savidis (2003). Propagation of elastic waves through heterogeneous geological structures causes reflection, refraction, diffraction, and scattering phenomena that are difficult to quantify. In addition, different types of discontinuities such as cracks, cavities, and inclusions further complicate the overall picture by acting as scatterers and stress concentrators. These heterogeneities may generate large amplification as well as spatial variations in the seismic motions, which have important repercussions in the analysis of large infrastructure such as lifelines (dams, bridges, pipelines, tunnels, etc.) The literature is quite rich in terms of results obtained for various cases of surface and subsurface topography, as well as for shallow sedimentary basins (Chen

et al. 2011; Gatmiri et al. 2008; Gatmiri and Arson 2008; Álvarez-Rubio et al. 2004, 2005; Trifunac 1971; Todorowska and Lee 1991), irregular soil layers (Liu et al. 2008, 2012; Bouchon and Courant 1994; Chen 1996), geological cracks (Rodríguez-Castellanos et al. 2005; Liu and Zhang 2001; Liu et al. 1997), buried cavities (Hall et al. 2010; Wang and Liu 2002; Lee and Manoogian 1995; Lee and Serif 1996), and finally tunnels (Lee et al. 1996, 2004b; Liang et al. 2007b; Liang and Liu 2009; Liang et al. 2004; Lee and Trifunac 1983; Manoogian 2000).

In particular, the presence of surface relief, non-parallel layers, internal and interface geological cracks, wave path inhomogeneity, and material poroelasticity on the seismic response of geological configurations need to be further examined (Dineva et al. 2006, 2012a, b) on account of its importance in many types of engineering and geophysical applications.

The scattering of plane harmonic P/SV and Rayleigh waves by a corrugated cavity and an inclusion embedded in the half-plane was investigated by Yu and Dravinski (2009) and Dravinski and Yu (2013), respectively, using the standard form of the direct BEM.

An effort to consider material anisotropy, Ahmad et al. (2001) used the displacement-based BEM to study steady-state vibrations of 2D surface foundations in an orthotropic subbase and Chuhan et al. (2004) to study the transient vibrations of underground structures in 2D orthotropic media. Also, free vibrations of anisotropic structural sheets were studied using the BEM by Albuquerque et al. (2003a, b), while Dravinski and Wilson (2001); Niu and Dravinski (2003a, b) examined time-harmonic response of 2D basins and 3D cavities in anisotropic media by an indirect version of the BEM. Next, integral equation methods have been employed for reconstructing the elastostatic field (Dong et al. 2004) and the time-harmonic wave field (Lee et al. 2004a) in the elastic half-plane containing anisotropic inclusions. Furthermore, Rubio-Gonzalez & Manzon (1999) presented an analysis of crack problems in orthotropic media under impact loads, while more recently, Albuquerque et al. (2004) developed a DR-BIEM for anisotropic dynamic fracture mechanics.

2.2.1.2 Inhomogeneous Media

The presence of an inhomogeneous background when considering elastic wave scattering by different type of heterogeneities presents a new set of difficulties, since wave signals no longer travel undisturbed, but are subjected to continuous amplitude changes and phase angle shifts. Functionally, graded materials are a special class of inhomogeneous media, mostly man-made but possibly naturally occurring. Their dynamic behavior has been the focus of much recent work, as for instance by Reddy and Cheng (2003) who employed 3D asymptotic theory in a transfer matrix setting to study harmonic vibrations of FGM plates. Furthermore, Santare et al. (2003) used the FEM for the analysis of elastic wave propagation through 2D continuously inhomogeneous structural components, while transient wave propagation in strips of FGM was studied by Berezovski et al. (2003) using an algorithm based on the numerical solution of systems of hyperbolic equations. In Aizikovich et al. (2002), a

combination of analytical solutions and dual integral equations is employed for the spherical indentation problem of a half-plane with depth-dependent properties. Finally, the generalized BEM with internal collocation points was also used for the static analysis of non-homogeneous solids under both 2D and 3D conditions by Chen et al. (2001). There is a relatively small amount of work dealing with the combined inhomogeneous and anisotropic material. At this point, we mention Ang et al. (2003), who employed the DR-BIEM for elastostatic problems.

More recently, analytical expressions for free-field motions in an inhomogeneous isotropic half-space with depth-dependent material properties were developed by Rangelov et al. (2010); Manolis et al. (2007, 2009); Rangelov and Dineva (2005), see Chap. 5, where restricted types of inhomogeneities of the quadratic and exponential type were considered.

As far as BVP solutions are concerned, Manolis (2003) studied the dynamic response of underground openings in an inhomogeneous continuum where both shear modulus and density vary proportionally in the vertical direction. This material model gives P and S wave velocities which are macroscopically constant. More specifically, a direct BEM formulation defined in the Laplace transform domain is used, and time dependence of the resulting displacements, tractions, and stresses is recovered through the inverse transformation. Specialized fundamental solutions are employed, which were derived following the method originally developed for a 3D continuum in Manolis and Shaw (1996). Also for non-homogeneous materials, Chen et al. (2001) developed a generalized BEM where the domain integral involves first-order derivatives of the displacement kernel. By using radial basis functions, the domain integral is converted into a boundary integral so that numerical implementation for 2D elastostatics is performed using a surface mesh plus relatively few internal collocation points. Also, Itagaki (2000) developed a DR-BIEM for Helmholtz-type equations with a space-dependent source term based on repeated application of particular solutions for the Poisson's equation. This scheme was subsequently expanded to iteratively solve problems involving non-uniform media. Similarly, Xu and Kamiya (1998) approximated the inhomogeneous term for nonlinear potential problems by polynomials whose coefficients were determined in a least square sense from a system of integral equations defined at both boundary and interior domains of the problem in question.

2.2.1.3 Poroelastic Media

Poroelasticity derives from continuum mechanics theory for materials comprising an elastic matrix with fluid-saturated pores. The presence of the fluid stiffens the material, but also results in diffusion between regions of higher to lower pressure. Since Biot (1956) proposed a phenomenological model for dynamic poroelasticity over fifty years ago, the problem of wave propagation in two-phase materials has been extensively studied. Regarding the use of the BIE-based methods in poroelasticity, we note the key role played by the fundamental solution (or Green's function) in reducing a given BVP into a system of integral equations along the boundaries.

Unsurprisingly, much research work has been directed toward the derivation of fundamental solutions for the governing partial differential equations of poroelasticity (Burridge and Vargas 1979; Norris 1985; Manolis and Beskos 1989; Cheng et al. 1991, Schanz and coworkers (Schanz and Antes 1997; Schanz 1999, 2001a,b; Schanz and Diebels 2003; Schanz and Pryl 2004; Schanz 2009), Gatmiri and Jabbari (2005a,b), Cheng (2016). These include derivations in either the time domain or transform domains and under both 2D and 3D conditions. Also, BIE formulations for poroelasticity appeared early on, e.g., Dominguez (1991, 1992). To summarize, the comprehensive state-of-the-art reviews by Gatmiri and Kamalian (2002); Gatmiri and Nguyen (2005); Seyrafiyan et al. (2006); Gatmiri and Eslami (2007) contain abundant information on the dynamic fundamental solutions derived for the porous media and their subsequent incorporation within coupled integral equation statements. Following numerical discretization for specific BVP, these BIEs yield non-symmetric systems of matrix equations; thus, effort has been expended at producing symmetric formulations, see Pan and Maier (1997). Application-type examples that serve as benchmarks can also be found in the literature, see for instance Theodorakopoulos and Beskos (2006) and Albers et al. (2012), where BIE and FEM results are compared for the classical poroelastic column example.

So far, BIE-based methods have seen limited application to seismic wave propagation in saturated heterogeneous geological media due to difficulties in accounting for the multiphase nature of the problem. There is, however, a marked similarity between poroelastic and viscoelastic materials in terms of their dynamic response, a fact that suggests the possibility to use a single-phase model with special, augmented properties in lieu of the multiphase one. This idea was promulgated by Bardet (1992, 1995); Morozhnik and Bardet (1996), who proposed an equivalent viscoelastic solid to model saturated poroelastic materials governed by Biot's theory. Specifically, the speed and attenuation of the longitudinal and transverse waves in soils described by viscoelastic isomorphism are related to Biot's material parameters in a way that equates the viscoelastic and poroelastic wave numbers. Parametric studies (Morozhnik and Bardet 1996) have shown that viscoelastic isomorphism gives solutions practically identical to those of the original poroelastic problem for a specific range of material parameters and in the low-frequency response range. More recently, a series of solutions conducted by Dineva et al. (2012a,b), see Chap. 10, using Bardet's model in conjunction with the displacement BEM show nearly identical results with those obtained by Biot (1956) model for benchmark type of BVP. The importance of Bardet's model used in conjunction with the BIE methods to study seismic wave propagation stems from the fact that it is possible to efficiently model non-homogeneous soil inclusions, the presence of non-parallel soil layers, free-surface relief, interface and internal cracks, plus the presence of a seismic source.

The problem of computing stress concentration effects becomes much more difficult when discontinuities occur in two-phase media such as water-saturated soils. As a consequence, relatively few results have been obtained, such as those of Kattis et al. (2003); Wang et al. (2005), who studied diffraction of elastic waves around cavities in a poroelastic medium, and of Liang et al. (2007a,b); Liang and Liu (2009), who presented results for a cavity in a poroelastic half-space under incident pressure and

shear waves based on Biot (1956) theory. The behavior of discontinuous poroelastic media is recognized as an area of high research interest in geomechanics, since it has important applications in energy extraction and recovery from geological formations, see Fjaer et al. (2008).

2.2.2 *Elastic Waves in Fractured Media*

The presence of cracks modifies the traveling elastic wave fields because of pure wave phenomena as diffraction and scattering, but also causes localized stress concentration phenomena. Due to this fact, numerical modeling of wave motion in a cracked continuum requires hybrid-type methodologies that combine elastic wave theory with dynamic fracture mechanics concepts. Crack analysis in elastodynamics has attracted the attention of researchers in fields as diverse as material science, acoustic emissions, ultrasonic nondestructive testing evaluation, seismology, and exploratory geophysics. In the latter case, observation of elastic waves generated by earthquakes or explosions may reveal the presence of structural discontinuities in the Earth's interior.

Since analytical solutions for dynamic crack problems can be obtained for simple cases only, numerical methods have to be applied to solve meaningful boundary-value problems. Among the computational methods used, BIE-based ones have become popular because fracture parameters such as SIF, COD, and ERR can be accurately determined.

In sum, BIE-based approaches can capture the rapid variation observed in the boundary stresses. More specifically, at the tip of an idealized crack, stresses may be expressed as an infinite series summation with a leading term inversely proportional to the square root of the distance from the crack-tip. Therefore, stresses are theoretically infinite at the tip, but the kernel functions used in BIE formulation are themselves singular, so this type of behavior can be reproduced. At the same time, some disadvantages of BIE crack formulations must be pointed out:

- (a) A crack in 2D solids is a line with two coincident faces, which leads to a degeneration of the displacement BIEM formulation;
- (b) Overall accuracy in the BIEM results is sensitive to the modeling of the crack-tip behavior and to the degree of precision with which both singular and regular boundary integrals are evaluated.

In order to overcome the above deficiencies, the following alternative BIE formulations have been developed:

- (a) the multidomain method (Lachat and Watson 1976; Dominguez and Gallego 1992), which introduces artificial boundaries in the elastic body by connecting the crack to a boundary in a way such that each region contains a single crack surface. These separate domains are then joined by imposing equilibrium and compatibility conditions;

- (b) the Green's function technique (Cruse 1988) uses specially derived Green's function satisfying the traction-free boundary condition on the crack surfaces, and thus, the displacement boundary integral equation approach can be applied without any discretization along the crack surface, but this technique is limited to simple geometries;
- (c) the displacement discontinuity method (Crouch and Starfield 1983) based on the Green's functions corresponding to a point dislocations. This method is quite suitable for crack problems in infinite domains, but for finite domains, the kernel functions involve singularities of an order higher than those occurring in the conventional displacement BEM;
- (d) the single domain approach utilizing a hypersingular traction BEM, see Gallego and Dominguez (1997); Ang and Park (1997); Ang et al. (1999); Dineva and Manolis (2001a, b); Zhang (2000, 2002b); Rangelov et al. (2003); Tan et al. (2005); Wunsche et al. (2009a, b). More specifically, traction boundary integral equations can be obtained by partial differentiation of the displacement BEM with the subsequent application of the elastic law. This results in a hypersingular BIE, because of the existence of the spatial derivatives of the stress tensor coming from the displacement fundamental solution. To circumvent this problem involving non-integrable singularities, different regularization techniques have been proposed, see Zhang and Gross (1998). The majority of these techniques employ partial integration, which shifts the spatial derivatives acting on the stress fundamental solution to the unknown crack opening displacement. Thus, the regularized traction BIE has the same order of singularities as the displacement BEM. The hypersingular traction BIE can be solved directly by the use of Galerkin's method instead of the standard nodal collocation procedure, see Guan and Norris (1992);
- (e) the non-hypersingular traction BIE that is based on the conservation integrals of the linear electrodynamics, see Zhang and Gross (1998); Manolis et al. (2004), see Chap. 8, Dineva et al. (2005), see also Chap. 5 in Dineva et al. (2014);
- (f) the dual BEM, where both displacement and traction BIE formulations are applied to the crack surfaces, see Portela et al. (1992); Chen and Chen (1995); Sollero and Aliabadi (1995); Aliabadi et al. (1998); Dellerba et al. (1998); Chen and Hong (1999); Albuquerque et al. (2004); Benedetti et al. (2009); Benedetti and Aliabadi (2009). An extension of the dual BEM can be found in Pan and Amadei (1996, 1999), where for finite cracked solids, the displacement BIE is applied on the uncracked boundary and the traction BIE along one side of the crack surface. Finally, an overview of the different approaches available in fracture analysis of elastic solids of finite size by BIE-based methods can be found in Zhang (2002b). The numerically computed COD can be post-processed to yield the SIF, which is one of the most important design parameters in fracture mechanics.

When examining transient fracture problems, the following formulations are available:

- (a) BIE methods based on the use of time-dependent fundamental solution, see Hirose (1989); Hirose et al. (2002); Zhang and Gross (1998); Zhang (2002b); Wang and Achenbach (1994);
- (b) BIE methods based on fundamental solutions defined in the Laplace or Fourier transform domain, with the concurrent use of the corresponding inverse integral transform, see Fedelinski et al. (1995a, 1996a,b); Albuquerque et al. (2003a); Zhang and Gross (1998); Dineva et al. (2004, 2005); Garcia-Sanchez et al. (2006);
- (c) Dual reciprocity-BEM based on the elastostatic fundamental solution with the inertial terms treated as external body forces, see Albuquerque et al. (2002, 2004). These formulations are derived from a reciprocal relation between static and dynamic states, in which the inertia forces are treated as body forces. They are useful when derivation of the fundamental solutions for the problem at hand is problematic;
- (d) Time-domain BIEM augmented with the convolution quadrature formula of Lubich (1988) for approximating of the Riemann time convolution integrals that arise when Laplace-domain fundamental solutions are used in lieu of time-domain ones. Note that this formulation leads to a time-stepping numerical scheme, see Schanz (1999, 2001b). The aforementioned hypersingular traction BIE methods defined in the Fourier, the Laplace, or the time domains have been compared and evaluated in terms of performance in Garcia-Sanchez and Zhang (2007).

In general, BIE methods in fracture mechanics were developed for isotropic and homogeneous elastic media in the 1990s and have since been extended to anisotropic and inhomogeneous solids over the last fifteen years.

More detailed reviews for the early period of BIE modeling of wave propagation in elastic solids with stationary cracks can be found in Beskos (1987a,b, 1997); Cruse (1996, 1978); Dominguez (1993); Pan (1997); Fedelinski et al. (1995a, 1996a); Aliabadi (2004, 1997); Zhang and Gross (1998); Chen and Hong (1999); Mukhopadhyay et al. (2000); Garcia-Sanchez and Zhang (2007); Liu et al. (2012); Guz et al. (2013). Specifically, a list of dynamic fracture mechanics problems solved by BIE can be found in Zhang and Gross (1998), while comparisons between time-domain, integral transform and dual reciprocity-BIE method in terms of computing time and memory storage requirements, as well as in terms accuracy and convergence, are presented in Fedelinski et al. (1995a,b). A state-of-the-art review on usage of the dual BEM can be found in Chen and Hong (1999), while some case studies were conducted by Agnatiaris et al. (1996). Also, a review of SIF computation and modeling of singularities in BIE methods can be found in Mukhopadhyay et al. (2000). Recent advances and emerging applications of BIE methods are discussed in Liu et al. (2012), while Guz et al. (2013) reviewed results for 3D problems in materials with interface cracks.

In what follows, we will list recent developments on wave propagation in homogeneous versus inhomogeneous, isotropic versus anisotropic, and continuously inhomogeneous (i.e., functionally graded) elastic materials.

2.2.2.1 Homogeneous and Isotropic or Anisotropic Cracked Media

Most numerically obtained results for wave propagation in discontinuous media are for the elastic, isotropic, and homogeneous case. In Zhang and Gross (1998), a plethora of results is collected for infinite media under both 2D (anti-plane/in-plane strain representations) and 3D conditions. The loads are either transient or time-harmonic, and the preferred method of solution is the traction BEM in both its hypersingular and non-hypersingular versions. In the former case, a Galerkin method is employed for proper modeling of the crack-tip behavior. The emphasis is on wave attenuation and dispersion phenomena in randomly cracked elastic solids with dilute micro-crack distribution. Although most of the early BIE implementations employed nodal collocation, nowadays the symmetric Galerkin boundary element formulation is primarily used, especially in 3D fracture mechanics, see Liu et al. (2012); Li et al. (1998); Frangi et al. (2002).

Non-singular, regularized traction BEM formulations defined in the Laplace transform domain can be found in Sladek and Sladek (2000), where the regularization procedure yields the same order of singularity in both elastodynamic and corresponding elastostatic kernel functions. Next, the influence of variable curvature in the 3D crack front on the dynamic SIF is studied in Sladek et al. (2003a) by using a novel BIE formulation obtained by mapping the crack area into a circular domain. A large volume of high-quality results were obtained for 2D and 3D problems involving cracks in isotropic and homogeneous, and infinite and finite solids under both transient and steady-state loads (Sladek and Sladek 1984, 1986, 1987; Sladek et al. 1986). In general, the number of papers dealing with the solution of crack problems in finite domains is rather limited. In this context, we mention the works of Chirino and Dominguez (1989); Dominguez and Gallego (1992); Fedelinski et al. (1995a, b); Gallego and Dominguez (1997) for 2D problems and Wen et al. (1968, 1999); Dominguez and Ariza (2000) and Ariza and Dominguez (2002) for 3D problems. In sum, the above group of results considers the crack to act as a stress concentrator rather than a scatterer. The exception is Zhang and Gross (1998), who simultaneously considered both facets of this type of phenomenon. Basically, there is a certain lack of results for complex multiple cracks configuration in finite solids with complex geometry and for high frequency spectra. The reason is that when conventional BIE methods are applied to large-scale problems of order N^2 , where N is the number of degrees of freedom of the problem, the CPU time required is of order N^3 when direct solvers are used. This has spurred the development of fast BEM, such as the following:

- (a) the fast multipole methods (Fujiwara 1998; Yoshida et al. 2000, 2001; Liu 2009; Nishimura et al. 1999);
- (b) the panel clustering methods (Hackbusch and Nowak 1989);
- (c) the mosaic-skeleton approximations (Tytyshnikov 1996);
- (d) the methods based on hierarchical matrices and adaptive cross-approximation (Hackbusch 1999; Bebendorf 1996).

Research work on cracks in anisotropic materials is based on the theoretical background developed by Sih et al. (1965), and early results obtained by Eshelby et al. (1953) also Le'khinski (1963) on the generalized 2D elastic anisotropic problem are often used in verification studies. Note that uncoupling of the equations of motion in anisotropic media that allows for separation of 3D problems into plane and anti-plane subproblems is possible only if the material is monoclinic, see Garcia-Sanchez (2005). Recent work on the application of BIE-based methods to wave motion in anisotropic solids can be found in Wang and Achenbach (1996); Wang et al. (1996); Saez et al. (1999); Saez and Dominguez (1999, 2000, 2001); Kogl and Gaul (2000); Zhang (2000, 2002a); Albuquerque et al. (2002, 2003a,b, 2004); Hirose et al. (2002); Niu and Dravinski (2003a); Sun et al. (2003); Garcia-Sanchez (2005); Garcia-Sanchez et al. (2004, 2006); Garcia-Sanchez and Zhang (2007); Dineva et al. (2005); Tan et al. (1992); Wunsche et al. (2009a,b).

Although BIE methods are well suited for the type of problems delineated here, few papers can be found on transient anisotropic fracture mechanics. The main difficulty stems from the complexity of the fundamental solutions and from their difficult numerical implementation.

BIE formulations for BVP require two basic ingredients, namely a reciprocal relation and a fundamental solution or a Green's function. The reciprocal relation for elastodynamics, as established by Graffi (1946) and extended by Wheeler and Sternberg (1968), is independent of anisotropic material behavior, see Pan and Amadei (1999). The most common techniques used in the derivation of fundamental solutions are based on Stroh's complex variable formalism, as extended by Le'khinski (1963), plus integral transformations methods based on the Fourier, Laplace, and Radon transforms. In particular, Wang and Achenbach (1994, 1995, 1996) derived fundamental solutions by using the Radon transform for the general anisotropic solid and presented some examples on transversely isotropic solids. These solutions are in the form of surface integrals over a unit sphere in 3D and of a contour integral over a unit circle in 2D. They are subsequently separated into singular part that corresponds to the elastostatic fundamental solution plus a regular part.

Similar types of fundamental solutions for transient and time-harmonic, 3D and 2D deformation states have also been used by Zhang (2000) and by Saez and Dominguez (1999, 2000, 2001). More specifically in Saez and Dominguez (1999), a BIE approach is developed for wave propagation problems in 3D transversely isotropic solids by transforming the fundamental solution obtained by Wang and Achenbach (1995) in order to obtain more manageable expressions for numerical computation. Their numerical implementation combined simplified fundamental solutions, the multidomain approach, and the use of special crack-tip BE. Thus, they were able to produce results for different types of wave-scattering problems in the frequency domain. A general, 3D BIE method for analysis of dynamic fracture mechanics problems for transversely isotropic media was subsequently presented in Saez and Dominguez (2001), by again using the frequency-domain formulation. Next, Ariza and Dominguez (2004) continued along the same lines to study frequency-domain crack problems in 3D transversely isotropic solid by introducing the traction-based BEM and using the quarter-point BE for discretization of the crack surfaces.

Next, Zhang (2000, 2002a) presented a hypersingular BEM formulation for fracture problems involving the general anisotropic continuum under anti-plane strain conditions and for the orthotropic plane, respectively. Furthermore, Galerkin's method is used in conjunction with the spatial discretization and, in order to integrate in time, the quadrature method of Lubich (1988) for Riemann convolutions along with a Laplace-domain fundamental solution. Also, Hirose et al. (2002) presented a comparison study between the aforementioned technique and the more standard time-domain formulation using fundamental solutions derived by Wang and Achenbach (1995) with a Galerkin-type spatial discretization. They obtained the dynamic SIF for a straight crack under a tensile impact load. A comparative study of three hypersingular traction BEM for transient dynamic crack analysis of 2D generally anisotropic solid is given in Garcia-Sanchez and Zhang (2007).

In terms of numerical results, Albuquerque et al. (2003a) computed SIF for a crack in a quasi-isotropic, infinitely long strip and for a rectangular orthotropic plate with a central crack plus a slanted edge crack. They applied the fundamental solution for 2D elastostatics to solve this transient type of problem and concurrently used the multidomain BEM formulation, the dual reciprocity method for computing volume integrals, and the singular quarter-point BE for modeling the surfaces of the cracks. Also, 3D anisotropic problems have been studied in Kogl and Gaul (2000) by the DR-BEM based on the elastostatic fundamental solution. In Niu and Dravinski (2003b), harmonic wave diffraction in an infinite 3D solid is studied, with applications focusing on the scattering of elastic waves by a spherical cavity in a triclinic medium. Their results showed that in addition to the frequency of the incident waves, the scattered waves strongly depend on the degree of anisotropy of the surrounding media.

Finally, 2D elastodynamic analysis for an in-plane finite crack placed within an anisotropic infinite plate is presented in Dineva et al. (2005) and Chap. 5 in Dineva et al. (2014), who used the non-hypersingular, traction BEM in the frequency domain. The fundamental solution used in this formulation was derived by the Radon transform for the general case of anisotropy. Results are obtained for the SIF, while the numerical scheme was validated by comparison with available literature results.

2.2.2.2 Inhomogeneous Cracked Media

Most studies on the fracture behavior of FGM are for quasi-static problems. As such, they are inapplicable to elastic wave propagation and/or impact loads, because inertia effects cannot be ignored. From a fracture mechanics viewpoint, the presence of a graded internal layer plays an important role in determining fracture resistance parameters. In addressing the issues pertaining to the fracture analysis of bonded media with such transitional interfacial properties, a series of solutions to benchmark problems were obtained, and firstly for the stress field singularities near the tip of a crack. In particular, Erdogan (1985, 1995), Eischen (1987), Hirai (1988), Konda and Erdogan (1994), Jin & Noda (1994), and Sladek et al. (2007) have all show that:

- (a) The asymptotic stress states (i.e., inverse square-root singularity) for homogeneous materials and for FGM are identical at the crack-tip if the material properties are continuous and piecewise continuously differentiable;
- (b) The dominant terms in the crack-tip stress field of an FGM are identical to those of a homogeneous material with the same material composition around the vicinity of the crack-tip;
- (c) In FGM that undergoes brittle fracture, the fracture criterion may be expressed in terms of either the SIF or the strain energy-release rates;
- (d) Although the structure of the asymptotic crack-tip field is not influenced by the material gradient, the SIF are dependent on the material gradient through solution of the corresponding BVP.

To a large extent, application of BIE techniques has therefore been limited to homogeneous or piecewise homogeneous media. In what follows, we present the existing literature on BIE numerical implementation schemes for elastodynamics in cracked, graded materials. For isotropic materials, we have the following methods:

- (a) The DR-BIEM appears as the most promising method for overcoming the aforementioned difficulties in modeling continuously inhomogeneous media, see Sladek et al. (1993); Katsikadelis (2003); Ang et al. (1999, 2003). This approach allows for an easy extension of the conventional direct BIEM for homogeneous solids to FGM, bypassing the need for exact fundamental solutions, since it uses those for homogeneous media. These methods, however, require domain discretization which cancels the purely surface-only discretization of the conventional formulations. More specifically, the DR-BIEM introduces a supplementary domain integral equation, which is then converted into a boundary integral equation by collapsing volume integrals to surface ones through global approximation by radial basis functions. Thus, the pure surface character of the method is maintained at the expense of accuracy;
- (b) An alternative approach is to use various types of functional transformations directly on the differential operator in an effort to convert the governing wave equation into one with constant coefficients. Then, fundamental solutions can be derived and conventional BIE formulations are possible, see Manolis and Shaw (1996, 1997, 2000); Manolis et al. (1999a,b); Manolis (2003). Such implementations are however possible for very specific material profiles that are in essence determined by the particular functional transformation used. An example of this is to assume that all material parameters vary proportionally so that the wave speeds become macroscopically constant. Alternatively, Poisson's ratio must be equal to 0.25, although Yue et al. (2003) have shown that Poisson's ratio values in FGM do not have significant influence on the SIF values.

In Manolis et al. (2004); Dineva et al. (2006, 2007), see Chap. 8, an efficient non-hypersingular, traction BIEM is developed with quadratic BE supplemented with special crack-type BE to study time-harmonic P and SV wave propagation in fractured media. Both the full space and the half-space are considered, and material variation is either quadratic or exponential in the depth coordinate. Parametric studies were

conducted regarding the influence of inclined cracks (with respect to the direction of material variation) on the wave fields that develop in the surrounding medium to the key problem parameters. Using the same approach, Manolis et al. (2012), see Chap. 6, considered the dynamic interaction between defects of different types, such as cracks and cavities, in an exponentially inhomogeneous and anisotropic full space subjected to incident SH waves. Also, Zhang and coworkers (see Zhang and Gross 1998; Zhang and Savidis 2003; Zhang et al. 2003a,b) developed a hypersingular, traction BIEM for solution of the following classes of transient fracture mechanics problems:

- (a) The cracked, exponentially inhomogeneous plane, where anti-plane strain conditions hold and for three different directions of the material gradient, namely parallel, normal, and arbitrary inclination with respect to the crack plane (Zhang et al. 2003a,b);
- (b) 3D crack analysis (Zhang and Savidis 2003). An alternative approach is to use localized BIE formulations for problems with variable coefficients, as in Mikhailov (2002).

In this type of method, a special class of localized matrices (in lieu of a proper fundamental solution) is constructed, which reduces the general BVP with variable coefficients to a localized integro-differential system of equations that becomes amenable to numerical treatment. Along these lines, we mention the work of Sladek et al. (2005a), who considered both inhomogeneous and anisotropic materials via a combination of local integral equation formulations with domain discretization for potential type of problems.

In recent years, meshless methods have been developed, whereby all information is collocated at the nodes of a discretized continuum using integral representations in conjunction with techniques such as the local Petrov–Galerkin approach, see Sladek et al. (2003b, 2005a, b, c) for anti-plane strain fracture problems and also Sladek et al. (2006) for in-plane fracture problems. In both cases, the material surrounding the cracks is isotropic and inhomogeneous, and the loads are transient ones. Extension of these meshless methods to anisotropic inhomogeneous media can be found in Sladek et al. (2007) for in-plane crack problems and in Sladek et al. (2005a) for anti-plane ones.

In closing, the structure of FGM is described by position-dependent material coefficients and moreover, owing to their composite structure, these material properties are directionally dependent, i.e., anisotropic. Relatively, few dynamic fundamental solutions have been derived for the combination of anisotropy and inhomogeneity in a medium, see Chen et al. (2002); Guo et al. (2004); Chen and Liu (2005); Sladek et al. (2005b); Watanabe and Payton (2006); Rangelov et al. (2005), see Chap. 4, who all obtained various forms of time-harmonic fundamental solutions for certain families of 2D media. Also, Daros (2008) obtained fundamental solutions for both transient and time-harmonic waves under anti-plane strain conditions, as well as for time-harmonic waves under in-plane conditions for transversely isotropic media. Furthermore, in Daros (2009, 2010), these fundamental solution was numerically implemented within the context of a non-hypersingular BIEM in order to solve BVP

problems for cracks in inhomogeneous (quadratic, exponential, and sinusoidal material parameter variation) and transversely isotropic infinite domains.

Basing on the above state of the art, the following conclusions can be drawn:

Homogeneous as well as inhomogeneous, isotropic as well as anisotropic, and cracked as well as uncracked media are all examined with emphasis placed on results that have been obtained from the late 1990s onward. It can be noted here that media which are no longer homogeneous and isotropic, nor unbounded, either because of their complex material structure or because of the presence of inclusions, of cracks, and of various boundaries, produce quite complex wave patterns when swept by traveling elastic waves. This is so because of continuous wave reflection, diffraction, and scattering phenomena. These phenomena are difficult to predict and require the use of advanced numerical techniques in order to produce results that can be used in engineering design or for scientific analysis purposes. In terms of fields of application for these techniques, we mention here geophysics, geomechanics, and material science as the primary beneficiaries.

In terms of numerical modeling, the basic computational tools based on the BIE, such as the multidomain technique, the displacement discontinuity approach, single-domain methods based on either the hypersingular or the non-hypersingular traction formulation, as well as various dual reciprocity schemes for handling domain integrals, were all developed prior to the late 1990s. Since then, the basic problem involving degeneration of the conventional displacement BEM formulation in fracture mechanics when considering isotropic, homogeneous, and infinite media was successfully addressed. This opened the door for extending BIE-based methods to model materials and structural elements with complex geometry and mechanical properties including anisotropy, layering, viscoelasticity, gradient elasticity, and the presence of various types of discontinuities such as cracks, cavities, and inclusions, all subjected to complex dynamic loads generated from natural or man-made sources.

In sum, BIE-based development for elastic wave motion in solids is an open-ended endeavor, as many issues in this field still need to be addressed. This includes the following:

- (a) new derivations of fundamental solutions and/or Green's function for various media such as functionally graded materials without restrictions on the type and profile of the material gradient;
- (b) new numerical schemes for computation of singular and hypersingular integrals;
- (c) new numerical schemes for accurately handling domain integrals;
- (d) development of fast BIE's for large-scale problems in geophysics;
- (e) development of fast BIE's for dynamic fracture problems and;
- (f) production of BIE software package with direct relevance to problems and applications that are relevant to modern high-tech industries such as those dealing with advanced composite materials.

Having in mind the presented here state of the art, we define the focus of the book as follows: development of high-performance computational tools based on 2D BIEM for modeling of seismic wave field and local zones of dynamic stress concentration in inhomogeneous and heterogeneous geological media. This is done by taking into

account the entire wave path from the seismic source to the site under consideration, which is the local geological region of interest.

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