Chapter 1
Introductory Remarks

Abstract This chapter presents some general introductory remarks and an outline of the topics which will be covered in this book.

1.1 Noncommutativity, Fuzziness and Matrices

It has been argued, by combining the principles of quantum mechanics and general relativity, that the manifold structure of spacetime will necessarily break down at the Planck scale, and that, at this scale, spacetime becomes quantized, expressed by the commutation relations [48, 49]

$$[x_\mu, x_\nu] = i\lambda_p^2 Q_{\mu\nu}. \tag{1.1}$$

This can be seen as follows. Measuring for example the coordinate x of an event with an accuracy a will cause, by the Heisenberg principle, an uncertainty in momentum of the order of $1/a$. An energy of the order of $1/a$ is transmitted to the system and concentrated at some time around x. This in turn will generate a gravitational field by Einstein's equations for the metric. The smaller the uncertainty a the larger the gravitational field which can then trap any possible signal from the event. At this scale localization looses thus its operational meaning, the manifold picture breaks down, and one expects spacetime uncertainty relations which in turn strongly suggest that spacetime has a quantum structure expressed by the above commutation relations (1.1). The geometry of spacetime at the very small is therefore noncommutative.

On the other hand, noncommutative geometry [38], see also [41, 61, 100, 110, 151] and [56], allows for the description of the geometry of arbitrary spaces in terms of their underlying C^*-algebras. Von Neumann called this "pointless geometry" meaning that there are no underlying points. The so-called Von-Neumann algebras can be viewed as marking the birth of noncommutative geometry.

Noncommutative geometry was also proposed, in fact earlier than renormalization, as a possible way to eliminate ultraviolet divergences in quantum field theories [143, 155]. This phenomena of regularization by quantization occurs also in quantum mechanics.

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Noncommutative field theory is by definition a field theory based on a noncommutative spacetime [50, 147]. The most studied examples in the literature are the Moyal-Weyl spaces $\mathbf{R}^d_\theta$ which correspond in (1.1) to the case $Q_{\mu\nu} = \theta_{\mu\nu}$ where $\theta_{\mu\nu}$ are rank 2 (or 1) antisymmetric constant tensors, i.e.

$$[x_\mu, x_\nu] = i\theta_{\mu\nu}. \tag{1.2}$$

This clearly breaks Lorentz symmetry. The corresponding quantum field theories are not UV finite [55], and furthermore they are plagued with the so-called UV-IR mixing phenomena [117]. This means in particular that the physics at very large distances is altered by the noncommutativity which is supposed to be relevant only at very short distances.

Another class of noncommutative spaces which will be important to us in these notes are fuzzy spaces [14, 125]. Fuzzy spaces, and their field theories and fuzzy physics, are discussed for example in [1, 16, 88, 99, 144, 156]. Fuzzy spaces are finite dimensional approximations to the algebra of functions on continuous manifolds which preserve the isometries and (super)symmetries of the underlying manifolds. Thus, by construction the corresponding field theories contain a finite number of degrees of freedom. The basic and original motivation behind fuzzy spaces is non-perturbative regularization of quantum field theory similar to the familiar technique of lattice regularization [70, 71]. Another very important motivation lies in the fact that string theory suggests that spacetime may be fuzzy and noncommutative at its fundamental level [2, 83]. A seminal example of fuzzy spaces is the fuzzy two-dimensional sphere $\mathbf{S}^2_N$ [84, 109], which is defined by three $N \times N$ matrices x_i, $i = 1, 2, 3$, playing the role of coordinates operators, satisfying $\sum_i x_i^2 = 1$, and the commutation relations

$$[x_i, x_j] = i\theta\epsilon_{ijk}x_k \ , \ \theta = \frac{1}{\sqrt{c_2}} \ , \ c_2 = \frac{N^2-1}{4}. \tag{1.3}$$

The fuzzy sphere, and its Cartesian products, and the Moyal-Weyl spaces are the main noncommutative spaces discussed in these lectures.

Original work on the connection between random matrix theory and physics dates back to Wigner, Dyson and then t'Hooft. More recently, random matrix theory was investigated, in fact quite extensively, with connection to discrete 2-dimensional gravity and dynamical triangulation of random surfaces. See for example [45] and references therein. In recent years, it has also become quite clear that the correct description of noncommutative field theory must be given in terms of matrix degrees of freedom.

Fuzzy spaces and their field theories are, by construction, given in terms of finite dimensional matrix models, whereas noncommutative Moyal-Weyl spaces must be properly thought of as infinite dimensional matrix algebras, not as continuum manifolds, and as such, they should be regularized by finite dimensional matrices. For example, they can be regularized using fuzzy spaces, or simply by just truncating the Hilbert space of the creation and annihilation operators.

However, these regularization are different from the usual, more natural one, adopted for Moyal-Weyl spaces, which is based on the Eguchi-Kawai model [51], and the noncommutative torus [6, 8, 9]. The so-called twisted Eguchi-Kawai model was employed as a non-perturbative regularization of noncommutative gauge theory on the Moyal-Weyl space in [24–26]. Another regulator providing a finite dimensional matrix model, but with boundary, is given by the fuzzy disc [104, 106–108].

There are two types of matrix field theories which are potentially of great interest. First, matrix Yang-Mills theories, with and without supersymmetries, which are relevant to noncommutative and fuzzy gauge theories, emergent geometry, emergent gravity and emergent time and cosmology. Second, matrix scalar field theories which are relevant to noncommutative, fuzzy and multitrace ϕ^4 models and their phase structure and renormalizability properties. The main theme, of these lectures, will be the detailed discussion of the phase structure of noncommutative ϕ^4, and noncommutative gauge theory, on Moyal-Weyl spaces and fuzzy projective spaces.

1.2 Noncommutativity in Quantum Mechanics

Spacetime noncommutativity is inspired by quantum mechanics. When a classical phase space is quantized we replace the canonical positions and momenta x_i, p_j with Hermitian operators $\hat{x}_i, \hat{p}_j$ such that

$$[x_i, p_j] = i\hbar\delta_{ij}. \tag{1.4}$$

The quantum phase space is seen to be fuzzy, i.e. points are replaced by Planck cells due to the basic Heisenberg uncertainty principle

$$\Delta x \Delta p \geq \frac{1}{2}\hbar. \tag{1.5}$$

The commutative limit is the quasiclassical limit $\hbar \longrightarrow 0$. Thus, phase space acquires a cell-like structure with minimum volume given roughly by $\hbar$. In this section we will rederive this result in an algebraic form in which the noncommutativity is established at the level of the underlying algebra of functions.

It is a textbook result that the classical atom can be characterized by a set of positive real numbers ν_i called the fundamental frequencies. The atom if viewed as a classical system will radiate via its dipole moment interaction until it collapses. The intensity of this radiation is given by

$$I_n \propto | < \nu, n > |^4$$
$$< \nu, n > = \sum_i n_i \nu_i, n_i \in Z. \tag{1.6}$$

It is clear that all possible emitted frequencies $< \nu, n >$ form a group Γ under the addition operation of real numbers

$$\Gamma = \{< n, \nu >; n_i \in Z\}. \tag{1.7}$$

Indeed, given two frequencies $< \nu, n >= \sum_i n_i \nu_i$ and $< \nu, n' >= \sum_i n'_i \nu_i$ in Γ it is obvious that $< \nu, n + n' >= \sum_i (n_i + n'_i)\nu_i$ is also in Γ.

The algebra of classical observables of this atom can be obtained as the convolution algebra of the abelian group Γ. To see how this works exactly one first recalls that any function on the phase space X of this atom can be expanded as (an almost) periodic series

$$f(q, p; t) = \sum_n f(q, p; n)e^{2\pi i<n,\nu>t}; n \equiv (n_1, \ldots, n_k). \tag{1.8}$$

The Fourier coefficients $f(q, p; n)$ are labelled by the elements $n \in \Gamma$. The convolution product is defined by

$$f * g(q, p; t; n) = \sum_{n_1+n_2=n} f(q, p; t; n_1)g(q, p; t; n_2) \tag{1.9}$$

$$f(q, p; t; n) = f(q, p; n) \exp(2\pi i < n, \nu > t). \tag{1.10}$$

This leads to the ordinary commutative pointwise multiplication of the corresponding functions $f(q, p; t)$ and $g(q, p; t)$, namely

$$fg(q, p; t) \equiv f(q, p; t)g(q, p; t) = \sum_n f_1 * f_2(q, p; t; n). \tag{1.11}$$

The key property leading to this result is the fact that Γ is an abelian group.

If we take experimental facts into account then we know that the atom must obey the Ritz-Rydberg combination principle which says that (a) rays in the spectrum are labeled with two indices and (b) frequencies of these rays obey the law of composition, viz

$$\nu_{ij} = \nu_{ik} + \nu_{kj}. \tag{1.12}$$

We write this as

$$(i, j) = (i, k) \circ (k, j). \tag{1.13}$$

The emitted frequencies ν_{ij} are therefore not parametrized by the group Γ but rather by the groupoid Δ of all pairs (i, j). It is a groupoid since not all frequencies can be composed to give another allowed frequency. Every element (i, j) has an inverse (j, i) and $\circ$ is associative.

The quantum algebra of observables is then the convolution algebra of the groupoid Δ and it turns out to be a noncommutative (matrix) algebra as one can see by rewriting (1.10) in the form

$$F_1F_{2(i,j)} = \sum_{(i,k)\circ(k,j)=(i,j)} F_{1(i,k)}F_{2(k,j)}. \tag{1.14}$$

One can easily check that $F_1F_2 \neq F_2F_1$ so $F's$ fail to commute.

To implement the element of the quantum algebra as matrices one should replace $f(q,p;t;n) = f(q,p;n)e^{2\pi i<n,\nu>t}$ by

$$F(Q,P;t)_{(i,j)} = F(Q,P)_{(i,j)}e^{2\pi i\nu_{ij}t}. \tag{1.15}$$

From here the Heisenberg's equation of motion, phase space canonical commutation relations, and Heisenberg's uncertainty relations follow in the usual way.

1.3 Matrix Yang-Mills Theories

The first indication that noncommutative gauge theory is related to Yang-Mills matrix models goes back to the early days of noncommutative field theories. Indeed, noncommutative gauge theories attracted a lot of interest originally because of their appearance in string theory [40, 137, 139]. For example, it was discovered that the dynamics of open strings, moving in a flat space, in the presence of a non-vanishing Neveu-Schwarz B-field, and with Dp-branes, is equivalent, to leading order in the string tension, to a gauge theory on a Moyal-Weyl space $\mathbf{R}^d_\theta$. The resulting action is

$$S = \frac{\sqrt{\det(\pi\theta B)}}{2g^2} Tr_{\mathcal{H}}\big(i[\hat{D}_i, \hat{D}_j] - \frac{1}{\theta}B^{-1}_{ij}\big)^2. \tag{1.16}$$

Extension of this result to curved spaces is also possible, at least in one particular instance, namely the case of open strings moving in a curved space with $\mathbf{S}^3$ metric. The resulting effective gauge theory lives on a noncommutative fuzzy sphere $\mathbf{S}^2_N$ [2, 3, 83].

This same phenomena happens already in quantum mechanics. Consider the following Lagrangian

$$\mathcal{L}_m = \frac{m}{2}(\frac{dx_i}{dt})^2 - \frac{dx_i}{dt}.A_i\ ,\ A_i = -\frac{B}{2}\epsilon_{ij}x_j. \tag{1.17}$$

After quantization the momentum space becomes noncommutative given by the commutation relations

$$[\pi_i, \pi_j] = iB\epsilon_{ij} \ , \ \pi_i = m\frac{dx_i}{dt}. \tag{1.18}$$

It is well known that spatial noncommutativity arises in the limit $m\longrightarrow 0$, i.e. from the following Lagrangian

$$\mathcal{L}_0 = -\frac{B}{2}\epsilon_{ij}\frac{dx_i}{dt}x_j. \tag{1.19}$$

In this case we have

$$[x_i, x_j] = i\theta\epsilon_{ij} \ , \ \theta = \frac{1}{B}. \tag{1.20}$$

The limit $m\longrightarrow 0$ keeping B fixed is the projection onto the lowest Landau level (recall that the mass gap is B/m). This projection is also achieved in the limit $B\longrightarrow\infty$ keeping m fixed.

The is precisely what happens in string theory. We get noncommutative gauge theories on Moyal-Weyl planes or fuzzy spheres depending on whether the strings are moving, in a Neveu-Schwarz B-field, in a flat or curved (with S^3 metric) backgrounds respectively. The corresponding limit is $\alpha'\longrightarrow 0$.

At almost around the same time, it was established that reduced Yang-Mills theories play a central role in the nonperturbative definitions of M-theory and superstrings. The BFSS conjecture [17] relates discrete light-cone quantization (DLCQ) of M-theory, to the theory of N coincident D0 branes which at low energy, small velocities and/or string coupling, is the reduction to $0 + 1$ dimension of the 10 dimensional $U(N)$ supersymmetric Yang-Mills gauge theory [154]. The BFSS model is therefore a Yang-Mills quantum mechanics which is supposed to be the UV completion of 11 dimensional supergravity.

As it turns out, the BFSS action is nothing else but the regularization of the supermembrane action in the light cone gauge [44].

The BMN model [20] is a generalization of the BFSS model to curved backgrounds. It is obtained by adding to the BFSS action a one-parameter mass deformation corresponding to the maximally supersymmetric pp-wave background of 11 dimensional supergravity. See for example [27, 28, 96]. We also note, in passing, that all maximally supersymmetric pp-wave geometries can arise as Penrose limits of $AdS_p \times S^q$ spaces [128].

The IKKT model [86] is, on the other hand, a Yang-Mills matrix model obtained by dimensionally reducing 10 dimensional $U(N)$ supersymmetric Yang-Mills gauge theory to $0+0$ dimensions. The IKKT model is postulated to provide a constructive definition of type II B superstring theory, and for this reason, it is also called type

IIB matrix model. The dynamical variables are d matrices of size N with action

$$S = -\frac{N}{4}Tr[X_\mu, X_\nu]^2 + Tr\bar{\psi}\Gamma_\mu[X_\mu, \psi]. \tag{1.21}$$

The supersymmetric analogue of the IKKT model also exists in dimensions $d = 3, 4$ and 6 while the partition functions converge only in dimensions $d = 4, 6$ and 10 [12, 97, 98]. In $d = 3, 4$ the determinant of the Dirac operator is positive definite [7, 98], and thus there is no sign problem. Mass deformations such as the Myers term [120] are essential in order to reproduce non-trivial geometrical backgrounds such as the fuzzy sphere in these Yang-Mills matrix models including the IKKT matrix model. Supersymmetric mass deformations in Yang-Mills matrix models and Yang-Mills quantum mechanics models are considered for example in [29, 91].

The IKKT Yang-Mills matrix models can be thought of as continuum Eguchi-Kawai reduced models as opposed to the usual lattice Eguchi-Kawai reduced model formulated originally in [51].

We point out here the similarity between the conjecture that, the lattice Eguchi-Kawai reduced model allows us to recover the full gauge theory in the large N theory, and the conjecture that, the IKKT matrix model allows us to recover type II B superstring.

The relation between the BFSS Yang-Mills quantum mechanics and the IKKT Yang-Mills matrix model is discussed at length in the seminal paper [40], where it is also shown that toroidal compactification of the D-instanton action, the bosonic part of the IKKT action, yields, in a very natural way, a noncommutative Yang-Mills theory on a dual noncommutative torus [39]. From the other hand, we can easily check that the ground state of the D-instanton action is given by commuting matrices, which can be diagonalized simultaneously, with the eigenvalues giving the coordinates of the D-branes. Thus at tree-level an ordinary spacetime emerges from the bosonic truncation of the IKKT action, while higher order quantum corrections will define a noncommutative spacetime.

The central motivation behind these proposals of using Yang-Mills matrix models and Yang-Mills quantum mechanics as non-perturbative definitions of M-theory and superstring theory lies in D-brane physics [131, 132, 148]. At low energy the theory on the $(p+1)$-dimensional world-volume of N coincident Dp-branes is the reduction to $p + 1$ dimensions of 10 dimensional supersymmetric Yang-Mills [154]. Thus we get a $(p+1)$ dimensional vector field together with $9-p$ normal scalar fields which play the role of position coordinates of the coincident N Dp-branes. The case $p = 0$ corresponds to D0-branes. The coordinates become noncommuting matrices.

The main reasons behind the interest in studying these matrix models are emergent geometry transitions and emergent gravity present in these models. Furthermore, the supersymmetric versions of these matrix models provide a natural non-perturbative regularization of supersymmetry which is very interesting in its own right. Also, since these matrix models are related to large N Yang-Mills theory, they are of paramount importance to the string/gauge duality, which would allow us to study non-perturbative aspects of gravity from the gauge side of the duality. See for example [76, 77, 87, 124].

In summary, Yang-Mills matrix models provide a non-perturbative framework for emergent spacetime geometry [138], and noncommutative gauge theories [10, 11]. Since noncommutativity is the only extension which preserves maximal supersymmetry, Yang-Mills matrix models will also provide a non-perturbative regularization of supersymmetry [123]. Indeed, Yang-Mills matrix models can be used as a non-perturbative regularization of the AdS/CFT correspondence [111]. This allows, for example, for the holographic description of a quantum black holes, and the calculation of the corresponding Hawking radiation [79]. This very exciting result can be found in [78]. Yang-Mills matrix models also allow for the emergence of $3 + 1$dimensional expanding universe [92] from string theory, as well as yielding emergent gravity [146].

Thus the connections between noncommutative gauge theories, emergent geometry, emergent physics and matrix models, from one side, and string theory, the AdS/CFT correspondence and M-theory, from the other side, run deep.

1.4 Noncommutative Scalar Field Theory

A noncommutative field theory is a non-local field theory in which we replace the ordinary local point-wise multiplication of fields with the non-local Moyal-Weyl star product [63, 119]. This product is intimately related to coherent states [93, 112, 129], Berezin quantization [21] and deformation quantization [94]. It is also very well understood that the underlying operator/matrix structure of the theory, exhibited by the Weyl map [152], is the singular most important difference with commutative field theory since it is at the root cause of profound physical differences between the two theories. We suggest [4] and references therein for elementary and illuminating discussion of the Moyal-Weyl product and other star products and their relations to the Weyl map and coherent states.

Noncommutative field theory is believed to be of importance to physics beyond the standard model and the Hall effect [50] and also to quantum gravity and string theory [40, 139].

Noncommutative scalar field theories are the most simple, at least conceptually, quantum field theories on noncommutative spaces. Some of the novel quantum properties of noncommutative scalar field theory and scalar phi-four theory are as follows:

1. The planar diagrams in a noncommutative ϕ^4 are essentially identical to the planar diagrams in the commutative theory as shown originally in [55].
2. As it turns out, even the free noncommutative scalar field is drastically different from its commutative counterpart contrary to widespread believe. For example, it was shown in [145] that the eigenvalues distribution of a free scalar field on a noncommutative space with an arbitrary kinetic term is given by a Wigner

semicircle law. This is due to the dominance of planar diagrams which reduce the number of independent contractions contributing to the expectation value $<\phi^{2n}>$ from $2^n n!$ to the number $N_{\mathrm{planar}}(2n)$ of planar contractions of a vertex with $2n$ legs. See also [121, 133, 149, 150] for an alternative derivation.

3. More interestingly, it was found in [117] that the renormalized one-loop action of a noncommutative ϕ^4 suffers from an infrared divergence which is obtained when we send either the external momentum or the non-commutativity to zero. This non-analyticity at small momenta or small non-commutativity (IR) which is due to the high energy modes (UV) in virtual loops is termed the UV-IR mixing.
4. We can control the UV-IR mixing found in noncommutative ϕ^4 by modifying the large distance behavior of the free propagator through adding a harmonic oscillator potential to the kinetic term [67]. More precisely, the UV-IR mixing of the theory is implemented precisely in terms of a certain duality symmetry of the new action which connects momenta and positions [101]. The corresponding Wilson-Polchinski renormalization group equation [90, 130] of the theory can then be solved in terms of ribbon graphs drawn on Riemann surfaces. Renormalization of noncommutative ϕ^4 along these lines was studied for example in [35, 36, 65, 67, 68, 74, 134]. Other approaches to renormalization of quantum noncommutative ϕ^4 can be found for example in [18, 19, 62, 73, 75, 140].
5. In two-dimensions the existence of a regular solution of the Wilson-Polchinski equation [130] together with the fact that we can scale to zero the coefficient of the harmonic oscillator potential in two dimensions leads to the conclusion that the standard non-commutative ϕ^4 in two dimensions is renormalizable [65]. In four dimensions, the harmonic oscillator term seems to be essential for the renormalizability of the theory [68].
6. The beta function of noncommutative ϕ^4 theory at the self-dual point is zero to all orders [46, 47, 66]. This means in particular that the theory is not asymptotically free in the UV since the RG flow of the coupling constant is bounded and thus the theory does not exhibit a Landau ghost, i.e. not trivial. In contrast the commutative ϕ^4 theory although also asymptotically free exhibits a Landau ghost.
7. Noncommutative scalar field theory can be non-perturbatively regularized using either fuzzy projective spaces $\mathbf{CP}^n$ [15] or fuzzy tori $\mathbf{T}^n$ [8]. The fuzzy tori are intimately related to a lattice regularization whereas fuzzy projective spaces, and spaces [16, 125] in general, provide a symmetry-preserving sharp cutoff regularization. By using these regulators noncommutative scalar field theory on a maximally noncommuting space can be rewritten as a matrix model given by the sum of kinetic (Laplacian) and potential terms. The geometry in encoded in the Laplacian in the sense of Connes [38, 56].

The case of degenerate noncommutativity is special and leads to a matrix model only in the noncommuting directions. See for example [64] where it was also shown that renormalizability in this case is reached only by the addition of the doubletrace term $\int d^D x (Tr\phi)^2$ to the action.

8. Another matrix regularization of non-commutative ϕ^4 can be found in [60, 102, 103] where some exact solutions of noncommutative scalar field theory in background magnetic fields are constructed explicitly. Furthermore, in order to obtain these exact solutions matrix model techniques were used extensively and to great efficiency. For a pedagogical introduction to matrix model theory see [32, 52, 89, 115, 141]. Exact solvability and non-triviality is discussed at great length in [69].
9. A more remarkable property of quantum noncommutative ϕ^4 is the appearance of a new order in the theory termed the striped phase which was first computed in a one-loop self-consistent Hartree-Fock approximation in the seminal paper [72]. For alternative derivations of this order see for example [33, 34]. It is believed that the perturbative UV-IR mixing is only a manifestation of this more profound property. As it turns out, this order should be called more appropriately a non-uniform ordered phase in contrast with the usual uniform ordered phase of the Ising universality class and it is related to spontaneous breaking of translational invariance. It was numerically observed in $d = 4$ in [5] and in $d = 3$ in [23, 116] where the Moyal-Weyl space was non-perturbatively regularized by a noncommutative fuzzy torus [8]. The beautiful result of Bietenholz et al. [23] shows explicitly that the minimum of the model shifts to a non-zero value of the momentum indicating a non-trivial condensation and hence spontaneous breaking of translational invariance.
10. Therefore, noncommutative scalar ϕ^4 enjoys three stable phases: (1) disordered (symmetric, one-cut, disk) phase, (2) uniform ordered (Ising, broken, asymmetric one-cut) phase and (3) non-uniform ordered (matrix, stripe, two-cut, annulus) phase. This picture is expected to hold for noncommutative/fuzzy phi-four theory in any dimension, and the three phases are all stable and are expected to meet at a triple point. The non-uniform ordered phase [30] is a full blown nonperturbative manifestation of the perturbative UV-IR mixing effect [117] which is due to the underlying highly non-local matrix degrees of freedom of the noncommutative scalar field. In [34, 72], it is conjectured that the triple point is a Lifshitz point which is a multi-critical point at which a disordered, a homogeneous (uniform) ordered and a spatially modulated (non-uniform) ordered phases meet [85].
11. In [34] the triple (Lifshitz) point was derived using the Wilson renormalization group approach [153], where it was also shown that the Wilson-Fisher fixed point of the theory at one-loop suffers from an instability at large non-commutativity. See [13, 95] for a pedagogical introduction to the subject of the functional renormalization group. The Wilson renormalization group recursion formula was also used in [37, 53, 54, 82, 122] to study matrix scalar models which, as it turns out, are of great relevance to the limit $\theta \longrightarrow \infty$ of noncommutative scalar field theory [22].
12. The phase structure of non-commutative ϕ^4 in $d = 2$ and $d = 3$ using as a regulator the fuzzy sphere was studied extensively in [43, 57, 58, 113, 114, 127, 158]. It was confirmed that the phase diagram consists of three phases: a

disordered phase, a uniform ordered phases and a non-uniform ordered phase which meet at a triple point. In this case it is well established that the transitions from the disordered phase to the non-uniform ordered phase and from the non-uniform ordered phase to the uniform ordered phase originate from the one-cut/two-cut transition in the quartic hermitian matrix model [32, 141]. The related problem of Monte Carlo simulation of noncommutative ϕ^4 on the fuzzy disc was considered in [105].

13. The above phase structure was also confirmed analytically by the multitrace approach of O'Connor and Saemann [126, 136] which relies on a small kinetic term expansion instead of the usual perturbation theory in which a small interaction potential expansion is performed. This is very reminiscent of the Hopping parameter expansion on the lattice [42, 118, 135, 142]. See also [157] for a review and an extension of this method to the noncommutative Moyal-Weyl plane. For an earlier approach see [145] and for a similar more non-perturbative approach see [121, 133, 149, 150]. This technique is expected to capture the matrix transition between disordered and non-uniform ordered phases with arbitrarily increasing accuracy by including more and more terms in the expansion. Capturing the Ising transition, and as a consequence the stripe transition, is more subtle and is only possible if we include odd moments in the effective action and do not impose the symmetry $\phi \longrightarrow -\phi$.
14. The multitrace approach in conjunction with the renormalization group approach and/or the Monte Carlo approach could be a very powerful tool in noncommutative scalar field theory. For example, multitrace matrix models are fully diagonalizable, i.e. they depend on N real eigenvalues only, and thus ergodic problems are absent and the phase structure can be probed quite directly. The phase boundaries, the triple point and the critical exponents can then be computed more easily and more efficiently. Furthermore, multitrace matrix models do not come with a Laplacian, yet one can attach to them an emergent geometry if the uniform ordered phase is sustained. See for example [161, 162]. Also, it is quite obvious that these multitrace matrix models lend themselves quite naturally to the matrix renormalization group approach of Brezin and Zinn-Justin [31], Higuchi et al. [80, 81], Zinn-Justin [164].
15. Among all the approaches discussed above, it is strongly believed that the renormalization group method is the only non-perturbative coherent framework in which we can fully understand renormalizability and critical behavior of noncommutative scalar field theory in complete analogy with the example of commutative quantum scalar field theory outlined in [163]. The Wilson recursion formula, in particular, is the oldest and most simple and intuitive renormalization group approach which although approximate agrees very well with high temperature expansions [153]. In this approximation we perform the usual truncation but also we perform a reduction to zero dimension which allows explicit calculation, or more precisely estimation, of Feynman diagrams. See [53, 54, 122]. This method was applied in [159] to noncommutative scalar ϕ^4 field theory at the self-dual point with two strongly noncommuting directions and in [160] to noncommutative $O(N)$ model.

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