Adaptive Position Tracking with Hard Constraints—Barrier Lyapunov Functions Approach

Jacek Kabziński, Przemysław Mosiołek and Marcin Jastrzębski

Abstract A servo control with unknown system parameters and constraints imposed on the maximal tracking error is considered. The barrier Lyapunov functions approach is applied to assure the preservation of constraints in any condition. The system's performance is examined for three methods of controller design based on: quadratic Lyapunov functions; on barrier Lyapunov functions if only position constraints are imposed; and on barrier Lyapunov functions if both position and velocity constraints are present. The tuning rules are discussed and several experiments demonstrating features of the proposed control and the influence of the parameters are presented.

Keywords Nonlinear control · Adaptive control · Servo control · Barrier Lyapunov functions

1 Introduction

Servo systems are commonly used in various branches of industrial automation and robotics. The basic aim of a servo system or a robot manipulator is to track the desired motion trajectory with sufficient precision. For numerous servo systems rigorous handling of constraints imposed on a position and/or speed during any dynamic transient is a necessary condition of safe operation. Any violation of the constraints can lead to damage or destruction of the drive system or destroy any objects that happen to be in collision with the actuator. For plentiful robot manipulators, such as medical robots, automatic welding machines, microelectromechanical systems and many others, operation inside constraints is a matter of safety. It is reasonable to assume that the desired trajectories are planned with sufficient security margins and hence rigorous constraints must be imposed on the

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tracking errors. In such cases any control method that assumes the constraints are ‘soft’ and that it is possible to neglect constraints in control design and circumvent the problem through adequate control-parameter selection must be rejected.

Several approaches to control nonlinear systems with constraints were reported. Among them, the nonlinear model predictive control seems to be promising [1]. From the recent reports the diffeomorphism-based control may be mentioned [2]. As the literature on nonlinear, constrained control is rich, it is out of the scope of this chapter to provide an exhaustive review.

Nonlinear adaptive control is widely applied to designing high-performance servo systems in the presence of unknown plant parameters. Usually the controller derivation is based on control Lyapunov functions (CLF) and backstepping techniques [3]. Quadratic Lyapunov functions (QLF) are commonly used to assure the stability of the system, but unfortunately with this approach the designer is not able to impose the hard constraints a priori and to guarantee the constraints are fulfilled during any transient conditions.

Recently, use of the so-called barrier Lyapunov functions (BLF) in control synthesis has been proposed for constraint handling in Brunovsky type systems [4], nonlinear systems in the strict feedback form [5] and with adaptive control [6, 7]. The BLF approach applies the backstepping technique and allows the system output (or all state variables) to be kept inside the predefined constraints. Although the theory of stability investigation by BLF is well established, only a few practical applications are reported [8, 9].

The aim of the presented chapter is to demonstrate the possibility of applications of BLF in servo systems design to provide a systematic description of the design procedure and to formulate some rules for the selection of the design parameters. Particular attention is given to illustrating the problem of interactions among position and velocity constraints. To present clear and compact derivation we concentrate on the simplest motion model with one degree of freedom, but generalizations to many more complex applications are straightforward.

2 Plant Model and Control Objectives

A linear servo is considered and its very simple model is described by:

$$\frac{d}{dt}x = v \tag{1}$$

$$m\frac{d}{dt}v = \varphi i - F_o, \tag{2}$$

where x, v are the forcer position and velocity, m is the forcer mass, φ represents the coefficient converting the motor current i into the thrust force and F_o is an external load force, acting against the motion. The motor current i is supplied by a PWM inverter working in a current control mode and it is assumed that this control loop is much faster than mechanical dynamics, so the motor current i is considered as the control input. The same approach and the analogous model may be used for the rotational motors, therefore the liner motion supposition is carried outdone without the loss of generality.

It is assumed that the parameters $m > 0$, $\varphi > 0$, are unknown, constant or slowly varying. Although the constant φ is usually provided by the motor manufacturer, this information is not accurate. This constant may vary with the motor temperature, PWM conduction mode or, for some tubular linear motors with permanent magnets built in the inner part, it may be noticeably lower if the forcer operates near the edge of the inner part. It is assumed that the load may be modelled as a nonlinear, memoryless function of the position and the velocity and that this model may be represented as a linear combination of known nonlinear functions ξ with unknown parameters A:

$$F_o(x, v) = A^T \xi(x, v). \tag{3}$$

Such models are natural if the load is approximated using any approximation technique: artificial neural networks, fuzzy modelling, polynomial approximation and so on. The number of unknown parameters and the approximation basis ξ may be chosen for the particular application.

Remark 1 For the sake of brevity, it is assumed that model (3) is accurate, but it is also possible to consider an inaccurate approximation with a bounded approximation error ε:

$$F_o(x, v) = A^T \xi(x, v) + \varepsilon. \tag{4}$$

The main consequence of using model (4) or (3) is that under assumption (3) it is possible to prove the asymptotic stability of the tracking errors system. In case of the load model (4), one has to introduce a switching control component to obtain the asymptotic stability, or to accept the stability in the sense of the uniform ultimate boundedness [10].

To remove the difficulties caused by the unknown control gain φ, the motion Eq. (2) is transferred into:

$$\mu \frac{d}{dt} v = i - A_o^T \xi(x, v), \tag{5}$$

where:

$$\mu = \frac{m}{\varphi}, \quad A_o = \frac{1}{\varphi} \mathrm{A}. \tag{6}$$

The control objective is that the motor position has to follow a smooth, bounded reference $x_d(t)$:

$$|x_d(t)| \leq x_{max}. \tag{7}$$

It is assumed that the derivatives of the reference are bounded as well. It is required that the tracking error denoted by:

$$e_x = x_d - x \tag{8}$$

is constrained for any t by a pre-defined, rigorous inequality:

$$|e_x(t)| < \Delta_{ex}, \tag{9}$$

for any initial conditions that assure $|e_x(0)| < \Delta_{ex}$. It follows from (9) that the position trajectory will be bounded by a hard constraint:

$$|x(t)| < \Delta_x \quad \Delta_x := \Delta_{ex} + x_{max}. \tag{10}$$

3 Quadratic and Barrier Lyapunov Functions

Lyapunov stability theory will be used to construct the stabilizing control for the discussed problem. For the sake of completeness, some preliminaries are given in this section.

Definition 1 Let $V : R^n \to R$ be a continuously differentiable, proper, and positive definite function defined with respect to the nonlinear system $\dot{x} = f(x,u)$. Let us denote $\dot{V}(x,u) = V_x^T f(x,u)$. $V(x)$ is a control Lyapunov function (CLF) for the system $\dot{x} = f(x,u)$ if, for all $x \neq 0$, there exists such u that $\dot{V}(x,u) < 0$. If $V(x) = x^T Px$ for some positive definite P, it is called a quadratic Lyapunov function (QLF).

Definition 2 [5] A Barrier Lyapunov Function (BLF) is a scalar function *V(x),* defined with respect to the system $\dot{x} = f(x)$ on an open region D containing the origin; that is continuous, positive definite, has continuous first-order partial derivatives at every point of D, has the property $V(x) \to \infty$ as x approaches the boundary of D, and satisfies condition: $\exists M, \forall t > 0 V(x(t)) < M$ along any system trajectory starting inside D.

Usually it is assumed that D is a hyper-rectangle defined by $D = \{x\colon |x_i| \leq \Delta_{xi}\}$.

Lemma 1 [5] *Consider a smooth dynamical system* $\dot{z} = f(t,x,w)$, *with the state variables* $z = [x,w]^T$. *Let* $V_i(x_i)$ *be a BLF satisfying* $V_i(x_i) \to \infty \quad \text{if} \quad x_i \to \pm\Delta_{xi}$, *let* $Q(w)$ *be a QLF. Let* $V = \sum_{i=1}^{\dim(x)} V_i(x_i) + Q(w)$. *If the inequality* $\dot{V} = \frac{\partial V^T}{\partial z} f \leq 0$ *holds anywhere in the set* $S = \{(x,w) : |x_i| < \Delta_{xi}\}$, *then any trajectory that fulfils the initial constraints* $\forall i |x_i(0)| < \Delta_{xi}$ *remains in* S *for any* t.

In Lemma 1 the state is split into the constrained variables x and the unconstrained variables w. For each x_i a BLF is constructed, while a QLF may be used for w.

It is well known that the selected Lyapunov function heavily influences the features of the resulting closed loop system. The commonly accepted form of a single variable BLF corresponding to the interval $D = (-\Delta, \Delta)$ is a logarithmic BLF [4–7]:

$$V(x) = \frac{1}{2}\log\left(\frac{1}{1-\left(\frac{x}{\Delta}\right)^2}\right). \tag{11}$$

The motion control with application of such BLFs was investigated in [11]. As the function (11) does not possess any parameters that may change the shape of the plot and therefore the resulting system properties, it is interesting and informative to investigate other possibilities of barrier function selection. In this chapter we consider BLFs based on trigonometric functions:

$$V(x) = \frac{\Delta}{\pi}\tan^2\frac{\pi x}{2\Delta} \tag{12}$$

and

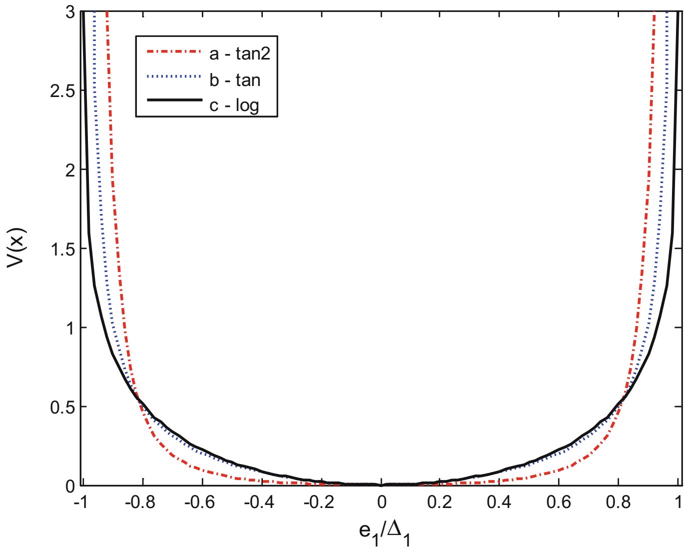


Fig. 1 Plots of **a** $\frac{\Delta}{\pi}\tan^2\frac{\pi x}{2\Delta}$, **b** $\frac{\Delta^2}{\pi}\tan\left(\frac{\pi}{2}\left(\frac{x}{\Delta}\right)^2\right)$, **c** $\frac{1}{2}\log\left(\frac{1}{1-\left(\frac{x}{\Delta}\right)^2}\right)$

$$V(x) = \frac{\Delta^2}{\pi} \tan\left(\frac{\pi}{2}\left(\frac{x}{\Delta}\right)^2\right). \tag{13}$$

The main difference between these Lyapunov functions is their behaviour near the constraint boundary. As it is shown in Fig. 1, if the scaling factors are ignored, the function (12) grows up most rapidly, (11) slowly, (13) is the moderate one. Therefore in the subsequent derivation we concentrate on BLF (13).

4 QLF Control Design

Let us forget for a moment about constraints (9) and design the controller using QLFs. The adaptive back-stepping scheme [12] will be used to design the controller. The velocity will be the 'virtual control' for position tracking. Let us consider the error equation:

$$\dot{e}_x = \dot{x}_d - v \tag{14}$$

and the desired 'virtual control' trajectory v_d with the tracking error defined as:

$$e_v = v_d - v. \tag{15}$$

The desired 'virtual control' v_d will be designed to guarantee the required convergence of the error e_x. Considering the following QLF:

$$V_1 = \frac{1}{2} e_x^2 \tag{16}$$

allows one to conclude that the desired 'virtual control' v_d:

$$v_d = \dot{x}_d + k_x e_x, \tag{17}$$

where $k_x > 0$ is a design parameter, will generate the tracking error dynamics:

$$\dot{e}_x = \dot{x}_d - \dot{x}_d - k_x e_x + e_v = -k_x e_x + e_v \tag{18}$$

and:

$$\dot{V}_1 = -k_x e_x^2 + e_x e_v \tag{19}$$

and so will assure stability if $v = v_d$.

During the second stage of the backstepping procedure the velocity error e_v is considered:

$$\mu\dot{e}_v = \mu\dot{v}_d - \mu\dot{v} = \mu\dot{v}_d - i + A_o^T\xi = -i + A_1^T\xi_1, \tag{20}$$

where the new variables are defined as:

$$A_1^T = \left[\mu, A_o^T\right], \quad \xi_1^T = \left[\dot{v}_d, \xi^T\right]. \tag{21}$$

The derivative of the reference speed is given by:

$$\dot{v}_d = \ddot{x}_d + k_x(-k_x e_x + e_v), \tag{22}$$

so, fortunately, it is available for the control algorithm, and hence ξ_1 in (21) is a known function. Parameters A_1 in (21) are not known, therefore they will be replaced by adaptive parameters $\widehat{A}_1^T = [\widehat{\mu}, \widehat{A}_o^T]$.

The control variable i will be designed using the QLF:

$$V_2 = V_1 + \frac{1}{2}\mu e_v^2 + \frac{1}{2}\widetilde{A}_1^T\Gamma^{-1}\widetilde{A}_1, \tag{23}$$

where:

$$\widetilde{A}_1 = A_1 - \widehat{A}_1 \tag{24}$$

denotes the adaptation error and positive definite Γ is the matrix of the design parameters of appropriate dimensions.

Plugging in (19) and (20) into:

$$\dot{V}_2 = \dot{V}_1 + e_v\mu\dot{e}_v + \widetilde{A}_1^T\Gamma^{-1}\dot{\widetilde{A}}_1 \tag{25}$$

Allows calculation of the Lyapunov function derivative:

$$\dot{V}_2 = -k_x e_x^2 + e_x e_v + e_v\left(-i + A_1^T\xi_1\right) + \widetilde{A}_1^T\Gamma^{-1}\dot{\widetilde{A}}_1. \tag{26}$$

The control variable i will be designed to compensate the unnecessary components in (26) and to introduce the stabilizing component, so:

$$i = e_x + \widehat{A}_1^T\xi_1 + k_v e_v, \tag{27}$$

where $k_v > 0$ is a design parameter. Such control allows the tracking error to be described by:

$$\mu\dot{e}_v = -k_v e_v - e_x - \widetilde{A}_1^T\xi_1 \tag{28}$$

and to represent the Lyapunov function derivative as:

$$\dot{V}_2 = -k_x e_x^2 - k_v e_v^2 + \widetilde{A}_1^T \left(e_v \xi_1 + \Gamma^{-1} \dot{\widetilde{A}}_1 \right). \tag{29}$$

As $\dot{\widetilde{A}}_1 = -\dot{\widehat{A}}_1$, the differential rule describing the adaptation may be used to guarantee that (29) is non-positive for any, unknown $\widetilde{A}_1$. The simplest way is to cancel the last component in (29) by imposing:

$$\dot{\widehat{A}}_1 = e_v \Gamma \xi_1. \tag{30}$$

By using the LaSalle-Yoshizawa theorem [12], (29, 30) guarantees that all errors $e_x, e_v, \widetilde{A}_1$ are uniformly bounded and e_v, e_x are regulated to zero. Since the reference x_d is bounded, x is bounded as well. The boundedness of v_d follows from the boundedness of $\dot{x}_d$ and e_x in (17). Combining this with (27), we find that the control is also bounded. Although the boundedness of state variables is proven using QLF, it is impossible to define the constraints a priori. The maximal value of each state variable depends on the design parameters and initial conditions.

Remark 2 It is well known that similar results may be obtained with some other adaptation rules. For example:

$$\dot{\widehat{A}}_1 = proj_\rho \left(\widehat{A}_1, e_v \Gamma \xi_1 \right), \tag{31}$$

where $proj_\rho(*,)$ is a projection operator assuring that $\|*\| \leq \rho$ [12]. Although (31) allows the maximal values of adaptive parameters to be influenced, it will not provide a priori constraints for the state variables.

Remark 3 The design parameter k_x influences not just the values of e_x, but also the 'virtual control' v_d in (17), and so the error e_v in (28). Therefore, the maximal value of the current (27) depends on both design parameters k_x and k_v, although only k_v is explicitly visible in (27).

Remark 4 State variables may be constrained by the initial value of the Lyapunov function. As $\dot{V}_2 \leq 0, V_2(t) \leq V_2(0)$ along any trajectory of the system (18, 28). Therefore, $\frac{1}{2} e_x^2 \leq V_2(0)$, so $|e_x| \leq \sqrt{2V_2(0)}$. Unfortunately, $V_2(0) = \left[\frac{1}{2} e_x^2 + \frac{1}{2} \mu e_v^2 + \frac{1}{2} \widetilde{A}_1^T \Gamma^{-1} \widetilde{A}_1 \right]_{t=0}$ depends on the initial guess of the unknown parameters and the obtained constraint is not informative.

Although QLF design allows the influence of the error system dynamics by a proper selection of design parameters, it will not provide any tool to impose hard constraints for position or velocity a priori.

5 BLF Design with Position Constraints

To satisfy the position error constraint (9), the BLF will be applied during the first stage of back-stepping:

$$V_1 = \frac{1}{2k}\tan\left(ke_x^2\right), k = \frac{\pi}{2\Delta_{ex}^2}. \tag{32}$$

The derivative of the BLF is given by:

$$\dot{V}_1 = e_x\dot{e}_x\left(1 + \tan^2\left(ke_x^2\right)\right), \tag{33}$$

hence plugging (18) into (33) gives:

$$\dot{V}_1 = e_x(\dot{x}_d - v_d + e_v)\left(1 + \tan^2\left(ke_x^2\right)\right). \tag{34}$$

At this moment of the design procedure we have two possibilities to select the desired velocity v_d: the nonlinear v_d, which makes the Lyapunov function derivative a quadratic function of the tracking error:

$$v_d = \dot{x}_d + \frac{k_x e_x}{1 + \tan^2\left(ke_x^2\right)} \Rightarrow \dot{V}_1 = -k_x e_x^2 + e_x e_v\left(1 + \tan^2\left(ke_x^2\right)\right), \tag{35}$$

or linear v_d, as in (17), which gives nonlinear $\dot{V}_1$:

$$v_d = \dot{x}_d + k_x e_x \Rightarrow \dot{V}_1 = -k_x e_x^2\left(1 + \tan^2\left(ke_x^2\right)\right) + e_v e_x\left(1 + \tan^2\left(ke_x^2\right)\right). \tag{36}$$

Both approaches give a theoretical possibility to derive stable control systems, but it must be noticed that in case of nonlinear v_d (35) the negative gain $Q(e_x) = \frac{k_x}{1+\tan^2\left(ke_x^2\right)}$, which is responsible for the stability of the tracking error dynamics:

$$\dot{e}_x = \dot{x}_d - v_d + e_v = -Q(e_x)e_x + e_v, \tag{37}$$

tends to zero if $|e_x| \to \Delta_{ex}$. Therefore, the linear form (36) of v_d is selected.

As the constraints are imposed only on the first state variable (the position tracking error), the control variable i will be designed using the Lyapunov function:

$$V_2 = V_1 + \frac{1}{2}\mu e_v^2 + \frac{1}{2}\widetilde{A}_1^T\Gamma^{-1}\widetilde{A}_1, \tag{38}$$

where V_1 defined in (32) is a BLF. Substitution of (20, 34) allows the calculation of the Lyapunov function derivative as:

$$\dot{V}_2 = -k_x e_x^2\left(1+\tan^2\left(ke_x^2\right)\right) + e_v e_x\left(1+\tan^2\left(ke_x^2\right)\right) \\ + e_v\left(-i + A_1^T\xi_1\right) + \tilde{A}_1^T\Gamma^{-1}\dot{\tilde{A}}_1. \tag{39}$$

The control variable i will be designed to compensate the unnecessary components in (39) and to introduce the stabilizing component, so:

$$i = \widehat{A}_1^T\xi_1 + e_x\left(1+\tan^2\left(ke_x^2\right)\right) + k_v e_v, \tag{40}$$

where $k_v > 0$ is a design parameter. This control gives the velocity tracking error:

$$\mu\dot{e}_v = -k_v e_v + \widetilde{A}_1^T\xi_1 - e_x\left(1+\tan^2\left(ke_x^2\right)\right) \tag{41}$$

and the Lyapunov function derivative fulfils:

$$\dot{V}_2 = -k_x e_x^2\left(1+\tan^2\left(ke_x^2\right)\right) - k_v e_v^2 + \widehat{A}_1^T\left(e_v\xi_1 + \Gamma^{-1}\dot{\tilde{A}}_1\right). \tag{42}$$

Either of the adaptation rules (30) or (31) assures that:

$$\dot{V}_2 \leq -k_x e_x^2\left(1+\tan^2\left(ke_x^2\right)\right) - k_v e_v^2 \leq 0. \tag{43}$$

The following corollary abstracts the main features of the obtained system.

Corollary 1 Consider the closed loop system (18), (41) with any of the adaptation laws (30) or (31) and the reference position trajectory, under all assumptions formulated above. Consider any trajectory with initial conditions fulfilling $|e_x(0)| < \Delta_{ex}$. Then the following properties hold along this trajectory:

1. The variables $e_x, e_v, \widetilde{A}_1^T$ remain inside a compact set and the tracking error e_x fulfils the constraint $|e_x(t)| < \Delta_{ex}$.
2. All closed loop signals are bounded.
3. The tracking errors e_x, e_v converge to zero asymptotically.

<u>Sketch of the proof:</u>

1. $V_2(0)$ is bounded and as $\dot{V}_2 \leq 0$, $V_2(t) \leq V_2(0)$ along the considered trajectory. Lemma 1 yields that $|e_x(t)| < \Delta_{ex}$. Another constraint for the tracking error may be obtained noticing that $\frac{1}{2k}\tan\left(ke_x^2\right) \leq V_2(0)$ and thus $|e_x| \leq \Delta_{ex}\sqrt{\frac{2\mathrm{atan}(2kV_2(0))}{\pi}}$. Similarly, we may derive that $|e_v| \leq \sqrt{\frac{2}{\mu}V_2(0)}$ and $\left\|\tilde{A}_1\right\| \leq \sqrt{\frac{2V_2(0)}{\lambda_{\min}\left(\Gamma^{-1}\right)}}$, where $\lambda_{\min}(*)$ denotes the smallest eigenvalue of the symmetric matrix $*$.
2. As $e_x, e_v, \widetilde{A}_1^T$ are bounded, $\widehat{A}_1$ are bounded also. From (17), (22), the desired 'virtual control' and its derivative are bounded as well. Therefore, the functions ξ_1 are bounded and from (40) the control is bounded.

3. The tracking error asymptotic convergence may be obtained by demonstrating that $\ddot{V}_2$ is bounded and making use of Barbalat's lemma [2].

Remark 5 The component $\tan^2\left(ke_x^2\right)$ in (40) suggests the control variable increases if $e_x \to \pm\Delta_{ex}$. Although the inequality $|e_x| < \Delta_{ex}$ is always fulfilled and the control is bounded, the maximal value of the control variable depends on all design parameters (similarly as it was explained in Remark 3) and initial conditions and requires careful investigation.

6 BLF Design with Position and Velocity Constraints

The hard constraint imposed on the position tracking may cause rapid variations of the trajectory near the constraint boundary and unacceptable velocity values. The maximal velocity may be restricted if (a) the tracking error of the desired velocity e_v will be limited by a hard constraint, and (b) the desired velocity v_d will be moderate. The constraints imposed on the velocity tracking error e_v may be preserved if a BLF will be used also during the second stage of the back-stepping design. Therefore, instead of (38), the Lyapunov function:

$$V_2 = V_1 + \frac{\mu}{2K}\tan\left(Ke_v^2\right) + \frac{1}{2}\widetilde{A}_1^T\Gamma^{-1}\widetilde{A}_1, \quad K = \frac{\pi}{2\Delta_{ev}^2} \tag{44}$$

will be used, where V_1 defined in (32) is a BLF and Δ_{ev} is a constraint imposed on the tracking error e_v. The Lyapunov function derivative may be represented as:

$$\dot{V}_2 = \dot{V}_1 + \mu e_v\left(1 + \tan^2\left(Ke_v^2\right)\right)\dot{e}_v + \widetilde{A}_1^T\Gamma^{-1}\dot{\widetilde{A}}_1 \tag{45}$$

and plugging in (36) and (20) results in:

$$\begin{aligned} \dot{V}_2 = & -k_x e_x^2\left(1 + \tan^2\left(ke_x^2\right)\right) + e_v e_x\left(1 + \tan^2\left(ke_x^2\right)\right) \\ & + e_v\left(1 + \tan^2\left(Ke_v^2\right)\right)\left(-i + A_1^T\xi_1\right) + \widetilde{A}_1^T\Gamma^{-1}\dot{\widetilde{A}}_1. \end{aligned} \tag{46}$$

Once again, the control variable i will be designed to compensate the unnecessary components in (46) and to introduce the stabilizing component, so:

$$i = \hat{A}_1^T\xi_1 + e_x\frac{1 + \tan^2\left(ke_x^2\right)}{1 + \tan^2\left(Ke_v^2\right)} + k_v e_v, \tag{47}$$

where $k_v > 0$ is a design parameter. This control gives the velocity tracking error:

$$\mu \dot{e}_v = -k_v e_v + \widetilde{A}_1^T \xi_1 - e_x \frac{1 + \tan^2\left(k e_x^2\right)}{1 + \tan^2\left(K e_v^2\right)} \tag{48}$$

and the Lyapunov function derivative fulfils:

$$\begin{aligned} \dot{V}_2 = & -k_x e_x^2 \left(1 + \tan^2\left(k e_x^2\right)\right) - k_v e_v^2 \left(1 + \tan^2\left(K e_v^2\right)\right) \\ & + \widetilde{A}_1^T \left(e_v \left(1 + \tan^2\left(K e_v^2\right)\right) \xi_1 + \Gamma^{-1} \dot{\widetilde{A}}_1\right). \end{aligned} \tag{49}$$

As $\dot{\widetilde{A}}_1 = -\dot{\hat{A}}_1$, the differential rule describing the adaptation may be used to guarantee that (49) is non-positive for any, unknown $\widetilde{A}_1$. The simplest way is to cancel the last component in (49) by selecting:

$$\dot{\hat{A}}_1 = e_v \left(1 + \tan^2\left(K e_v^2\right)\right) \Gamma \, \xi_1, \tag{50}$$

which results in:

$$\dot{V}_2 = -k_x e_x^2 \left(1 + \tan^2\left(k e_x^2\right)\right) - k_v e_v^2 \left(1 + \tan^2\left(K e_v^2\right)\right). \tag{51}$$

Therefore:

$$\dot{V}_2 \leq 0 \,\text{in}\, S = \left\{ \left(e_x, e_v, \widetilde{A}_1\right) : |e_x| < \Delta_{ex}, \ |e_v| < \Delta_{ev} \right\}. \tag{52}$$

The following corollary summarizes the main features of the obtained system.

Corollary 2 Consider the closed loop system (18), (48) with adaptation laws (50). Assume there is a trajectory with initial conditions fulfilling $|e_x(0)| < \Delta_{ex}$, $|e_v(0)| < \Delta_{ev}$. Then the following properties hold along this trajectory:

1. The variables $e_x, e_v, \widetilde{A}_1^T$ remain inside a compact set and state variables fulfil the constraints $|e_x(t)| < \Delta_{ex}$, $|e_v(t)| < \Delta_{ev}$.
2. All closed loop signals are bounded.
3. The tracking errors e_x, e_v converge to zero asymptotically.

Sketch of the proof:

1. $V_2(0)$ is bounded and as $\dot{V}_2 \leq 0$, hence $V_2(t) < V_2(0)$ along the considered trajectory. Lemma 1 yields that $|e_x(t)| < \Delta_{ex}$ and $|e_v(t)| < \Delta_{ev}$. Another constraint for the tracking error may be obtained noticing that $\frac{1}{2k}\tan\left(k e_x^2\right) \leq V_2(0)$ and thus $|e_x| \leq \Delta_{ex}\sqrt{\frac{2\mathrm{atan}(2kV_2(0))}{\pi}}$. Similarly, $\frac{\mu}{2K}\tan\left(K e_v^2\right) \leq V_2(0)$ and so $|e_v| \leq \Delta_{ev}\sqrt{\frac{2\mathrm{atan}(2KV_2(0)/\mu)}{\pi}}$. Using analogical reasoning, it can be derived that

$\widetilde{A}_1 \leq \sqrt{\frac{2V_2(0)}{\lambda_{\min}(\Gamma^{-1})}}$, where $\lambda_{\min}(*)$ denotes the smallest eigenvalue of the symmetric matrix $*$.

2. As $e_x, e_v, \widetilde{A}_1^T$ are bounded, $\widehat{A}_1$ are also bounded. The desired 'virtual control' and its derivative are bounded as well. Therefore the functions ξ_1 are bounded and from (47) the control is bounded.
3. The tracking error asymptotic convergence may be obtained by demonstrating that $\ddot{V}_2$ is bounded and making use of the Barbalat's lemma [2].

Remark 6 The property (52), and therefore Corollary 2 will hold with other adaptation rules, corresponding to (31), for example:

$$\dot{\widehat{A}}_1 = proj_\rho\left(\widehat{A}_1, e_v\left(1 + \tan^2\left(Ke_v^2\right)\right)\Gamma\xi_1\right), \tag{53}$$

where $proj_\rho(*, \cdot)$ is a projection operator assuring that $\|*\| \leq \rho$.

7 Feasibility Conditions

The set of design parameters that must be selected by the designer consists of $\Delta_{ex}, \Delta_{ev}, k_x, k_v, \Gamma$ and initial conditions for adaptive parameters $\widehat{A}_1$. The constraints Δ_{ex}, Δ_{ev} are imposed by the designer to limit the tracking errors, the gains k_x, k_v are responsible for tracking error convergence and Γ is in charge of the adaptation speed. The Corollary 2 is derived under the feasibility condition that 'there exists a trajectory with initial conditions fulfilling $|e_x(0)| < \Delta_{ex}$, $|e_v(0)| < \Delta_{ev}$'. This condition implies that k_x and Δ_{ev} cannot be selected independently. As:

$$e_v(0) = \dot{x}_d(0) + k_x e_x(0) - v(0), \tag{54}$$

Δ_{ev} must be chosen according to the inequality:

$$|\dot{x}_d(0) + k_x e_x(0) - v(0)| \leq |\dot{x}_d(0) - v(0)| + k_x\Delta_{ex} < \Delta_{ev}. \tag{55}$$

Similarly, as imposing Δ_{ex} allows $|x(t)|$ to be constrained by the inequality (10), the choice of Δ_{ev} provides constraints for the maximal velocity:

$$|v(t)| \leq |\dot{x}_d(t) + k_x e_x(t)| + |e_v(t)| \leq \max_{t>0}|\dot{x}_d(t)| + k_x\Delta_{ex} + \Delta_{ev} := \Delta_v. \tag{56}$$

The designer may be interested in constraining the gap between the desired position derivative and the actual velocity:

$$E(t) = \dot{x}_d(t) - v(t). \tag{57}$$

As:

$$|E| = |\dot{x}_d - v| = |v_d - v - k_x e_x| = |e_v - k_x e_x|, \tag{58}$$

it may be bounded by a proper choice of Δ_{ex} and Δ_{ev}:

$$|E| \leq \Delta_{ev} + k_x \Delta_{ex}. \tag{59}$$

Therefore it is evident that a larger k_x, which allows the faster convergence of the tracking error, requires greater maximal velocity and a bigger gap between the desired position derivative and the actual velocity. Also the maximal value of the control variable (47) depends on the selected design parameters and must be checked carefully before implementation in a real drive.

8 Numerical Experiments

The linear actuator with the parameters m = 8 kg, and φ = 39 N/A is supposed to track the desired position provided by the filtered sinusoid:

$$x_d(t) = \mathcal{L}^{-1}\left\{\frac{1}{T^2 s^2 + 2Ts + 1}\mathcal{L}\{0.3\ \sin(3t)\}\right\} \quad [\mathrm{m}], \tag{60}$$

where T = 0.1 s and $\mathcal{L}$ denotes the Laplace transform. The desired trajectory is presented in Fig. 2.

It is assumed that the actuator works against the load described by:

$$F_o(x, v) = Av^2 sign(v) + B|v|\sin(2\pi x)\ [\mathrm{N}]. \tag{61}$$

with unknown coefficients A and B, hence, according to (21):

$$\begin{aligned} A_1^T &= \frac{1}{\varphi}[m, A, B] \\ \xi_1^T &= \left[\dot{v}_d, v^2 sign(v), |v|\sin(2\pi x)\right]. \end{aligned} \tag{62}$$

If it is assumed that none of the motor or load parameters are known, the initial values of adaptive parameters are selected as:

$$\widehat{A}_1^T = \frac{1}{\varphi}[0.5\,\mathrm{m}, 0, 0], \tag{63}$$

while the real values are $\widehat{A}_1^T = \frac{1}{\varphi}[m, A, B] = \frac{1}{39}[8, 10, 6]$.

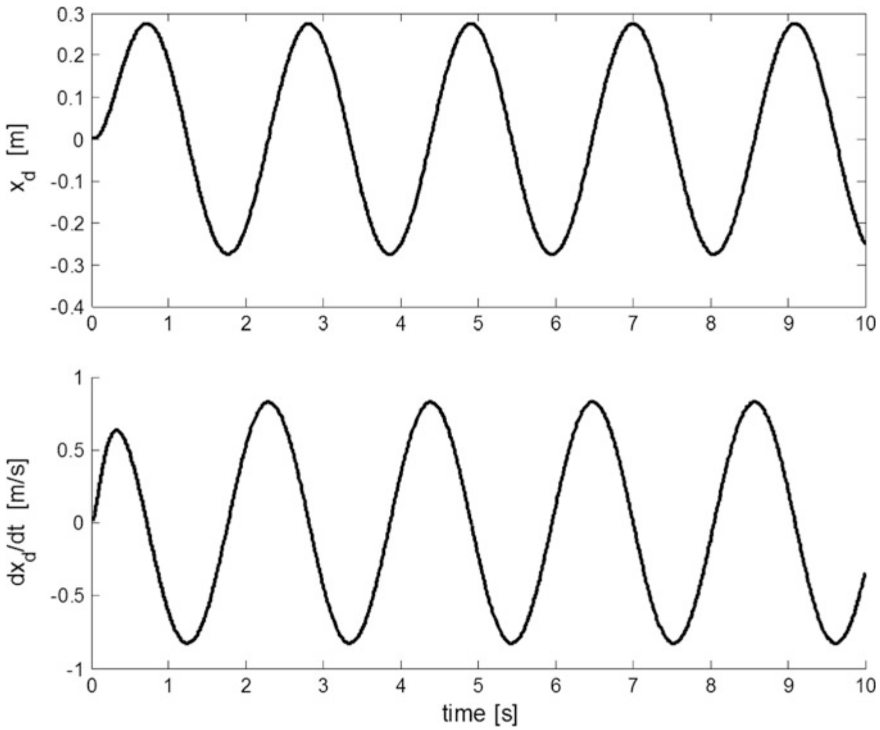


Fig. 2 Desired position trajectory x_d and its derivative $\dot{x}_d$

8.1 Known-Parameter Case

If all parameters A_1 are known exactly, the problem is simplified by taking $\widehat{A}_1 = A_1$ and $\Gamma^{-1} = 0$. The adaptive loop is inactive. The two-dimensional state-space allows simple comparisons of the system behaviour under all discussed control strategies. The final Lyapunov functions for the non-adaptive case are:

$$V_2 = \frac{1}{2}e_x^2 + \frac{1}{2}\mu e_v^2 \tag{64}$$

for the QLF approach;

$$V_2 = \frac{1}{2k}\tan\left(ke_x^2\right) + \frac{1}{2}\mu e_v^2, \quad k = \frac{\pi}{2\Delta_{ex}^2} \tag{65}$$

for the position constrained BLF approach; and

$$V_2 = \frac{1}{2k}\tan\left(ke_x^2\right) + \frac{\mu}{2K}\tan\left(Ke_v^2\right), \quad k = \frac{\pi}{2\Delta_{ex}^2}, \quad K = \frac{\pi}{2\Delta_{ev}^2} \tag{66}$$

for the position and velocity constrained BLF approach.

The control signal is given by:

$$i = A_1^T\xi_1 + e_x + k_v e_v \tag{67}$$

$$i = A_1^T\xi_1 + e_x\left(1 + \tan^2\left(ke_x^2\right)\right) + k_v e_v \tag{68}$$

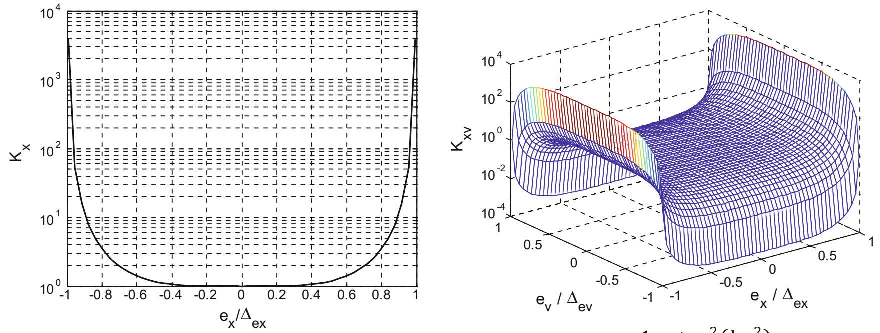
$$i = A_1^T\xi_1 + e_x\frac{1 + \tan^2\left(ke_x^2\right)}{1 + \tan^2\left(Ke_v^2\right)} + k_v e_v \tag{69}$$

for the discussed strategies respectively, where $e_v = \dot{x}_d + k_x e_x - v$. The main difference between the approaches (67–69) lies in the coefficient of e_x. The 'gain' of e_x equals $K_{QLF} = 1$, $K_x = 1 + \tan^2\left(ke_x^2\right)$ and $K_{xv} = \frac{1 + \tan^2\left(ke_x^2\right)}{1 + \tan^2\left(Ke_v^2\right)}$ respectively. The gains are plotted in Fig. 3.

If the position tracking error is the only constrained variable, the control effort increases illimitably if the error approaches the constraint boundary. If both errors are restricted, the current increase is moderated if the velocity error is large.

In Fig. 4 the state trajectories of the tracking error system under all discussed control strategies are compared for the same control gains k_x, k_v. The trajectories are plotted on the background of constant-level curves of the Lyapunov function. The plots illustrate how the barrier Lyapunov function acts to preserve the constraints.

Figure 5 demonstrates the time history under QLF and BLF control starting with the same initial conditions $e_x(0) = -0.018$, $e_v(0) = -0.04$, lying inside the constraints $|e_x| < 0.02$, $|e_v| < 0.05$. It is visible that even in the non-adaptive case the



$K_x = 1 + \tan^2(ke_x^2)$

$K_{xv} = \frac{1 \text{ a } \tan^2(ke_x^2)}{1 + \tan^2(Ke_v^2)}$

Fig. 3 The 'gain' of e_x for BLF strategies

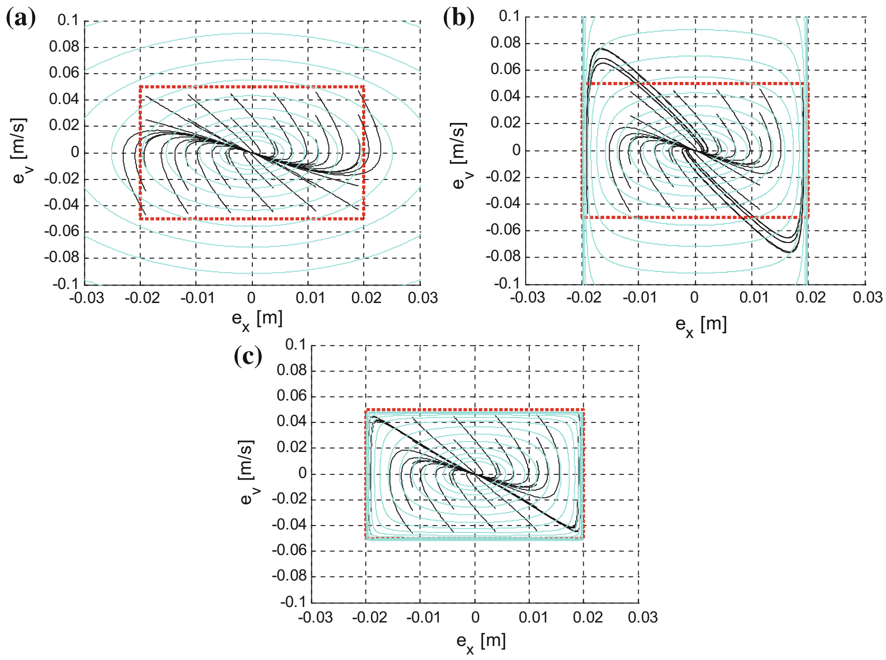


Fig. 4 State trajectories of the tracking error: **a** the QLF design, **b** position constrained BLF design and **c** position and velocity constrained BLF approach

QLF-based control does not guarantee that the position tracking error will remain inside the predefined bounds. Therefore the BLF approaches will be investigated for the adaptive case.

8.2 BLF-Based Adaptive Control with Position Constraints

The command (67) is presented under initial conditions $x(0) = -0.019$ [m] and $v(0) = -0.09$ [m/s], so $e_x(0) = 0.019$, $E(0) = 0.09$, $e_v(0) = 0.019k_x + 0.09$ according to (54). The control (40) and the adaptive law (30) with $\Gamma = diag(1, 10, 1000)$ were applied with design coefficients $k_x = 0.1$, $k_v = 1$. The influence of the imposed constraint on the system performance was tested in three cases: $\Delta_{ex} = 0.04, \Delta_{ex} = 0.03$ and $\Delta_{ex} = 0.02$, close to the initial condition $e_x(0) = 0.019$ (Fig. 6). The asymptotic stability of all errors is observed. The imposed constraint is preserved in all cases with a sufficient, save distant between the extremal value and the barrier.

If the constraint gets tighter more rapid movements near the boundary are observed and thus the maximal 'virtual control' tracking error and the 'velocity gap'

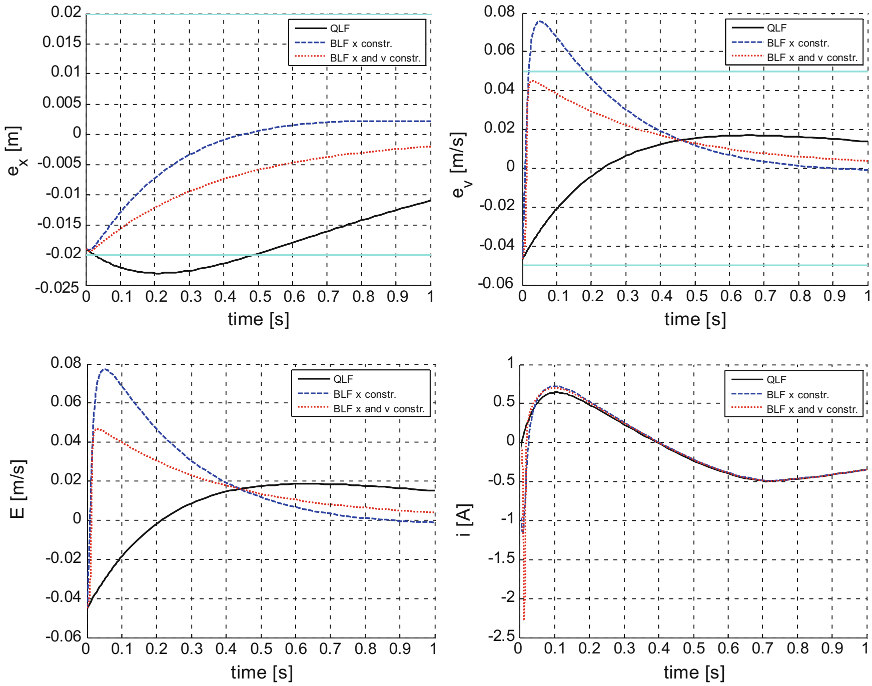


Fig. 5 Time history under QLF and BLF control

increase. The maximal current increases significantly, which is not surprising. Because large oscillations of e_v are observed, this system requires a careful tuning of adaptation gains in (30). Therefore, considering all the above, the application of a velocity constraint that will contribute to current restriction and will damp the oscillations is highly recommended.

8.3 BLF-Based Adaptive Control with Position and Velocity Constraints

The control (47) and the adaptive law (50) were applied with design coefficients $k_x = 0.1$, $k_v = 1$. For the initial conditions we can select any $\Delta_{ex} > e_x(0) = 0.019$ and $\Delta_{ev} > e_v(0) = 0.019k_x - (-0.09) = 0.0919$. We select $\Delta_{ex} = 0.02$ [m] and $\Delta_{ev} = 0.2; 0.15; 0.1$ [m/s]. As demonstrated in Fig. 7, in all the cases the imposed bounds are preserved during the transient. Tightening the bound Δ_{ev} allows to decrease the 'velocity gap' and the maximal current as well.

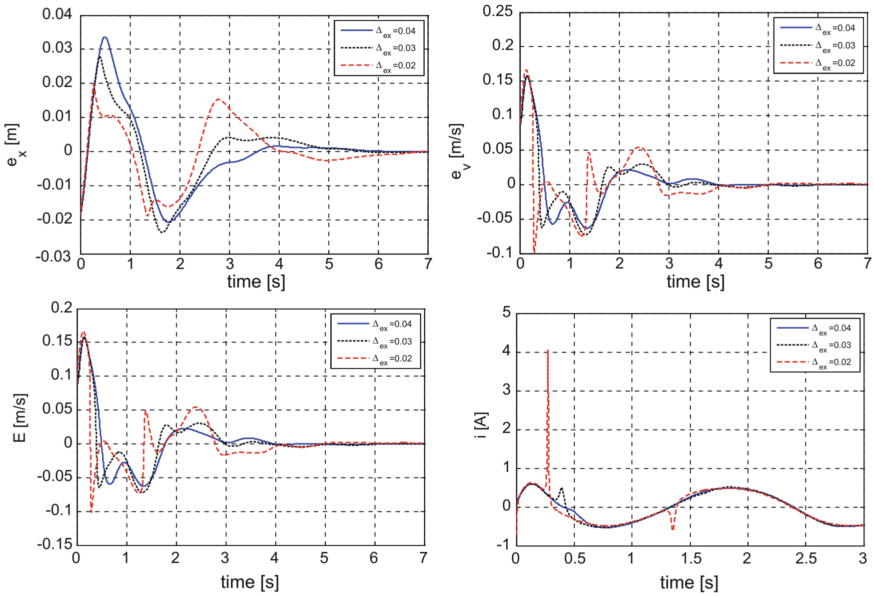


Fig. 6 Position tracking error e_x, 'virtual control' tracking error e_v, 'velocity gap' E and control i for different constraints Δ_{ex} (k_x = 0.1, k_v = 1)

8.4 System Performance with Bounded Adaptive Parameters

All systems tested above demonstrate robustness against imperfect reconstruction of the load. Selection of the adaptive gains Γ in (30) was not difficult and the maximal values of adaptive parameters were reasonable. In spite of this, the system performance with a priori constraints imposed on adaptive parameters was tested. The control strategies (27), (40), (47) were applied with constraints $\Delta_{ex} = 0.02$ [m] and Δ_{ev} = 0.1 [m/s] with appropriate adaptive laws (30), (50). The adaptive loop coefficients were $\Gamma = diag(1, 10, 1000)$ as previously, also the design coefficients k_v and k_x remained unchanged. Estimated parameters were bounded to be larger than 0 and smaller than 300 % of their exact value. The results are plotted in Figs. 8, 9 and 10. The adaptive parameters' bounds are respected and the tracking errors are asymptotically stable for any applied controller. As in the non-adaptive case, the QLF design does not assure the position tracking error respects the constraint. The BLF designed system with position constraints demonstrates rapid changes of velocity near the constraint boundary accompanied by high values of the current. To sum up, the worse transient of the position error is observed for the QLF-designed system and the best transient is obtained for the BLF-designed system with position and velocity constraints.

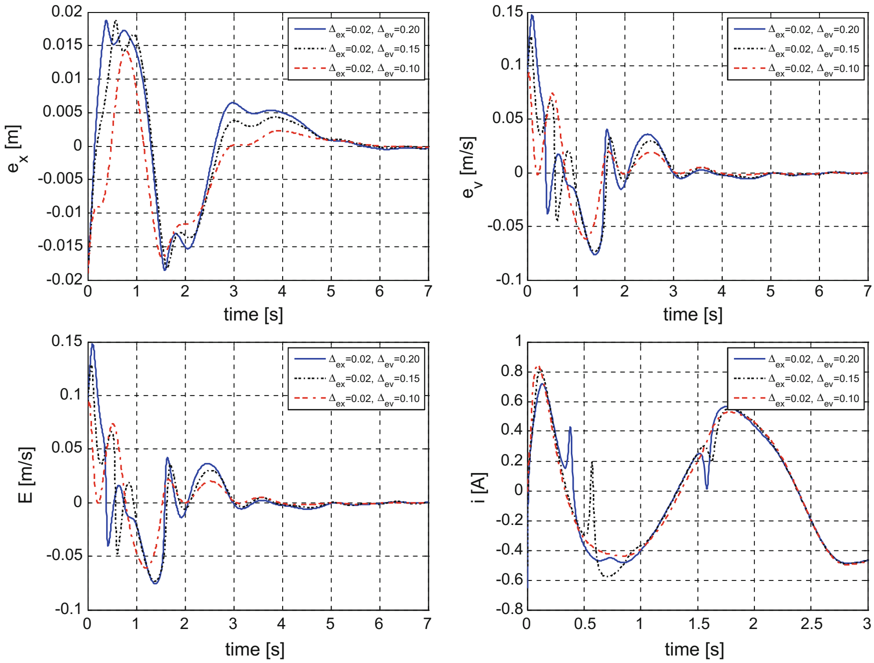


Fig. 7 Position tracking error e_x, 'virtual control' tracking error e_v, 'velocity gap' E and control i for different constraints Δ_{ex} (k_x = 0.1, k_v = 1, $\Delta_{ex} = 0.02$)

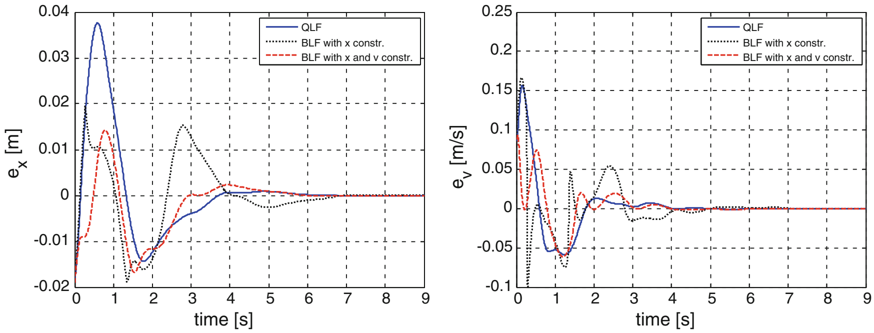


Fig. 8 Position tracking error e_x and 'virtual control' tracking error e_v

9 Real Plant Experiment

The application of the proposed controller design to a real plant allows the checking of the robustness against several phenomena that were not consider during the theoretical derivation. The effects of discrete in time controller implementation,

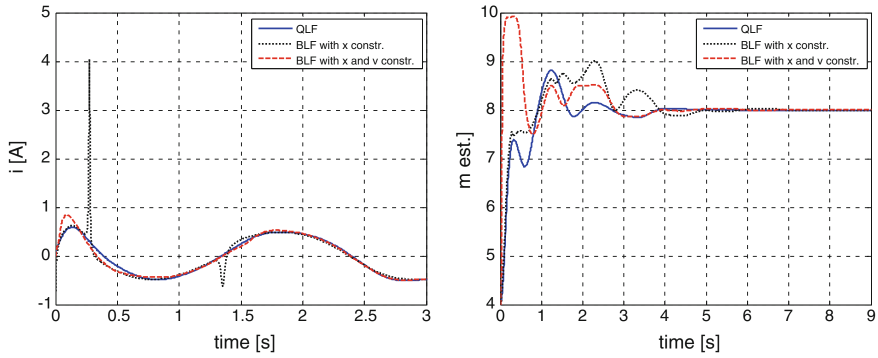


Fig. 9 Control i and estimate of parameter m (the first component of $\widehat{A}_1$ multiplied by 39)

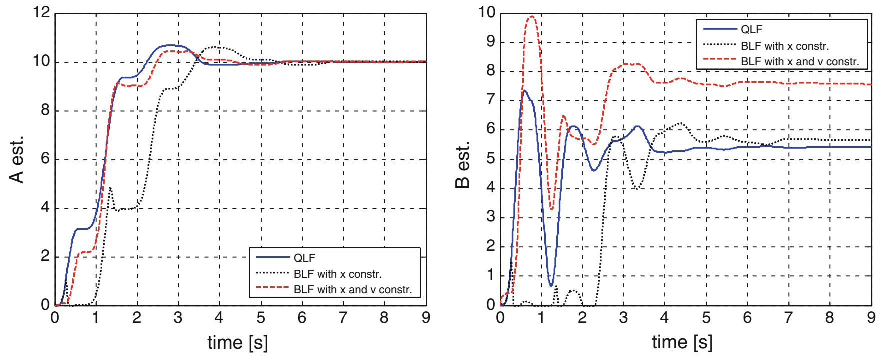


Fig. 10 Estimates of load parameters A, B (the second and the third component of $\widehat{A}_1$ multiplied by 39)

error of quantization during any digital measurement of an analogous signal, measurement noise and outliers are present in any real drive. The fast part of the drive dynamics—the current generation—was neglected during the derivation, but, most importantly, the load model (3) is idealized. Therefore the BLF design was tested with a real drive.

The electric linear drive (Fig. 11) system includes: TB2510 linear motor, total weight: $m = 8$ kg, long-term force 104 N, current-force coefficient $\varphi = 39$ N/A, 1 μm resolution encoder, XTL-230-18 Xenus PWM inverter with built-in current controller, modular set based on DS1006 dSpace processor card for control and data acquisition.

Fig. 11 Linear drive

The load is mostly the friction and pulling forces from the cables. It is very difficult to propose a realistic model for such resistance force, hence it will be described by the simplest possible friction model:

$$F_o(v) = A^T \xi(v) = S \cdot sign(v) + b \cdot v \tag{70}$$

with unknown parameters S, b for static and Coulomb friction respectively. A very rough guess for these parameters is 5 N and 25Ns/m respectively. The desired position trajectory is:

$$x_d(t) = \mathcal{L}^{-1}\left\{\frac{1}{0.1^2 s^2 + 0.2s + 1}\mathcal{L}\{0.3\ \sin(2t)\}\right\} \quad [\mathrm{m}]. \tag{71}$$

The control strategies (27), (40), (47) with appropriate adaptive laws (30), (49) were applied with the design coefficients $k_x = 1$; $k_v = 1$, constraints $\Delta_{ex} = 0.01$ [m] and $\Delta_{ev} = 0.05$ [m/s]. The initial conditions for the presented plots were $e_x(0) = 0.008$ and $e_v(0) = 0.008k_x = 0.008$. The adaptive loop gains were $\Gamma = diag(1, 10, 1000)$. Estimated parameters $\widehat{A}$ were constrained to be positive and smaller than 300 % of their initial value. The results of experiments are plotted in Figs. 12, 13, 14, 15, 16 and 17.

As is visible in Figs. 12, 13 and 14, the constraints are violated under the QLF design and respected if the BLF approach is used. If the position tracking constraint is applied alone, the system requires high control effort (high motor current), while

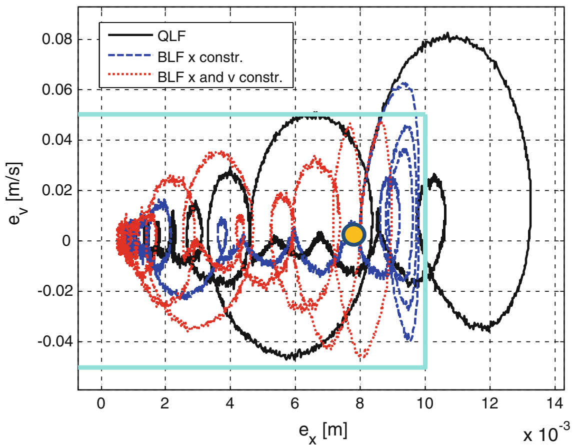


Fig. 12 The error system state-space trajectories: 'virtual control' tracking error e_v versus position tracking error e_x, is the initial condition

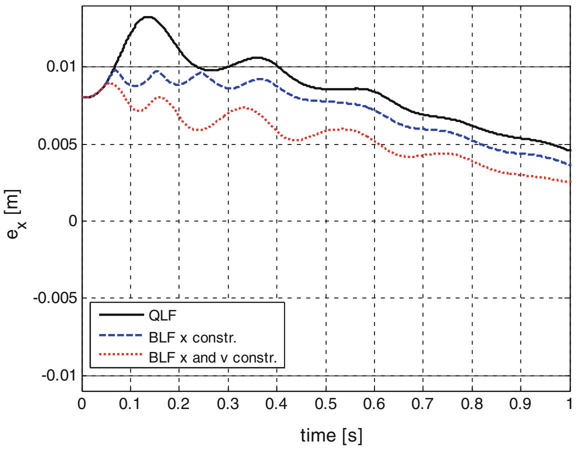


Fig. 13 The position tracking error e_x time history

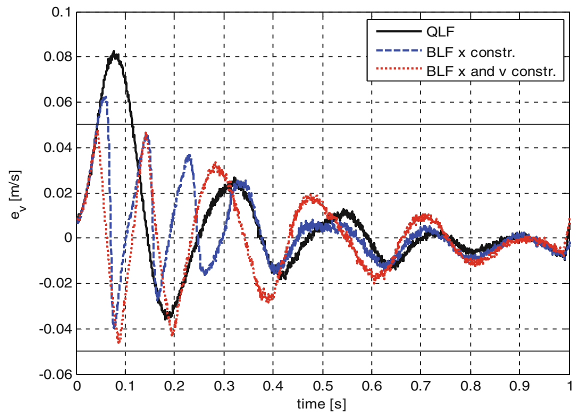


Fig. 14 The 'virtual control' tracking error e_v time history

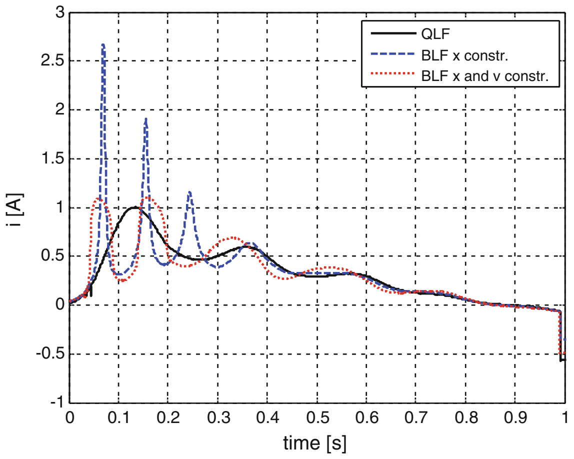


Fig. 15 The control i (motor current) time history

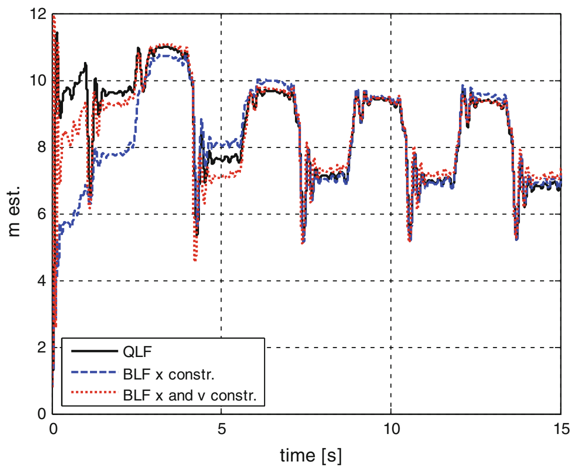


Fig. 16 Estimate of parameter m (the first component of $\widehat{A}_1$ multiplied by 39)

changing the direction of movement near the constraint boundary (see Fig. 15). Position and velocity constraints applied together assure the best performance. The resistance force is changing (because of movement of cables mostly), so no steady state of the entire system is achieved. The adaptive parameters are continuously trying to fit to the inexact model (70) (Figs. 16, 17) and this is visible in the oscillations in the tracking errors (Fig. 13). As a matter of fact, the uniform ultimate boundedness is achieved (see Remark 1) rather than the asymptotic stability.

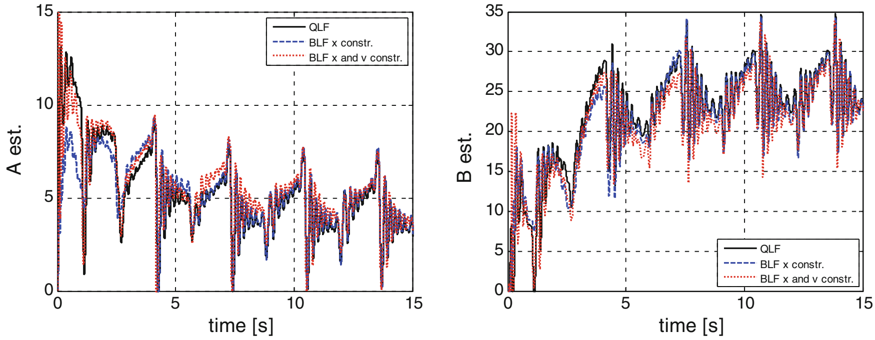


Fig. 17 Estimates of friction parameters A, B (the second and the third component of $\widehat{A}_1$ multiplied by 39)

10 Conclusions

A systematic motion controller design procedure based on barrier Lyapunov functions was described. The method ensures the tracking errors remain inside the predefined constraints in spite of unknown system parameters and along any trajectory starting inside the constraints. Even if preliminary requirements impose the bounds for the position tracking error only, it is advantageous to constrain the velocity tracking error as well. This provides a smoother trajectory and allows the avoidance of rapid movements near the position-constraint boundary, requiring high values of the control. Proper choice of control system parameters is easier in the case of coexisting position and velocity constraints.

The experiment with real linear drives demonstrates that the application of the proposed approach in real servos is possible and all expected features of the control system are preserved.

The proposed approach may be easily generalized to include inexact models like in (4) and cover motion control with many degrees of freedom, particularly control of robotic manipulators.

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